OLTS-2021-T8-FT-IV-KVPY-CLASS-XII
FULL TEST – IV

PART – I
MATHEMATICS

1. Let $a_n = \sum_{k=1}^{n} \frac{1}{k(n+1-k)}$, then for $n \geq 2$

(A) $a_{n+1} > a_n$
(B) $a_{n+1} < a_n$
(C) $a_{n+1} = a_n$
(D) $a_{n+1} - a_n = \frac{1}{n}$

Ans. B

Sol. We have $a_n = \sum_{k=1}^{n} \left(\frac{1}{k} + \frac{1}{n+1-k}\right)$

$= \frac{2}{n+1} \sum_{k=1}^{n} \left(\frac{1}{k} + \frac{1}{n+1-k}\right)$

$= \frac{2}{n+1} \sum_{k=1}^{n} \frac{1}{k}$

For $n \geq 2$

$\frac{1}{2}(a_n - a_{n+1}) = \frac{1}{n+1} \sum_{k=1}^{n} \frac{1}{k} - \frac{1}{n+2} \sum_{k=1}^{n+1} \frac{1}{k}$

$= \left(\frac{1}{n+1} - \frac{1}{n+2}\right) \sum_{k=1}^{n} \frac{1}{k} - \frac{1}{(n+1)(n+2)}$

$= \frac{1}{(n+1)(n+2)} \sum_{k=2}^{n} \frac{1}{k} > 0$

$\Rightarrow a_n > a_{n+1}$

2. If $\cos\theta_1 + 2\cos\theta_2 + 3\cos\theta_3 = 6$ then $\tan\theta_1 + \tan\theta_2 + \tan\theta_3$ equals to

(A) 1/2
(B) 6
(C) 0
(D) 3

Ans. C

Sol. Since, $\cos\theta \leq 1$.

If $\cos\theta_1 + 2\cos\theta_2 + \sec\theta_3 = 6$, then

$\cos\theta_1 = \cos\theta_2 = \cos\theta_3 = 1 \Rightarrow \tan\theta_1 = \tan\theta_2 = \tan\theta_3 = 0$

3. The number of non – negative integral solutions of $x_1 + x_2 + x_3 + 4x_4 = 20$ is

(A) 530
(B) 532
(C) 534
(D) 536

Ans. D

Sol. Coefficient of x^{20} in $(1-x)^{-3}(1-x^4)^{-1}$

Coefficient of x^2 in

$\left(1+{}^3C_1x+{}^4C_2x^2+......\right)\left(1+x^4+x^8+x^{12}+x^{16}+x^{20}\right)$

$=1+{}^6C_4+{}^{10}C_8+{}^{14}C_{12}+{}^{18}C_{16}+{}^{22}C_{20}$

$=1+{}^6C_2+{}^{10}C_2+{}^{14}C_2+{}^{18}C_2+{}^{21}C_2$

$=1+15+45+91+153+231$

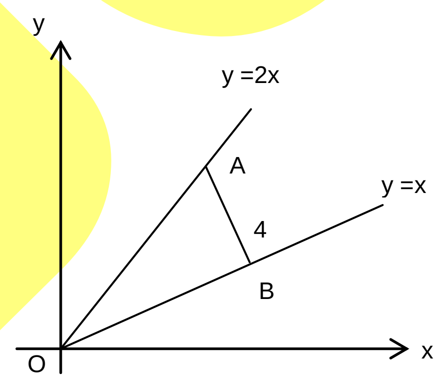
= 536

4. Let AB be a line segment of length 4 unit with the point A on the line $y=2x$ and B on the line $y=x$. Then locus of middle point of all such line segment is

(A) a parabola
(B) an ellipse
(C) a hyperbola
(D) a circle

Ans. B

Sol. Let $B=(\alpha,\alpha)$ and middle point AB is (h, k)



Then, $A=(2h-\alpha,\ 2k-\alpha)$

lies on $y=2x$

then, $(2k-\alpha)=2(2h-\alpha)$ ...(i)

$\therefore$ $\alpha=4h-2k$

$\Rightarrow$ $\dfrac{|AB|=4}{\sqrt{(2h-2\alpha)^2+(2k-2\alpha)^2}}=4$

or $(h-\alpha)^2+(k-\alpha)^2=4$

or $\left[h-(4h-2k)\right]^2+\left[k-(4h-2k)\right]^2=4$

$\Rightarrow$ $(-3h+2k)^2+(-4h+3k)^2=4$

or $25h^2+13k^2-36hk=4$

Required locus is $25x^2+13y^2-36xy-4=0$

Here, $h^2<ab$ and $\Delta\neq 0$

$\therefore$ ellipse

5. If $(1+x)^n=\sum\limits_{r=0}^{n}a_rx^r$ and $b_r=1+\dfrac{a_r}{a_{r-1}}$ and $\prod\limits_{r=1}^{n}b_r=\dfrac{(101)^{100}}{100!}$, then n is

(A) 99
(B) 100
(C) 101
(D) 102

Ans. B

Sol. $(1+x)^n=\sum\limits_{r=0}^{n}{}^nC_r\,x^r=\sum\limits_{r=0}^{n}a_rx^r$ (given)

$\therefore \quad a_r = {}^nC_r$

Also, $b_r = 1 + \frac{a_r}{a_{r-1}} = \frac{a_r}{a_{r-1}} = 1 + \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{{}^{n+1}C_r}{{}^nC_{r-1}}$

$b_r = \left(\frac{n+1}{r}\right)$

$\because \quad \prod_{r=1}^{n} b_r = \prod_{r=2}^{n}\left(\frac{n+1}{r}\right) = \frac{(n+1)^n}{n!}$

$= \frac{(101)^{100}}{100!}$ (given)

$\therefore \quad$ n = 100

6. The line which intersect the skew lines $y = mx,\ z = c;\ y = -mx,\ z = -c$ and the x – axis lie on the surface

(A) $cz = mxy$ (B) $cy = mxz$

(C) $xy = cmz$ (D) none of these

Ans. B

Sol. Equation of the planes through $y = mx,\ z = c$ and $y = -mx,\ z = -c$ are respectively

$(y - mx) + \lambda_1(z - c) = 0$ ……(i)

and $(y + mx) + \lambda_2(z + c) = 0$ ……(ii)

It meets at x – axis i.e. $y = 0 = z$

$\therefore \quad \lambda_2 = \lambda_1$

From equation (i) and (ii),

$$\frac{y - mx}{z - c} = \frac{y + mx}{z + c}$$

$\therefore \quad cy = mzx$

7. The number of six digit numbers that can be formed from the digits 1, 2, 3, 4, 5, 6 and 7, so that digits do not repeat and the terminal digits are even is

(A) 144 (B) 72

(C) 288 (D) 720

Ans. D

Sol. Terminal digits are the first and last digits.

$\because$ Terminal digits are even

$\therefore$ 1st place can be filled in 3 ways and last place can be filled in 2nd ways and remaining places can be filled in ${}^5P_4 = 120$ ways

Hence, the number of six digit numbers, the terminal digits are even, is

$= 3 \times 120 \times 2 = 720$

8. Three players A, B, C in this order, cut a pack of cards, and the whole pack is reshuffled after each cut. If the winner is one who first draws a diamond, then C" s chance of winning is

(A) $\frac{9}{28}$ (B) $\frac{9}{37}$

(C) $\frac{9}{64}$ (D) $\frac{27}{64}$

Ans. B

Sol. Probability of diamond card $= \frac{^{13}C_1}{^{52}C_1} = \frac{1}{4}$

Probability of C's wining

$= P(\bar{A})P(\bar{B})P(C) + P(\bar{A})P(\bar{B})P(\bar{C})P(\bar{A})P(\bar{B})P(C) + ...$

$$= \frac{P(\bar{A})P(\bar{B})P(\bar{C})}{1 - P(\bar{A})P(\bar{B})P(\bar{C})} = \frac{\frac{3}{4} \times \frac{3}{4} \times \frac{1}{4}}{1 - \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4}} = \frac{9}{37}$$

9. A three digit number, which is multiple of 11, is chosen at random. The probability that the number so chosen is also a multiple of 9 is equal to

(A) $\frac{1}{9}$ (B) $\frac{2}{9}$

(C) $\frac{1}{100}$ (D) $\frac{9}{100}$

Ans. A

Sol. The number of three digit numbers, which are multiple 11 = 90 –9 = 81. Again the number, which are divisible of 9 also, are divisible by 99, whose number is 10 – 1 = 9.

So, required probability $= \frac{9}{81} = \frac{1}{9}$.

