OLTS-2021-T9-FT-V-KVPY-CLASS-XII
FULL TEST – V

PART – I
MATHEMATICS

1. The solution set of the in equation $\log_{1/3}\left(x^2+x+1\right)+1>0$ is

(A) $(-\infty,-2)\cup(1,+\infty)$ (B) $[-1,2]$

(C) (–2, 1) (D) $(-\infty,+\infty)$

Ans. C

Sol. $\log_{1/3}\left(x^2+x+1\right)>-1$

$=\log_{1/3}\left(\frac{1}{3}\right)^3 \Rightarrow x^2+x+1<\left(\frac{1}{3}\right)^{-1}$

$\Rightarrow x^2+x-2<0$. Use sign scheme.

2. The number of solutions of the equation $z^2+|z|^2=0$, where $z\in C$ is:

(A) one (B) two

(C) three (D) infinitely many

Ans. D

Sol. $z^2+|z|^2=0,$

$\Rightarrow \quad z^2+z\bar{z}=0$

$\Rightarrow \quad z(z+\bar{z})=0$

$\therefore z=0$ and $\operatorname{Re}(z)=0$

If $z=a+ib$

$\therefore$ Solutions are $z=0, ib\,(b\in R)$

3. Number of solutions of the equations $y=\frac{1}{3}\left[\sin x+\left[\sin x+[\sin x]\right]\right]$ and $\left[y+[y]\right]=2\cos x$, where [.] denotes the greatest integer function is:

(A) 0 (B) 1

(C) 2 (D) infinite

Ans. A

Sol. $\because y=\frac{1}{3}\left[\sin x+\left[\sin x+[\sin x]\right]\right]$

$=\frac{1}{3}\left([\sin x]+[\sin x]+[\sin x]\right)$

$= [\sin x]$ and $[y + [y]] = 2\cos x$

$\Rightarrow \quad 2[y] = (2\cos x)$

$\Rightarrow \quad [y] = \cos x$

Case 1 : When $-1 \le \sin x < 0$

$\Rightarrow \quad [\sin x] = -1$

Then, $\quad y = -1$

i.e., $\quad [-1] = \cos x$

$\Rightarrow \quad \cos x = -1$

$\Rightarrow \quad \sin x = 0$ (impossible)

Case II : When $0 \le \sin x < 1$

$\therefore \quad [\sin x] = 0$

then, $\quad y = 0$

i.e., $\quad 0 = \cos x$

$\Rightarrow \quad \sin x = 1$ (impossible)

Case III $\sin x = 1$

$\therefore \quad [\sin x] = 0$

then, $\quad y = 0$

i.e. $\quad 0 = \cos x$

$\Rightarrow \quad \sin x = 0$ (impossible)

Hence, no solution

i.e. no of solutions is zero.

4. In any triangle $\cos A + \cos B + \cos C$ is always equal to:

(A) $\frac{R}{r}$ (B) $1 + \frac{R}{r}$

(C) $1 + \frac{r}{R}$ (D) $\frac{r}{R}$

Ans. C

Sol. $\cos A + \cos B + \cos C$

$= 4\sin\frac{A}{2}.\sin\frac{B}{2}.\sin\frac{C}{2} + 1 = 1 + \frac{r}{R}$

5. If $\frac{x^2}{f(4a)} + \frac{y^2}{f(a^2 - 5)}$ represents an ellipse with major axis as y – axis and f is a decreasing function, then:

(A) $a \in (-\infty, 1)$ (B) $a \in (5, \infty)$

(C) $a \in (1, 4)$ (D) $a \in (-1, 5)$

Ans. D

Sol. Since y – axis is major axis

$\Rightarrow \quad f(4a) < f(a^2 - 5)$

$\Rightarrow \quad 4a > a^2 - 5 \qquad (\because f \text{ is decreasing})$

$\Rightarrow \quad a^2 - 4a - 5 < 0$

$\Rightarrow \quad a \in (-1, 5)$

6. If all the roots of $z^3 + az^2 + bz + c = 0$ are of unit modulus, then

(A) $|a| \le 3$

(B) $|b| > 3$

(C) $|c| \le 3$

(D) none of these

Ans. A

Sol. $z^3 + az^2 + bz + c = 0$

if roots are α, β, γ, then $\alpha + \beta + \gamma = -a$, $\alpha\beta + \beta\gamma + \gamma\alpha = b$, $\alpha\beta\gamma = c$

$|-a| = |\alpha + \beta + \gamma|$

$|a| \le 1 + 1 + 1$

$|a| \le 3$

$|b| = |\alpha\beta + \beta\gamma + \gamma\alpha|$

$|b| \le |\alpha||\beta| + |\beta||\gamma| + |\gamma||\alpha|$

$|b| \le 1.1 + 1.1 + 1.1$

$|b| \le 3$

$|c| = |\alpha\beta\gamma|$

$= |\alpha||\beta||\gamma|$

$= 1.1.1 = 1$

7. $\left(1\frac{2}{3}\right)^2 + \left(2\frac{1}{3}\right)^2 + \left(3\frac{2}{3}\right)^2 + \ldots\ldots..$ to 10 terms, the sum is

(A) $\frac{1390}{9}$

(B) $\frac{1790}{9}$

(C) $\frac{1990}{9}$

(D) none of these

Ans. D

Sol. $\left(1\frac{2}{3}\right)^2 + \left(2\frac{1}{3}\right)^2 + 3^2 + \left(3\frac{2}{3}\right)^2 + \ldots..$ to 10 terms

$= \left(\frac{5}{3}\right)^2 + \left(\frac{7}{3}\right)^2 + \left(\frac{9}{3}\right)^2 + \left(\frac{11}{3}\right)^2 + \ldots\ldots + \left(\frac{23}{3}\right)^2$

$= \frac{5^2 + 7^2 + 9^2 + \ldots\ldots\ldots(23)^2}{9} = \frac{\sum_{r=2}^{11}(2r+1)^2}{9}$

$$= \frac{\sum_{r=1}^{10}(2r+3)^2}{9}$$

$$= \frac{1}{9}\left\{4\sum_{r=1}^{10} r^2 + 12\sum_{r=1}^{10} r + 9\sum_{r=1}^{10} 1\right\}$$

$$= \frac{1}{9}\left\{4 \times \frac{10\times 11\times 21}{6} + \frac{12\times 10\times 11}{2} + 9\times 10\right\} = \frac{2290}{9}$$

8. If a, b, c, d be four consecutive coefficients in the binomial expansion of $(1+x)^n$, then the value of the expression $\left\{\left(\frac{b}{b+c}\right)^2 - \frac{ac}{(a+b)(c+d)}\right\}$ (where $x > 0$) is

(A) < 0
(B) > 0
(C) = 0
(D) 2

Ans. B

Sol. Let $a = {}^nC_{r-1},\ b = {}^nC_r,\ c = {}^nC_{r+1}$ and $d = {}^nC_{r+2}$

$\therefore\ a+b = {}^{n+1}C_r, b+c = {}^{n+1}C_{r+1}, c+d = {}^{n+1}C_{r+2}$

$\Rightarrow \frac{a+b}{a} = \frac{{}^{n-1}C_r}{{}^nC_{r-1}} = \frac{n+1}{r} \Rightarrow \frac{a}{a+b} = \frac{r}{n+1}$ and $\frac{b+c}{b} = \frac{{}^{n+1}C_{r+1}}{{}^nC_r} = \frac{n+1}{r+1} \Rightarrow \frac{b}{b+c} = \frac{r+1}{n+1}$ and

$\frac{c+d}{c} = \frac{{}^{n+1}C_{r+2}}{{}^nC_{r+1}} = \frac{n+1}{r+2} \Rightarrow \frac{c}{c+d} = \frac{r+2}{n+1}$

$\therefore \frac{a}{a+b}, \frac{b}{b+c}, \frac{c}{c+d}$ are in AP.

$\because$ AM > GM

$$\Rightarrow \quad \frac{b}{b+c} > \sqrt{\frac{ac}{(a+b)(c+d)}}$$

or $$\left\{\left(\frac{b}{b+c}\right)^2 - \frac{ac}{(a+b)(c+d)}\right\} > 0$$

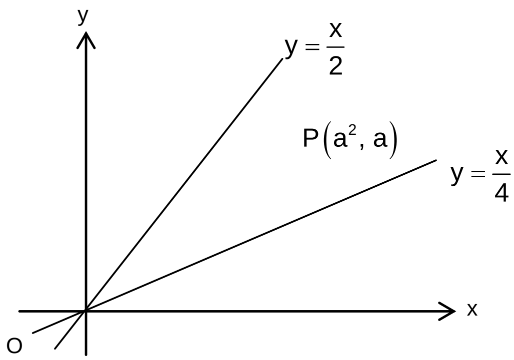
9. If the point $P(a^2, a)$ lies in the region corresponding to the acute angle between the lines $2y = x$ and $4y = x$, then

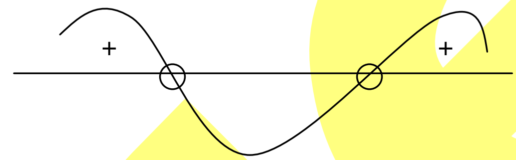
(A) $a \in (2, 4)$
(B) $a \in (2, 6)$
(C) $a \in (4, 6)$
(D) $a \in (4, 8)$

Ans. A

Sol. We have, $\frac{a^2 - 2a}{a^2 - 4a} < 0$

$\Rightarrow \quad \dfrac{a-2}{a-4} < 0$





$\therefore \quad a \in (2, 4)$

10. If the normals from any point to the parabola $x^2 = 4y$ cuts the line y = 2 in points whose abscissas are in AP, then the slopes of the tangents at the three conormal points are in

(A) AP (B) GP
(C) HP (D) none of these

Ans. B

Sol. Let any point on the parabola $x^2 = 4y$ is $(2t, t^2)$

$\therefore$ tangent at $(2t, t^2)$ is $x(2t) = 2(y + t^2)$

or $tx = y + t^2$

$\therefore$ Slope of tangent is t ...(i)

$\therefore$ Slope of normal is $-\dfrac{1}{t}$

Equation of normal at $(2t, t^2)$ is $y - t^2 = -\dfrac{1}{t}(x - 2t)$

$\Rightarrow \quad ty - t^3 = -x + 2t$

or $\quad t^3 - (y-2)t - x = 0$ ...(ii)

$\therefore \quad t_1 + t_2 + t_3 = 0,\ t_1 t_2 t_3 = x$

Solving $y = 2$ and equation (ii), then we get $x = t^3$

Given t_1^3, t_2^3, t_3^3 are in AP $2t_2^3 = t_1^3 + t_3^3$

$= (t_1 + t_3)^3 - 3t_1t_3(t_1 + t_3)$

$= (-t_2)^3 - 3t_1t_3(-t_2)$

$\Rightarrow 3t_2^3 = 3t_1t_2t_3$

$\Rightarrow \ t_2^2 = t_1t_3$

$\therefore \ t_1, t_2, t_3$ are in GP i.e. slopes of the tangents at the three co – normal points are in GP.

