FTRE
on
December 27, 2020

CBSE PART TEST – I

ALL X^{TH} STUDYING BATCHES

MATHS

Time: 1:30 Hours **Max Marks: 40**

Instructions:

1. The question paper consist of 20 questions divided into four sections. **A**, **B**, **C** and **D.**
2. Section A contains 10 questions of 1 mark each. Section B contains 3 questions of 2 marks each. Section C contains 4 questions of 3 marks each. Section D contains 3 questions of 4 marks each.
3. All questions are **compulsory**. However, internal choices are given.
4. Use of calculator is not permitted.

Name of the Candidate : ....

Enroll Number : ....

Date of Examination : ....

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Section - A
1 Mark Questions

1. If two tangents inclined at an angle 60° are drawn to a circle of radius 3 cm, then the length of each tangent is:

(A) $\frac{3\sqrt{3}}{2}$ cm (B) $2\sqrt{3}$ cm
(C) $3\sqrt{3}$ cm (D) 6 cm

2. The number of terms in 5, 8, 11, 14,…..95 is
(A) 31 (B) 32
(C) 33 (D) 34

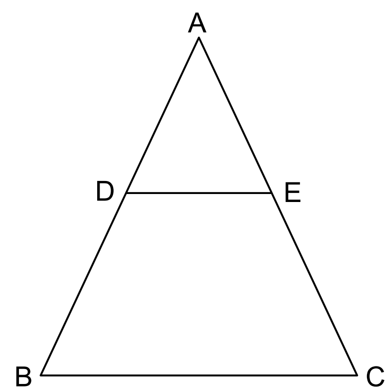
OR

Which term the A.P. 21, 42, 63, 84,……….is 210?
(A) 9^{th} (B) 10^{th}
(C) 11^{th} (D) 12^{th}

3. For what value of k does the equation $x^2 + 2x + k^2 + 1 = 0$ has real and equal roots?
(A) 0 (B) 1
(C) 2 (D) 3

4 The perimeter of two similar triangles ABC and PQR are respectively 27 cm and 15 cm. If PQ = 5 cm, then find AB
(A) 9 cm (B) 20 cm
(C) 11 cm (D) 5 cm

5. In the figure, $DE \parallel BC$ and AD : DB = 2 : 3, then $ar(\Delta ADE) : ar(\Delta ABC)$ is
(A) 2 : 5 (B) 2 : 3
(C) 3 : 5 (D) 4 : 25



6. If –4 is a zero of the polynomial $x^2 - x - (2k + 2)$ then value of k is _______.

OR

The quadratic polynomial whose zeroes are $3 + \sqrt{2}$ and $3 - \sqrt{2}$ is ________.

7. If the n^{th} term of an A.P. is 7 – 4n then its common difference is ________

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8. If $x = \frac{2}{3}$ and x = –3 are roots of the quadratic equation $ax^2 + 7x + b = 0$, find the value of a and b

9. If ΔABC and ΔDEF are similar such that 2AB = DE and BC = 8 cm, then find value of EF

10. If one root of quadratic equation $2x^2 + kx - 6 = 0$ is 2 then find other root

Section -B
2 Mark Questions

11. If α and β are zeroes of the quadratic polynomial $x^2 - 6x + a$ find the value of 'a' if $3\alpha + 2\beta = 20$

12. Two circular pieces of equal radii and maximum area, touch each other are cut from a rectangular card board of dimensions 14 cm x 7 cm. Find the area of the remaining card board. [Use $\pi = \frac{22}{7}$]

13. If 2 is the root of $x^2 + bx + 12 = 0$ and the equation $x^2 + bx + q = 0$ has equal roots, then find value of q

OR

Through the mid point M of the side CD of a parallelogram ABCD, the line BM is drawn intersecting AD produced at E. Prove that AE = 2BC.

