FIITJEE

CBSE PART TEST – II

ALL XTH STUDYING BATCHES

MATHS

Time: 1:30 Hours **Max Marks: 40**

Instructions:

1. The question paper contains two parts A and B.
2. Both Part – A and Part – B have internal choices.

PART – A

i. It consists of two sections – I and II.

ii. Section – I have 8 questions internal choices given in any three questions.

iii. Section – II has two case study – based questions. Each case study has 5 case – based sub – parts. A student is to attempt any 4 out of 5 sub parts.

PART – B

i. Question number 11 to 12 are very short answer type questions of 2 marks each.

ii. Question number 13 to 17 are short answer type questions of 3 marks each.

iii. Question number 18 are long answer type question of 5 marks.

iv. Internal choice has been provided in few questions.

Name of the Candidate : .....

Enroll Number : .....

Date of Examination : .....

PART – A

SECTION – I

(8 question, 1 mark each)

1. If $\tan\theta = \frac{1}{\sqrt{3}}$, find the value of $\sin(90^\circ - \theta)$

OR

If $\theta = 45^\circ$, the value of $\text{cosec}^2\theta$ is

2. Find the value of a so that the point (3, a) lies on the line represented by $2x - 3y = 5$.

3. If a = $(2^2 \times 3^3 \times 5^4)$ and b = $(2^3 \times 3^2 \times 5)$ then HCF (a, b) = ?

OR

LCM of $(2^3 \times 3 \times 5)$ and $(2^4 \times 5 \times 7)$ is:

4. If the HCF of 85 and 153 is expressible in the form 85n – 153, then value of n is:

5. The value of k for which the pair of linear equations $4x + 6y - 1 = 0$ and $2x + ky - 7 = 0$ represents parallel lines is

6. Find the angle of elevation of the sun when shadow of a pole x m high is $\sqrt{3}$ x m long.

7. What is the value of $(\cos^2 67^\circ - \sin^2 23^\circ)$ _____.

8. The value of $\tan 1^\circ . \tan 2^\circ . \tan 3^\circ ..... \tan 89^\circ$ is

OR

If A and B are complementary angles (0 < A < 90°) and $\sin A = \frac{1}{2}$, then the value of (cos A sin B – sin A cos B) is

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SECTION – II

(2 question, 4 mark each)

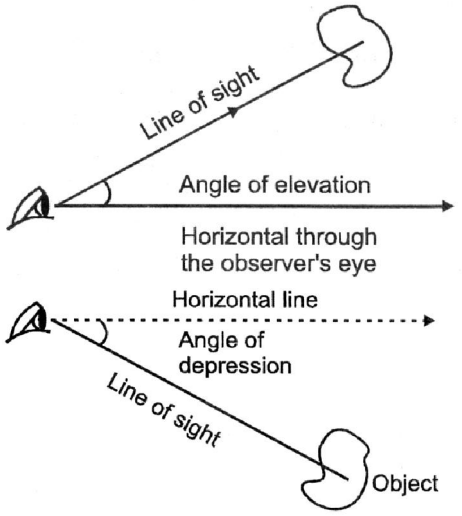
(Out of 5 sub-parts, attempt any four)

9.

PARAGRAPH – 1

ANGLE OF ELEVATION :

In order to see an object which is at a higher level compared to the ground level we are to look up. The line joining the object and the eye of the observer is known as the line sight and the angle which this line of sight makes with the horizontal drawn through the eye of the observer is known as the angle of elevation. (**figure)**



ANGLE OF DEPRESSION :

When the object is at a lower level tan the observer's eyes, he has to look downwards to have a view of the object. It that case, the angle which the line of sight makes with the horizontal thought the observer's eye is known as the **angle of depression (Figure).**

1. Sara is standing on the deck of a ship, which is 8 m above water level. She observes the angle of elevations of the top of a hill as 60° and the angle of depression of the base of the hill as 30°. Calculate the height of the hill.

(A) 34 m (B) 38 m

(C) 32 m (D) 36 m

2. A vertical tower stands on a horizontal plane and is surmounted by vertical flag staff of height 5 meters. At a point on the plane, the angle of elevation of the bottom and the top of the flag staff are respectively 30° and 60° find the height of tower.

