BATCH – AII 1820

PHYSICS, CHEMISTRY & MATHEMATICS

CPT2 | **CODE:** | **PAPER - 2**

Time Allotted: 3 Hours **Maximum Marks: 312**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.
- You are not allowed to leave the Examination Hall before the end of the test.

INSTRUCTIONS

Caution: Question Paper CODE as given above MUST be correctly marked in the answer OMR sheet before attempting the paper. Wrong CODE or no CODE will give wrong results.

A. General Instructions

1. Attempt ALL the questions. Answers have to be marked on the OMR sheets.
2. This question paper contains Three Sections.
3. **Section-I** is Physics, **Section-II** is Chemistry and **Section-III** is Mathematics.
4. Each section is further divided into two part: **Part A & B**
5. Rough spaces are provided for rough work inside the question paper. No additional sheets will be provided for rough work.
6. Blank Papers, clip boards, log tables, slide rule, calculator, cellular phones, pagers and electronic devices, in any form, are not allowed.

B. Filling of OMR Sheet

1. Ensure matching of OMR sheet with the Question paper before you start marking your answers on OMR sheet.
2. On the OMR sheet, darken the appropriate bubble with ***Blue/Black Ball Point Pen*** for each character of your Enrolment No. and write in ink your Name, Test Centre and other details at the designated places.
3. OMR sheet contains alphabets, numerals & special characters for marking answers.

C. Marking Scheme For All Three Parts.

(i) **PART-A (01 – 08)** contains 8 Multiple Choice Questions which have Only One Correct answer. Each question carries **+4 marks** for correct answer and **–2 marks** for wrong answer.

PART-A (09 – 16) contains 4 Paragraphs. Based upon each paragraph, 2 Multiple Choice Questions have to be answered. Each question has Only One Correct answer and carries **+3 marks** for the correct answer and **–1 mark** for a wrong answer.

(ii) **PART-B (01 – 04)** contains 4 Matrix Match Type Question which have statements given in 2 columns. Statements in the first column have to be matched with statements in the second column. There may be One or More Than One Correct choices. Each question carries **+12 marks** for all correct answer however for each correct row **+3 marks** will be awarded and **–1 mark** for each row matched incorrectly.

Name of the Candidate :_____

Batch :_____ **Date of Examination :**_____

Enrolment Number :_____

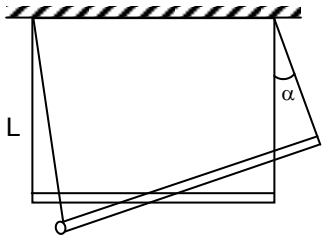
SECTION-I: PHYSICS

Part-A: Only One Option Correct Type

This section contains **8 Multiple Choice Questions.** Each question has 4 choices (A), (B), (C) and (D), out of which **Only One Option is correct.**

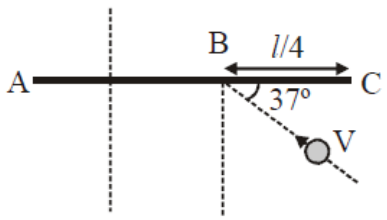
1. Two glass hemispheres of refractive index μ and radius R are placed with their plane surfaces opposed separated by a distance d.
   (A) If the combination is achromatic for rays parallel to the axis when their planes surfaces are facing towards each other, then the value of 'd' = $\frac{R}{\mu-1}$
   (B) If the combination is achromatic for rays parallel to the axis when their planes surfaces are facing towards each other, then the value of 'd' = $\frac{2R}{\mu-1}$.
   (C) If the combination is achromatic for rays parallel to the axis when their curved surfaces are facing towards each other, then the value of 'd' = $\frac{R}{\mu-1}$.
   (D) If the combination is achromatic for rays parallel to the axis when their curved surfaces are facing towards each other, then the value of $d=\frac{R(2\mu-1)}{\mu-1}$.

2. A uniform rod AB of mass m suspended by two equal parallel thread of length L was turned though an angle about a vertical axis passing through its centre C. The threads deviate in the process through an angle α. Then the rod was released.



   (A) energy of the oscillating rod = $\frac{mgL\alpha^2}{2}$
   (B) energy of the oscillating rod = $\frac{mgL\alpha^2}{4}$
   (C) period of oscillation of the rod = $\sqrt{\frac{3g}{L}}$
   (D) period of oscillation of the rod = $\sqrt{\frac{3g}{2L}}$

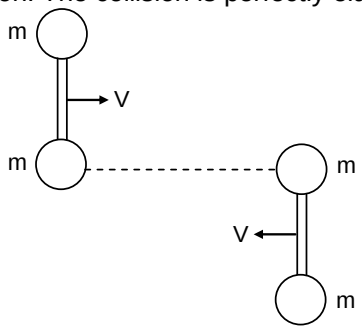
3. A uniform rod AC of length ℓ and mass m is kept on a horizontal smooth plane. It is free to rotate and move A particle of same mass m moving on the plane with velocity v strikes the rod elastically as shown. Then after collision:



   (A) The angular velocity of the rod will be $\frac{72V}{25\ell}$
   (B) The centre of the rod will travel a distance $\frac{\pi\ell}{3}$ in which it make half rotation.
   (C) Impulse of the impact force is $\frac{24mV}{55}$
   (D) None of these

space for rough work

4. Two identical dumb-bells move towards each other on a horizontal frictionless surface as shown. Each can be considered to be two point mass 'm' joined by a massless rod of length 2ℓ. Initially they are given only translation motion. The collision is perfectly elastic.