11. a, b, c are the length of sides BC, CA, AB respectively of ΔABC satisfying

$$\log\left(1+\frac{c}{a}\right)+\log a-\log b=\log 2$$

Also, the quadratic equation $a\left(1-x^2\right)+2bx+c\left(1+x^2\right)=0$ has two equal roots.

The value of $\left(\sin A+\sin B+\sin C\right)$ is equal to

(A) $\frac{5}{2}$ (B) $\frac{12}{5}$

(C) $\frac{8}{3}$ (D) 2

Ans. B

Sol. Equal roots $\Rightarrow C=90^o$

and $\sin A+1=2\sin B\Rightarrow \cos B=2\sin B-1$ $\left(\because A+B=90^o\right)$

$\Rightarrow 1-\sin^2 B=4\sin^2 B-4\sin B+1$

$\Rightarrow \sin B=\frac{4}{5}$

$\Rightarrow \sin A+\sin B+\sin C=3\sin B=\frac{12}{5}$

12. $\lim\limits_{n\to\infty}\left[1-\frac{1}{2^2}\right]\left[1-\frac{1}{3^2}\right]\left[1-\frac{1}{4^2}\right]\ldots\ldots\ldots\ldots\left[1-\frac{1}{n^2}\right]$ equals:

(A) $\frac{3}{8}$ (B) $\frac{1}{2}$

(C) $\frac{1}{4}$ (D) $\frac{1}{8}$

Ans. B

Sol. $t_n=1-\frac{1}{n^2}=\frac{(n+1)}{n}\cdot\frac{(n-1)}{n}$, $n\geq 2$

Given product $=\frac{3}{2}\cdot\frac{4}{3}\ldots\ldots\frac{(n+1)}{n}\cdot\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{3}{4}\ldots\frac{(n-1)}{n}$

$=\frac{(n+1)}{2}\cdot\frac{1}{n}=\lim\limits_{n\to\infty}\frac{(n+1)}{2n}=\frac{1}{2}$

13. f is an odd function. It is also known that $f(x)$ is continuous for all values of x and is periodic with period 2. If $g(x)=\int_0^x f(t)dt$, then:

(A) $g(x)$ is odd (B) $g(n)=0, n\in N$

(C) $g(2n)=0, n\in N$ (D) $g(x)$ is non – periodic

Ans. C

Sol. $g(x)=\int_0^x f(t)dt$, so, $g(-x)=\int_0^{-x} f(t)dt=-\int_0^x f(-t)dt$

$g(-x)=\int_0^x f(t)dt$, as $f(-t)=-f(t)$, so, $g(-x)=g(x)$

Also $g(x+2)=\int_0^{x+2} f(t)dt=\int_0^2 f(t)dt+\int_2^{2+x} f(t)dt$

$g(x+2)=g(2)+\int_0^x f(t+2)dt$

$=g(2)+\int_0^x f(t)dt=g(2)+g(x)$

Now, $g(2)=\int_0^2 f(t)dt=\int_0^1 f(t)dt+\int_1^2 f(t)dt=\int_0^1 f(t)dt+\int_{-1}^0 f(t+2)dt$

$g(2)=\int_0^1 f(t)dt+\int_{-1}^0 f(t)dt=\int_{-1}^1 f(t)dt=0$ as $f(t)$ is odd

$\Rightarrow \quad g(2)=0\Rightarrow g(x+2)=g(x)$

$\Rightarrow g(x)$ is periodic with period 2.

$\Rightarrow \quad g(4)=0$

$\Rightarrow \quad f(6)=0,\ g(2n)=0,\ n\in N$

14. The sum of all real roots of the equation $x^6+4x^4-10x^3+4x^2+1=0$ is

(A) 2
(B) 0
(C) 1
(D) 4

Ans. A

Sol. Clearly $x=0$ is not a solution.

$\therefore$ Dividing by x^3

$\left(x^3+\frac{1}{x^3}\right)+4\left(x+\frac{1}{x}\right)-10=0$

Let $x+\frac{1}{x}=z$

$\Rightarrow z^3+z-10=0\Rightarrow (z-2)\left(z^2+2z+5\right)=0$

For $z^2+2z+5=0$ Discriminant < 0

$\therefore$ Real roots of $x+\frac{1}{x}=2$

$\therefore (x-1)^2=0\Rightarrow x=1,1$

Sum = 2

15. A car is to be driven 200kms on a highway at an uniform speed of x km/hr (speed rules of the highway require $40\le x\le 70$). The cost of diesel is Rs 30/ litre and is consumed at the rate of $100+\frac{x^2}{60}$ litres per hour. If the wage of the driver is Rs200 per hour then the most economical speed to drive the car is

(A) 55.5
(B) 70
(C) 40
(D) 80

Ans. B

Sol. Let cost incurred to travel 200kms be c(x)

$\Rightarrow c(x) = \left(100 + \frac{x^2}{60}\right)\frac{200}{x} \times 30 + 200 \times \frac{200}{x} = \frac{640000}{x} + 100x$

$\Rightarrow c'(x) < 0 \text{ in } x \in [40, 70]$

$\Rightarrow$ c(x) is minimum for x=70 in $x \in [40, 70]$

16. If $(1+x)(1+x^2)(1+x^4)..........(1+x^{128}) = \sum_{r=0}^{n} x^r$, then 'n' is equal to:

(A) 256
(B) 255
(C) 254
(D) none of these

Ans. B

Sol. $\frac{(1+x)(1+x^2)(1+x^4)(1+x^8)....(1+x^{2^7})(1-x^2)(1+x^2)(1+x^2)(1+x^4)(1+x^8)..(1+x^{2^7})}{(1-x)}$

$= \frac{1}{(1-x)}(1-x^4)(1+x^4)....(1+x^{2^7})$

$= \frac{1-x^{2^8}}{1-x} \Rightarrow n = 255$

17. Let A, B, C, D be (not necessarily square) real matrices such that $A^T = BCD : B^T = CDA; C^T = DAB,$ and $D^T = ABC$ for the matrix S = ABCD; consider the two statements :

I : $S^3 = S$ II : $S^2 = S^4$

(A) II is true but not I
(B) I is true but not II
(C) both I and II are true
(D) Both I and II are false

Ans. C

Sol. $S = ABCD = A(BCD) = AA^T$ …(i)

$S^3 = (ABCD)(ABCD)(ABCD)$

$= (ABC)(DAB)(CDA)(BCD)$

$= D^T C^T B^T A^T$

$= (ABCD)^T = (BCD)^T A^T$

$= AA^T$ ...(ii)

From equation (i) and (ii)

$S = S^3$

$S^2 = S^4$

18. The number of positive integral solutions $\frac{x^2(3x-4)^3(x-2)^4}{(x-5)^5(2x-7)^6} \le 0$ is

(A) four
(B) three
(C) two
(D) only one

Ans. B

Sol. Since, $\frac{x^2(3x-4)^3(x-2)^4}{(x-5)^5(2x-7)^6} \le 0$

$\therefore \quad x \in \left[\frac{4}{3}, \frac{7}{2}\right) \cup \left(\frac{7}{2}, 5\right) \qquad \therefore \quad x = 2, 3, 4.$

19. The sides of a triangle are in the ratio 3 : 5 and the third side is 16, if the largest possible area of the triangle is k, then $\frac{k}{30}$ is ________________

(A) 2
(B) 4
(C) 6
(D) 8

Ans. B

Sol. $S = \frac{16+3x+5x}{2} = 8+4x$

Now $3x+5x > 16 \Rightarrow x > 2$

$3x+16 > 5x \Rightarrow x < 8 \Rightarrow x \in (2, 8)$

$5x+16 > 3x \Rightarrow x > -8$

Now $A^2(x) = (8+4x)(4x-8)(x+8)(8-x) = \left(16x^2-64\right)\left(64-x^2\right)$

Let $x^2 = t \Rightarrow t \in (4, 64)$, $f(t) = 16(t-4)(64-t) = 16\left(64t - t^2 - 256 + 4t\right)$

$\therefore f(t) = -16\left(t^2 - 68t + 256\right), f'(t) = 2t - 68 = 0 \Rightarrow t = 34$

$f''(t) = -32 < 0$

Maxima occurs at t = 34

F(34) = 16(30) (30)

$\therefore$ largest possible area = 120.