(A) 2.5 m (B) 3 m

(C) 3.5 m (D) 4 m

3. The angles of depressions of the top and bottom of 8m tall building from the top of a multistoried building are 30° and 45° respectively. Find the distance between the two buildings.

(A) $4(3-\sqrt{3})$ metres (B) $4(3+\sqrt{3})$ metres

(C) $2(3+\sqrt{3})$ metres (D) $2(3-\sqrt{3})$ metres

4. The angle of elevation of an aeroplane from a point on the ground is 45^0. After a flight of 15 sec, the elevation changes to 30^0. If the aeroplane is flying at a height of 3000 metres, find the speed of the aeroplane $\left(\text{Use } \sqrt{3}=1.732\right)$.

(A) 597.08 km/hr (B) 623.50 km/hr

(C) 480.20 km/hr (D) 527.04 km/hr

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5. A boy is standing on the ground and flying a kite with 100 m of string at an elevation of 30^0. Another boy is standing on the roof of a 10 m high building and is flying his kite at an elevation of 45^0. Both the boys are on opposite sides of both the kites. Find the length of the string that the second boy must have so that the two kites meet.

(A) $50\sqrt{3}$ m (B) $50\sqrt{2}$

(C) $40\sqrt{2}$ m (D) $40\sqrt{3}$ m

10.

PARAGRAPH – 2

If **'a'** and **'b'** are positive integers such that **a = bq + r,** then every common divisor of **'a'** and **'b'** is a common divisor of **'b'** and **'r'** and vice-versa.

1. Find the H.C.F. of 196 and 38318.

(A) 88 (B) 94

(C) 86 (D) 98

2. If the H.C.F. of 657 and 963 is expressible in the form 657x + 963 × (-15), find x.

(A) 22 (B) 24

(C) 26 (D) 28

3. What is the largest number that divides 626, 3127 and 15628 and leaves remainders of 1, 2 and 3 respectively.

(A) 475 (B) 625

(C) 575 (D) 525

4. 144 cartons of coke cans and 90 cartons of Pepsi cans are to be stacked is a canteen. If each stack is of same height and is to contains cartons of the same drink, what would be the greatest number of cartons each stack would have?

(A) 14 (B) 16

(C) 12 (D) 18

5. What is the largest number that divides 245 and 1029, leaving remainders 5 in each case?

(A) 12 (B) 14

(C) 16 (D) 18

PART – B

2 Questions (2 Marks Each)

11. Solve for x and y: $31x + 29y = 33,\ 29x + 31y = 27$

OR

Find c if the system of equations $cx + 3y + (3 - c) = 0;\ 12x + cy - c = 0$ has infinitely may solutions?

12. Evaluate : $\dfrac{\sin^2 73^\circ + \sin^2 17}{\cos^2 37 + \cos^2 53^\circ}$

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5 Questions (3 Marks Each)

13. A two digit number is such that the product of its digits is 18. When 63 is subtracted from the number, the digits interchange their places. Find the number.

OR

Solve for x: $\frac{3}{x+1}+\frac{4}{x-1}=\frac{29}{4x-1}; x \neq 1,-1,\frac{1}{4}$

14. If $\sec\theta = x+\frac{1}{4x}$, prove that $\sec\theta+\tan\theta = 2x$ or $\frac{1}{2x}$

15. Use Euclid's Division Lemma to show that the cube of any positive integer is of the form 9m, 9m + 1 or 9 m + 8, for some integer q.

OR

Prove that the square of any positive integer of the form 5q + 1 is of the same form.

16. Seven years ago Varun's age was five times the square of Swati's age. Three years hence Swati's age will be two-fifth of Varun's age. Find their present ages.

17. Show that $3(\sin\theta-\cos\theta)^4+6(\sin\theta+\cos\theta)^2+4(\sin^6\theta+\cos^6\theta)$ is independent of θ.

1 Question (5 Marks)

18. Solve the following system of linear equations graphically: x - y = 1, 2x + y = 8. Find the area bounded by these two lines and y-axis.