Choose the correct option(s).

(A) After first collision each dumb-bell rotates with $\omega = \frac{V}{2\ell}$.

(B) Each dumb-bell stops translating after first collision.

(C) The second collision occur after time $t = \frac{\pi\ell}{V}$.

(D) After second collision both dumb-bells move with velocity V without rotation.

5. A non-conducting ring of mass m and radius R has a charge Q uniformly distributed over its circumference. The ring is placed on a rough horizontal surface such that plane of the ring is parallel to the surface. A vertical magnetic field $B = B_0t^2$ tesla is switched on. After 2s from switching on the magnetic field the ring is just about to rotate about vertical axis through its centre:

(A) Friction coefficient between the ring and the surface is $\frac{2B_0RQ}{mg}$.

(B) Friction coefficient between the ring and the surface is $\frac{B_0RQ}{2mg}$.

(C) If magnetic field is switched off after 4s, then the angle rotated by the ring before coming to stop after switching off magnetic field is $\frac{2B_0Q}{m}$.

(D) If magnetic field is switched off after 4s, then the angle rotated by the ring before coming to stop after switching off magnetic field is $\frac{B_0Q}{m}$.

space for rough work

6. A square loop of side l = 10 cm with its sides parallel to x and y axes is kept in a magnetic field which is in positive z-direction. It moves with velocity $v = -8\hat{i}$ m/s If the resistance of the loop is $2 \times 10^{-3}\ \Omega$

(A) If the magnetic field decreases along negative x-axis wit a gradient $\frac{\partial B}{\partial x}$ = 0.1 T/m then emf induced is 8×10^{-3} V.

(B) If the magnetic field increases with time at a rate of $\frac{\partial B}{\partial t}$ = 0.1T/s then emf induced is 10^{-3} V.

(C) If the magnetic field decreases along negative x-axis wit a gradient $\frac{\partial B}{\partial x}$ = 0.1 T/m and simultaneously increases with time at a rate of $\frac{\partial B}{\partial t}$ = 0.1T/s then emf induced is 7×10^{-3} V.

(D) If the resistance of the loop is $2 \times 10^{-3}\ \Omega$ in option C then current in the loop is 3.5 amp anticlockwise.

7. A small circular loop of radius a carrying a current i_0 is placed co-axially with the solenoid of radius R, length 2R and current I flowing through its windings i.e. along its surface. If the loop is displaced from the centre C of the solenoid to the end of the solenoid, find the magnitude of work done by the external agent.

(A) $\frac{\mu_0 i_0 i\pi a^2}{2R}\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{5}}\right)$ (B) $\frac{\mu_0 i_0 i\pi a^2}{2R}\left(\frac{1}{\sqrt{5}} - \frac{1}{\sqrt{2}}\right)$

(C) $\frac{\mu_0 i_0 i\pi a^2}{R}\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{5}}\right)$ (D) zero

8. A rocket is launched from and returns to a spherical planet of the radius R is in such a way that its velocity vector on return is parallel to its launch vector and is oppositely directed. The angular separation at the centre of the planet between the launch and arrival points is θ. The maximum distance of the rocket above the surface of planet is

(A) $R\cos\frac{\theta}{2}$ (B) $R\sin\frac{\theta}{2}$ (C) $R\cos\theta$ (D) R

space for rough work

Paragraph Type Questions

This section contains **4 paragraphs.** Based upon each paragraph, 2 Multiple Choice Questions have to be answered. Each question has **Only One Correct** answer.

Paragraph for Question Nos. 9 to 10

If a plane simple harmonic wave is traveling towards positive x axis its wave function is given by

$$S = S_0 \sin(\omega - kx)$$

Here S is longitudinal displacement of any particle. If we take a plane parallel to yz plane all particles in this plane have same phase. This plane is called wave front.

If a plane wave is traveling along direction $\hat{n}$ its wave function can be written as:

$$S = S_0 \sin(\omega t - k\hat{n}.\vec{r})$$

Let's call $k\hat{n} = \vec{k}$ (a wave vector)

$$S = S_0 \sin(\omega t - \vec{k}.\vec{r})$$

Wave vector points in direction of phase velocity. In other words, the wave vector points in the normal direction to the wavefront.

9. Consider a wave function:
$S = S_0 \sin(\omega t - 2x + 2y + z)$ Here all variables are in SI units.
Select correct statement(s)
(A) Wave length of the wave is $\frac{2\pi}{3}$
(B) Particle (0, 0, 0) and (1, 1, 2) are on same wavefront
(C) The wave vector is at angle $\cos^{-1}\left(\frac{1}{3}\right)$ from x – axis
(D) The wave vector is at equal angle from positive x-axis and positive y-axis

10. Consider following wave function
$S_1 = S_0 \sin(\omega t - 2x + 2y + z)$
$S_2 = S_0 \sin(\omega t + 2x - 2y - z)$
An observer is traveling towards +x axis with speed π m/s. The beat frequency observed by the observer is:
(A) $\frac{1}{2}$ (B) 2 (C) 1 (D) $\frac{3}{2}$

Paragraph for Question Nos. 11 to 12

Three small bodies with the mass ratio 3 : 4 : 5 (the mass of the lightest body is m) are kept at three different points on the inner surface of a smooth hemispherical cup of radius r. The cup is fixed at its lower point on a horizontal surface. The three bodies are released at the same given instant. All the collisions are perfectly inelastic.