10. If x, y, z are integers in AP, lying between 1 and 9 and x51, y41 and z31 are three digit numbers, then the value of $\begin{vmatrix} 5 & 4 & 3 \\ x51 & y41 & z31 \\ x & y & z \end{vmatrix}$ is

(A) $x + y + z$ (B) $x - y + z$

(C) 0 (D) $x + 2y + z$

Ans. C

Sol. Let $\Delta = \begin{vmatrix} 5 & 4 & 3 \\ x51 & y41 & z31 \\ x & y & z \end{vmatrix}$

$$\Delta=\begin{vmatrix}5 & 4 & 3\\ 100x+50+1 & 100y+40+1 & 100z+30+1\\ x & y & z\end{vmatrix}$$

Applying $R_2 \to R_2-(100R_3+10R_1)$, then

$$\Delta=\begin{vmatrix}5 & 4 & 3\\ 1 & 1 & 1\\ x & y & z\end{vmatrix}$$

Applying $C_2 \to C_2-\frac{1}{2}(C_1+C_3)$, then $\Delta=\begin{vmatrix}5 & 0 & 3\\ 1 & 0 & 1\\ x & y-\frac{1}{2}(x+z) & z\end{vmatrix}$

$$=\begin{vmatrix}5 & 0 & 3\\ 1 & 0 & 1\\ x & 0 & z\end{vmatrix}=0 \ (\because \text{ x, y, z are in AP})$$

11. For the equations; $x+2y+3z=1, 2x+y+3z=2, 5x+5y+9z=4$

(A) there is only one solution
(B) there exists infinitely many solutions
(C) there is no solution
(D) none of the above

Ans. A

Sol. $\because \begin{vmatrix}1 & 2 & 3\\ 2 & 1 & 3\\ 5 & 5 & 9\end{vmatrix}=1(9-15)-2(18-15)+3(10-5)$

$=-6-6+15=3\neq 0$

i.e. only one solution.

12. $f(x)=1+\sin x[\cos x], 0<x\le\frac{\pi}{2}$ ([.] denotes the greatest integer function)

(A) is continuous in $\left(0,\frac{\pi}{2}\right)$
(B) is strictly decreasing in $\left(0,\frac{\pi}{2}\right)$
(C) is strictly increasing in $\left(0,\frac{\pi}{2}\right)$
(D) has global maximum value 2

Ans. A

Sol. $\because 0<x<\frac{\pi}{2}$ $\quad\therefore 0\le\cos x<1$ then $[\cos x]=0$

$\therefore f(x)=1$

Hence, $f(x)$ is continuous in $\left(0,\frac{\pi}{2}\right)$

13. The solution of the differential equation $\frac{dy}{dx} = \frac{1}{xy\left(x^2 \sin y^2 + 1\right)}$ is

(A) $x^2\left(\cos y^2 - \sin y^2 - ce^{-y^2}\right) = 2$ (B) $y^2\left(\cos x^2 - \sin y^2 - ce^{-y^2}\right) = 2$

(C) $x^2\left(\cos y^2 - \sin y^2 - e^{-y^2}\right) = 4c$ (D) none of these

Ans. A

Sol. $\frac{dy}{dx} = \frac{1}{xy\left(x^2 \sin y^2 + 1\right)} \Rightarrow \frac{dx}{dy} = xy\left(x^2 \sin y^2 + 1\right)$

$\Rightarrow \quad \frac{1}{x^3}\frac{dx}{dy} - \frac{1}{x^2}y = y \sin y^2 \quad$ ....(1)

Put $-\frac{1}{x^2} = u$.

$\Rightarrow \quad \frac{du}{dy} + 2uy = 2\, y \sin y^2$

This is a linear differential equation.

14. $\int \frac{\operatorname{cosec}^2 x - 2005}{\cos^{2005} x} dx$ is equal to

(A) $\frac{\cot x}{(\cos x)^{2005}} + C$ (B) $\frac{\tan x}{(\cos x)^{2005}} + C$

(C) $-\frac{\tan x}{(\cos x)^{2005}} + C$ (D) $-\frac{\cot x}{(\cos x)^{2005}} + C$

Ans. D

Sol. $I = \int \frac{\operatorname{cosec}^2 x - 2005}{\cos^{2005} x} dx$

$= \int (\cos x)^{-2005} \operatorname{cosec}^2 x\, dx - 2005 \int \frac{dx}{\cos^{2005} x}$

$= (\cos x)^{-2005}(-\cot x) - \int (-2005)(\cos x)^{-2006}(-\sin x)(-\cot x)dx - 2005\int \frac{dx}{\cos^{2005} x}$

$= -\frac{\cot x}{(\cos x)^{2005}} + C$

15. Number of common tangents with finite slope to the curves $xy = c^2$ and $y^2 = 4ax$ is

(A) 0 (B) 1

(C) 2 (D) 4

Ans. B

Sol. Equation of PQ is $yt - x = at^2$ ......(1)

$\frac{x}{t'} + yt' = 2c$ ......(2)

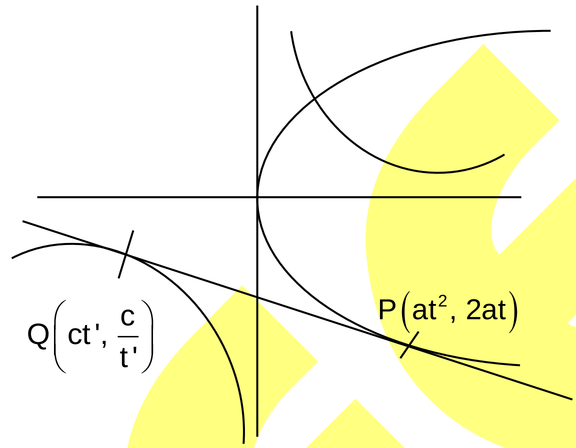
$\because$ (1) and (2) are identical

$\Rightarrow \quad -t' = \frac{t}{t'} = \frac{at^2}{2c}$

$\Rightarrow \frac{a(t')^4}{2c} = -t'$

$\Rightarrow (t')^3 = -\frac{2c}{a}$

$\Rightarrow$ Only one real value of 't' exists.



16. If the roots of $x^5 - 40x^4 + Px^3 + Qx^2 + Rx + S = 0$ are in G.P. and the sum of their reciprocals is 10, then $|S|$ is equal to

(A) 4 (B) 6
(C) 8 (D) 32

Ans. D

Sol. The roots of the equation $x^5 - 40x^4 + Px^3 + Qx^2 + Rx + S = 0$ are in G.P. Let the roots be a, ar, ar^2, ar^3, ar^4.

$\therefore \; a + ar + ar^2 + ar^3 + ar^4 = 40$

and $\frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \frac{1}{ar^3} + \frac{1}{ar^4} = \frac{1 + r + r^2 + r^3 + r^4}{ar^4} = 10$

$\Rightarrow a^2r^4 = 4$ or $ar^2 = \pm 2$

Now, –S = product of roots $= a^5r^{10} = (ar^2)^5 = \pm 32$

$\therefore |S| = 32$.

17. Let $f(x) = -x^3 + px^2 + qx + 6\,\mathrm{sgn}(x^2 + x + 1)$, where $p, q \in R$ and sgn stands for signum function. If the largest possible interval in which $f'(x)$ is positive is $\left(-\frac{5}{3}, 1\right)$, then $(p + q)$ equals

(A) 6 (B) 4
(C) –4 (D) –6

Ans. B

Sol. $f(x) = -x^3 + px^2 + qx + 6$

$f'(x) = -3x^2 + 2px + q > 0$

$3x^2 - 2px - q < 0$

$x^2 - \frac{2p}{3}x - \frac{q}{3} < 0$

Given, $\left(x-\left(-\frac{5}{3}\right)\right)(x-1)<0$

$x^2+\frac{2}{3}x-\frac{5}{3}<0$

$p=-1;\ q=5$

18. If the line $x+y-1=0$ is a tangent to a parabola with focus (1, 2) at A and intersects the directrix at B and the tangent at the vertex at C, then AC. BC is equal to

(A) 2 (B) 1

(C) $\frac{1}{2}$ (D) $\frac{1}{4}$

Ans. A

Sol. $(BC)(AC)=(CS)^2$

19. If α,β,γ are the angle of a triangle and the system of equations

$\cos(\alpha-\beta)x+\cos(\beta-\gamma)y+\cos(\gamma-\alpha)z=0$

$\cos(\alpha+\beta)x+\cos(\beta+\gamma)y+\cos(\gamma+\alpha)z=0$

$\sin(\alpha+\beta)x+\sin(\beta+\gamma)y+\sin(\gamma+\alpha)z=0$

has non – trivial solutions, then the triangle is necessarily

(A) equilateral (B) isosceles

(C) right angled (D) acute angled

Ans. B

Sol. Let $\Delta=\begin{vmatrix}\cos(\alpha-\beta) & \cos(\beta-\gamma) & \cos(\gamma-\alpha)\\ \cos(\alpha+\beta) & \cos(\beta+\gamma) & \cos(\gamma+\alpha)\\ \sin(\alpha+\beta) & \sin(\beta+\gamma) & \sin(\gamma+\alpha)\end{vmatrix}$

It is clear that either $\alpha=\beta$ or $\beta=\gamma$ or $\gamma=\alpha$ is sufficient to make $\Delta=0$. It is not necessary that the triangle be equilateral.