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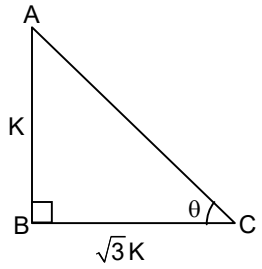
HINTS AND SOLUTIONS

1. $\tan\theta = \dfrac{1}{\sqrt{3}}$

$\sin(90-\theta) = \cos\theta$

$\therefore \quad AC = \sqrt{3K^2 + K^2} = 2K$

$\cos\theta = \dfrac{BC}{AC} = \dfrac{\sqrt{3}}{2}$



OR

1. $\theta = 45°$

$\therefore \quad \text{cosec}\,\theta = \dfrac{1}{\sin\theta} = \sqrt{2}$

$\therefore \quad \text{cosec}^2\theta = 2$

2. $2(3) - 3(a) = 5$

$6 - 5 = 3a \Rightarrow a = \dfrac{1}{3}$

3. $a = 2^2 \times 3^3 \times 5^4$
$b = 2^3 \times 3^2 \times 5$
HCF (a, b) $= 2^2 \times 3^2 \times 5$
$= 4 \times 9 \times 5$
$= 180$

OR

3. $p = 2^3 \times 3 \times 5$
$q = 2^4 \times 5 \times 7$
LCM (p, q) $= 2^4 \times 3 \times 5 \times 7$
$= 48 \times 35$
$= 1680$

4. HCF of (85, 153) $\Rightarrow 17$
$85n - 153 = 17 \Rightarrow n = 2$

5. $\dfrac{4}{2} = \dfrac{6}{k}$
$k = 3$

6. $\dfrac{x}{\sqrt{3}x} = \tan\theta$

$\theta = \dfrac{\pi}{6}$

7. $\cos^2 67° - \sin^2 23°$

$\Rightarrow \quad \cos^2(90° - 23°) - \sin^2 23°$

$\Rightarrow \quad \sin^2 23° - \sin^2 23° = 0$

8 $\Rightarrow \quad \tan1°\tan2°\tan3°.....\tan(90°-3°)\tan(90°-2°)\tan(90°-1°)$

$\Rightarrow \quad \tan1°\tan2°\tan3°.....\tan45°.....\cot3°\cot2°\cot1°$

$\Rightarrow \quad \tan1°\tan2°\tan3°.....\tan45°.....\dfrac{1}{\tan1°}\cdot\dfrac{1}{\tan2°}\cdot\dfrac{1}{\tan3°} = 1$

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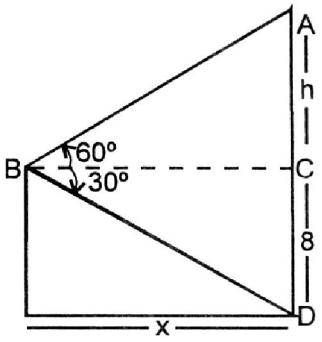
OR

8. sin B = cos A, cos B = sin A
$\therefore \cos^2 A - \sin^2 A = 1/2$

9.

1. C

Sol. Let x be distance of hill from man and h + 8 be height of hill which is required.
is right triangle ACB.



$\Rightarrow \quad \tan 60^0 = \frac{AC}{BC} = \frac{h}{x}$

$\Rightarrow \quad \sqrt{3} = \frac{h}{x}$

In right triangle BCD.

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{8}{x} \qquad \Rightarrow \qquad x = 8\sqrt{3}$

$\therefore$ Height of hill = h + 8 = $\sqrt{3}.x + 8 = \left(\sqrt{3}\right)\left(8\sqrt{3}\right) + 8 = 32\,m.$

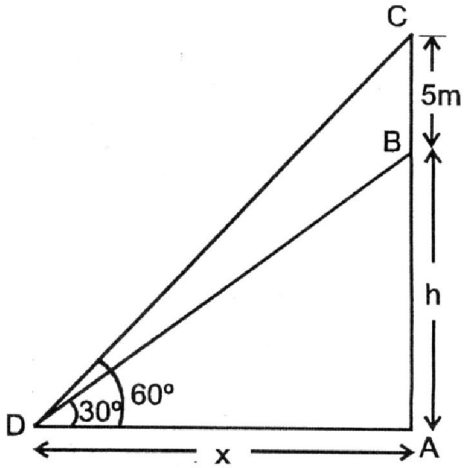
2. A

Sol. Let AB be the tower of height h metre and BC be the height of flag staff surmounted on the tower, Let the point of the place be D at a distance x meter from the foot of the tower in Δ ABD