11. The maximum amount of heat Q that can be liberated in such a system is
(A) mgr (B) 2 mgr (C) 4 mgr (D) 3 mgr

space for rough work

12. If the amount of heat generated in the system to be maximum the initial distance between the lightest and the heaviest ball is

(A) $\frac{4r}{\sqrt{5}}$ (B) $\frac{2r}{\sqrt{5}}$ (C) $\frac{r}{\sqrt{5}}$ (D) $\frac{4r}{\sqrt{3}}$

Paragraph for question 13 to 14

A rubber balloon filled with helium gas goes up high into the sky where pressure and temperature decreases with height. Assume the shape of the balloon remains spherical and temperature of helium gas inside the balloon is same as at that of ambient air and treat all the gases as ideal. Molecular mass of helium and air are M_{He} and M_A.

13. Let the pressure of the ambient air be P and temperature be T. the pressure inside the balloon (P: + ΔP) is higher then outside due to surface tension of the balloon. If there are 'n' moles of helium gas inside balloon, then buoyant force acting on the balloon as a function of P and ΔP is

(A) $M_A ng\frac{P}{P+\Delta P}$ (B) $M_A g\frac{P}{P+\Delta P}$ (C) $M_A ng\frac{P+\Delta P}{P}$ (D) $M_{He} ng\frac{P}{P+\Delta P}$

14. When a rubber balloon of spherical shape with unstretched radius r_0 is inflated to a sphere of radius r ($\geq r_0$) the balloon surface contains extra elastic energy due to stretching. The elastic energy at temperature T can be expressed by

$$U = 4\pi r_o^2 kRT\left(2\lambda^2+\frac{1}{\lambda^4}-3\right)$$

Where $\lambda=\frac{r}{r_0}$ is size inflation ratio, k is constant in units of mol/m^2.

ΔP in terms of given parameters is

(A) $\frac{4kRT}{r_0}\left(\frac{1}{\lambda}+\frac{1}{\lambda^2}\right)$ (B) $\frac{4kRT}{r_0}\left(\frac{1}{\lambda}-\frac{1}{\lambda^2}\right)$

(C) $\frac{2kRT}{r_0}\left(\frac{1}{\lambda}-\frac{1}{\lambda^2}\right)$ (D) $\frac{4kRT}{r_0}\left(\frac{1}{\lambda}-\frac{1}{\lambda^3}\right)$

Paragraph for question 15 to 16

A block of mass 6 kg is kept on a rough horizontal surface and is pulled by time varying force F = Kt where K = 5Ns^{-1}, acting at an angle 37° with horizontal. It is observed that motion of block has started at t = 4 sec. and just after that acceleration of block is 2/3 m/s^2 (g = 10 m/s^2).

15. Co-efficient of static friction between block & surface is

(A) 2/3 (B) 1/3 (C) 4/5 (D) 3/4

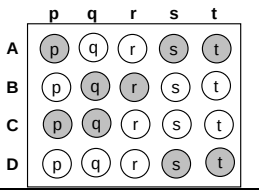
16. Velocity of block when it breaks off the surface is equal to

(A) 218 m/s (B) 128 m/s (C) 112 m/s (D) $\frac{233}{2}$ m/s

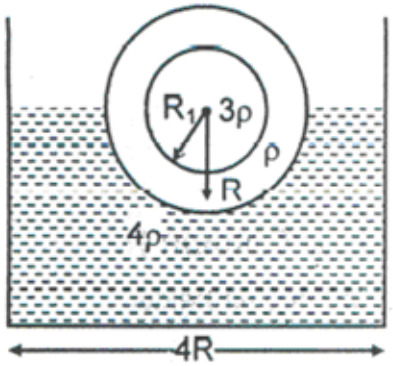
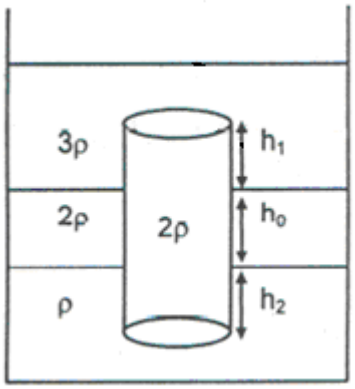
space for rough work

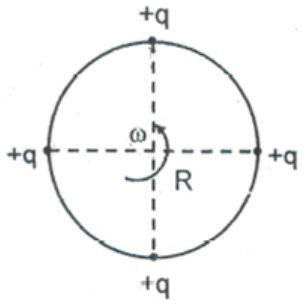
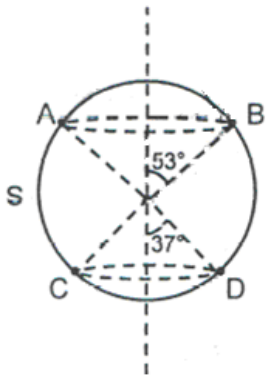
Part-B : Matrix-Match Type Questions

This Section contains **4 Matrix Match Type Questions.** Each question has **four statements** (A, B, C and D) given in **Column I** and **five statements** (p, q, r, s and t) in **Column II.** Any given statement in Column I can have correct matching with **ONE** or **MORE** statement(s) given in Column II. For example, if for a given question, statement B matches with the statements given in q and r, then for the particular question, against statement B, darken the bubbles corresponding to q and r in the ORS.