20. Let $f:[0,1]\to R$ be a continuous function and assumes only rational values. If $f(0)=2$, then the value of $\tan^{-1}\left(f\left(\frac{1}{2}\right)\right)+\tan^{-1}\left(\frac{3}{2}f\left(\frac{1}{2}\right)\right)$ is

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{6}$

(C) $\frac{3\pi}{4}$ (D) π

Ans. C

Sol. $f(0) = 2 \Rightarrow f\left(\frac{1}{2}\right) = 2$

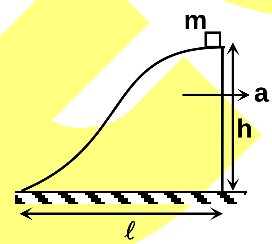
Required expression $= \tan^{-1}(2) + \tan^{-1}\left(\frac{3}{2}.\,2\right)$

$= \tan^{-1}(2) + \tan^{-1}(3) = \pi + \tan^{-1}\left(\frac{2+3}{1-2.3}\right)$

$= \pi - \tan^{-1}(1) = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

PHYSICS

21. A block of mass m is placed on the top of a wedge having undefined smooth curved surface. If the wedge is now accelerated horizontally with acceleration a, then the speed of block with respect to wedge when it reaches the bottom of wedge is



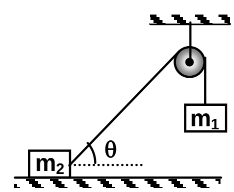
(A) $\sqrt{2gh}$
(B) $\sqrt{2(a+g)h}$
(C) $\sqrt{2(a\ell - gh)}$
(D) $\sqrt{2(a\ell + gh)}$

Ans. D

Sol. $ma\ell + mgh = \frac{1}{2}mv^2$

$\therefore\ v = \sqrt{2(a\ell + gh)}$

22. At an instant m_1 and m_2 are having speed v_1 and v_2 and acceleration a_1 and a_2. Then



(A) $v_1 = v_2 \sec\theta$
(B) $a_1 = a_2 \cos\theta$ only at an instant when the blocks starts from rest
(C) $a_1 = a_2 \cos\theta$ for all instant
(D) $v_1 = v_2 \sin\theta$

Ans. B

Sol. differentiating the relation $v_1 = v_2 \cos\theta$ with respect to time.

23. A particle is projected with speed u at an angle θ with ground. The radius of curvature at highest point is

(A) $\frac{u^2\cos^2\theta}{g}$
(B) $\frac{u^2\sin^2\theta}{g}$
(C) $\frac{u^2\cos^2\theta}{2g}$
(D) $\frac{u^2\sin^2\theta}{2g}$

Ans. A

Sol. $R = \dfrac{v_{net}^2}{a_r} \Rightarrow \vec{a}_r \perp \vec{v}_{net}$

24. If the change in the value of 'g' at a height h above the surface of the earth is the same as a depth x below it, then (both x and h being much smaller than the radius of the earth):

(A) x = h
(B) x = 2 h
(C) x = 1/2 h
(D) $x = h^2$.

Ans. B

Sol. Above surface of earth at height $h \ll R$

$$g' = g\left[1 - \frac{2h}{R}\right] \Rightarrow \frac{\Delta g}{g} = \frac{2h}{R}$$

surface below earth $x \ll R$.

$$g' = g\left[1 - \frac{x}{R}\right] \Rightarrow \frac{\Delta g}{g} = \frac{x}{R}.$$

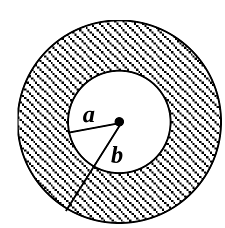
25. A system consists of two stars of equal masses that revolve in a circular orbit about a centre of mass midway between them. Orbital speed of each star is v & period is T. Find the mass M of each star: (G is gravitational constant)

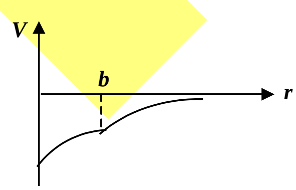
(A) $\dfrac{2Gv^3}{\pi T}$
(B) $\dfrac{v^3 T}{\pi G}$
(C) $\dfrac{v^3 T}{2\pi G}$
(D) $\dfrac{2Tv^3}{\pi G}$

Ans. D

Sol. $\dfrac{GM^2}{(2r)^2} = \dfrac{MV^2}{r}$ *Or* $M = \dfrac{4v^2 r}{G}$ & $T = \dfrac{2\pi r}{v}$, $r = \dfrac{Tv}{2\pi}$

26. A sphere of mass *M* and radius *b* has a concentric cavity of radius *a* as shown in figure. The graph showing variation of gravitational potential *V* with distance *r* from the center of sphere is



(A)


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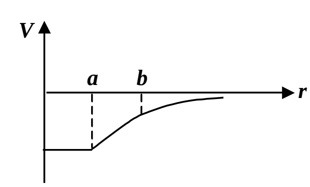
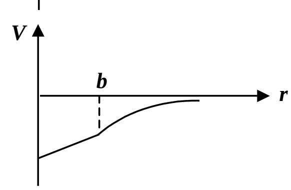
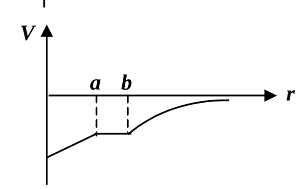


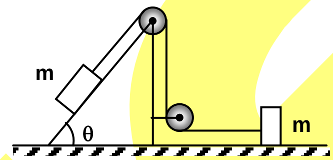
(C)


(D)


Ans. B

Sol. Gravitational potential inside a shell = constant

27. In the figure shown, the pulley is light, wedge is fixed and all the surfaces are smooth. Tension in the string is



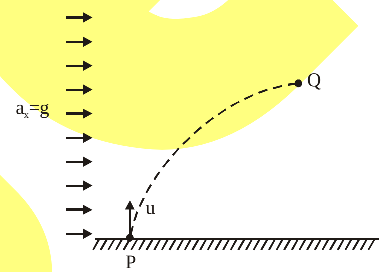
(A) mg sin θ
(B) $\frac{mg\sin\theta}{2}$
(C) $mg\frac{\sin\theta}{3}$
(D) mg

Ans. B

Sol. $mg\sin\theta - T = ma$

$T = ma$

28. Air is blowing and is providing a constant horizontal acceleration a_x = g to the particle as shown in the figure. Particle is projected from point P with a velocity u in upward direction. Let Q be the highest point of particle. Speed of the particle at highest point Q is



(A) $\sqrt{2}u$
(B) u
(C) $u/\sqrt{2}$
(D) None

Ans. B

Sol. $v_x = u_x + a_x t$

$v_x = 0 + gt$ ….. (1)

$v_y = u_y + a_y t$

$0 = u - gt$

$t = \frac{u}{g}$ …..(2)

By (1) and (2) we get

$v_x = u$ and $v_y = 0$

Hence net velocity = u

29. When the current in a coil changes from 8A to 2A in 3 × 10^{-2} seconds, the e.m.f induced in the coil is 2 volts. The self inductance of the cell in millihenery is

(A) 1
(B) 5
(C) 20
(D) 10

Ans. D

Sol. $\varepsilon = L\frac{dI}{dt} \Rightarrow 2 = L \times \frac{6A}{3\times10^{-2}}$

L = 10^{-2} Henry = 10 mH

30. Electric charge q, q and – 2q are placed at the corners of an equilateral triangle ABC of side L. The magnitude of electric dipole moment of the system is:

(A) qL (B) 2qL

(C) $\sqrt{3}$ qL (D) 4qL

Ans. C

Sol. Two dipoles will be formed at angle 60º to each other.

$\therefore P_{net} = \sqrt{(qL)^2 + (qL)^2 + 2(qL)^2 \cos 60^o} = \sqrt{3}qL$

31. A uniform magnetic field of 30 mT exists in the + X direction. A particle of charge + e and mass 1.67×10^{-27} kg is projected through the field in the + Y direction with a speed of 4.8×16^6 m/s. Radius of the circular path followed by the particle is

(A) 6.17 m (B) 1.67 m

(C) 1.76 m (D) 1.77 m

Ans. B

Sol. (B) $F = qVB \sin\theta$

$= (1.6 \times 10^{-19})\,(4.8 \times 10^6)\,(30 \times 10^{-3}) \sin 90^0$

$= 230.4 \times 10^{-6}$ N.

The direction of the force is in the (-z) direction.

(b) If the particle were negatively charged, the magnitude of the force will be the same but the direction will be along (+z) direction.

(c) As $V \perp B$, the path described is a circle

$R = mV/qB$

$= (1.67 \times 10^{-27})\cdot(4.8 \times 10^6)/(1.6 \times 10^{-19}) \cdot (30 \times 10^{-3}) = 1.67$ m.

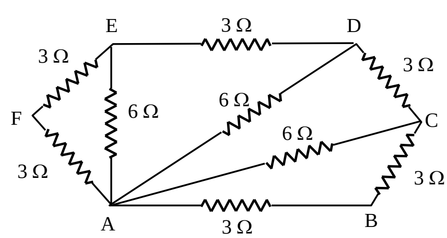
32. The effective resistance between the point A and B will be

(A) 4 Ω

(B) 2Ω

(C) 6Ω

(D) 8 Ω



Ans. B

Sol. Resistors AF and FE are in series with each other. Therefore, network AEF reduces to a parallel combination of two resistors of 6 Ω each.

