# OLTS-2021-T6-FT-II-KVPY-CLASS-XII
## FULL TEST – II

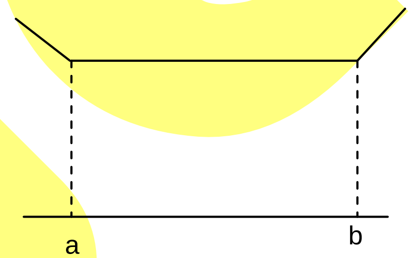
## PART – I
## MATHEMATICS

1. If a, b $\in$ R satisfying
$|a - 2020| + |b - 2020| = |a - 2019| + |b - 2019| = \ldots\ldots = |a + 2019| + |b + 2019| = |a + 2020| + |b + 2020|$
then minimum value of $|a-b| =$

(A) 4032
(B) 2020
(C) 1010
(D) 4040

Ans. D

Sol. Let $a < b$ and $f(x) = |x-a| + |x-b|\ \forall x \in R$
here $f(2020) = f(2019)$........
$= f(-2019) = f(-2020)$
$\{-2020, -2019 .....2020\} \in [a, b]$




2. If the minimum value of $(\tan C - \cos A)^2 + (\cot C - \sin A)^2$ be $b - a\sqrt{a}$, then the value of $b - a + 2$ is

(A) 0
(B) 2
(C) $2\sqrt{2}$
(D) 3

Ans. D

Sol. It can be interpreted as distance between the points $(\tan C, \cot C)$ and $(\cos A, \sin A)$.
Its minimum value is the square of the minimum distance between the curve $xy = 1$ and $x^2 + y^2 = 1$
$\therefore\ b - a\sqrt{a} = 3 - 2\sqrt{2}$
$\therefore\ b - a + 2 = 3$

3. The minimum distance of the surface $xyz^2 = 2$ from the origin is equal to

(A) 0
(B) 1
(C) 2
(D) 5

Ans. C

Sol. Let $d = x^2 + y^2 + z^2$.

$$d = x^2 + y^2 + \frac{2}{xy} = (x-y)^2 + 2xy + \frac{2}{xy} \geq 2xy + \frac{2}{xy} \geq 4$$

4. Let $A = [-2, 4), B = \{x \mid x^2 - ax - 4 \leq 0\}$. If $B \subseteq A$, then the range of real a is

(A) $[-1, 2)$

(B) $[-1, 2]$

(C) $[0, 3]$

(D) $[0, 3)$

Ans. D

Sol. $x^2 - ax - 4 = 0$ has two rots:

$x_1 = \frac{a}{2} - \sqrt{4 + \frac{a^2}{4}},\ x_2 = \frac{a}{2} + \sqrt{4 + \frac{a^2}{4}}$

We have $B \subseteq A \Leftrightarrow x_1 \geq -2$ and $x_2 < 4$.

This mean that $\frac{a}{2} - \sqrt{4 + \frac{a^2}{4}} \geq -2,\ \frac{a}{2} + \sqrt{4 + \frac{a^2}{4}} < 4$

From the above we get $0 \leq a < 3$

5. If $\log\left(x^3 + \frac{y^3}{3} + \frac{1}{9}\right) = \log x + \log y$, then the value of $x^3 + y^3$ is

(A) 4/9

(B) 9/4

(C) Data is in-sufficient

(D) log4/9

Ans. A

Sol. use $a^3 + b^3 + c^3 - 3abc = 0$ then a+b+c=0 or $\sum (a-b)^2 = 0$

6. Let f(x) is a polynomial of degree 4 such that $f(x) = x$ has no real roots, then minimum possible number of real roots of $f(f(f(x))) + x^2 + x + 2020 = 0$ is

(A) 2

(B) 1

(C) 6

(D) 0

Ans. D

Sol. Let $f(x) - x > 0 \forall x$ .....(i)

$\Rightarrow f(f(x)) - f(x) > 0$ .....(ii)

$\Rightarrow f(f(f(x))) - f(f(x)) > 0$ ....(iii)

From (i), (ii) & (iii)