1. In Column-I a system is shown in which we have defined 2 numbers x and y. Column-II has some mathematical relations between x and y. Match the proper entries from Column-II to column-I.

Column - I		Column - II	
(A)	 A solid sphere of radius R is made up by freezing two liquids one of density 3ρ and another of density ρ, core of sphere of radius R_1 is made u by liquid of density 3ρ and rest by another liquid. Sphere is floating in a cylindrical container having liquid of density 4ρ, diameter 4R and sufficient height. It is half submerged. A ratio x is defined as : $x = \frac{R^3}{R_1^3}$ Now temperature is raised, neglect expansion due to increase in temperature everywhere, when whole freezed liquid is melt its found that height of liquid column in container raises by Δh. A ratio y is defined as : $y = \frac{R}{\Delta h}$	(P)	$x + y = 8$
(B)	 A uniform solid cylinder of density 2ρ height h is floating inside 3-liquids as shown. All the three liquids are perfectly immiscible. There are movable separators between each interface and so they stay at their place. These separators doesn't apply any tangential force on cylinder, pressure variation doesn't change due to separator. A quantity X is defined as : $x = \frac{2h_2}{h_1}$ Now we define a number α, such that if we displace the cylinder slightly downwards and then if it further goes down on its own, value of α is 1 and if its comes back then value of α is 2. Similarly we define another number β, such that when we	(Q)	$\lvert x - y \rvert = 1$

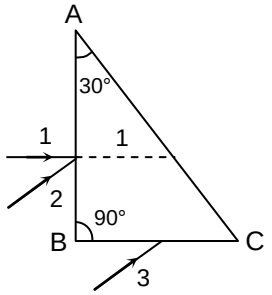
	displace the cylinder slightly upward and then if it further goes upward on its own value of β is 3 and if its comes back then value of β is 4. Another number y is defined as : $y = \alpha\beta$		
(C)	 For identical charges are fixed on the periphery of anon conducting ring of radius R as shown. Ring is rotated with constant angular velocity ω w.r.t. an axis perpendicular to plane of the ring and passing through the centre of the ring. Force acting on any of the charge particle only due to the magnetic interaction between the charges is $F = \frac{\mu_0 q^2 \omega^2}{\alpha\pi}\left[\sqrt{\beta} + \frac{1}{\beta}\right]$. Here α & β are integers. x is defined as $x = \frac{3\alpha}{8}y$ is defined as $y = \frac{5\beta}{2}$	(R)	$\frac{x}{y} > 1$
(D)	 We have a spherical bubble of radius R, made up of liquid having surface tension S. AB and CD are two parallel planes as shown in figure. Net force of surface tension on part ABDC due to rest of the part is $\frac{\alpha\pi RS}{\beta}$. There is no common factor in α and β. x is defined as $x = \frac{3\alpha}{14}y$ is defined as $y = \frac{\beta}{5}$	(S)	$\frac{x}{y} < 1$
		(T)	$\frac{x+y}{\lvert x-y\rvert}$ = odd number

space for rough work

2. If a real point object is moving with speed V_o away from a mirror perpendicular to its reflecting surface then its image is moving with speed v_i. u, v and R are object distance, image distance and radius of curvature of the mirror respectively. Column-I has the information about image and column-II has information about position of object. Then match the column-I and column-II.

Column-I		Column-II	
(A)	Real image and $v_i > v_o$	(P)	$O < \|u\| < \left\|\frac{R}{2}\right\|$
(B)	Real image and $v_i < v_o$	(Q)	$\left\|\frac{R}{2}\right\| < \|u\| < \|R\|$
(C)	Virtual image and $v_i > v_o$	(R)	$\|R\| < \|u\|$
(D)	Real image and $v_i = v_o$	(S)	$\|u\| = \|R\|$

3. ABC is a right-angled prism kept in air. A ray (1) is incident on the face AB along the normal. Refractive index of the material of prism should be greater than μ such that will be required so that ray (1) undergoes total internal reflection at the face AC. Another ray (2) is incident on the face AB such that it emerges from face AC along the normal to AC (considering $\mu_{prism} = 2$). A third ray (3) falls on the face BC and emerges from face AC (considering $\mu_{prism} = 2$) such that its angle of emergence is the same as that of incidence. Assuming light (1), (2) and (3) have the same wavelength, then match the following.



Column-I		Column-II	
(A)	μ is equal to	(P)	120
(B)	Angle of incidence in degree of ray (2) is	(Q)	90
(C)	Deviation in degree suffered by ray (2) is	(R)	2
(D)	Deviation in degree suffered by ray (3) is	(S)	60
		(T)	$\sqrt{3}/2$

4. Column II contains a list of some physical quantities or expressions and Column I lists some dimensional formula. Match the entries in Column I with all the entries in Column II.