20. If $y = \tan^{-1}\left(\frac{1}{1+x+x^2}\right) + \tan^{-1}\left(\frac{1}{x^2+3x+3}\right) + \tan^{-1}\left(\frac{1}{x^2+5x+7}\right) + \ldots\ldots +$ upto n terms, then y'(0) is equal to:

(A) $-\frac{1}{1+n^2}$
(B) $\frac{-n^2}{1+n^2}$
(C) $\frac{n}{1+n^2}$
(D) none of these

Ans. B

Sol. $y=\tan^{-1}\left\{\frac{1}{1+x(1+x)}\right\}+\tan^{-1}\left\{\frac{1}{1+(x+1)(x+2)}\right\}+\left\{\frac{1}{1+(x+2)(x+3)}\right\}+\ldots$

$+\tan^{-1}\left\{\frac{1}{1+(x+n-1)(x+n)}\right\}$

$=\sum_{r=1}^{n}\tan^{-1}\left\{\frac{1}{1+(x+r-1)(x+r)}\right\}$

$=\sum_{r=1}^{n}\tan^{-1}\left\{\frac{(x+r)-(x+r-1)}{1+(x+r-1)(x+r)}\right\}$

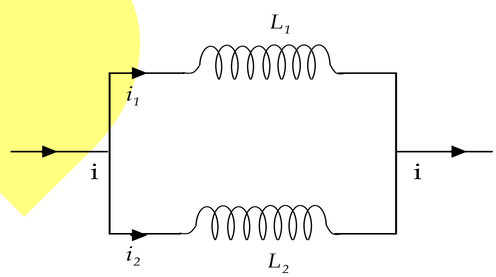
$=\sum_{r=1}^{n}\left\{\tan^{-1}(x+r)-\tan^{-1}(x+r-1)\right\}$

$=\tan^{-1}(x+n)-\tan^{-1}x$

$y'=\frac{1}{1+(x+n)^2}-\frac{1}{1+x^2}\quad\Rightarrow\quad y'(0)=\frac{1}{1+n^2}-1=\frac{-n^2}{1+n^2}$

PHYSICS

21. Two inductors L_1 and L_2 are connected in parallel and a time varying current flows as shown. The ratio of currents i_1/i_2 at any time t is:



(A) L_1/L_2
(B) L_2/L_1
(C) $L_1^2/(L_1+L_2)^2$
(D) $L_2^2/(L_1+L_2)^2$

Ans. B

Sol. $L_1\frac{dI_1}{dt}=L_2\frac{dI_2}{dt}$

Integrating both sides

$L_1I_1=L_2I_2$

22. A rigid circular loop of radius r and mass m lies in the x-y plane of a flat table and has a current I flowing in it. At this particular place the earth's magnetic field is $B=B_x\hat{i}+B_2k$. The value of i so that the loop start tilting is

(A) $\frac{mg}{\pi r\sqrt{B_x^2+B_z^2}}$

(B) $\frac{mg}{\pi rB_x}$

(C) $\frac{mg}{\pi rB_z}$

(D) $\frac{mg}{\pi r\sqrt{B_xB_z}}$

Ans. B

Sol. $i\pi r^2 B_x = mgr$

23. In an L-C-R series AC circuit the voltage across L, C and R is 10 V each. If the inductor is short circuited, the voltage across the capacitor would become

(A) 10 V
(B) $\frac{20}{\sqrt{2}}$ V
(C) $20\sqrt{2}$V
(D) $\frac{10}{\sqrt{2}}$ V

Ans. D

Sol. $X_L = X_c = R$ and $V_R = V_S = 10$ V

$V_R^2 + V_C^2 = V_S^2$

$V_R = V_C$

24. If a man at the equator would weight (3/5)th of his weight, then the angular speed of the earth would be

(A) $\sqrt{\frac{2}{5}\frac{g}{R}}$
(B) $\sqrt{\frac{g}{R}}$
(C) $\sqrt{\frac{R}{g}}$
(D) $\sqrt{\frac{2}{5}\frac{R}{g}}$

Ans. A

Sol. At equator, $g' = g - \omega^2 R \quad \Rightarrow \frac{3}{5}g = g - \omega^2 R \quad \Rightarrow \omega = \sqrt{\frac{2g}{5R}}$

25. The pitch of a screw gauge is 1 mm and there are 100 division on its circular scale. When nothing is put in between its jaws, the zero of the circular scale lies 4 divisions below the reference line. When a steel wire is placed between the jaws, two main scale divisions are clearly visible and 67 divisions on the circular scale are observed. The diameter of the wire is

(A) 2.71 mm
(B) 2.67 mm
(C) 2.63 mm
(D) 2.65 mm

Ans. C

Sol. p = 1mm, N = 100

Least count, $C = \frac{P}{N} = \frac{1mm}{100} = 0.01mm$

The instrument has a positive zero error $e = +NC = +4 \times 0.01 = +0.04$ mm

Main scale reading is 2 × (1 mm) = 2 mm

Circular scale reading is 67 (0.01) = 0.67 mm

$\therefore$ observed reading is $R_0 = 2 + 0.67 = 2.67\,\text{mm}$

So true reading = $R_0 - e$ = 2.63 mm

26. A particle moves on the x axis according to equation $x = x_0 \sin^2 \omega t$. The motion is

(A) periodic but not simple harmonic
(B) simple harmonic with a periodic $2\pi/\omega$ and amplitude x_0
(C) simple harmonic with a periodic π/ω and amplitude $2x_0$
(D) simple harmonic with a periodic π/ω and amplitude $x_0 / 2$

Ans. D

Sol. $x = x_0\left[\frac{1-\cos 2\omega t}{2}\right]$

$= \frac{x_0}{2} - \frac{x_0}{2}\cos 2\omega t$

Time period $\frac{2\pi}{2\omega} = \frac{\pi}{\omega}$

Amplitude $\frac{x_0}{2}$ m

27. A heater of power 2000 kW is switched on inside a body, so that its surface temperature is maintained at 27^0C. The surrounding temperature is zero kelvin. If the voltage is dropped by 19%, the new equilibrium temperature of body is (consider heat loss due to radiation only)

(A) 300^0C (B) -3^0C
(C) 270^0C (D) -6^0C

Ans. B

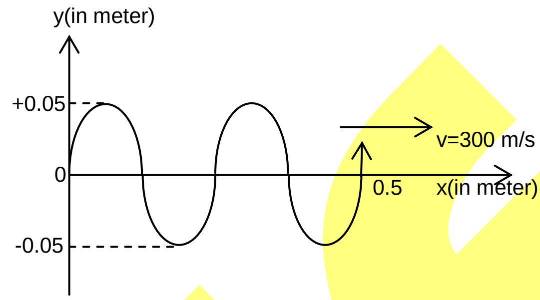
Sol. $\frac{\Delta Q}{\Delta t} = 2\times 10^6 = \sigma A (300)^4$ … (i)

when potential drop by 19 % then new power is

$\left(\frac{\Delta Q}{\Delta t}\right)' = 2\times 10^6\left(\frac{81}{100}\right)^2 = \sigma A (T^4)$ … (ii)

Dividing (i) by (ii) T = 270 K = -3^0C

28. Shape of a plane progressive wave at t=0, is shown in the adjoining phase diagram. The wave equation of this wave is



(A) $y = 0.05\sin 8\pi(x - 300t)$

(B) $y = 0.05\sin 2\pi(300t + x)$

(C) $y = 0.05\sin 8\pi(300t + x)$

(D) $y = 0.05\sin 8\pi(300t - x)$

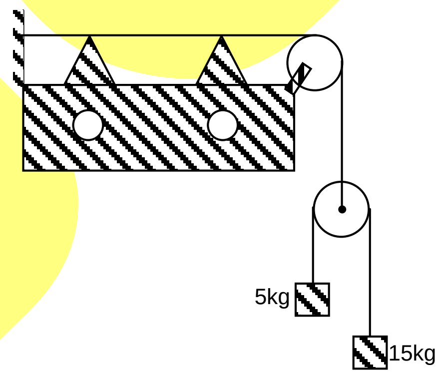
Ans. A

Sol. A = 0.05 m, $\lambda = \frac{0.5}{2}$ m

$f = \frac{v}{\lambda} = \frac{300}{.25} = 1200$ Hz

$y = A\sin(kx - \omega t)$

29. In a sonometer wire, When the tension is maintained by suspending a 20 kg mass from the free end of the wire, the fundamental frequency of vibration is 300 Hz. If the tension is provided by two masses of 5 kg and 15 kg suspended form a pulley (massless and friction less) as shown in the figure, the fundamental frequency will



(A) 300 Hz
(B) 490 Hz
(C) 184 Hz
(D) 260Hz

Ans. D

Sol. $300 = f = \frac{1}{2L}\sqrt{\frac{20g}{\mu}}, f' = \frac{1}{2L}\sqrt{\frac{4\times 5\times 15g}{(5+15)\mu}}$

$\Rightarrow \frac{f'}{300} = \sqrt{\frac{15}{20}} \Rightarrow f' = 259.80$ Hz

30. The force F is given in terms of time t and displacement x by the equation $F = A\cos Bx + C\sin Dt$. The dimensions of $\frac{D}{B}$ are

(A) $M^0L^0T^0$ (B) $M^0L^0T^{-1}$

(C) $M^0L^{-1}T^0$ (D) $M^0L^1T^{-1}$

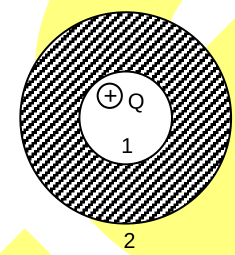
Ans. D

Sol. $F = A\cos Bx + C\sin Dt$

$[D]=[T^{-1}]$; $[B]=[L^{-1}]$

$$\left[\frac{D}{B}\right]=\left[\frac{T^{-1}}{L^{-1}}\right]=\left[M^0L^1T^{-1}\right]$$

31. The positive charge Q is placed at the centre in the cavity of the spherical conductor.

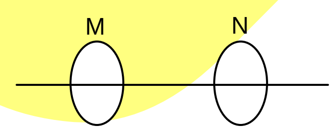


(A) the induced charge at the surface 1 is –Q and on the surface 2 is +2Q
(B) the surface charge density at inner surface is uniform
(C) the surface charge density at the outer surface is non-uniform
(D) the charge Q will experience electric force.

Ans. B

Sol. Apply Gauss law and property of conductor.