$f(f(f(x))) - x > 0$

7. Maximum number of real roots of $1+2x+3x^2+a_3x^3+a_4x^4+........+a_{2022}x^{2022}=0$, $a_i$ are real & $a_{2016} \neq 0$

(A) 2
(B) 0
(C) 2020
(D) 2022

Ans. C

Sol. Since the roots cannot be zero

So, replace $x \to \frac{1}{x}$

We get $x^{2022}+2x^{2021}+3x^{2020}+.......+a_{2022}=0$

Let its roots are $\alpha_i$

$\therefore \sum \alpha_i = -2 \quad \sum \alpha_1\alpha_2 = 3$

Now, $\sum \alpha_i^2 = \left(\sum \alpha_i\right)^2 - 2\sum \alpha_1\alpha_2$

$= 4 - 6 < 0$

So, all the $\alpha_i$ are not real

Hence all the $\frac{1}{\alpha_i}$ are also not real

8. Value of $\int_0^{\infty} \frac{f\left(x^{2020n}+x^{2019n}+..............+x^n+x^{-n}+x^{-2n}+......x^{-2020n}\right)}{1+x^2} \ln x \, dx$ is ____

(A) 0
(B) $\frac{1}{2}$
(C) $\frac{1}{2019}$
(D) cannot be evaluated

Ans. A

Sol. Put $\ell nx = t$ and use the property of odd function. As the whole integrand becomes odd.

9. If $u = \cot^{-1}\sqrt{\cos\alpha} - \tan^{-1}\sqrt{\cos\alpha}$, then $\sin u$ is equal to

(A) $\tan^2\left(\frac{\alpha}{2}\right)$
(B) $\cot^2\left(\frac{\alpha}{2}\right)$
(C) $\tan^2\alpha$
(D) $\cot^2\alpha$

Ans. A

Sol. Let $\sqrt{\cos\alpha} = \tan y$. Then $\tan^{-1}\sqrt{\cos\alpha} = y$ and $\cot^{-1}\sqrt{\cos\alpha} = \frac{\pi}{2} - y$. Therefore,

$u = \left(\frac{\pi}{2}\right) - y - y$

$\Rightarrow \sin u = \cos 2y$

$$= \frac{1-\tan^2 y}{1+\tan^2 y} = \frac{1-\cos\alpha}{1+\cos\alpha} = \frac{2\sin^2\left(\frac{\alpha}{2}\right)}{2\cos^2\left(\frac{\alpha}{2}\right)} = \tan^2\frac{\alpha}{2}$$

10. If $\left|z^2 + \frac{1}{z^3}\right| \le 2$, then $\left|z + \frac{1}{z}\right|$ cannot exceed

(A) 2 (B) 1
(C) $\sqrt{2}$ (D) $\sqrt{2}-1$

Ans. A

Sol. Let $\left|z + \frac{1}{z}\right| = a$

Now, the identity, $z^3 + \frac{1}{z^3} = \left(z + \frac{1}{z}\right)^3 - 3\left(z + \frac{1}{z}\right)$

gives us $\left|z + \frac{1}{z}\right|^3 \le \left|z^3 + \frac{1}{z^3}\right| + 3\left|z + \frac{1}{z}\right|$

$\Rightarrow\ a^3 \le 2 + 3a \Rightarrow 3a \Rightarrow a^3 - 3a - 2 \le 0$

$\Rightarrow (a-2)(a+1)^2 \le 0 \Rightarrow a - 2 \le 0$

$\Rightarrow\ a \le 2$

11. Let $k = \lim_{n\to\infty} n^2 \int_{-1/n}^{1/n} (2012\sin x + 2013\cos x)\ |x|\ dx$. The value of $k - 2012$ is equal to

(A) –1 (B) 0
(C) 1 (D) 2012

Ans. C

Sol. $\int_{-1/n}^{1/n} (2012\sin x + 2013\cos x)\ |x|\ dx = 4026\left(\frac{1}{n}\sin\frac{1}{n} + \cos\frac{1}{n} - 1\right) = 4026\left(\frac{1}{n}\sin\frac{1}{n} - 2\sin^2\frac{1}{2n}\right)$