Column-I		Column-II	
(A)	$\sqrt{\frac{ch}{G}}$	(P)	Mass
(B)	$\sqrt{\frac{Gh}{c^5}}$	(Q)	Time
(C)	$\frac{c^5}{G}$	(R)	Torque
(D)	$\sqrt{\frac{c^5h}{G}}$	(S)	$(\text{Current})^2 \times \text{Resistance}$

space for rough work

SECTION-II: CHEMISTRY

Part-A: Only One Option Correct Type

This section contains **8 Multiple Choice Questions.** Each question has 4 choices (A), (B), (C) and (D), out of which **Only One Option is correct.**

1. Which of the following forms a purple colour complex with alcoholic α-napthol in the presence of H_2SO_4

(A) Carbohydrates (B) Thiols (C) Nitro Compounds (D) Amines

2. Which of the following amino acid does not give blue product with indane-1, 2, 3 trione?

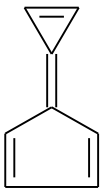
(A) Glycine (B) Alanine (C) Proline (D) Histidine

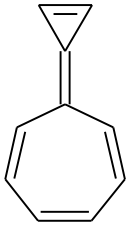
3. $RCOOAg \xrightarrow[\text{Reflux}]{X_2(CCl_4)} RX$

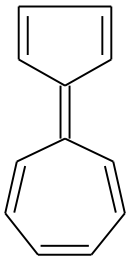
Which of the following X_2 can't be used in the above reaction?

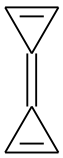
(A) Cl_2 (B) Br_2 (C) I_2 (D) All of these

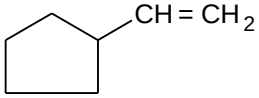
4. Which of the following is most soluble in water?

(A) 

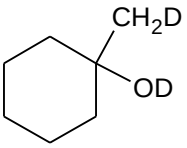
(B) 

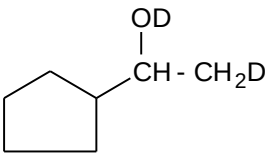
(C) 

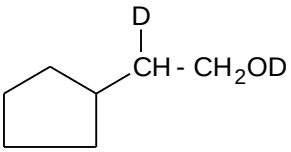
(D) 

5. 

Identify (P)

(A) 

(B) 

(C) 

(D) None of these

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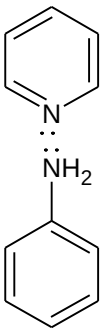
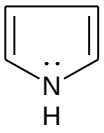
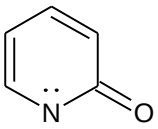
6. Which of the following is most basic?

(A) 

(B) 

(C)

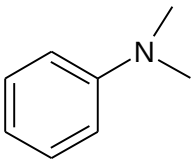
(D) 

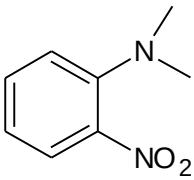
7. $H_2\overset{1}{C}=\overset{2}{C}H-\overset{3}{C}\equiv\overset{4}{C}H \xrightarrow{E^{\oplus}}$

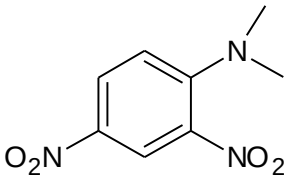
Which carbon is most reactive towards the attack of electrophile?

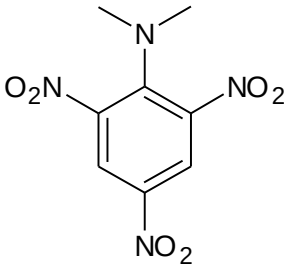
(A) 1 (B) 2 (C) 3 (D) 4

8. Which of the following is most basic in aqueous medium?

(A) 

(B) 

(C) 

(D) 

Paragraph Type Questions

This section contains **4 paragraphs.** Based upon each paragraph, 2 Multiple Choice Questions have to be answered. Each question has **Only One Correct** answer.

Paragraph for Question Nos. 9 to 10

The lower oxidation states of N in its oxide are neutral and sparingly soluble in water while higher O.S are acidic and more soluble in water.

9. Which of the following is/are mixed anhydride of N?

(A) NO (B) NO_2 (C) N_2O_3 (D) N_2O_5

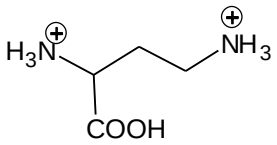
10. What products is not formed when N_2O_3 is treated with H_2O_2?

(A) O_2 (B) H_2O (C) HNO_3 (D) None of these

space for rough work

Paragraph for Question Nos. 11 to 12

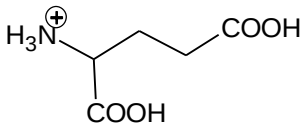
Isoelectric point is an intermediate pH at which conc. of zwitter ion is maximum as well as it is temperature dependent only.

11. 

$K_{a_1} = 10^{-2}$
$K_{a_2} = 10^{-5}$ at 25°C
$K_{a_3} = 10^{-9}$

What is isoelectric point of the above amino acid at 25°C?

(A) 3.5 (B) 7 (C) 5.5 (D) 8

12. 

$K_{a_1} = 10^{-2}$
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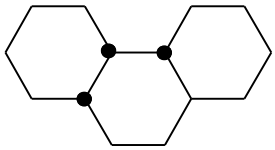
What is isoelectric point of the above amino acid at 25°C?

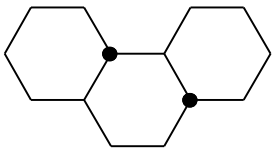
(A) 3.5 (B) 7 (C) 5.5 (D) 8

Paragraph for question 13 to 14

The nomenclature of perhydrophenathrene is done in such a way that the prefixed cis and trans refer to stereochemistry of terminal rings fusion to centre ring whereas syn and anti are used to denote the orientation of the terminal rings with respect to each other. A heavy dot indicates a hydrogen in front of the plane whereas the absence of dot denotes a hydrogen behind the plane. Like



Cis-syn-Trans

13. 