32. Two identical circular coils M and N are arranged coaxially as shown in the figure. Separation between the coils is large as compared to their radii. The arrangement is viewed from left along the common axis. The sign convention adopted is that currents are taken to be positive when they appear to flow in clockwise direction. Then which of this is incorrect.

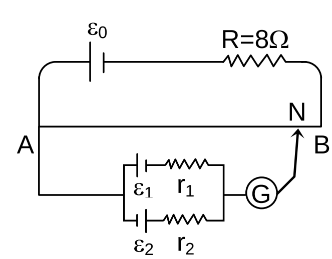


(A) if M carries a constant positive current and is moved towards N, a positive current is induced in N
(B) if M carries a constant positive current and N is moved towards M, a negative current is induced in N
(C) if a positive current in M is switched off, a positive current is momentarily induced in N
(D) if both coils carry positive currents, they will attract each other

Ans. A

Sol. Apply Faraday law for direction of induced current.

33. A battery of emf $\varepsilon_0=24V$ is connected across a 4m long uniform wire having resistance 4Ω/m. The cells of small emfs $\varepsilon_1=2V$ and $\varepsilon_2=4V$ having internal resistance 2Ω and 6Ω respectively are connected as shown in the figure. If galvanometer shows no deflection at the point N, the distance of point N from the point A is equal to :



(A) $\frac{1}{6}$m
(B) 12.5 cm
(C) 25 cm
(D) 50 cm

Ans. B

Sol. $4 \times \ell \frac{24}{24} = \frac{\varepsilon_1 r_2 - r_1 \varepsilon_2}{r_1 + r_2}$

34. The image of an object, formed by a plano-convex lens at a distance of 8 m behind the lens, is real and is one-third the size of the object. The wavelength of light inside the lens is $\frac{2}{3}$ times the wavelength in free space. The radius of the curved surface of the lens is

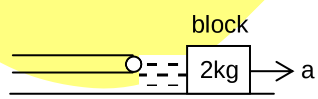
(A) 1 m
(B) 2 m
(C) 3 m
(D) 6 m

Ans. C

Sol. $\mu = \frac{3}{2}$; $V = 8$; $m = -\frac{1}{3}$

$1 - m = \frac{V}{f} = \frac{V}{2R}$

35. A block of metal weighing 2 kg is resting on a frictionless plane as shown in figure. It is struck by a jet releasing water at the rate of 1 kg/s and at speed of 5 m/s. The magnitude of initial acceleration of the block is



(A) 2.5 m/s^2
(B) 5 m/s^2
(C) 7.5 m/s^2
(D) 10 m/s^2

Ans. A

Sol. Force applied by water jet = rate of change in momentum of water = $1 \times 5 = 5N$
So acceleration = 5/2 m/s^2

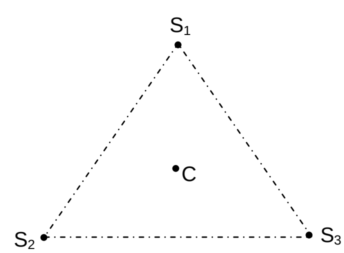
36. The threshold frequency for a metallic surface corresponds to an energy of 6.2 eV, and the stopping potential for a radiation incident on this surface 5 V. The incident radiation lies in

(A) X-ray region
(B) ultra-violet region
(C) infra-red region
(D) visible region

Ans. B

Sol. $\lambda = \frac{1240\,eV \quad nm}{11.2} \approx 1100\,\text{Å}$. Ultraviolet region

37. Three identical sources S_1, S_2 and S_3 are placed at the vertices of an equilateral triangle. If they have intensity I_0 each at centroid c of triangle. The resulting intensity of sound at c will be



(A) $3I_0$
(B) $6I_0$
(C) zero
(D) $9I_0$

Ans. D

Sol. $I_r = \left(\sqrt{I_o} + \sqrt{I_o} + \sqrt{I_o}\right)^2 = 9I_o$

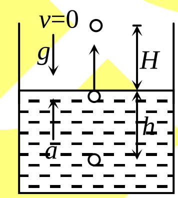
38. A ball of mass m and density ρ is immersed in a liquid of density 3 ρ at a depth h and released. To what height will the ball jump up above the surface of liquid? (neglect the resistance of water and air, radius of ball << h).

(A) h (B) h/2

(C) 2h (D) 3h

Ans. C

Sol. Volume of ball $V = \frac{m}{\rho}$



Acceleration of ball inside the liquid

$$a = \frac{F_{net}}{m} = \frac{\text{upthrust} - \text{weight}}{m}$$

or $$a = \frac{\left(\frac{m}{\rho}\right)(3\rho)(g) - mg}{m} = 2g \text{ (upwards)}$$

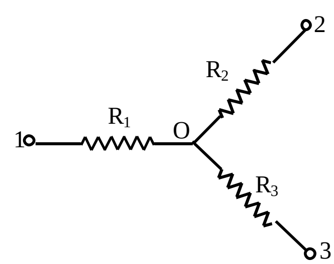
$\therefore$ Velocity of ball while reaching at surface

$$v = \sqrt{2ah} = \sqrt{4gh}$$

$\therefore$ The ball will jump to a height

$$H = \frac{v^2}{2g} = \frac{4gh}{2g} = 2h$$

39. Find the current flowing through the resistance R_1 of the circuit shown in figure if the resistances are equal to R_1 = 10Ω. R_2 = 20Ω and R_3 = 30Ω and the potentials of the points 1, 2, and 3 are equal to φ_1 = 10V, φ_2 = 6V and φ_3 = 5V



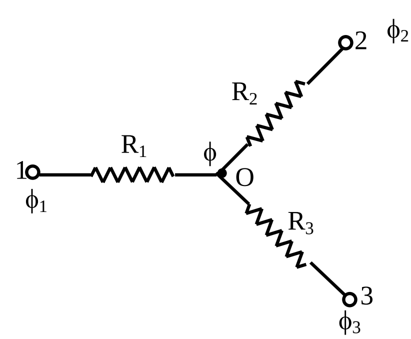
(A) 0.1 Amp (B) 0.4 Amp

(C) 0.3 Amp (D) 0.2 Amp

Ans. D

Sol. Let potential at point "O" be ϕ

Apply KCL at junction "O"



$$\frac{\phi - \phi_1}{R_1} + \frac{\phi - \phi_2}{R_2} + \frac{\phi - \phi_3}{R_3} = 0$$

$$\Rightarrow \phi\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) = \frac{\phi_1}{R_1} + \frac{\phi_2}{R_2} + \frac{\phi_3}{R_3}$$

$$\Rightarrow \phi\left(\frac{1}{10} + \frac{1}{20} + \frac{1}{30}\right) = \frac{10}{10} + \frac{6}{20} + \frac{5}{30}$$

$$\Rightarrow \phi\left(\frac{6+3+2}{60}\right)=\left(\frac{60+18+10}{60}\right)$$

$$\Rightarrow \phi=\frac{88}{11}$$

$$\Rightarrow \phi = 8 \text{ volts}$$

$\therefore$ current through R_1

$$I_1 = \frac{\phi_2-\phi}{R_1}=\frac{10-8}{10}=\frac{2}{10}=0.2$$

$\therefore\ I_2 = 0.2$ Amp.

40. A conducting liquid bubble of radius a and thickness $t(t<<a)$ is charged to potential V. If the bubble collapses to a droplet, find the potential on the droplet.

(A) $V\left(\frac{a}{3t}\right)^{1/3}$

(B) $Va^{1/3}$

(C) $V\left(\frac{a^2}{t^{2/3}}\right)$

(D) $\frac{Va^{1/3}}{3g^3}$

Ans. A

Sol. $V = \frac{1}{4\pi\varepsilon_o}\frac{q}{a}$ (for bubble)

For droplet :- $\frac{4}{3}\pi r^3=\frac{4}{3}\pi(a+t)^3-\frac{4}{3}\pi a^3$

$\Rightarrow r^3 = 3a^2t \Rightarrow r = \left(3a^2t\right)^{1/3}$

$$V_{droplet}=\frac{1}{4\pi\varepsilon_o}\frac{q}{r}=V\left[\frac{a}{3t}\right]^{1/3}$$

CHEMISTRY

41. For a hypothetical reaction

$A+3B\longrightarrow P\ \Delta H=-2x\,kJ/\text{mole of } A\ \&M\longrightarrow 2Q+R\ \ \Delta H=+x\ kJ/\text{mole of } M$

If these reactions are carried simultaneously in a reactor such that temperature is not changing. If rate of disappearance of B is y M sec^{-1} then rate of formation (in M sec^{-1}) of Q is

(A) $\frac{2}{3}y$

(B) $\frac{3}{2}y$

(C) $\frac{4}{3}y$

(D) $\frac{3}{4}y$

Ans. C

Sol. No change in temperature so, that net ΔH for the reaction in which both reactions are carried out must be zero.

$$A + 3B \longrightarrow P \qquad \Delta H = -2x\,kJ/mole$$

$$2H \longrightarrow 4Q + 2R \qquad \Delta H = +2x$$

$$\frac{1}{3}\frac{d[B]}{dt} = \frac{1}{4}\frac{d[A]}{dt} \rightarrow \boxed{\frac{4}{3}y = \frac{dQ}{dt}}$$

42. Consider the following first order competing reactions:

$$X \xrightarrow{k_1} A + B \text{ and } Y \xrightarrow{k_2} C + D$$

If 50% of the reaction of X was completed when 96% of the reaction of Y was completed, the ratio of their rate constants(k_2/k_1) is

(A) 4.06 (B) 0.215
(C) 1.1 (D) 4.65

Ans. D

Sol. for 1st reaction

$$k_1 = \frac{\ell n2}{t_{1/2}} \qquad (t_{96\%}) = \frac{1}{k_2}\ell n\left(\frac{100}{100-96}\right)$$

$$(t_{1/2}) = \frac{\ell n2}{k_1} \qquad (t_{96\%}) = \frac{1}{k_2}\ell n(25)$$

$$\frac{\ell n2}{k_1} = \frac{\ell n(25)}{k_2}$$

$$\frac{k_2}{k_1} = \ell n\left(\frac{25}{2}\right)$$

$$\frac{k_2}{k_1} = 4.65$$

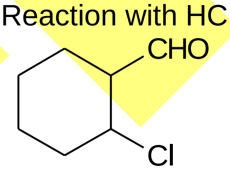
43. How many lattice points are there in the cubic primitive unit cell?