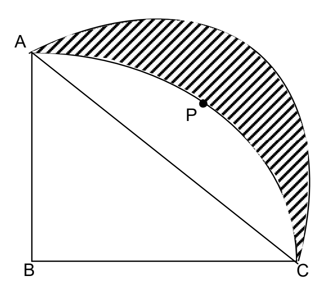
Section - C
3 Mark Questions

14. Solve the following quadratic equation for x:

$$x^2 + \left(\frac{a}{a+b} + \frac{a+b}{a}\right)x + 1 = 0$$

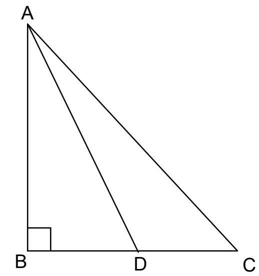
OR

In figure ABC is a quadrant of a circle of radius 14 cm and a semi – circle is drawn with BC as diameter. Find the area of the shaded region.



15. If the sum to first n terms of an A.P. is given by $S_n = n(n+1)$, find the 20^{th} term of the A.P.

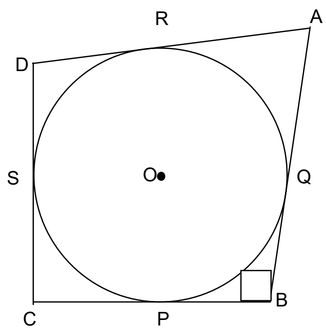
16. In figure, ΔABC is right angled at B and D is the mid point of BC. Prove that $AC^2 = 4AD^2 - 3AB^2$



OR

Solve the following quadratic equation for x:
$4x^2 - 4a^2x + (a^4 - b^4) = 0$.

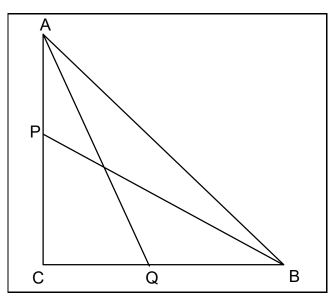
17. In figure, a circle with centre O is inscribed in a quadrilateral ABCD such that, it touches the sides BC, AB, AD and CD at points P, Q, R and S respectively. If AB = 29 cm, AD = 23, $\angle B = 90^\circ$ and DS = 5 cm, then find the radius of the circle.



Section - D
4 Mark Questions

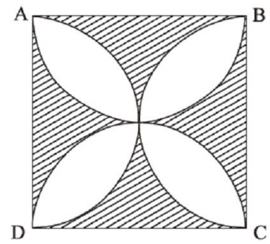
18. If the sum of first four terms of an AP is 40 and that of first 14 terms is 280. Find the sum of its first n terms.

19. In figure, P and Q are the midpoints of the sides CA and CB respectively of ΔABC right angled at C. Prove that $4(AQ^2 + BP^2) = 5AB^2$

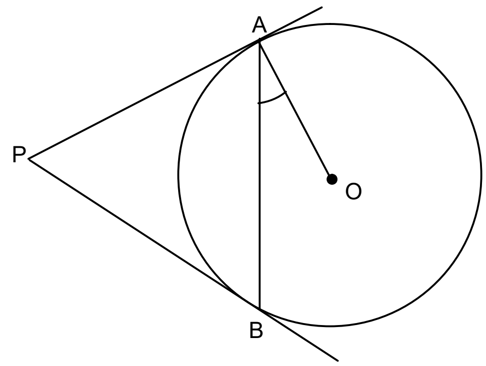


OR

In figure, ABCD is a square of side 14 cm. Semi – circles are drawn with each side of square as diameter. Find the area of the shaded region. (Use $\pi = \frac{22}{7}$)

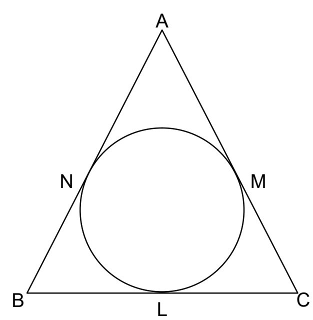


20. Two tangents PA and PB are drawn to a circle with centre O from an external point P. Prove that $\angle APB = 2\angle OAB$.