What is orientation in the given compound?

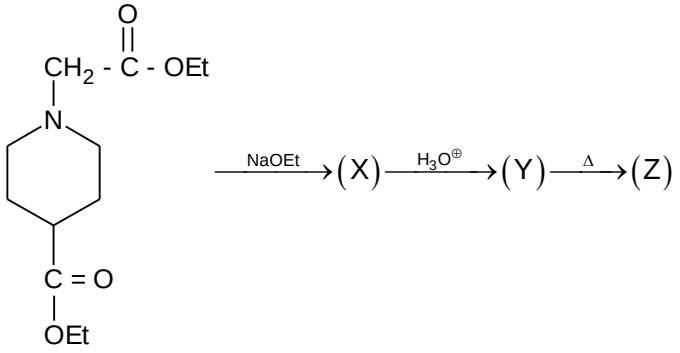
(A) cis-syn-cis (B) trans-anti-trans (C) cis-anti-trans (D) cis-anti-cis

space for rough work

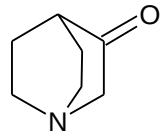
14. Total how many stereoisomers of perhydrophenanthrene are possible?
(A) 10 (B) 8 (C) 5 (D) None of these

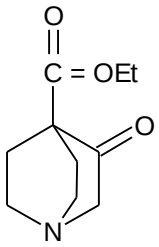
Paragraph for question 15 to 16

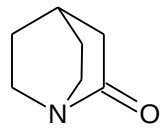
Intramolecular claisen ester condensation is called as Dieckmann condensation which takes place on treating a diester with strong base. The driving force of the reaction is stability of ring.

15. 

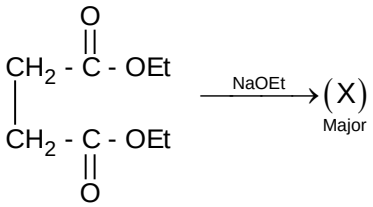
Identify(Z)

(A) 

(B) 

(C) 

(D) None of these

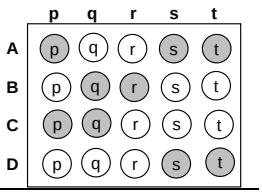
16. 

Total how many mole of CH_3MgX are required to react with 1 mole of (X)
(A) 2 (B) 4 (C) 8 (D) None of these

space for rough work

Part-B: Matrix-Match Type Questions

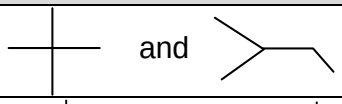
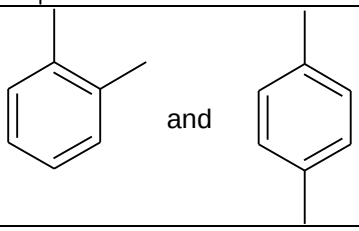
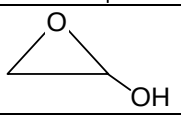
This Section contains **4 Matrix Match Type Questions.** Each question has **four statements** (A, B, C and D) given in **Column I** and **five statements** (p, q, r, s and t) in **Column II.** Any given statement in Column I can have correct matching with **ONE** or **MORE** statement(s) given in Column II. For example, if for a given question, statement B matches with the statements given in q and r, then for the particular question, against statement B, darken the bubbles corresponding to q and r in the ORS.



1. Match the following:

Column – I Type of solid		Column – II 2nd nearest neighbour of cation	
(A)	NaCl	(P)	a
(B)	ZnS(Zinc blend)	(Q)	$a/\sqrt{2}$
(C)	CaF_2	(R)	$\sqrt{3}a/2$
(D)	CsCl	(S)	$\sqrt{3}a/4$

2. Match the following:

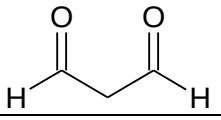
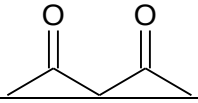
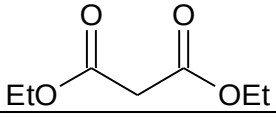
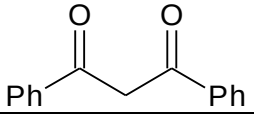
Column – I Type of isomerism		Column – II ef	
(A)	Position	(P)	
(B)	Chain	(Q)	
(C)	Ring Chain	(R)	$OHCCH_2OH$ and 
(D)	Functional group	(S)	HCN & HNC

3. Match the following:

Column – I Possible shape		Column – II Type of hybridization	
(A)	Pyramidal	(P)	sp^3d
(B)	Linear	(Q)	sp^3d^2
(C)	Sea-saw	(R)	sp^3d^3
(D)	Capped octahedral	(S)	sp^3

space for rough work

4. Match the following:

Column – I Compounds		Column – II Properties	
(A)		(P)	Maximum enolic content
(B)		(Q)	Minimum enolic content
(C)		(R)	Least acidic
(D)		(S)	% of Keto is more than enol in protic solvent

SECTION-III: MATHEMATICS

Part-A: Only One Option Correct Type

This section contains **8 Multiple Choice Questions.** Each question has 4 choices (A), (B), (C) and (D), out of which **Only One Option is correct.**

1. Let $f(x) = x^2 - (k^2 - 3k + 2)x + 4 - 4k,\ k \in R$ and $g(x) = \ell n\left(\frac{1 - x + x^2}{1 + x + x^2}\right)$. If α and β are two distinct real roots of the equation $f(x) = 0$ such that $g(\alpha) + g(\beta) = 0$, then sum of all possible values of k is