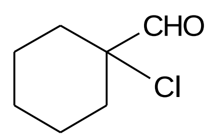
(A) 14 (B) 12
(C) 8 (D) 9

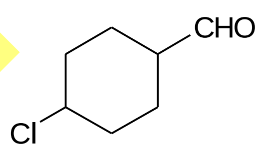
Ans. C

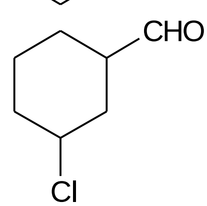
Sol. Only corners are occupied in primitive unit cell.

44. Which of the following aldehyde is most reactive towards nucleophilic addition reactions (e.g Reaction with HCN)?

(A) 

(B) 

(C) 

(D) 

Ans. B

Sol. Cl is very nearer to CHO group.

45. Teflon is a/an

(A) ester
(B) ketone
(C) polymer
(D) amino acid

Ans. C

Sol. Teflon is $\left(CF_2 - CF_2 \right)_n$

46. Which of the following is the strongest acid?

(A) $CH_3CH_2CH_2CH_2COOH$

(B) $(H_3C)(CH_3)CH - CH_2 - COOH$

(C) $CH_3 - C(CH_3)_2 - COOH$

(D) $CH_3CH_2CH(CH_3) - COOH$

Ans. A

Sol. $CH_3CH_2CH_2CH_2COOH$ experience the lowest +I effect.

47. The hybridization of xenon in XeO_3 is:

(A) sp^3d^2
(B) sp^3d
(C) sp^3d^3
(D) none of these

Ans. D

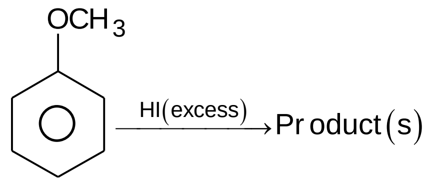
Sol. The hybridization of Xe is sp^3.

48. Which of the following metal is extracted by Mond's process?

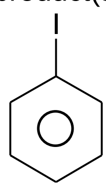
(A) Cu
(B) Ni
(C) Ti
(D) Zn

Ans. B

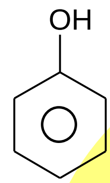
Sol. Ni is extracted by Mond's process.

49. 

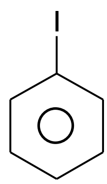
The product(s) of above reaction is/are

(A) 

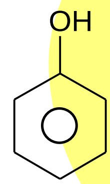
and CH_3OH

(B) 

and CH_3I

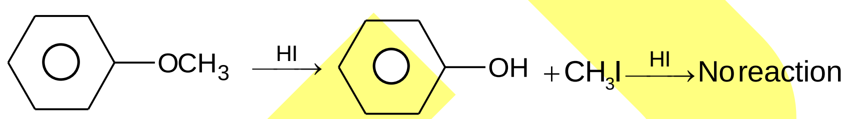
(C) 

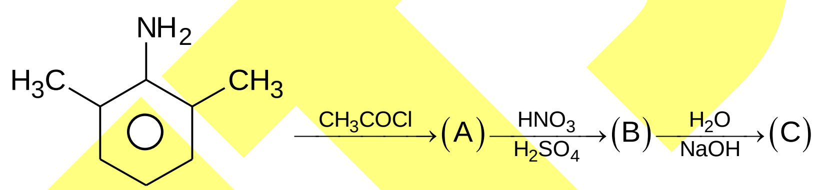
and CH_3I

(D) 

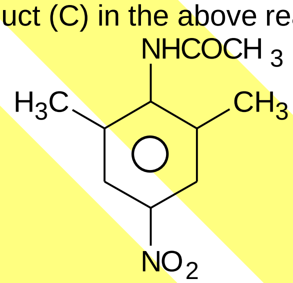
and CH_3OH

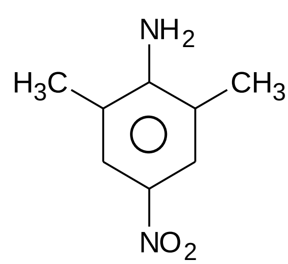
Ans. B

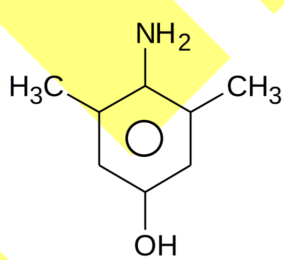
Sol. 

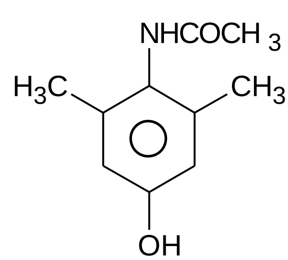
50. 

Product (C) in the above reaction is:

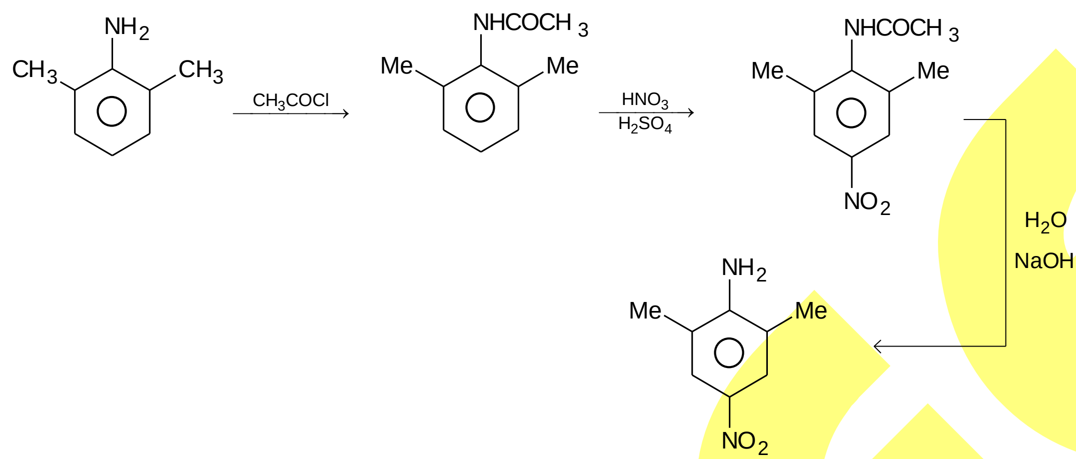
(A) 

(B) 

(C) 

(D) 

Ans. B

Sol. 

51. Which of the following does not give iodoform reaction?

(A) $C_2H_5CH(OH)CH_3$ (OH on the CH carbon)

(B) CH_3CH_2CHO

(C) CH_3CHO

(D) $CH_3COCH_2CH_3$

Ans. B

Sol. CH_3CH_2CHO does not give iodoform reaction due to absence of $(CH_3-\overset{O}{\overset{||}{C}}-)$ or $(CH_3\overset{OH}{\overset{|}{C}H})$ groups.

52. Which one of the following polymers of glucose is stored by animals?

(A) Cellulose
(B) Amylose
(C) Amylopectin
(D) Glycogen

Ans. D

Sol. Glycogen is stored by animals.

53. Pure nitrogen gas is obtained by heating

(A) $Pb(NO_3)_2$
(B) NH_4NO_2
(C) $Ba(N_3)_2$
(D) NH_4NO_3

Ans. C

Sol. $Ba(N_3)_2 \xrightarrow{\Delta} Ba + 3N_2$

54. Which of the following electronic configuration of the metal ion, in its octahedral complex, provides maximum paramagnetic property according to crystal field theory?

(A) $t_{2g}^6e_g^2$
(B) $t_{2g}^4e_g^2$
(C) $t_{2g}^3e_g^2$
(D) $t_{2g}^5e_g^2$

Ans. C

Sol. $t_{2g}^3 e_g^2$ contains five unpaired electrons.

55. $NiCl_2(aq) + \underset{excess}{NH_3(aq)} \longrightarrow 'P'$

'P' is

(A) Blue, d^2sp^3
(B) Red, d^2sp^3
(C) Blue, sp^3d^2
(D) Red, sp^3d^2

Ans. C

Sol. CFSE α charge on metal ions '5' heated rings are present in $[(Fedta)^-]$

56. The osmotic pressure of a dilute aqueous solution of sucrose at 300 K is 2.47 atm; the molar volume of water at this temperature is 18 cm^3. Calculate the elevation of boiling point of this solution. Given for H_2O, $\Delta_{vap}H_{1,m}$ = 37.3 kJ mol^{-1} and T_b = 373 K

(A) 0.056 K
(B) 0.037 K
(C) 0.025 K
(D) 0.018 K

Ans. A

Sol. Substituting data with proper units in the expression

$$\Delta T_b = \frac{\Pi\, V\, T_b^2}{T.\, \Delta_{vap} H_{1,m}} = 0.056K$$

57. Element 'X' belongs to the fourth period. The magnetic moment of X^{3+} ion is 5.92 B.M. Therefore, 'X' is

(A) Ni
(B) Fe
(C) Mn
(D) Co

Ans. B

Sol. $\sqrt{n(n+2)}$ = 5.92

On solving n = 5

$\therefore$ The atom contains 5 unpaired electrons in +3 oxidation state

$\therefore$ The ion is Fe^{2+} and X is Fe

58. Which of the following compound(s) can form aldehyde on hydrolysis?

(I) $HC \equiv C - CH_2 - C \equiv CH$
(II) $CH_3C \equiv C - CH_2 - C \equiv CH$
(III) $HC \equiv C - C \equiv CH$
(IV) $HC \equiv CH$