$Req. = \frac{6 \text{ x } 6}{6+6} = 3\Omega.$

Similarly, the resistance between A and D is given

$\frac{6 \text{ x } 6}{6+6} = 3\Omega.$

Now, resistor AC is in parallel with the series combination of AD and DC. Therefore, the resistance between A and C is $\frac{6 \times 6}{6+6} = 3\Omega$.

AC + CB = 3 + 3 = 6 Ω. Since they are in series resistance between A and B is given by $\frac{1}{R} = \frac{1}{6} + \frac{1}{3} = \frac{3}{6}$ or R_{AB} = 2Ω.

33. Potential in the x-y plane is given as $V = 5(x^2 + xy)$ volts. The electric field at the point (1, –2) will be

(A) $3\hat{j}$ V/m
(A) $-5\hat{j}$ V/m
(C) 5J V/m
(D) $-3\hat{j}$ V/m

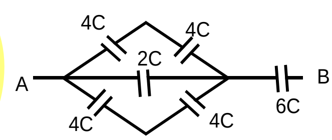
Ans. B

Sol. $E_x = -\frac{\partial V}{\partial x} = -(10x + 5y) = -10 + 10 = 0$

$E_y = -\frac{\partial V}{\partial x} = -5x = -5$

$\therefore \ \vec{E} = -5\hat{j} \ \ V/m.$

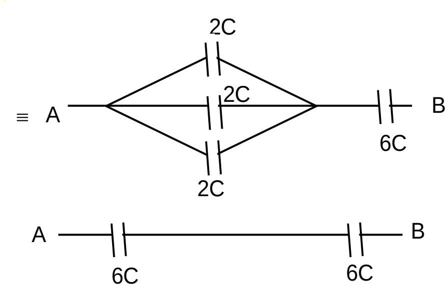
34. The equivalent capacitance between points A and B of the circuit will be



(A) 12C
(B) 6 C
(C) 3 C
(D) 24 C

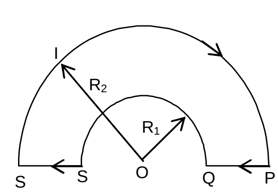
Ans. C

Sol. Equivalent circuit of the above figure can be drawn as



$C_{AB} = 3C$

35. The wire loop PQRSP formed by joining two semi-circular wires of radii R_1 and R_2 carries a current I as shown in figure. What is the magnetic induction at the centre O and magnetic moment of the loop?



(A) $\frac{\pi I}{2}\left(R_2^2 - R_1^2\right)$ into the page
(B) $\frac{\pi I}{2}\left(R_1^2 - R_2^2\right)$ into the page
(C) $\frac{\pi I}{2}\left(R_2^2 - R_1^2\right)$ out to the page
(D) $\frac{\pi I}{2}\left(R_2^2 + R_1^2\right)$ out to the page

Ans. A

Sol. As the point O is along the length of the straight wire, so the field at O due to them will be zero and hence

$$\vec{B} = \frac{\mu_o}{4\pi}\left[\frac{\pi \quad I}{R_2} \otimes + \frac{\pi \quad I}{R_1}\right] \odot$$

$$\Rightarrow \vec{B} = \frac{\mu_o}{4\pi}\pi \quad I\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \text{ out of the page}$$

and $\vec{M} = I\left[\frac{1}{2}\pi R_2^2 \otimes + \frac{1}{2}\pi R_1^2\right] \odot$

$= \frac{1}{2}\pi \quad I\left[R_2^2 - R_1^2\right]$ into the page

36. The mass of the three wires of copper are in the ratio 1 : 3 : 5. and their lengths are in ratio 5 : 3 : 1 . The ratio of their electrical resistance is

(A) 1 : 3 : 5
(B) 5 : 3 : 1
(C) 1 : 15 : 125
(D) 125 : 15 : 1

Ans. D

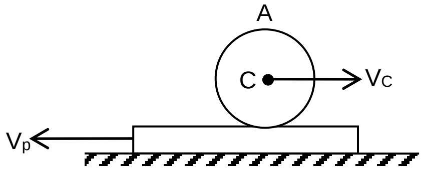
Sol. $R = \frac{\rho\ell}{A} = \frac{\rho\ell^2}{A\ell} = \frac{\rho\ell^2}{V} = \frac{\rho\ell^2}{m/d}$

$R = \frac{\rho d\ell^2}{m}$ or $R \propto \frac{\ell^2}{m}$

$R_1 : R_2 : R_3 = \frac{\ell_1^2}{m_1} : \frac{\ell_2^2}{m_2} : \frac{\ell_3^2}{m_3}$

$= \frac{25}{1} : \frac{9}{3} : \frac{1}{5} = 125 : 15 : 1$

37. The velocities are in ground frame and the cylinder is performing pure rolling on the plank. Velocity of point 'A' would be



(A) $2V_c$
(B) $2V_c + V_p$
(C) $2V_c - V_p$
(D) $2(V_c + V_p)$

Ans. C

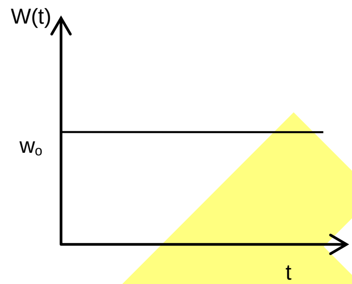
Sol. $V_{Ap} = 2V_c$
$V_{Ag} = V_{Ap} + V_{Pg} = 2V_c - V_p$

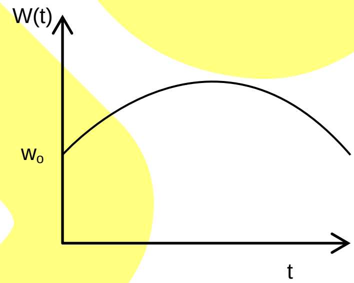
38. A cylinder rolls up an inclined plane, reaches some height and then rolls down (without slipping throughout these motions) The directions of the frictional force acting on the cylinder are
(A) up the incline while ascending and down the incline while descending
(B) up the incline while ascending and up the incline while descending
(C) Down the incline while ascending and up the incline while descending.
(D) Down the incline while ascending as well as descending.

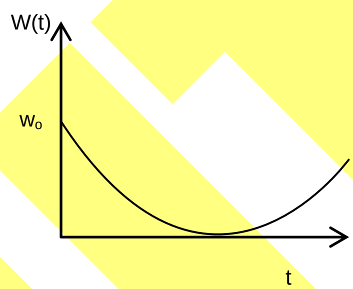
Ans. B

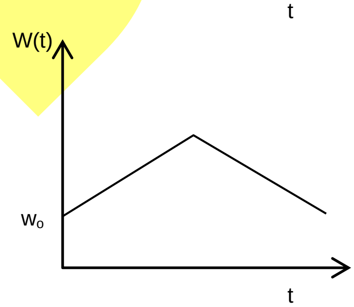
Sol. Friction force always acts up the incline irrespective of direction of motion

39. A circular platform is free to rotate on a horizontal plane about a vertical axis passing through its centre. A tortoise is sitting at the edge of the platform. Now the platform is given an angular velocity ω_o. When the tortoise moves along a chord of the platform with a constant velocity (with respect to the platform), the angular velocity of the platform w(t) will vary with time t as

(A)


(B)


(C)


(D)


Ans. B

Sol. Conserve angular momentum

40. The image of an object, formed by a plano-convex lens at a distance of 8 m behind the lens, is real and is one-third the size of the object. The wavelength of light inside the lens is $\frac{2}{3}$ times the wavelength in free space. The radius of the curved surface of the lens is
(A) 1 m
(B) 2 m
(C) 3 m
(D) 6 m

Ans. C

Sol. $\mu = \frac{3}{2}$; V = 8 ; $m = \frac{1}{3}$

$1 + m = \frac{V}{f} = \frac{V}{2R}$

CHEMISTRY

41. The magnitude of an orbital angular momentum vector is $\sqrt{6}\frac{h}{2\pi}$. Into how many components will the vector split if a magnetic field is applied on it?

(A) 3 (B) 5
(C) 7 (D) 9

Ans. B

Sol. Orbital angular momentum = $\sqrt{l(l+1)}\frac{h}{2\pi} = \sqrt{6}\frac{h}{2\pi}$

$\therefore$ l = 2 $\rightarrow$ represents d-orbitals which splits into five components in presence of magnetic field.

42. Which of the following compound does not exist?

(A) CF_4 (B) SF_4
(C) OF_4 (D) XeF_4

Ans. C

Sol. Oxygen can't form more than two covalent bonds.