(A) $\frac{17}{4}$ (B) 2 (C) 3 (D) $\frac{13}{4}$

2. If $N = 2^3 \times 3^4 \times 5^2 \times 7^1$ then product of all divisors of N which are divisible by 5 is

(A) N^{20} (B) N^{40} (C) $(5N)^{20}$ (D) $(5N)^{40}$

3. If the sum of all values of y satisfying the equation $e^y([x] - 2) = [x] - 1$ where $x \in (3, 100)$ is S, then [S] is equal to

(A) 2 (B) 3 (C) 4 (D) 5

[Note : [m] denote greatest integer less than or equal to m]

4. The maximum integral value of the function

$f(x) = 2\sqrt{3}\sin\left(x + \frac{\pi}{3}\right) - (\sqrt{3} - 1)\sin x - x^2 + 2(\cot^{-1}3)x + 4 - \left(\text{cosec}^{-1}\sqrt{10}\right)^2$ is

(A) 3 (B) 5 (C) 6 (D) 7

space for rough work

5. The number of integral values of y for which
$\left(y^4-4y^2\right)\left(\int_{-\sin^{-1}x}^{\sin^{-1}x} e^{\cos t}\left(\sin^3 t\right)dt+x^2-4x+8\right)<65\,\forall\, x\in[-1,1]$ is

(A) 4 (B) 5 (C) 6 (D) 7

6. Let z be a complex number such that $\mathrm{Re}(z)=2$. If area of a triangle whose sides are represented by $i\left(z^2-2z\right),\left(2z-z^2\right)$ and $(1-i)\left(2z-z^2\right)$ is 16 square units, then $|z|$ is equal to [Note $z=x+iy$ and $i=\sqrt{-1}$]

(A) 2 (B) $2\sqrt{2}$ (C) 4 (D) $4\sqrt{2}$

7. If x_1 and x_2 are the point of local minima of the function $f(x)=(x-2)^2\left(x^2-3x+2\right)\left(x^2-5x+6\right)$, then $[x_1]+[x_2]$ is equal to

(A) 0 (B) 2 (C) 3 (D) 5

[Note : [k] denote greatest integral value less than or equal to k]

8. The probability for which $\lim_{x\to 0}\frac{\ell n\left((\cos x)^a\right)}{x^b}$ exist finitely, where (a, b) is the possible outcome of throwing a pair of normal fair dice, is

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$

Paragraph Type Questions

This section contains **4 paragraphs.** Based upon each paragraph, 2 Multiple Choice Questions have to be answered. Each question has **Only One Correct** answer.

Paragraph for Question Nos. 9 to 10

Let $f(x)(x\ge 1)$ be a differentiable function satisfying $f(x)=(\log_e x)^2-\int_1^e \frac{f(t)}{t}dt$

9. $\lim_{x\to e}\left(f(x)+\frac{1}{6}\right)^{\frac{1}{x-e}}$ is equal to

(A) $\frac{2}{e}$ (B) $\frac{1}{e}$ (C) $e^{\frac{1}{e}}$ (D) $e^{\frac{2}{e}}$

10. Area bounded by tangent line of $y=f(x)$ at the point (e, f(e)), the curve $y=f(x)$ and the line $x=1$, is

(A) $e+\frac{1}{e}$ (B) $e+\frac{1}{e}-1$ (C) $e+\frac{1}{e}-2$ (D) $e+\frac{1}{e}-3$

space for rough work

Paragraph for Question Nos. 11 to 12

Consider the line, $L: \frac{x-1}{2} = \frac{y-2}{-1} = \frac{z}{2}$ in space.

11. The equation of plane containing the line L and at a distance of 3 unit from the point (2, –1, –1) are
(A) $x-2y-2z+3$ and $2x-14y-5z-30=0$
(B) $x-2y-2z-3=0$ and $2x+14y+5z-30=0$
(C) $x-2y-2z+3=0$ and $2x+14y+5z-30=0$
(D) $x-2y-2z-3=0$ and $2x-14y+5z+30=0$

12. If the equation of plane passing through (2, 1, 1) and containing the line L has the equation $ax+by+cz=1$, then the value of $|a|+|b|+|c|$ is equal to
(A) 2 (B) 3 (C) 5 (D) 8

Paragraph for question 13 to 14

A coin is weighted such that probability of it showing head (H) is $\frac{2}{3}$ and that of tail (T) is $\frac{1}{3}$ when it is tossed. If head appears, then a number from the first 9 natural numbers is selected at random, otherwise a number from 1, 2,3, 4, 5 is selected. Let E be the event of getting an even number.

13. The P (E) is equal to
(A) $\frac{54}{135}$ (B) $\frac{17}{45}$ (C) $\frac{58}{135}$ (D) $\frac{38}{45}$

14. The $P\left(\frac{H}{E}\right)$ is equal to;
(A) $\frac{11}{29}$ (B) $\frac{20}{29}$ (C) $\frac{14}{29}$ (D) $\frac{22}{29}$

Paragraph for question 15 to 16

Let matrix $M=\begin{bmatrix}1&0&0\\1&0&1\\0&1&0\end{bmatrix}$ satisfies $M^n = M^{n-2}+M^2-I$ for $n=3,4,5,6.....$, where I is an identity matrix of order 3. Also U_1, U_2, U_3 are column matrices satisfying $M^{50}U_1=\begin{bmatrix}1\\25\\22\end{bmatrix}$, $M^{50}U_2=\begin{bmatrix}0\\1\\0\end{bmatrix}$, $M^{50}U_3=\begin{bmatrix}0\\0\\1\end{bmatrix}$ and U is a 3×3 matrix whose columns are U_1, U_2, U_3.