Hence, $k = 4026\ \lim_{n\to\infty}\left(\frac{\sin\frac{1}{n}}{\frac{1}{n}} - \frac{\sin^2\frac{1}{2n}}{2\left(\frac{1}{2n}\right)^2}\right) = 2013$

12. The quadrilateral ABCD is formed by $5x + 3y = 9$, $x = 3y$, $y = 2x$, $x + 4y + 2 = 0$ then circum radius of ABCD is ____________

(A) $\frac{25}{18}$ (B) $\frac{5}{6}$
(C) $\frac{5}{3}$ (D) $\frac{25}{6}$

Ans. A

Sol. $(5x+3y-9)(y-2x)+\lambda(x+4y+2)(x-3y)=0$

coeff. of $x^2$ = coeff. of $y^2$

13. If $x=\sum_{r=1}^{90} 2r\sin(2r^\circ)$, then the value of $x$ is equal to

(A) $90\cot 1^\circ \operatorname{cosec} 1^\circ$
(B) $90\sec 1^\circ$
(C) $90\cot 1^\circ$
(D) none of these

Ans. C

Sol. $S=1\sin 2^\circ+2\sin 4^\circ+3\sin 6^\circ+...+89\sin 178^\circ$

$S=89\sin 178^\circ+88\sin 176^\circ+87\sin 174^\circ+...+1\sin 2^\circ$

Adding the two, we get

$$S=90(\sin 2^\circ+\sin 4^\circ+...+\sin 178^\circ)=90\left(\frac{\sin 89^\circ}{\sin 1^\circ}\right)\sin 90^\circ=90\cot 1^\circ$$

14. Consider the polynomial

$x^n+a_1x^{n-1}a_2x^{n-2}+.........+a_n=0$ with coefficients $a_1, a_2$............. an belongs to $\{1,-1\}$

then maximum value of n is __________

(A) 2
(B) 3
(C) 4
(D) 5

Ans. B

Sol. If $\alpha_1, \alpha_2.........\alpha_n$ are roots

then

$$\sum_{i=1}^{n}\alpha_i^2=a_1^2-2a_2=1-2a_2$$

Also $\frac{\sum\alpha_i^2}{n}\geq\left(\alpha_i^2\right)^{1/n}=\left(a_n^2\right)=1$

$1-2a_2\geq n$

15. The number of integral points on the hyperbola $x^2-y^2=(2000)^2$ are (an integral point is a point both of whose co – ordinates are integer) is equal to

(A) 0
(B) 98
(C) 48
(D) 1 96

Ans. B

Sol. $(x+y)(x-y)=2^8 5^6$

$y=0 \Rightarrow x=\pm 2000$

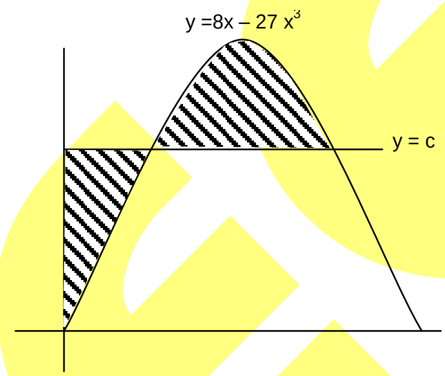
$|x|>|y|$

Total cases for the (x, y)

$= \frac{7 \times 7 - 1}{2} = 24$

Total cases = $24 \times 4 + 2 = 98$

16. The value of c such that areas of shaded region are equal.

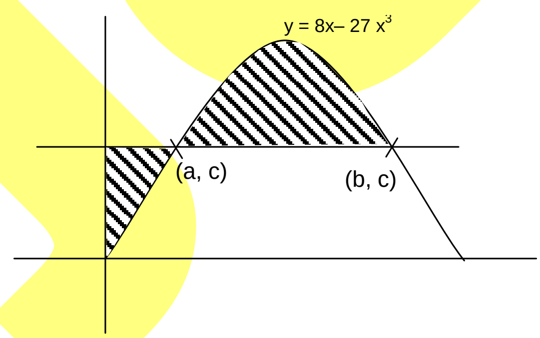



(A) $\frac{32}{27}$ (B) $\frac{32}{9}$

(C) $\frac{16}{27}$ (D) $\frac{8}{27}$

Ans. A

Sol. $\int_0^a \left(c - \left(8x - 27x^3\right)\right)dx = \int_a^b \left(\left(8x - 27x^3\right) - c\right)dx$