43. Which of the following solution mixture exerts common ion effect?

(A) HCl + NaCl (B) NaHS + H_2S
(C) $NaNO_3 + HNO_3$ (D) $Na_2S + H_2SO_4$

Ans. B

Sol. $H_2S \rightleftharpoons HS^- + H^+$

$NaHS \longrightarrow Na^+ + HS^-$

44. The half-life of a chemical reaction is expressed as: $t_{½} = \frac{\sqrt{3}-K}{2}C_O^{-2}$ where C_O is the initial concentration of the reactant. What is the order of the reaction?

(A) Zero (B) 1
(C) 2 (D) 3

Ans. D

Sol. $t_{1/2} = \frac{\sqrt{3}-K}{2}C_o^{-2}$ = K C_o^{-2}

Since $t_{1/2} \propto C_o^{1-n}$

$\therefore 1 - n = -2 \Rightarrow n = 3$

45. Consider the following equilibrium constants

$$A_{(g)} \rightleftharpoons B_{(g)}; K_1$$

$$B_{(g)} \rightleftharpoons C_{(g)}; K_2$$

$$C_{(g)} \rightleftharpoons D_{(g)}; K_3$$

$$A_{(g)} \rightleftharpoons D_{(g)}; K_4$$

The correct relation among the above equilibrium constants is:

(A) $K_4 = K_1 + K_2 + K_3$

(B) $K_4 = K_1 \times K_2 \times K_3$

(C) $K_4 = \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3}$

(D) $K_4 = \frac{1}{K_1} \times \frac{1}{K_2} \times \frac{1}{K_3}$

Ans. B

Sol. $K_4 = K_1 \times K_2 \times K_3 \times K_4$

$$= \frac{[B]}{[A]} \times \frac{[C]}{[B]} \times \frac{[D]}{[C]} = \frac{[D]}{[A]} = K_4$$

46. Element 'X' belongs to the fourth period. The magnetic moment of X^{3+} ion is 5.92 B.M. Therefore, 'X' is

(A) Ni

(B) Fe

(C) Mn

(D) Co

Ans. B

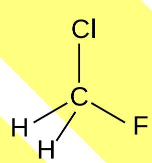
Sol. $\sqrt{n(n+2)} = 5.92$

On solving n = 5

$\therefore$ The atom contains 5 unpaired electrons in +3 oxidation state

$\therefore$ The ion is Fe^{2+} and X is Fe

47.



The largest bond angle observed in the above molecule is

(A) $\angle$ HCCl

(B) $\angle$ HCF

(C) $\angle$ HCH

(D) $\angle$ FCCl

Ans. C

Sol. Carbon is more electronegative than hydrogen and F and Cl are more electronegative than carbon.

48. Which of the following pair of compounds are not isomorphous to each other?
(A) $ZnSO_4.7H_2O$ and $MgSO_4.7H_2O$
(B) KNO_3 and $KClO_3$
(C) Cu_2S and Ag_2S
(D) None of these

Ans. B

Sol. Hybridization of N in NO_3^- is sp^2 and that of Cl in ClO_3^- is sp^3

49. If a blue litmus paper is dipped in HClO solution, it turns
(A) green
(B) red
(C) blue
(D) colourless

Ans. D

Sol. $HClO \rightleftharpoons HCl + [O]$

Coloured substance + [O] $\rightarrow$ Colourless substance.

50. In metallurgy Serpec's process is used to purify
(A) Haematite
(B) Bauxite
(C) Siderite
(D) Calamine

Ans. B

Sol. It is used for the purification of bauxite.

51. $\underset{(I)}{[Fe(CO)_3(PH_3)_3]}$ $\underset{(II)}{[Fe(CO)_3(PH_3)_2(NH_3)]}$

$\underset{(III)}{[Fe(CO)_3(PH_3)(NH_3)_2]}$ $\underset{(IV)}{[Fe(CO)_3(NH_3)_3]}$

Arrange the above complexes in decreasing order of Fe – C bond energy.
(A) I > II > III > IV
(B) IV > III > II > I
(C) IV > II > III > I
(D) I > III > IV > II

Ans. B

Sol. CO and PH_3 form back bond with metal ion.

52. How many geometrical isomers are possible for the following complex?

$[Pt(NH_3)_2(Py)_2Cl_2]^{2+}$

(A) 2
(B) 4
(C) 5
(D) 6

Ans. C

Sol. Five geometrical isomers are formed.

53. Which of the following electronic transition needs highest amount of energy?

(A) $2s^2 2p^2 \rightarrow 2s^1 2p^3$

(B) $3s^2 3p^2 \rightarrow 3s^1 3p^3$

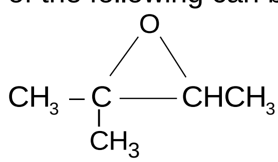
(C) $4s^2 3d^4 \rightarrow 4s^1 3d^5$

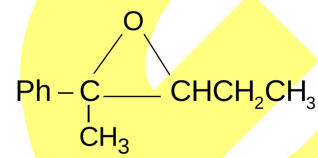
(D) all the above transitions need same amount of energy

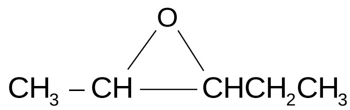
Ans. A

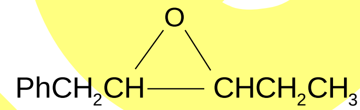
Sol. Orbitals with lower value of 'n' requires higher energy for electron excitation.

54. Which of the following can be easily cleaved with dil.HCl?

(A) 

(B) 

(C) 

(D) 

Ans. B

Sol. It will form the most stable carbocation.

55. $CH_3COONH_{4(aq)} \rightleftharpoons CH_3COO^-_{(aq)} + NH^+_{4(aq)}$

Ammonium acetate is most soluble in

(A) Water

(B) Dil.HCl

(C) Aq.NaCl

(D) Aq.KNO_3

Ans. B

Sol. $CH_3COO^- + H^+ \longrightarrow CH_3COOH$

$\therefore$ Forward reaction is driven up.

56. The molecular mass of a polyhydric alcohol increases by 336 amu if it reacts with CH_3COCl in presence of pyridine to form the corresponding ester. How many OH groups are present in the alcohol?

(A) 6

(B) 10

(C) 8

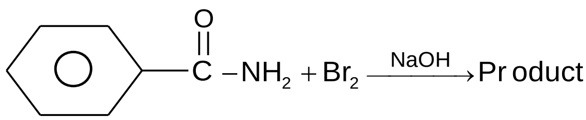
(D) 12

Ans. C

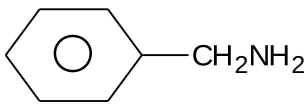
Sol. $ROH + CH_3COCl \longrightarrow R-O-COCH_3 + HCl$

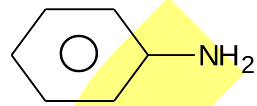
CH_3CO(mol. mass = 43) replaces one H-atom. Therefore the mass gain by alcohol is 42 for one OH group.

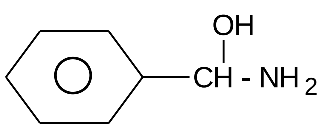
$\therefore$ No. of OH groups = $\frac{336}{42} = 8$

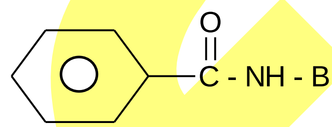
57. 

The product of above reaction is

(A) 

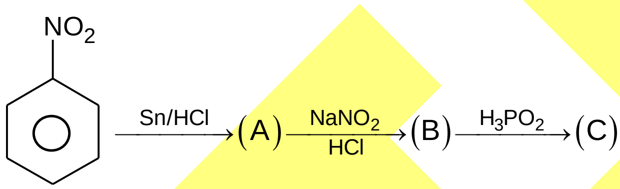
(B) 

(C) 

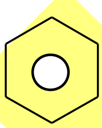
(D) 

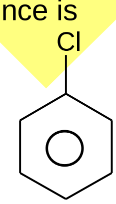
Ans. B

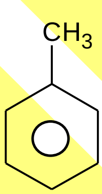
Sol. Hofmann's bromamide reactions.

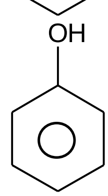
58. 

The end product (C) in the above reaction sequence is

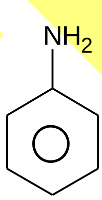
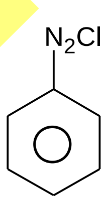
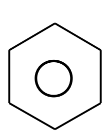
(A) 

(B) 

(C) 

(D) 

Ans. A

Sol. A =  , B =  , C = 

59. How many mole of phenyl hydrazine can completely react with one mole of glucose to form glucosazone?

(A) 2

(B) 3

(C) 4 (D) 6

Ans. B

Sol. 3 moles

60. Two flasks A and B of equal volumes maintained at temperature 300 K and 700 K contain equal mass of He(g) and N_2(g) respectively. What is the ratio of total translational kinetic energy of gas in flask A to that in flask B?