OR

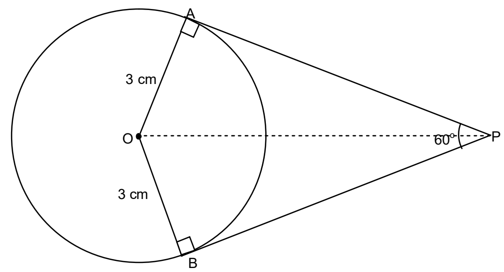
Prove that the lengths of tangents drawn from an external point to a circle are equal. Using the above, prove the following:
ABC is an isosceles triangle in which
AB = AC, circumscribed about a circle, as shown in figure. Prove that the base is bisected by the point of contact.



HINTS AND SOLUTIONS

1. C

1.

From figure, $\angle AOB = 120^o$ Joint O to P. then $\Delta OAP \cong \Delta OBP$ $\Rightarrow \angle AOP = \angle BOP = 60^o$ Now in ΔOAP, $\frac{AP}{OA} = \tan 60^o$ $\Rightarrow AP = 3\sqrt{3}$ cm	

2. A

2. $95 = 5 + (n-1)3 \Rightarrow n = 31$

OR

2. B

2. First term = 21

Common difference $= 42 - 21 = 21$

$210 = 21 + (n-1)21$

$(n-1) \times 21 = 210 - 21 \Rightarrow (n-1) = \frac{189}{21}$

$\Rightarrow (n-1) = 9$

$n = 10$

3. A

3. For equal roots $D = 0$

$b^2 - 4ac = 0$

$4 - 4(k^2 + 1) = 0$

$-4k^2 - 4 + 4 = 0$

k = 0

4 A

4 Given $\frac{AB + BC + CA}{PQ + QR + PR} = \frac{27}{15}$

Since $\Delta ABC \sim \Delta PQR$

$\Rightarrow \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{AB + BC + CA}{PQ + QR + PR}$

$\Rightarrow \frac{AB}{5} = \frac{27}{15}$

$\Rightarrow AB = 9$

5. D

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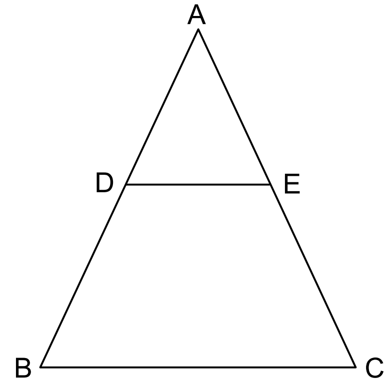
5. $DE \parallel BC$

$\therefore \Delta ADE \sim \Delta ABC$

$$\frac{AD}{DB} = \frac{2}{3}$$

$$\Rightarrow \frac{AD^2}{AB^2} = \frac{[ADE]}{[ABC]}$$

$$= \frac{4}{25} = \frac{[ADE]}{[ABC]}$$



6. 9

6. Put x = –4

$16 + 4 - (2k + 2) = 0$

$2k + 2 = 20$

$k = 9$

OR

6. $k(x^2 - 6x + 7)$

6. $f(x) = k(x^2 -$ (sum of roots)$x +$ product of roots)

$f(x) = k(x^2 - 6x + 7)$

7. Common difference = –4.

7. $T_n = 7 - 4n$

$T_1 = 7 - 4(1) = 3$

$T_2 = 7 - 4(2) = -1$

d = $T_2 - T_1 = -1 - 3 = -4$

8. $a = 3,\ b = -6$

8. $a\left(\frac{2}{3}\right)^2 + 7\left(\frac{2}{3}\right) + b = 0$

$\frac{4}{9}a + \frac{14}{3} + b = 0 \Rightarrow 4a + 42 + 9b = 0$

$a(-3)^2 + 7(-3) + b = 0$

$9a - 21 + b = 0$

$81a - 189 + 9b = 0$

$\underline{\underset{-}{4a} \underset{-}{+} 42 \underset{-}{+} 9b = 0}$

77a = 231

a = 3

$b = 21 - 9a \Rightarrow -6$

$a = 3,\ b = -6$

9. **16**

9. $2AB = DE$ **and** $BC = 8\,cm,\ EF = ?$

$\because \Delta ABC \sim \Delta DEF \Rightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