(A) I, II
(B) I, II & III
(C) II, IV
(D) Only IV

Ans. D

Sol. $HC \equiv CH \xrightarrow[H_2SO_4]{HgSO_4} CH_3CHO$

59. How many mole of phenyl hydrazine can completely react with one mole of glucose to form glucosazone?

(A) 2
(B) 3
(C) 4
(D) 6

Ans. B

Sol. 3 moles

60. Which of the following is one of the products of the following reaction?

$SO_2 + H_2S \longrightarrow Products$

(A) H_2
(B) S
(C) SO_3
(D) $H_2S_2O_3$

Ans. B

Sol. $SO_2 + 2H_2S \longrightarrow 3S + 2H_2O$

PART – II

MATHEMATICS

61. The general solution of the differential equation $2xy^2\,dx = e^x(dy - ydx)$, is equal to:

(A) $x^2y + e^x = cy$ (B) $x^2y - e^x = cy$

(C) $y^2x + e^x = cx$ (D) $y^2x - e^x = cx$

Ans. A

Sol. $y(5x^4y + 1)dx + (x + 2y + 2x^5y)dy = 0$

$\Rightarrow x\,dy + y\,dx + 5x^4\,y^2dx + 2x^5\,y\,dy + 2y\,dy = 0$

$\Rightarrow d(xy) + d(x^5\,y^2) + d(y^2) = 0$

$\Rightarrow xy + x^5y + y^2 = c$

62. $\lim_{n\to\infty} \frac{1}{n}\sum_{r=n+1}^{2n} \ln\left(1+\frac{r}{n}\right)$ equals:

(A) $\ln\left(\frac{27}{4e}\right)$ (B) $\ln\left(\frac{27}{e^2}\right)$

(C) $\ln\left(\frac{4}{e}\right)$ (D) none of these

Ans. A

Sol. Let $P = \lim_{n\to\infty} \frac{1}{n}\sum_{n+1}^{2n} \log_e\left(1+\frac{r}{n}\right)$

$= \int_1^2 \log_e(1+x)\,dx$

Put $1 + x = t$

$\therefore dx = dt$

Then $P = \int_2^3 \log_e t\;dt$

$= [\log_e t.t - t]_2^3$

$= (3\log_e 3 - 3) - (2\log_e 2 - 2)$

$= \log_e\left(\frac{27}{4}\right) - 1$

$= \log_e\left(\frac{27}{4}\right) - \log_e e$

$= \log_e\left(\frac{27}{4e}\right)$

63. The least value of 'a' for which $\frac{4}{\sin x}+\frac{1}{1-\sin x}=a$, has at least one solution in the interval $\left(0, \frac{\pi}{2}\right)$ is:

(A) 9
(B) 8
(C) 4
(D) 1

Ans. A

Sol. $f(x)=\frac{4}{\sin x}+\frac{1}{1-\sin x}=a$ ...(i)

$f'(x)=\frac{4\cos x}{\sin^2 x}+\frac{\cos x}{(1-\sin x)^2}$

$=\cos x\left(\frac{1}{(1-\sin x)^2}-\frac{4}{\sin^2 x}\right)$

$\therefore f'(x)=0$

$\Rightarrow \frac{1}{(1-\sin x)^2}-\frac{4}{\sin^2 x}=0$ as $\cos x \neq 0$ in $\left(0, \frac{\pi}{2}\right)$

This gives $\sin x=\frac{2}{3}$

Substituting this in equation (i), we get a = 9

64. If $\int_0^{\infty} e^{-x^2}\,dx=\frac{\sqrt{\pi}}{2}$ and $\int_0^{\infty} e^{-ax^2}\,dx$, $a>0$ is

(A) $\frac{\sqrt{\pi}}{2}$
(B) $\frac{\sqrt{\pi}}{2a}$
(C) $2\frac{\sqrt{\pi}}{a}$
(D) $\frac{1}{2}\sqrt{\frac{\pi}{a}}$

Ans. D

Sol. Let $I=\int_0^{\infty} e^{-ax^2}\,dx$

Put $\sqrt{a}x=t$ $\therefore$ $dx=\frac{dt}{\sqrt{a}}$

Then $I=\frac{1}{\sqrt{a}}\int_0^{\infty} e^{-t^2}\,dt$

$=\frac{1}{\sqrt{a}}\int_0^{\infty} e^{-t^2}\,dx$ (by property)

$=\frac{1}{\sqrt{a}}\cdot\frac{\sqrt{\pi}}{2}$ (given)

$=\frac{1}{2}\sqrt{\left(\frac{\pi}{a}\right)}$

65. The equation of the curve in which the portion of y – axis cut off between the origin and the tangent varies as the cube of the abscissa of the point of contact is

(A) $y = \frac{k\,x^3}{3} + c\,x$

(B) $y = \frac{k\,x^2}{2} + c$

(C) $y = \frac{k\,x^3}{2} + c\,x$

(D) $y = \frac{k\,x^3}{3} + \frac{cx^2}{2}$

(k is constant of proportionality)
(where c is arbitrary constant)

Ans. C

Sol. Equation of tangent at (x, y) is $Y - y = \frac{dy}{dx}(X - x)$

for y – axis X =0,

Then, $Y = y - x\frac{dy}{dx}$

Given, $\left(y - x\frac{dy}{dx}\right) \propto x^3$

$\Rightarrow \quad y - x\frac{dy}{dx} = kx^3$

or $\quad \frac{dy}{dx} - \frac{y}{x} = -kx^2$

IF= $e^{\int \frac{1}{x}dx} = e^{-\ln x} = e^{\ln\left(\frac{1}{x}\right)} = \frac{1}{x}$

Then, solution is $y.\left(\frac{1}{x}\right) = \int \frac{-kx^2}{x}dx$

$\Rightarrow \quad \frac{y}{x} = \frac{kx^2}{2} + c$

or $\quad y = -\frac{kx^3}{2} + cx$

66. The area bounded by the curves $y = \sin^{-1}|\sin x|$ and $y = \left(\sin^{-1}|\sin x|\right)^2$, $0 \le x \le 2\pi$ is

(A) $\left(\frac{\pi^3}{3} + \frac{4}{3}\right)$ sq. unit

(B) $\left(\frac{\pi^3}{6} - \frac{\pi^2}{2} + \frac{4}{3}\right)$ sq. unit

(C) $\left(\frac{\pi^2}{2} - \frac{4}{3}\right)$ sq unit

(D) $\left(\frac{\pi^2}{6} - \frac{\pi}{4} + \frac{4}{3}\right)$ sq. unit

Ans. B

Sol. $\because \sin^{-1}|\sin x| = \begin{cases} x, & 0 \le x \le \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} \le x \le \pi \\ -\pi + x, & \pi \le x \le \frac{3\pi}{2} \\ 2\pi - x, & \frac{3\pi}{2} \le x \le 2\pi \end{cases}$

$\therefore \quad \left(\sin^{-1}|\sin x|\right)^2 = \begin{cases} x^2, & 0 \le x \le \frac{\pi}{2} \\ (\pi - x)^2, & \frac{\pi}{2} \le x \le \pi \\ (x - \pi)^2, & \pi \le x \le \frac{3\pi}{2} \\ (2\pi - x)^2, & \frac{3\pi}{2} \le x \le 2\pi \end{cases}$

$\therefore$ Required area $= 4\left\{\int_0^1 \left(x - x^2\right)dx + \int_1^{\pi/2}\left(x^2 - x\right)dx\right\}$

$= 4\left\{\left[\left(\frac{x^2}{2} - \frac{x^3}{3}\right)\right]_0^1 + \left[\left(\frac{x^3}{3} - \frac{x^2}{2}\right)\right]_1^{\frac{\pi}{2}}\right\}$

$= 4\left\{\left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{\pi^3}{24} - \frac{\pi^2}{8}\right) - \left(\frac{1}{3} - \frac{1}{2}\right)\right\}$

$= 4\left\{1 - \frac{2}{3} + \frac{\pi^3}{24} - \frac{\pi^2}{8}\right\}$

$= \left(\frac{\pi^3}{6} - \frac{\pi^2}{2} + \frac{4}{3}\right)$ sq. unit

67. The equation of the line passing though the point (1, 1, –1) and perpendicular to the plane $x - 2y - 3z = 7$ is

(A) $\frac{x-1}{-1} = \frac{y-1}{2} = \frac{z+1}{3}$

(B) $\frac{x-1}{-1} = \frac{y-1}{-2} = \frac{z+1}{3}$

(C) $\frac{x-1}{1} = \frac{y-1}{-2} = \frac{z+1}{-3}$

(D) none of these

Ans. C

Sol. Lines is parallel to the normal of the plane $x - 2y - 3z = 7$

$\therefore$ Equation of line through $(1, 1, -1)$ is $\frac{x-1}{1} = \frac{y-1}{-2} = \frac{z+1}{-3}$

68. $\int \frac{\left(2x^{12}+5x^{9}\right)}{\left(x^{5}+x^{3}+1\right)^{3}}dx$ is equal to:

(A) $\frac{x^{2}+2x}{\left(x^{5}+x^{3}+1\right)^{2}}+C$

(B) $\frac{x^{10}}{2\left(x^{5}+x^{3}+1\right)^{2}}+C$

(C) $\ln\left|x^{5}+x^{3}+1\right|+\sqrt{\left(2x^{7}+5x^{4}\right)}+C$

(D) none of these

Ans. B

Sol. Let $I=\int \frac{2x^{12}+5x^{9}}{\left(x^{5}+x^{3}+1\right)^{3}}dx$

$$=\int \frac{\left(\frac{2}{x^{3}}+\frac{5}{x^{6}}\right)}{\left(1+\frac{1}{x^{2}}+\frac{1}{x^{5}}\right)^{3}}dx$$