$\tan 30^0 = \frac{AB}{AD}$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x}$

$\Rightarrow \quad x = \sqrt{3}h \qquad$ .....(i)



In Δ ABD

$\tan 60° = \frac{AC}{AD}$

$\Rightarrow \quad \sqrt{3} = \frac{5+h}{x}$

$\Rightarrow \quad x = \frac{5+h}{\sqrt{3}} \qquad$ .....(ii)

From (i) and (ii)

$\Rightarrow \quad \sqrt{3}\ h = \frac{5+h}{\sqrt{3}}$

$\Rightarrow \quad 3h = 5 + h$

$\Rightarrow \quad 2h = 5$

$\Rightarrow \quad h = \frac{5}{2} = 2.5m$

So, the height of tower = 2.5 m

3. B

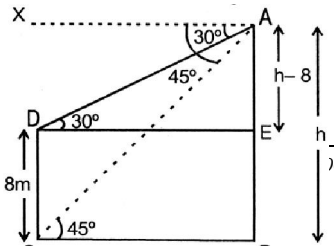
Sol. Let AB be the multistoried building of height h and let the distance between two buildings be x meters.

$\angle XAC = \angle ACB = 45^0$ [Alternate angles $\because$ AX $\|$ DE]

$XAD = ADE = 30^0$ [Alternate angles $\because$ AX $\|$ BC]

In Δ ADE

$\tan 30^0 = \frac{AE}{ED}$



$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h-8}{x}$ ($\because$ CB = DE = x)

$\Rightarrow \quad x = \sqrt{3}(h-8)$ .....(i)

In Δ ACB tan $45^0 = \frac{h}{x}$

$\Rightarrow \quad 1 = \frac{h}{x}$

$\Rightarrow \quad x = h$ .....(ii)

Form (i) and (ii)

$\sqrt{3}(h-8) = h \quad \Rightarrow \quad \sqrt{3}h - 8\sqrt{3} = h$

$\Rightarrow \quad \sqrt{3}h - h = 8\sqrt{3}$

$\Rightarrow \quad h(\sqrt{3}-1) = 8\sqrt{3}$

$\Rightarrow \quad h = \frac{8\sqrt{3}}{\sqrt{3}-1} \times \frac{(\sqrt{3}+1)}{\sqrt{3}+1}$

$\Rightarrow \quad h = \frac{8\sqrt{3}(\sqrt{3}+1)}{2}$

$\Rightarrow \quad h = 4\sqrt{3}(\sqrt{3}+1)$

$\Rightarrow \quad h = 4(3+\sqrt{3})$ metres

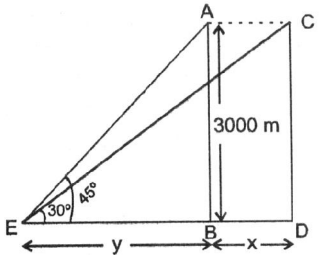
Form (ii) x = h

So, x = 4(3 + $\sqrt{3}$) metres

Distance between two building = 4(3 + $\sqrt{3}$) metres

4. D

Sol. Let the point on the ground is E which is y metres from point B and let after 15 sec flight it covers x metres distance.



In Δ AEB.

tan 45^0 = $\frac{AB}{EB}$

$\Rightarrow \quad 1 = \frac{3000}{y}$

$\Rightarrow \quad y = 3000\,m$ .....(i)

In ΔCED

$\Rightarrow \quad \tan 30^0 = \frac{CD}{ED}$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{3000}{x+y}$ ($\because$ AB = CD)

$\Rightarrow \quad x + y = 3000\ \sqrt{3}$ .....(ii)

From equation (i) and (ii)