Given that 2AB = DE $\Rightarrow \frac{AB}{DE} = \frac{1}{2}$

$\Rightarrow \frac{1}{2} = \frac{8}{EF} \Rightarrow EF = 16$

10. $\frac{-3}{2}$

10. $2x^2 + kx - 6 = 0$ has one root $= 2$

$\therefore 2(2)^2 + 2k - 6 = 0 \Rightarrow k = -1$

$\therefore 2x^2 - x - 6 = 0$

$\Rightarrow 2x^2 - 4x + 3x - 6 = 0$

$\Rightarrow 2x(x-2) + 3(x-2) = 0$

$\Rightarrow (x-2)(2x+3) = 0 \Rightarrow x = 2, \frac{-3}{2}$

11. $a = -16$

11. $\alpha + \beta = 6$

$\alpha\beta = a$

$3\alpha + 2\beta = 20$ ...(1)

$3\alpha + 3\beta = 18$ ...(2)

(1) – (2) $-\beta = 2$

$\beta = -2$

$\alpha = 8$

$a = -16$

12. 21 cm^2

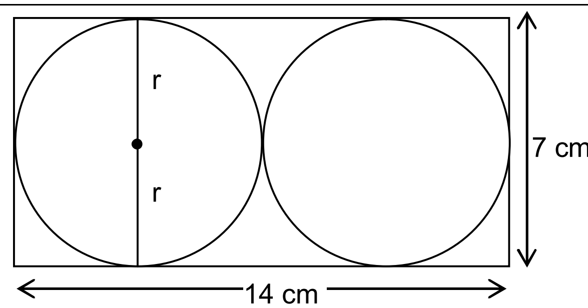
12. From figure $2r = 7$

$\Rightarrow r = \frac{7}{2}$ cm

Area of remaining cardboard

$= 14 \times 7 - \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 2$

= 21 cm^2



13. 16

13. $\because$ 2 is root of equation $x^2 + bx + 12 = 0$

$\therefore (2)^2 + 2b + 12 = 0 \Rightarrow b = -8$

$x^2 + bx + q = 0$

$\Rightarrow x^2 - 8x + q = 0$ has equal roots.

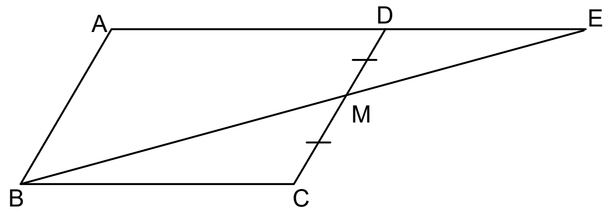
$\Rightarrow D = 0$

$\therefore 64 - 4q = 0$

$q = 16$

OR

13. $\because$ ABCD is a parallelogram.



$\Rightarrow AD \parallel BC \Rightarrow \angle ADM = \angle BCM$ (alternate)

In ΔDME and ΔBMC

$\Rightarrow DM = MC$

$\angle MDE = \angle MCB$ (alternate angle)

$\angle DME = \angle BMC$ (V.O.A)

$\Rightarrow \Delta DME \cong \Delta CMB$

$\Rightarrow DE = BC$

$\because AD = BC$ (IIgm ABCD) $\Rightarrow BC = AD = DE$

$\Rightarrow AE = 2BC$

14. $x = \frac{-a}{a+b}, \frac{-(a+b)}{a}$

14. $x^2 + \left(\frac{a}{a+b} + \frac{a+b}{a}\right)x + 1 = 0$

$$x^2 + \frac{a}{a+b}x + \frac{a+b}{a}x + 1 = 0$$

$$x\left(x + \frac{a}{a+b}\right) + \frac{a+b}{a}\left(x + \frac{a}{a+b}\right) = 0$$