Put $1+\frac{1}{x^{2}}+\frac{1}{x^{5}}=t$

$\therefore \left(-\frac{2}{x^{3}}-\frac{5}{x^{6}}\right)dx=dt$

Then $I=-\int \frac{dt}{t^{3}}=\frac{1}{2t^{2}}+C$

$$=\frac{1}{2\left(1+\frac{1}{x^{2}}+\frac{1}{x^{5}}\right)^{2}}+C$$

$$=\frac{x^{10}}{2\left(x^{5}+x^{3}+1\right)^{2}}+C$$

69. If $F(x)=\frac{1}{x^{2}}\int_{4}^{x}\left\{4t^{2}-2F'(t)\right\}dt$, then $F'(4)$ equals

(A) $\frac{32}{9}$

(B) $\frac{64}{3}$

(C) $\frac{64}{9}$

(D) none of these

Ans. A

Sol. $\because F(x)=\frac{1}{x^{2}}\int_{4}^{x}\left\{4t^{2}-2F'(t)\right\}dt$

or $x^{2}F(x)=\int_{4}^{x}\left\{4t^{2}-2F'(t)\right\}$

Differentiating both sides w.r.t. x,

then $x^2F'(x)+F(x).2x=4x^2-2F'(x)$

Put $x=4$

$16F'(4)+8F(4)=64-2F'(4)$

$\therefore\ 18F'(4)+0=64 \quad [\because F(4)=0, \text{ from equation (i)}]$

$\therefore\ F'(4)=\frac{32}{9}$

70. If $f'(x)=|x|-\{x\}$, where {x} denotes the fractional part of x, then $f(x)$ is decreasing in

(A) $\left(-\frac{1}{2}, 0\right)$

(B) $\left(-\frac{1}{2}, 2\right)$

(C) $\left(-\frac{1}{2}, 2\right]$

(D) $\left(\frac{1}{2}, \infty\right)$

Ans. A

Sol. $\because f'(x)=|x|-\{x\}$

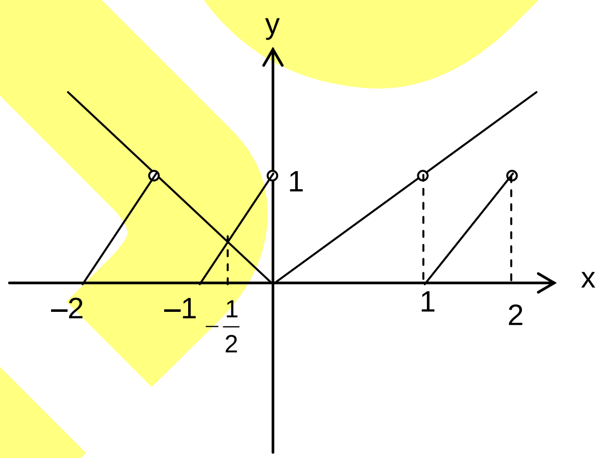
$\because f(x)$ is decreasing

$\therefore\ f'(x)<0$

$\Rightarrow\ |x|-\{x\}<0$

$\Rightarrow\ |x|<\{x\}$

It is clear from the figure $x\in\left(-\frac{1}{2}, 0\right)$



PHYSICS

71. A charge particle enters into a region containing uniform electric field (*E*) and uniform magnetic field (B) along *x*-axis and *y*-axis respectively. If it passes the region undeviated, the velocity of charge particle is given by

(A) $2\hat{i}+\frac{E}{B}\ \hat{k}$

(B) $2\hat{j}\ +\ \frac{E}{B}\hat{k}$

(C) $2\hat{i}-\frac{E}{B}\hat{k}$

(D) none of these

Ans. B

Sol. If $\vec{V} = V_x\hat{i} + V_y\hat{j} + V_z\hat{k}$, $\vec{E} = E\hat{i}$, $\vec{B} = B\hat{j}$

$\therefore\ \vec{F} = q\left[E\hat{i} + (V_x\hat{i} + V_y\hat{j} + V_z\hat{k}) \times B\hat{j}\right]$

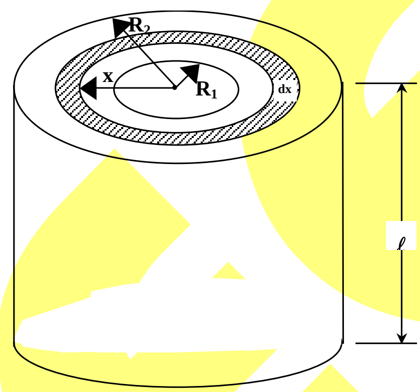
$= qE\hat{i} + qV_xB\hat{k} + 0 - qV_zB\hat{i}$

$= (E - V_z B)\,\hat{i} + V_x B\,\hat{k}$

For no deviation net force should either be zero or in the direction of velocity of particle.

$\therefore$ For $F = 0$, $V_Z = E/B$, $V_x = 0$, $V_y \rightarrow$ has any value

72. A cylindrical conductor of length ℓ and inner radius R_1 and outer radius R_2 has specific resistance ρ. A cell of emf ε is connected across the two lateral faces of the conductor. Find the current drown from the cell.



(A) $\dfrac{2\pi\ell\varepsilon}{\rho \ln \dfrac{R_2}{R_1}}$

(B) $\dfrac{2\pi\ell\varepsilon}{\rho\,(R_2^2 - R_1^2)}$

(C) $\dfrac{2\ell\varepsilon}{\rho\pi(R_2^2 - R_1^2)}$

(D) None of thee

Ans. A

Sol. Consider the differential element of the cylinder as shown in the figure.

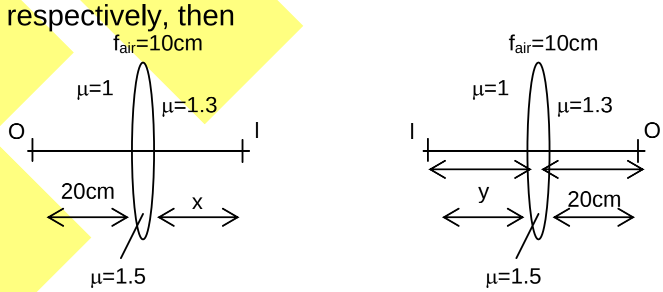
$\therefore \quad R = \int_{R_1}^{R_2} \rho \dfrac{dx}{2\pi x l} \qquad (\because R = \rho \dfrac{l}{a})$

$\Rightarrow R = \dfrac{\rho}{2\pi l} \ln\left(R_2/R_1\right)$

$I = \dfrac{\varepsilon}{R}$

$\Rightarrow I = \dfrac{2\pi l \varepsilon}{\rho \ln\left(\dfrac{R_2}{R_1}\right)}$

73. An equiconvex lens made up of material of refractive index 1.5 has focal length of 10 cm. when placed in air, as shown in the figure. One side of medium is replaced by a medium of refractive index 1.3. If x, y are the image distance when object is placed at a distance of 20 cm from pole in medium with refractive index 1 and refractive index 1.3 respectively, then



(A) $x > 1.3\,y$

(B) $x < 1.3\,y$

(C) $x = 1.3\,y$

(D) cannot determine

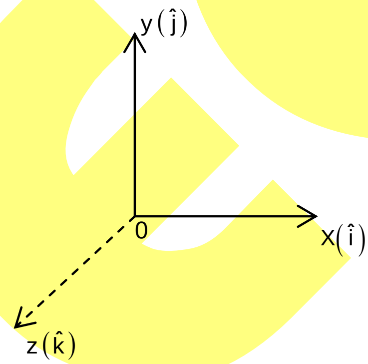
Ans. B

Sol. For 1st lens:-

(I) $\frac{1.3}{x} - \frac{1}{(-20)} = \frac{(\mu - 1)}{R_1} + \frac{(1.3 - \mu)}{-R_2}$

(II) $\frac{1.3}{20} - \frac{1}{-y} = \frac{(\mu - 1)}{R_1} + \frac{(1.3 - \mu)}{-R_2}$

74. A particle is projected from origin with velocity $\vec{u} = \left(\hat{i} + \hat{j} + \sqrt{2}\hat{k}\right)$ m/s. Horizontal surface lies in X – Y plane, then (take g = 10 m/sec^2) Choose the incorrect option.



(A) Time of flight = $\frac{\sqrt{2}}{5}$ sec

(B) horizontal range = $\frac{2}{5}$ m

(C) Maximum height $\frac{1}{10}$ m

(D) Maximum height = $\frac{1}{5}$ m

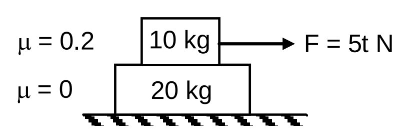
Ans. C

Sol. T = $\frac{2 \times \sqrt{2}}{10} = \frac{\sqrt{2}}{5}$

Range R = $\frac{V^2 \sin 2\theta}{g} = \frac{(2)^2 \sin(2 \times 45^o)}{10} = \frac{2}{5}$

H = $\frac{V^2 \sin^2\theta}{2g} = \frac{(2)^2 \times \left(\frac{1}{\sqrt{2}}\right)^2}{2 \times 10} = \frac{1}{10}$

75. The speed of 10 kg block at the end of six second.



(A) $\frac{10}{3}$ m/s
(B) 3 m/s
(C) 2 m/s
(D) None of these

Ans. B

Sol. $(F_s)_{max}$ = 20 N. a_{max} of 20 kg black = 1 m/s^2

∴ both black move together till 6 sec.

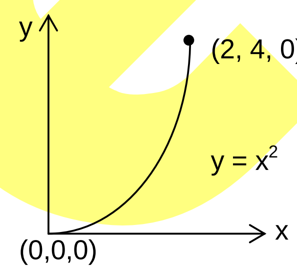
∴ $\frac{dv}{dt} = \frac{5t}{30}$ $\Rightarrow$ $\int dv = \frac{1}{6}\frac{t^2}{2}$, V = $\frac{6^2}{12}$ = 3 m/s

76. Hydrogen gas absorbs radiations of wavelength λ_0 and consequently emit radiations of 6 different wavelengths of which two wavelengths are longer than λ_0. Choose the incorrect statement.
(A) The final excited state of the atom is n = 4.
(B) The initial state of the atom may be n = 2.
(C) The initial state of the atom may be n = 3.
(D) There are three transitions belonging to Lyman series.