$\Rightarrow \quad x + 3000 = 3000\sqrt{3}$

$\Rightarrow \quad x = 3000\sqrt{3} - 3000$

$\Rightarrow \quad x = 3000(\sqrt{3}-1)$

$\Rightarrow \quad x = 3000 \times (1.732 - 1)$

$\Rightarrow \quad x = 2196\,m$

Speed of Aeroplane $= \frac{\text{Distance covered}}{\text{Time taken}}$

$= \frac{2196}{15}\,m/sec. = 146.4\ m/sec.$

$= \frac{2196}{15} \times \frac{18}{5}\,Km/hr$

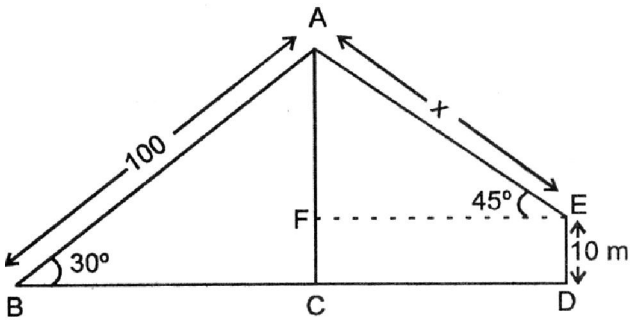
= 527.04 Km/hr

Hence, the speed of aeroplane is 527.04 Km/hr.

5. C

Sol. Let the length of second string b x m.

In $\triangle$ ABC



$\text{Sin } 30^0 = \frac{AC}{AB}$

$\frac{1}{2} = \frac{AC}{100} \Rightarrow AC = 50\,m$

In ΔAEF

$\text{Sin } 30^0 = \frac{AF}{AE}$

$\frac{1}{\sqrt{2}} = \frac{AC - FC}{x}$

$\frac{1}{\sqrt{2}} = \frac{50 - 10}{x}$ [$\therefore$ AC = 50 m, FC = ED = 10 m]

$\frac{1}{\sqrt{2}} = \frac{40}{x}$

$x = 40\sqrt{2}m$

(So the length of string that the second boy must have so that the two kites meet = $40\sqrt{2}$ m.)

10.

1. D

Sol. Applying Euclid's division lemma to 196 and 38318.

38318 = 195 × 196 + 98

196 = 98 × 2 + 0

The remainder at the second stage is zero. So, the H.C.F. of 38318 and 196 is 98.

2. A

Sol. Applying Euclid's division lemma on 657 and 963.

963 = 657 × 1 + 306

657 = 306 × 2 + 45

306 = 45 × 6 + 36

45 = 36 × 1 + 9

36 = 9 × 4 + 0

So, the H.C.F. of 657 and 963 is 9.

Given : 657x + 963 × (-15) = H.C.F. of 657 and 963.

657 x + 963 × (-15) = 9

657 x = 9 + 963 × 15

657 x = 14454

$x = \frac{14454}{657} = 22.$

3. B

Sol. Clearly, the required number is the H.C.F. of the number 626 - 1 = 625, 3127 - 2 = 3125 and 15628 – 3 = 15625.

Using Euclid's division lemma to find the H.C.F. of 625 and 3125.

3125 = 625 × 5 + 0

Clearly, H.C.F. of 625 and 3125 is 625.

Now, H.C.F. of 625 and 15625

15625 = 625 × 25 + 0

So, the H.C.F. of 625 and 15625 is 625.

Hence, H.C.F. of 625, 3125 and 15625 is 625.

Hence, the required number is 625.

4. D

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Sol. In order to arrange the cartons of the same drink is the same stack, we have to find the greatest number that divides 144 and 90 exactly. Using Euclid's algorithm, to find the H.C.F. of 144 and 90.

$144 = 90 \times 1 + 54$

$90 = 54 \times 1 + 36$

$54 = 36 \times 1 + 18$

$36 = 18 \times 2 + 0$

So, the H.C.F. of 144 and 90 is 18.

Number of cartons in each stack = 18.

5. C

Sol. It is given that the required number when divides 245 and 1029, the remainder is 5 in each case. This means that 245 – 5 = 240 and 1029 – 5 = 1024 are completely divisible by the required number.

It follows from this that the required number is a common factor of 240 and 1024. It is also given that the required number is the largest number satisfying the given property. Therefore, it is the HCF of 240 and 1024.

Let us now find the HCF of 240 and 1024 by Prime factorization method (refer above image).

2	240
2	120
2	60
2	30
3	15
5	5
	1

2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5$

$1024 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

Clearly, HCF of 240 and 1024 is common divisor i.e., $2 \times 2 \times 2 \times 2 = 16$.