(A) 1:3 (B) 3:1

(C) 3:49 (D) None of these

Ans. B

Sol. $$\frac{\text{K.E of He}}{\text{K.E of } N_2} = \frac{\frac{3}{2}n_1RT_1}{\frac{3}{2}n_2RT_2} = \frac{\frac{3}{2}\times\frac{w}{4}\times R\times 300}{\frac{3}{2}\times\frac{w}{28}\times R\times 700} = 3:1$$

PART – II

MATHEMATICS

61. In the isosceles triangle ABC, $\left|\vec{AB}\right| = \left|\vec{BC}\right| = 8$, a point E divides AB internally in the ratio 1 : 3, then the cosine of the angle between $\vec{CE}$ and $\vec{CA}$ is $\left(\text{where } \left|\vec{CA}\right| = 12\right)$

(A) $-\frac{3\sqrt{7}}{8}$
(B) $\frac{3\sqrt{8}}{17}$
(C) $\frac{3\sqrt{7}}{8}$
(D) $\frac{-3\sqrt{8}}{17}$

Ans. C

Sol. $\cos A = \frac{8^2 + 12^2 + 8^2}{2(8)(12)} = \frac{3}{4} \Rightarrow CE^2 = AE^2 + AC^2 - 2AE.AC.\cos A$

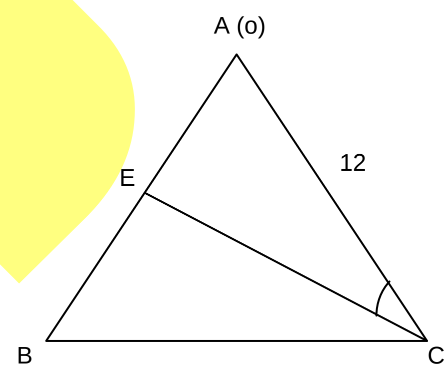
Now use cosine formula

$\left|\vec{b} - \vec{c}\right| = \left|\vec{b}\right| = 8$

$b^2 + c^2 - 2\vec{b}.\vec{c} = b^2 = 64$

$c^2 = 2\vec{b}.\vec{c}$

$\vec{b}.\vec{c} = 72$



62. Let $f(x) = x^3 - \frac{3x^2}{2} + x + \frac{1}{4}$. The value of $\int_{1/4}^{3/4} f(f(x))dx$

(A) $\frac{1}{4}$
(B) $\frac{1}{2}$
(C) 0
(D) $\frac{3}{4}$

Ans. A

Sol. $4f(x) = x^4 - (1-x)^4 + 2$

$\Rightarrow f(x) + f(1-x) = 1$

63. The minimum value of $f(x) = 8^x + 8^{-x} - 4\left(4^x + 4^{-x}\right)\ \forall x \in R$.

(A) –1
(B) –2
(C) –3
(D) 1

Ans. C

Sol. Let $u = 2^x + 2^{-x}$

$4^x + 4^{-x} = 4^u - 2$

$8^x + 8^{-x} = u^3 - 3u$

$\Rightarrow f(x) = u^3 - 3u - 4(u^2 - 2) = u^3 - 4u^2 - 3u + 8$

Let $g(u) = u^3 - 4u^2 - 3u + 8;\ u \ge 2$

$g'(u) = (3u+1)(u-3) \Rightarrow u = 3$

$g''(u) = 6u - 8 \qquad \Rightarrow g''(3) > 0 \quad \Rightarrow u = 3$ is point of minimum

$g(3) = 27 - 36 - 9 + 8 = -10$

64. The slope of the normal at the point with abscissa $x = -2$ of the graph of the function $f(x) = |x^2 - x|$ is

(A) $\frac{-1}{6}$ (B) $\frac{-1}{3}$

(C) $\frac{1}{6}$ (D) $\frac{1}{3}$

Ans. D

Sol. For $x < 0$

$f(x) = |x^2 + x| = |x||x+1|$

For $x < -1$

$f(x) = (-x)(-x-1) = x^2 + x$

$\therefore\ f'(x) = 2x + 1$

Slope of tangent $= 2(-2) + 1 = -3$

$\therefore$ Slope of normal $= \frac{1}{3}$

65. If $f : R \to R$ is the function defined by $f(x) = \frac{e^{x^2} - e^{-x^2}}{e^{x^2} + e^{-x^2}}$, then

(A) $f(x)$ is an increasing function (B) $f(x)$ us a decreasing function

(C) $f(x)$ is onto (surjective) (D) none of these

Ans. D

Sol. $\because\ f(x) = \frac{e^{x^2} - e^{-x^2}}{e^{x^2} + e^{-x^2}} \quad \therefore\ f'(x) = \frac{8x}{\left(e^{x^2} + e^{-x^2}\right)^2}$

$$= \begin{cases} > 0, & x > 0 \\ < 0, & x < 0 \\ 0, & x = 0 \end{cases}$$

66. A cylindrical gas container is closed at the top and open at the bottom; if the iron plate of the top is $\frac{5}{4}$ times as thick as the plate forming the cylindrical sides. The ratio of the radius to the height of the cylinder using minimum material for the same capacity is

(A) $\frac{2}{3}$ (B) $\frac{1}{2}$

(C) $\frac{4}{5}$ (D) $\frac{1}{3}$

Ans. C

Sol. $V = \pi r^2 h$

If k be the thickness of the sides, then that of the top will be $\left(\frac{5}{4}\right)k$.

$\therefore S = (2\pi rh)k + (\pi r^2)\left(\frac{5}{4}\right)k$ ('S is volume of material used)

or $S = 2\pi rk.\frac{V}{\pi r^2} + \frac{5}{4}\pi r^2 k = k\left(\frac{2V}{r} + \frac{5}{4}\pi r^2\right)$

$\therefore \frac{ds}{dr} = k\left(-\frac{2V}{r^2} + \frac{5}{2}\pi r\right)$ $\therefore r^3 = \frac{4V}{5\pi}$

$\frac{d^2S}{dr^2} = k\left(\frac{4V}{r^3} + \frac{5}{2}n\right) = \pi k\left(5 + \frac{5}{2}\right) = +ive$

when $r^3 = \frac{4V}{5\pi}$ or $5\pi r^3 = 4\pi r^2 h$

$\therefore \frac{r}{h} = \frac{4}{5}$

67. Let ABC be a triangle. Let A be the point (1, 2), y = x be the perpendicular bisector of AB and $x - 2y + 1 = 0$ be the angle bisector of angle C. If the equation of BC is given by $ax + by - 5 = 0$, then the value of $a + b$ is

(A) 1 (B) 2

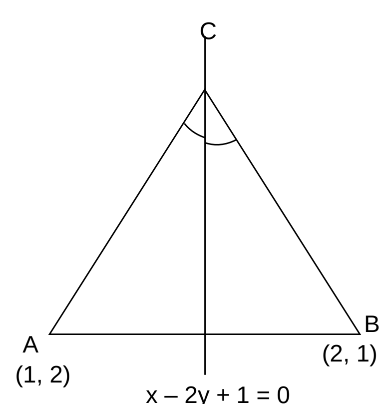
(C) 3 (D) 4

Ans. B

Sol. Image of A, say A', w.r.t $x - 2y + 1 = 0$ lies on BC.

$\frac{x-1}{1} = \frac{y-2}{-2} = -2\frac{(1-4+1)}{1+4} = \frac{4}{5}$

$A' = \left(\frac{9}{5}, \frac{2}{5}\right)$



68. Let $f(x)$ be a non – constant twice differentiable function on R such that $f(2+x)=f(2-x)$ and $f'\left(\frac{1}{2}\right)=f'(1)=0$. The minimum number of roots of the equation $f''(x)=0$ in (0, 4) is

(A) 2
(B) 4
(C) 5
(D) 6

Ans. B

Sol. $f(2+x)=f(2-x)$ and $f'\left(\frac{1}{2}\right)=f'(1)=0$

It is symmetric w.r. to the line $x=2$, hence $f'(2)=0$.

$f'\left(\frac{1}{2}\right)=f'\left(2-\frac{3}{2}\right)=-f'\left(2+\frac{3}{2}\right)$

$f'(1)=f'(2-1)=-f'(2+1)$

Thus, $f'(x)=0$ has roots at $x=\frac{1}{2},1,2,3,\frac{7}{2}$.

$f'(x)=0$ has five roots

So, $f''(x)=0$ will have four roots (at the least).