Ans. C

Sol. Since only 6 different wavelength are excited, therefore highest excited stat is n = 4.
Two wavelengths are longer than λ_0, initially atoms were in excited state n = 2
Corresponding transitions are $4 \to 3$, $4 \to 2$, $4 \to 1$, $3 \to 2$, $3 \to 1$, $2 \to 1$.

77. By applying a force $\vec{F} = (3xy - 5z)\hat{j} + 4z\hat{k}$ a particle is moved along the path $y = x^2$ from point (0, 0, 0) to (2, 4, 0). The work done by the force F on the particle is



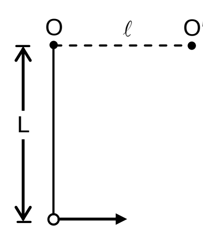
(A) $\frac{280}{5}$ unit (B) $\frac{140}{5}$ unit
(C) $\frac{232}{5}$ unit (D) $\frac{192}{5}$ unit

Ans. D

Sol. $w = \int \vec{F}.d\vec{r}$

$= \int (3xy - 5z)\,dy$; $\int_0^4 3\sqrt{y}\; y dy = \frac{192}{5}$

78. A particle is hanging from a fixed pint 0 by means of a string of length L. There is a small nail O′ in the same horizontal line with O at a distance ℓ(<L) from O. The minimum velocity with which particle should be projected from its lowest position in order that it may make a complete revolution round the nail.



(A) $\sqrt{3gL}$ (B) $\sqrt{5gL}$
(C) $\sqrt{g(5L - 3\ell)}$ (D) $\sqrt{g(5\ell - 3L)}$

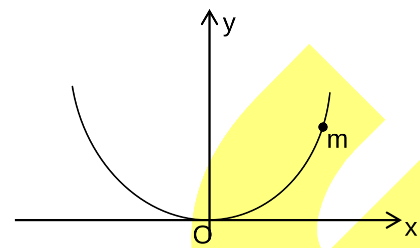
Ans. C

Sol. From conservation of mechanical energy.

$\frac{1}{2}mv^2 = mgL + \frac{1}{2}m3(L - \ell)g$

$\therefore \quad v = \sqrt{g(5L - 3\ell)}$

79. A bead of mass m is located on parabolic wire with its axis vertical and vertex at the origin as shown in figure and whose equation is $x^2 = 4ay$. The wire frame is fixed and bead can slide on it without friction. The bead is released from the point y = 4a on the wire frame from rest. The tangential acceleration of the bead when it reaches the position given by y = a is



(A) $\frac{g}{2}$
(B) $\frac{\sqrt{3}}{2}g$
(C) $\frac{g}{\sqrt{2}}$
(D) $\frac{g}{\sqrt{5}}$

Ans. C

Sol. At y = a, x = 2a, $\frac{dy}{dx} = 1 = 45°$ $a_f = g \cos 45 = \frac{g}{\sqrt{2}}$.

80. Two soap bubbles of radii 2mm and 4mm are brought in contact. If the surface tension of liquid is 7×10^{-2} Nm^{-1}. Then the radius of the common surface is
(A) 2×10^{-3}m
(B) 4×10^{-3}m
(C) 6×10^{-3}m
(D) 8×10^{-3}m

Ans. B

Sol. $P_{convex} = P_{concave} - \frac{4s}{R}$

$$P_o + \frac{4s}{R_1} = P_o + \left(\frac{4s}{R_2}\right) - \frac{4s}{R}$$

$$\Rightarrow \quad R = \frac{R_1 R_2}{R_1 - R_2} = 4mm$$

CHEMISTRY

81. Which one is used as an air purifier in space craft?
(A) quick lime
(B) slaked lime
(C) KO_2
(D) anhydrous $CaCl_2$

Ans. C

Sol. KO_2 absorbs CO_2 and releases O_2.

82. Identify the compound A and B in the following reaction sequence

$$CaC_2\,(s) + H_2O\,(l) \longrightarrow A(g) \xrightarrow{\text{dil. } H_2SO_4,\ Hg^{2+}} (B)\,(l)$$

(A) A is ethylene, B is acetaldehyde
(B) A is acetylene, B is propionaldehyde
(C) A is ethane, B is ethanol
(D) A is acetylene, B is ethanal

Ans. D

Sol. $CaC_2\,(s) + H_2O\,(l) \longrightarrow HC \equiv CH \xrightarrow{\text{dil. } H_2SO_4,\ Hg^{2+}} CH_3CHO$

83. The cell reaction for the given cell is spontaneous if
Pt, $Br_2(P_1$ atm | Br^- (1M) || $Br_2(P_2$ atm), Pt
(A) $P_1 > P_2$ (B) $P_2 > P_1$
(C) $P_1 = P_2$ (D) $P_1 = 1$ atm

Ans. B

Sol. E_{Cell} will be positive if $P_2 > P_1$.

84. Which of the following will have the highest value of ($\wedge_\infty$) equivalent conductance at infinite dilution if one gram of each is dissolved in sufficient water ?
(A) NaCl (B) NH_4Cl
(C) KCl (D) $CaCl_2$

Ans. B

Sol. NH_4Cl is acidic salt it will produce H^+ ion in solution which is strongly conducting

85. What is the hybridization of 'Cu' in $[Cu(NH_3)_4]SO_4$.
(A) sp^3 (B) sd^3
(C) sp^2d (D) dsp^2

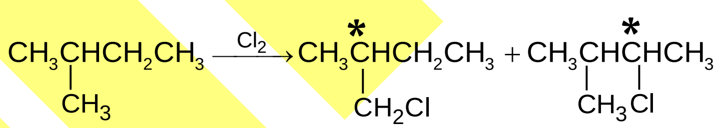
Ans. B

Sol. It is a square planar complex.

86. The number of possible enantiometric pairs that can be produced during monochlorination of 2-Methylbutane is
(A) 2 (B) 3
(C) 4 (D) 1

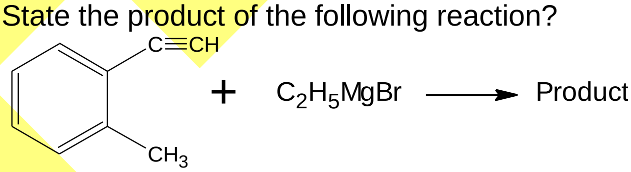
Ans. A

Sol.



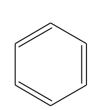
Two enantiomeric pairs are formed.

87. State the product of the following reaction?



(A) C_2H_5OH

(B)

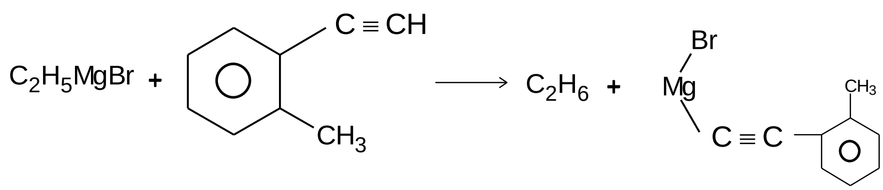


(C) C_2H_6

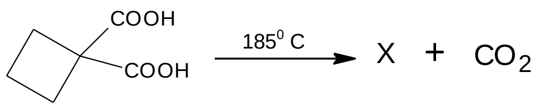
(D) C_2H_4

Ans. C

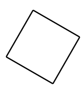
Sol.



88. Consider the reaction

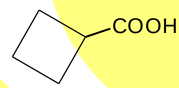


Which of the following is X ?

(A) 

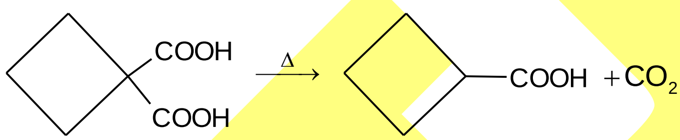
(B) $CH_3CH_2CH_2COOH$

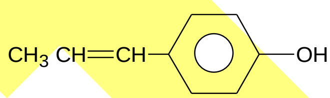
(C) $HOOCCH_2CH_2CH_2COOH$

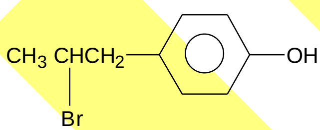
(D) 

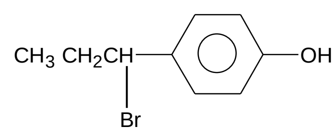
Ans. D

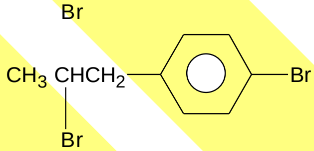
Sol.

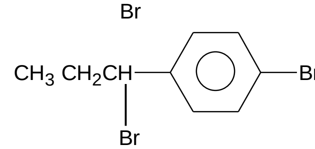


89. The reaction of  with HBr gives :

(A) 

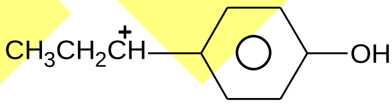
(B) 

(C) 

(D) 

Ans. B

Sol. The stable carbocation formed is



90. Two mole of an ideal gas is expanded irreversibly and isothermally of 37°C until its volume is doubled and 3.41 KJ heat is absorbed from surrounding. ΔS_{total} (Sys + surrounding) is:

(A) -0.52 J/K
(B) 0.52 J/K
(C) 22.52 J/K
(D) 0

Ans. B

Sol. Hint $\Delta S_{system} = nTln\frac{V_2}{V_1} = 2 \times Rxln_2$

= 11.52 J/K

$\Delta S_{surrounding} = \frac{-3.41 \times 1000}{310} = -11J/K$

$\Delta S_{total} = 11.52 + (-11) = 0.52\,J/K$