11. $31x + 29y = 33$ …(i)

$29x + 31y = 27$ …(ii)

(i) + (ii)

$60x + 60y = 60$

$x + y = 1$ ...(iii)

(i) – (ii)

$2x - 2y = 6$

$x - y = 3$ …(iv)

(iii) + (iv)

$2x = 4$

$x = 2,\ y = -1$

OR

11. System of equations infinitely many solutions

$\therefore \quad \frac{c}{12} = \frac{3}{c} = \frac{3-c}{-c}$

$\Rightarrow c^2 = 36 \Rightarrow c = 6$ or $c = -6$ .....(1)

Also $-3c = 3c - c^2 \Rightarrow c = 6$ or $c = 0$ ……..(2)
From equation (1) and (2)
c = 6.

12. We have

$$\frac{\sin^2 73° + \sin^2 17}{\cos^2 37 + \cos^2 53°} = \frac{\sin^2(90° - 17°) + \sin^2 17°}{\cos^2 37° + \cos^2(90° - 37°)}$$

$$= \frac{\cos^2 17° + \sin^2 17°}{\cos^2 37° + \sin^2 37°} \qquad [\because \sin(90° - \theta) = \cos\theta \text{ and } \cos(90° - \theta) = \sin\theta]$$

$$= \frac{1}{1} = 1$$

13. Number xy
$x \times y = 18$ …(i)
$10x + y - 63 = 10y + x$
$9x - 9y = 63$
$x - y = 7$ …(ii)
$x^2 + y^2 = 49 + 2 \times 18 = 85$
$x + y = \sqrt{x^2 + y^2 + 2xy} = \sqrt{85 + 36} = \sqrt{121} = 11$
x = 9, y = 2
No = 92

OR

13. $x = 4, -7$

$$\frac{3}{x+1} + \frac{4}{x-1} = \frac{29}{4x-1}$$

$$\frac{3x - 3 + 4x + 4}{(x+1)(x-1)} = \frac{29}{4x-1}$$

$$(7x+1)(4x-1) = 29(x^2 - 1)$$

$$28x^2 - 7x + 4x - 1 = 29x^2 - 29$$

$$x^2 + 3x - 28 = 0$$

$$x^2 + 7x - 4x - 28 = 0$$

$$x(x+7) - 4(x+7) = 0$$

$$(x+7)(x-4) = 0 \Rightarrow x = 4, -7$$

14. We have, $\sec\theta = x + \frac{1}{4x}$

$$\Rightarrow \sec^2\theta = \left(x + \frac{1}{4x}\right)^2 \Rightarrow 1 + \tan^2\theta = x^2 + \frac{1}{16x^2} + \frac{1}{2}$$

$$\Rightarrow \tan^2\theta = x^2 + \frac{1}{16x^2} + \frac{1}{2} - 1 \Rightarrow \tan^2\theta = x^2 + \frac{1}{16x^2} - \frac{1}{2}$$

$$\Rightarrow \tan^2\theta = \left(x - \frac{1}{4x}\right)^2 \Rightarrow \tan\theta = \pm\left(x - \frac{1}{4x}\right)$$

$$\Rightarrow \tan\theta = x - \frac{1}{4x} \text{ or } \tan\theta = -\left(x - \frac{1}{4x}\right)$$

When $\tan\theta = x - \frac{1}{4x}$, we have $\sec\theta + \tan\theta = x + \frac{1}{4x} + x - \frac{1}{4x} = 2x$

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When $\tan\theta = -\left(x - \frac{1}{4x}\right)$, we have $\sec\theta + \tan\theta = \left(x + \frac{1}{4x}\right) - \left(x - \frac{1}{4x}\right) = \frac{1}{2x}$

Hence, $\sec\theta + \tan\theta = 2x$ or $\frac{1}{2x}$

15. Let x be any positive integer. Then, it is of the form 3q or, 3q + 1 or, 3 + 2.
Case - I When x = 3q
$\Rightarrow x^3 = (3q)^3 = 27q^3 = 9(3q^3) = 9m$, where $m = 9q^3$
Case - II when x = 3q + 1
$\Rightarrow x^3 = (3q + 1)^3$
$\Rightarrow x^3 = 27q^3 + 27q^2 + 9q + 1$
$\Rightarrow x^3 = 9q\,(3q^2 + 3q + 1) + 1$
$\Rightarrow x^3 = 9m + 1$, where $m = q\,(3q^2 + 3q + 1)$.
Case – III when x = 3q + 2
$\Rightarrow x^3 = (3q + 2)^3$
$\Rightarrow x^3 = 27q^3 + 54q^2 + 36q + 8$
$\Rightarrow x^3 = 9q(3q^2 + 6q + 4) + 8$
$\Rightarrow x^3 = 9m + 8$, where $m = q(3q^2 + 6q + 4)$
Hence, x^3 is either of the form 9m of 9m + 1 or 9m + 8.