69. The value of $\sum_{r=0}^{20} r(20-r)\left({}^{20}C_r\right)^2$ is equal to

(A) $400\times{}^{39}C_{20}$
(B) $400\times{}^{40}C_{19}$
(C) $400\times{}^{39}C_{19}$
(D) $400\times{}^{38}C_{20}$

Ans. D

Sol. $\sum_{r=0}^{20} r(20-r)\left({}^{20}C_r\right)^2=\sum_{r=0}^{20}\left(r\times{}^{20}C_r\right)\left((20-r)\times{}^{20}C_{20-r}\right)$

$=\sum_{r=0}^{20} 20\times{}^{19}C_{r-1}\times 20\times{}^{19}C_{19-r}$

$=400\sum_{r=0}^{20} {}^{19}C_{r-1}\times{}^{19}C_{19-r}$

= 400 x coefficient of x^{18} in $(1+x)^{19}(1+x)^{19}$

$=400\times{}^{38}C_{18}$

$=400\times{}^{38}C_{20}$

70. All the roots of the equation $11z^{10}+10iz^9+10iz-11=0$ lie

(A) inside $|z|=1$
(B) on $|z|=1$
(C) outside $|z|=1$
(D) cannot say

Ans. B

Sol. $11z^{10}+10iz^{9}+10iz-11=0$

or $z^{9}\left(11z+10i\right)=11--10iz$

or $z^{9}=\dfrac{11-10iz}{11z+10i}$

or $\left|z^{9}\right|=\dfrac{|11i-10z|}{|11z+10i|}$

Now $|11i-10z|^{2}-|11z+10i|^{2}=21\left(1-|z|\right)$

For $|z|<1$

$|11zi-10z|^{2}-|11z+10i|^{2}>0$

$\Rightarrow \left|z^{2}\right|=\dfrac{|11i-10z|}{|11z+10i|}>1$

i.e. $\left|z^{9}\right|>1$ which contradicts with $|z|<1$

For $|z|>1$ we get $\left|z^{9}\right|<1$

$\Rightarrow |z|=1$

PHYSICS

71. A conducting liquid bubble of radius a and thickness $t\left(t<<a\right)$ is charged to potential V. If the bubble collapses to a droplet, find the potential on the droplet.

(A) $V\left(\dfrac{a}{3t}\right)^{1/3}$

(B) $Va^{1/3}$

(C) $V\left(\dfrac{a^{2}}{t^{2/3}}\right)$

(D) $\dfrac{Va^{1/3}}{3g^{3}}$

Ans. A

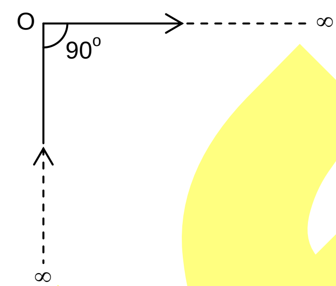
Sol. $V=\dfrac{1}{4\pi\varepsilon_{o}}\dfrac{q}{a}$ (for bubble)

For droplet :- $\dfrac{4}{3}\pi r^{3}=\dfrac{4}{3}\pi\left(a+t\right)^{3}-\dfrac{4}{3}\pi a^{3}$

$\Rightarrow r^{3}=3a^{2}t \Rightarrow r=\left(3a^{2}t\right)^{1/3}$

$V_{droplet}=\dfrac{1}{4\pi\varepsilon_{o}}\dfrac{q}{r}=V\left[\dfrac{a}{3t}\right]^{1/3}$

72. A very long wire carrying a current 10A is bent at right angle at O. Find the magnetic induction at a point P lying on the perpendicular to the wire at O. The distance of the point P from O is 35cm.



(A) 0.404×10^{-5} T
(B) 0.405×10^{-5} T
(C) 0.408×10^{-5} T
(D) 0.454×10^{-5} T

Ans. A

Sol. $B_1=\frac{\mu_o i}{4\pi R}$

$B_2=\frac{\mu_o i}{4\pi R}$

B = $\sqrt{B_1^2+B_2^2}$

= $B_1\sqrt{2}$

= $\frac{\mu_o i\sqrt{2}}{4\pi R}$

= $\frac{4\pi\times10^{-7}\times10\sqrt{2}}{4\pi\times35\times10^{-2}}$

B = 0.404×10^{-5} T

73. A uniform electric field of magnitude E = 100 kV/m is directed upward. Perpendicular to E and directed into the page there exists a uniform magnetic field of magnitude B = 0.5T. A beam of particles of charge +q enters this region. What should be the chosen speed of particles for which the particles will not be deflected by the electric and magnetic field?

(A) 2×10^{-5} m/s
(B) 3×10^{-5} m/s
(C) 5×10^{-5} m/s
(D) 6×10^{-5} m/s

Ans. A

Sol. $\vec{F}=q\vec{E}+q\left(\vec{V}\times\vec{B}\right)$

$\vec{F}=q\left(\vec{E}+\vec{V}\times\vec{B}\right)$

If there has to be no deflection of beam then

$\vec{F}=0$

$\vec{E}+\left(\vec{V}\times\vec{B}\right)=0$

$\vec{V}\times\vec{B}=\vec{E}$

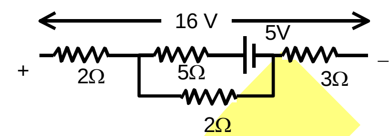
$\vec{V}\times\vec{B}=-\vec{E}$

VBsin90° = E

V = $\frac{E}{B}$ = $\frac{100\times10^{3}}{0.5}$

V = 2×10^{5} m/sec

74. In the circuit shown below, determine the current through 5V cell

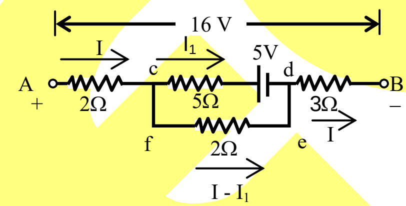


(A) $\frac{1}{15}$ Amp (B) $\frac{4}{15}$ Amp

(C) $\frac{1}{17}$ Amp (D) $\frac{4}{17}$ Amp

Ans. A

Sol. $\phi_A - \phi_B = 16$ ...(i)



Apply KVL on loop (cd efc)

$0 - 5I_1 - 5 + 2(I - I_1) = 0$

$\Rightarrow I_1 = \frac{1}{7}(2I - 5)$ ...(ii)

Apply KVL on path (AC d B)

$\phi_A - 2I - 5I_1 - 5 - 3I = \phi_B$

$\Rightarrow \phi_A - \phi_B = 5I + 5I_1 + 5$

$\Rightarrow 16 = 5I + 5\left(\frac{2I - 5}{7}\right) + 5$

$\Rightarrow 16 - 5 + \frac{25}{7} = I\left(5 + \frac{10}{7}\right)$

$\Rightarrow I = \frac{34}{15}$ Amp

(i) and (ii) give

$I_1 = -\frac{1}{15}$ Amp

Current through battery (5V) = $\frac{1}{15}$ Amp

75. What amount of heat will be generated in a coil of resistance R due to a charge q passing through it if the current in the coil decreases down to zero uniformly during a time interval Δt?

(A) $\frac{4}{3}\frac{q^2R}{\Delta t}$ (B) $\frac{2}{3}\frac{q^2R}{\Delta t}$

(C) $\frac{3}{4}\frac{q^2R}{\Delta t}$ (D) $\frac{3}{2}\frac{q^2R}{\Delta t}$

Ans. A

Sol. Suppose initial current is i_0, then

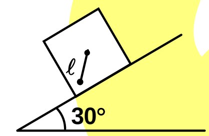
$i(t) = i_0\left(1 - \frac{t}{\Delta t}\right)$

$q = i_0\int_0^{\Delta t}\left(1 - \frac{t}{\Delta t}\right)dt$

So, $i_0 = \dfrac{2q}{\Delta t}$

$$H = \int_0^{\Delta t} \left\{ \frac{2q}{\Delta t}\left(1 - \frac{t}{\Delta t}\right) \right\}^2 R\,dt$$

76. A cabin is accelerating up the incline with acceleration g m/s^2. A simple pendulum of length $\sqrt{3}$ meter is hanging from the vertical wall of the cabin. The minimum speed given to the bob so that it performs vertical circular motion with respect to cabin is (g = 10 m/s^2)



(A) 110 m/s (B) $\sqrt{15}$ m/s
(C) $\sqrt{150}$ m/s (D) 12 m/s

Ans. C

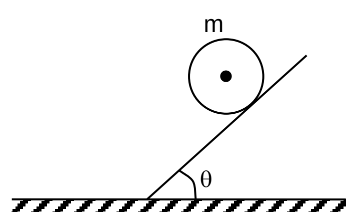
Sol. Apply work energy theorem and Newton's second law which will give $u = \sqrt{150}$ m/s.