$$\left(x + \frac{a}{a+b}\right)\left(x + \frac{a+b}{a}\right) = 0$$

$$x = \frac{-a}{a+b}, \frac{-(a+b)}{a}$$

OR

14. $98\,cm^2$

14. Since ABCPA is a quadrant of a circle of radius 14 cm; we have

$AB = BC = 14$ cm and $\angle ABC = 90°$

$\therefore$ $AC = \sqrt{AB^2 + BC^2}$ [By Pythagoras' theorem]

$$= \sqrt{(14)^2 + (14)^2}\,cm = \sqrt{2 \times 196}\ cm = 14\sqrt{2}\,cm$$

$\therefore$ radius of the semi circle $= \frac{1}{2}AC = 7\sqrt{2}\,cm$

Required area

= (area of the semicircle with AC as diameter) + (area of ΔABC) – (area of the quadrant with r = 14 cm)

$$= \left[\frac{1}{2} \times \frac{22}{7} \times \left(7\sqrt{2}\right)^2 + \frac{1}{2} \times 14 \times 14 - \frac{1}{4} \times \frac{22}{7} \times 14 \times 14\right] cm^2$$

$$= (154 + 98 - 154)\,cm^2 = 98\,cm^2$$

15. 40

15. $a_{20} = S_{20} - S_{19} = 20 \times 21 - 19 \times 20 = 40$

16. In triangle ABD, $AD^2 = AB^2 + BD^2 \Rightarrow AD^2 - AB^2 = BD^2$

In triangle ABC, $AC^2 = AB^2 + (2BD)^2 = AB^2 + 4\left(AD^2 - AB^2\right)$

$\Rightarrow 4AD^2 - 3AB^2 = AC^2$

OR

16. $x = \frac{a^2 - b^2}{2}, \frac{a^2 + b^2}{2}$

16. $4x^2 - 4a^2x + a^4 - b^4 = 0$

$4x^2 - 2(a^2 + b^2)x - 2(a^2 - b^2)x + a^4 - b^4 = 0$

$2x(2x - (a^2 + b^2) - (a^2 - b^2)(2x - (a^2 + b^2) = 0$

$(2x - (a^2 - b^2))\ (2x - (a^2 + b^2) = 0$

$x = \frac{a^2 - b^2}{2}, \frac{a^2 + b^2}{2}$

17. 11

17. We know that the tangents drawn from an external point to a circle are equal.

$\therefore$ $AQ = AR, BP = BQ, CS = CP$ and DR = DS.

Now, DR = DS = 5 cm.

AR = (AD – DR) = (23 – 5) cm = 18 cm.

$\therefore$ $AQ = AR = 18$ cm

$BQ = (AB - AQ) = (29 - 18)\text{cm} = 11\text{cm}$

We know that the tangent to a circle is perpendicular to the radius through the point of contact.

$\therefore$ $\angle OQB = \angle OPB = 90^\circ$ and $\angle B = 90^\circ$ (given)

$OQ = OP = r$ (given)

$\therefore$ OQBP is a square.

$\therefore$ $r = OP = OQ = BQ = 11$ cm

Hence, radius of the circle is r = 11 cm.

18. $n^2 + 6n$

18. $S_4 = 40 \Rightarrow 2(2a + 3d) = 40 \Rightarrow 2a + 3d = 20$

$S_{14} = 280 \Rightarrow 7(2a + 13d) = 280 \Rightarrow 2a + 13d = 40$

Solving to get d = 2 and $a = 7$

$\therefore$ $S_n = \frac{n}{2}\left[14 + (n-1) \times 2\right]$

$= n(n + 6) = (n^2 + 6n)$

19. $AQ^2 = AC^2 + \frac{BC^2}{4}$ ……..(i) (Pythagoras Theorem)

$BP^2 = \frac{AC^2}{4} + BC^2$ ……..(ii)

(i) + (ii) we get $4(AQ^2 + BP^2) = 5(AC^2 + BC^2)$

$4(AQ^2 + BP^2) = 5AB^2$

OR

19. 84 cm^2

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19. Radius of semi circle, $r = \frac{\text{side}}{2} = \frac{14}{2} = 7$ cm

Since the radius of all the semi – circles is same.