OR

15. Let x be any positive's integer of the form 5q + 1.
When x= 5q + 1
$x^2 = 25q^2 + 10q + 1$
$x^2 = 5q(5q + 2) + 1$
Let m = q (5q + 2).
$x^2 = 5m + 1$.
Hence, x^2 is of the same form i.e. 5m + 1.

16. Let Varun's present age be x years and Swati's present age be y years.
Case-I: 7 years ago
Varun's age was (x – 7) years and Swati's age was (y – 7) years
ATQ $(x - 7) = 5(y - 7)^2 \Rightarrow x = 5\,(y - 7)^2 + 7$ …(i)
Case – II: 3 years hence
Varun's age will be (x + 3) years and Swati's age will be (y + 3) years

ATQ $y + 3 = \frac{2}{5}(x + 3)$

$\Rightarrow$ $y + 3 = \frac{2}{5}\left[\left(5(y - 7)^2 + 7\right) + 3\right]$ [Using eq. (i)]

$\Rightarrow$ $y + 3 = \frac{2}{5} \times 5(y - 7)^2 + \frac{2}{5} \times 10 \Rightarrow y + 3 = 2\left(y^2 - 14y + 49\right) + 4$

$\Rightarrow$ $y + 3 = 2y^2 - 28y + 98 + 4 \Rightarrow 2y^2 - 29y + 99 = 0$

$\Rightarrow$ $2y^2 - 18y - 11y + 99 = 0 \Rightarrow 2y\,(y - 9) - 11(y - 9) = 0$

$\Rightarrow$ $(y - 9)\,(2y - 11) = 0 \Rightarrow y = 9, y = \frac{11}{2}(\text{rejecting})$

$\therefore$ $y = 9$ $\therefore x = 5(9 - 7)^2 + 7$ [From (i)]

$\therefore$ Present age of Swati = 9 years and Varun = 27 years.

17. $3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\left(\sin^6\theta + \cos^6\theta\right)$

$= 3\left[(\sin\theta - \cos\theta)^2\right]^2 + 6\left(\sin^2\theta + \cos^2\theta + 2\sin\theta.\cos\theta\right)$

$+4\left[\left(\sin^2\theta + \cos^2\theta\right)^3 - 3\sin^2\theta.\cos^2\theta\left(\sin^2\theta + \cos^2\theta\right)\right]$

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$= 3\left[\sin^2\theta + \cos^2\theta - 2\sin\theta.\cos\theta\right]^2 + 6\left[1 + 2\sin\theta.\cos\theta\right] + 4\left(1 - 3\sin^2\theta.\cos^2\theta\right)$

$= 3\left(1 + 4\sin^2\theta.\cos^2\theta - 4\sin\theta\cos\theta\right) + 6 + 12\sin\theta.\cos\theta + 4 - 12\sin^2\theta\cos^2\theta$

$= 13 \Rightarrow$ independent of θ.

18. (i) x - y = 1

x	0	1	2
y	-1	0	1

(ii) 2x + y = 8

(ii) 2 x + y = 8

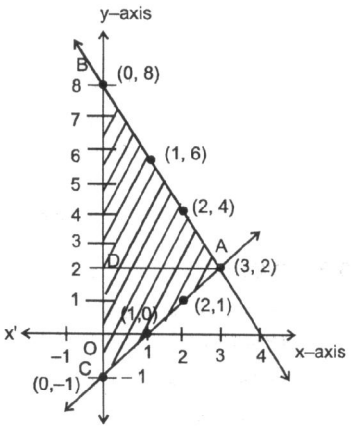
y = 8 - 2x

x	0	1	2
y	8	6	4

Solution is x = 3 and y = 2

Area of $\Delta ABC = \frac{1}{2} \times BC \times AD$

$= \frac{1}{2} \times 9 \times 3 = 13.5$ Sq. unit.



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