77. A particle moves along x-axis. The position of the particle at time t is given as
$x = t^3 - 9t^2 + 24t + 1$
The distance traveled in first 5 seconds is
(A) 20 m (B) 10 m
(C) 18 m (D) 28 m

Ans. D

Sol. Distance Travelled = $\int_0^5 |\vec{v}|\,dt = \int_0^5 |3t^2 - 18t + 24|\,dt$ = 28

78. A sphere of mass m has to purely roll on a rough inclined plane of coefficient plane of coefficient of friction 'μ'. The friction force acting on the sphere is



(A) $\mu mg\cos\theta$
(B) $\dfrac{2mg\sin\theta}{7}$ downward
(C) $\dfrac{2mg\sin\theta}{7}$ upward
(D) $\dfrac{5mg\sin\theta}{7}$ downward

Ans. C

Sol. $f = \dfrac{mg\sin\theta}{1 + \dfrac{R^2}{K^2}}$

79. A body of mass 1kg is suspended from a massless spring having force constant 600 N/m. Another body of mass 0.5 kg moving vertically upwards hits the suspended body with a velocity 3 m/sec and gets embedded in it. The frequency of oscillation and the amplitude of motion are

(A) $\frac{5}{\pi}$Hz, 10Cm
(B) $\frac{10}{\pi}$Hz, 5Cm
(C) $\frac{5}{\pi}$Hz, 5Cm
(D) $\frac{5}{\pi}$Hz, 10Cm

Ans. B

Sol. $V = \frac{0.5 \times 3}{(0.5+1)} = 1\,m/s \Rightarrow K_{max} = U_{max}$

$\frac{1}{2} \times 1.5 \times 1^2 = \frac{1}{2} \times 600 \times A^2$

A= 5 cm

$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{600}{1.5}} = \frac{10}{\pi}$

$f = \frac{10}{\pi}$ Hz

80. When the electron in a hydrogen atom jumps from the second orbit to the first orbit, the wavelength of the radiation emitted is λ. When the electron jumps from the third to the first orbit, the wavelength of the radiation emitted as

(A) $\frac{9}{4}\lambda$
(B) $\frac{4}{9}\lambda$
(C) $\frac{27}{32}\lambda$
(D) $\frac{32}{27}\lambda$

Ans. C

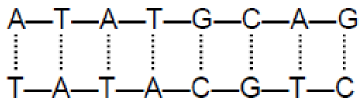
Sol. $\frac{hc}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3}{4}R$ ...(i)

$\frac{hc}{\lambda'} = R\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = \frac{8}{9}R$ ...(ii)

$\frac{\lambda'}{\lambda} = \frac{3}{4} \times \frac{9}{8}, \quad \lambda' = \frac{27}{32}\lambda$

CHEMISTRY

81. The average energy of each hydrogen bond in A-T pair is x kcal mol^{-1} and that in G-C pair is y kcal mol^{-1}. Assuming that no other interaction exists between the nucleotides, the approximate energy required in kcal mol^{-1} to split the following double stranded DNA into two single strands is



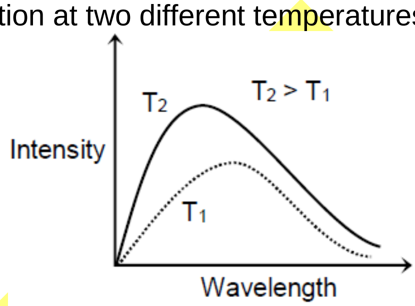
[Each dashed line may represent more than one hydrogen bond between the base pairs]

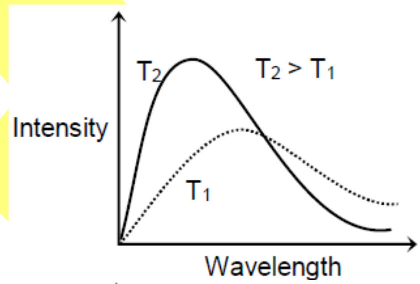
(A) 10x + 9y (B) 5x + 3y
(C) 15x + 6y (D) 5x + 4.5y

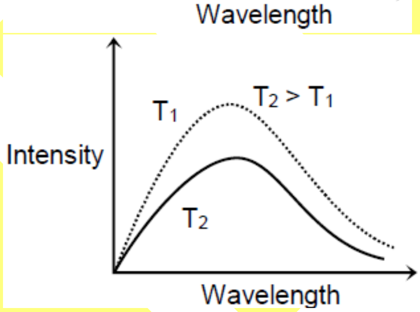
Ans. A

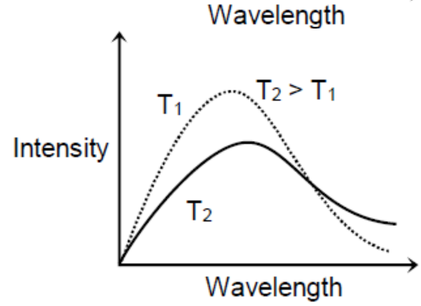
Sol. Number of H-bond is A–T pair = 2, while no of H-bond in G–C pair is 3. Therefore
(i) Total number of A–T. H-bond = number of A–T pair × Number of H bond = 5 × 2 = 10
(ii) Total number of G–C H-bond = number of G–C pair × number of H bond = 3 × 3= 9
Total energy required to dissociate the stand = 10x + 9y Kcal mol^{-1}

82. The correct representation of wavelength intensity relationship of an ideal blackbody radiation at two different temperatures T_1 and T_2 is

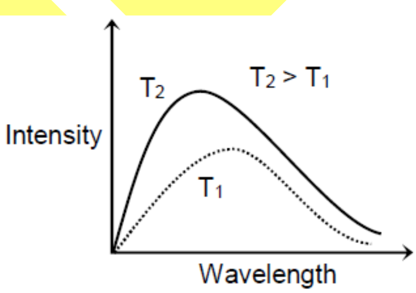
(A)


(B)


(C)


(D)


Ans. A

Sol.


83. A gas at atmospheric pressure is heated from $0^\circ C$ to $546^\circ C$ and simultaneously compressed to one-third of its original volume. Hence final pressure is.

(A) 6 atm (B) 9 atm
(C) 18 atm (D) 27 atm

Ans. B

Sol. $P_1 = 1$ atm
$T_1 = 273$ K
Let volume at this condition is V L and
$P_2 = P$ atm
$T_2 = 819$ K
$V_2 = V/3$ L

Then, $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

$\Rightarrow \frac{1 \times V}{273} = \frac{P \times V}{3 \times 819}$

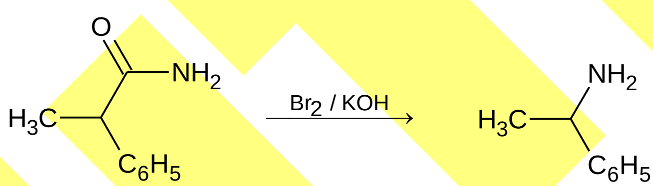
$\Rightarrow P = 9$ atm
Hence, the correct answer is option B.

84. Alkanamide which on Hoffmann's reaction gives 1-phenyl ethyl amine is:

(A) 2 – phenylpropanamide (B) 3 – phenylpropanamide
(C) 2 – phenylethanamide (D) N – phenylethanamide

Ans. A

Sol.



Hence, the correct answer is option A.

85. The order of pK_a of these carboxylic acids in water is

(I) CH_3COOH (II) C_2H_5COOH
(III) C_3H_7COOH
(A) I > II > III (B) II > I > III
(C) III > II > I (D) III > I > II

Ans. C

Sol. The electron releasing substituents (+I) decrease the acidic strength of carboxylic acid by destabilizing the carboxylate ion. The strength of the acid is expressed in terms of the dissociation constant (K_a). A stronger acid has higher Ka but lesser pK_a value. Hence, the correct answer is option C.

86. X mL of O_2 effuses through a hole in a container in 20 seconds. The time taken for the effusion of the same volume of the gas specified below under identical conditions is

(A) 10 seconds : He
(B) 5 seconds : H_2
(C) 25 seconds : CO
(D) 55 seconds : CO_2

Ans. B

Sol. $\frac{r_{Gas}}{r_{O_2}} = \sqrt{\frac{M_{O_2}}{M_{Gas}}}$, where r is rate of effusion and M is molecular mass of gas.

$\Rightarrow \frac{V_{Gas}}{t_{Gas}} \times \frac{t_{O_2}}{V_{O_2}} = \sqrt{\frac{M_{O_2}}{M_{Gas}}}$, where t is time for effusion of V is volume of gas effused.

If $V_{O_2} = V_{Gas} = X$ mL

$\Rightarrow \frac{t_{O_2}}{t_{Gas}} = \sqrt{\frac{M_{O_2}}{M_{Gas}}}$

$\frac{20}{t_{Gas}} = \sqrt{\frac{32}{M_{Gas}}}$

Hence, the correct answer is option B.

87. How many Faradays of electricity is needed to oxidise one mole of H_2O completely to dioxygen gas?

(A) 1
(B) 2
(C) 4
(D) 1.5

Ans. B

Sol. One mole H_2O contains one mole O^{2-}

$O^{2-} - 2e \longrightarrow o = \frac{1}{2}O_2$

88. Which of the following atom has most stable electronic configuration in +2 oxidation state? [At. No. of Cr = 24, Mn = 25, Fe = 26, Cu = 29]

(A) Cr
(B) Mn
(C) Fe
(D) Cu

Ans. B

Sol. Mn($4s^2 3d^5$)

Mn^{2+} contains the stable $3d^5$ half-filled electron configuration.

89. Which of the following releases SO_2 gas when treated with dil.H_2SO_4?

(A) Na_2SO_4
(B) Na_2SO_3
(C) Na_2S
(D) All are correct

Ans. B

Sol. $SO_3^{2-} + 2H^+ \longrightarrow H_2O + SO_2 \uparrow$

90. Which of the following complex displays ionization isomerism?

(A) $[Cr(NH_3)_4(Br)_2]Br$ (B) $[Zn(NH_3)_4]Br_2$

(C) $[Co(NH_3)_3(Br)_3]$ (D) $[Mn(NH_3)_4(Br)_2]Cl$

Ans. D

Sol. In(D) the Br^- and Cl^- ligands exchange their positions.