Area of 4 semicircles

$= 4\left[\frac{1}{2}\pi r^2\right]$

$= 4\left[\frac{1}{2} \times \frac{22}{7} \times 7 \times 7\right] = 308 \text{ cm}^2$

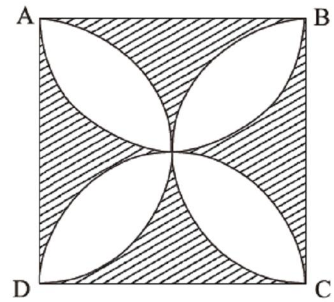
Area of Square, ABCD = $(\text{Side})^2$

$= (14)^2 = 196 \text{ cm}^2$

Area of shaded region

= 2 (Area of square) – 4 (Area of semi – circle)

$= 2 \times 196 - 308 = 392 - 308 = 84 \text{ cm}^2$



20. **Given** A circle with centre O and PA, PB are the tangents on it from a point P outside it.

To Prove : $\angle APB = 2\angle OAB$

Proof : Let $\angle APB = x^\circ$

We know that the tangents to a circle from an external point are equal. So, PA = PB

Since the angle opposite to the equal sides of a triangle are equal, so PA = PB

$\Rightarrow \angle PBA = \angle PAB$.

Also, the sum of the angles of a triangle is 180°.

$\therefore \quad \angle APB + \angle PAB + \angle PBA = 180^\circ$

$\Rightarrow \; x^\circ + 2\angle PAB = 180^\circ \qquad [\because \angle PBA = \angle PAB]$

$\Rightarrow \; \angle PAB = \frac{1}{2}(180^\circ - x^\circ) = \left(90^\circ - \frac{1}{2}x^\circ\right)$

But, PA is a tangent and OA is the radius of the given circle.

$\therefore \quad \angle OAB + \angle PAB = 90^\circ$

$\Rightarrow \quad \angle OAB + \left(90^\circ - \frac{1}{2}x^\circ\right) = 90^\circ \Rightarrow \angle OAB = \frac{1}{2}x^\circ = \frac{1}{2}\angle APB$

$\Rightarrow \quad \angle APB = 2\angle OAB$.

OR

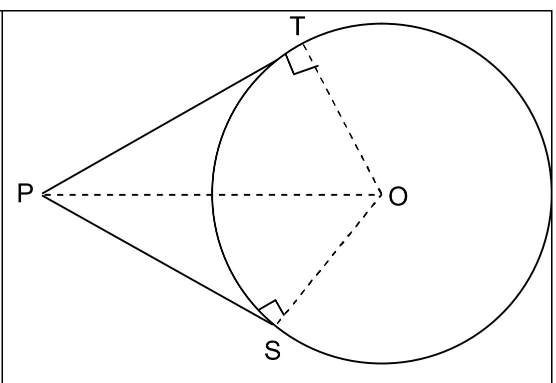
20. Given: PT and PS are tangents from an external point P to the circle with centre O.

To prove : PT = PS

Const: Join O to P, T and S.

Proof : In ΔOTP and ΔOSP

OT = OS	[radii of same circle]
OP = OP	[Common]
$\angle OTP = \angle OSP$	[each 90°]
$\therefore \Delta OTP \cong \Delta OSP$	[R.H.S.]
$\therefore PT = PS$	[c.p.c.t.]



Let the given incircle touches side AB in N and side AC in M. Since the tangent to a circle from an exterior point are equal in length $\Rightarrow AN = AM, BN = BL$ and CL = CM, On adding, we get $AN + BN + CL = AM + BL + CM$ $\Rightarrow (AN + BN) + CL = (AM + CM) + BL$ $\Rightarrow AB + CL = AC + BL$ $\Rightarrow CL = BL$ [Given : AB = AC] i.e. The point L bisects BC.