15. The $\det\left(\text{adj}M^{50}\right)$ is equal to
(A) 0 (B) 1 (C) 25 (D) 12

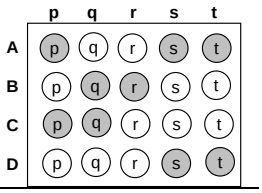
space for rough work

16. Let $X=\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B=\begin{bmatrix} 0 \\ 2 \\ 4 \end{bmatrix}$ be two matrices then the system of equations

(A) UX = B is inconsistent
(B) UX = B has infinitely many solutions
(C) UX = B has a unique solution
(D) UX = B has exactly three solutions

Part-B : Matrix-Match Type Questions

This Section contains **4 Matrix Match Type Questions.** Each question has **four statements** (A, B, C and D) given in **Column I** and **five statements** (p, q, r, s and t) in **Column II.** Any given statement in Column I can have correct matching with **ONE** or **MORE** statement(s) given in Column II. For example, if for a given question, statement B matches with the statements given in q and r, then for the particular question, against statement B, darken the bubbles corresponding to q and r in the ORS.



1. Match the following

Column-I		Column-II	
(A)	Let $f:R\to R$ be defined as $f(x)=2x^3+7x-5$ and $g(x)=f^{-1}(x)$. If $g'(4)=\frac{a}{b}$ where a and b are relatively prime positive integers then $(a+b)$ is equal to	(P)	12
(B)	A box contains 24 balls of which 12 are black and 12 are white. The balls are drawn at random from the box one at a time with replacement. The probability that a white ball is drawn for the fourth time on the seventh draw is $\frac{m}{n}$, where m and n are relatively prime, then (m + n) equals	(Q)	14
(C)	If α is a real constant A, B and C are variable angles such that $\sqrt{\alpha^2-4}\tan A+\alpha\tan B+\sqrt{\alpha^2+4}\tan C=6\alpha$, then the least value of $(\tan^2 A+\tan^2 B+\tan^2 C)$ is equal to	(R)	25
(D)	A has (n + 1) fair coins and B has n fair coins. They both toss their coins. If probability that A gets more heads than B be p, then 24p is equal to	(S)	37

space for rough work

2. Match the following

List - I		List - II	
(A)	If the distance between two parallel tangents having slope m drawn to the hyperbola $\frac{x^2}{9}-\frac{y^2}{49}=1$ is 2, then $\frac{2}{5}\lvert m\rvert$ is equal to	(P)	5
(B)	If a variable line has its intercepts on the coordinate axes e and e', where $\frac{e}{2}$ and $\frac{e'}{2}$ are the eccentricities of a hyperbola and its conjugate hyperbola, then the line always touches the circle $x^2+y^2=r^2$, where r is equal to	(Q)	4
(C)	If the equation of line touching both parabola $y^2-4x=0$ and $x^2+32y=0$ is $ax-2y+b=0(a,b\in R)$, then $(a+b)$ is equal to	(R)	2
(D)	If the mid – point of a chord of the ellipse $\frac{x^2}{16}+\frac{y^2}{25}=1$ is $M(0,3)$ and the length of chord is $\frac{8p}{5}$, then p is equal to	(S)	1

3. Match the following

List - I		List - II	
(A)	Let z,ω,α be complex numbers such that $\lvert z\rvert=\lvert\omega\rvert=4$ and $\alpha=\frac{z-\bar{\omega}}{16+z\omega}$, then $\mathrm{Re}(\alpha)$ is equal to	(P)	0
(B)	If $x=p+iq$ is a complex number such that $x^2=3+4i$ and $x^3=2+11i$ where $i=\sqrt{-1}$, then $(p+q)$ is equal to	(Q)	3
(C)	Number of complex numbers z satisfying the equation $\bar{z}=iz^2$, where $i=\sqrt{-1}$ is equal to	(R)	4
(D)	If $z\in C$ satisfies $\lvert z+2-i\rvert=5$ then the maximum value of $\frac{\lvert 3z+9-7i\rvert}{4}$ is equal to	(S)	5

4. Match the following

List - I		List - II	
(A)	Let $f:R\to R$ and $g:R\to R$ be two functions defined as $f(x)=(3\sin x-4\cos x-10)$ and $g(x)=(3\sin x+4\cos x-10)$ then the minimum value of $\sqrt{f(x)g(x)}$ is equal to	(P)	2
(B)	Number of integral values of $k\in[-1,4]$ for which the equation $\sin x+\cos(k+x)+\cos(k-x)=2$ has real solution is	(Q)	3
(C)	If $\log_\alpha\beta=2, \log_\beta\gamma=2$ and $\log_3\gamma=3+\log_3\alpha$, then the value of $\left(\frac{\gamma}{\alpha\beta}\right)$ is equal to	(R)	6
(D)	In triangle ABC, if a = 4, b = 3 and $\cos\frac{A}{2}=\sqrt{\frac{b+c}{2c}}$, then area of triangle ABC is equal to [Note : All symbols used have usual meaning in triangle ABC.]	(S)	7

space for rough work