$O = 4b^2 - 27b^4 - bc$

$O = 4b^2 - \frac{27}{4} - b\left(8b - 27b^3\right)$

$b^4\left(\frac{81}{4}b^2 - 4\right) = 0$

$b > 0 \;\; b^2 = \frac{4^2}{81} \qquad b = \frac{4}{81}$

$C = 8b - 27b^3$

$= \frac{32}{27}$

17. If the points of intersection of the curves $x^2 - y^2 = a^2$ and $y = x^2$ lie on a unique circle, then 'a' belongs to

(A) (–1, 1) (B) (0, 1)

(C) (–1, 0) (D) $\left(-\frac{1}{2}, \frac{1}{2}\right)$

Ans. D

Sol. The points of intersection lies on $(x^2 - y^2 - a^2) + \lambda(x^2 - y) = 0$

It represents a circle if $\lambda = -2$

$\therefore$ equation of circle is $x^2 + (y - 1)^2 = 1 - a^2$

$\Rightarrow 1 - a^2 > 0 \Rightarrow a \in (-1, 1)$

But both curves will intersect in real points if $y^2 - y + a^2 = 0$ for some real y

i.e. $a \in \left(-\frac{1}{2}, \frac{1}{2}\right)$.

18. ABCD is a trapezium in which AB and CD are parallel and AD is perpendicular to AB. E is the mid-point of AD. If AD = 1 unit and BEC is an equilateral triangle, then the length of the side of the triangle is equal to

(A) 1 unit (B) 2 units
(C) 3 units (D) 4 units

Ans. A

Sol. It is possible if and only if the trapezium becomes a rectangle because we have ED = AE and BE = EC

$\Rightarrow$ AB = CD $\Rightarrow$ BE = $\frac{1}{2}$ cosec30° = 1.

19. Consider the equation $\frac{2}{x}+\frac{5}{y}=\frac{1}{3}$, where $x, y \in N$. The number of solutions of the equation is

(A) 6 if both x and y are even (B) 0 if both x and y are odd
(C) 4 if x is even and y is odd (D) 2 if x is odd and y is even

Ans. A

Sol. $(x-6)(y-15)=2\times3^2\times5$

When both x and y are even, (x – 6) and (y – 15) are even and odd respectively. So, 2 must be used with the first bracket. Number of ways = 6.

20. Let P(x) be a polynomial with integer coefficients. It is known that P(x) takes the value 2013 for four distinct integers. The number of integral values of x for which P (x) equals 2020.

(A) 0 (B) 2
(C) 4 (D) 2020

Ans. A

Sol. $P(x)-2013=q(x)(x-x_1)(x-x_2)(x-x_3)(x-x_4)$

Let $P(\alpha)=2020$

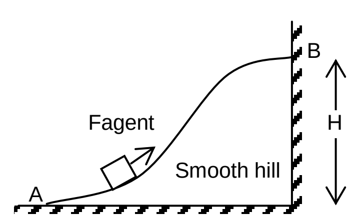
$\Rightarrow P(\alpha)-2013=q(\alpha)(\alpha-x_1)(\alpha-x_2)(\alpha-x_3)(\alpha-x_4)$

$\Rightarrow 7=q(\alpha)(\alpha-x_1)(\alpha-\alpha_2)(\alpha-\alpha_3)(\alpha-\alpha_4)$

Impossible since 7 is prime.

## PHYSICS

21. A external agent moves the block m slowly from A to B, along a smooth hill such that every time he applies the force tangentially. Find the work done by agent in this interval.

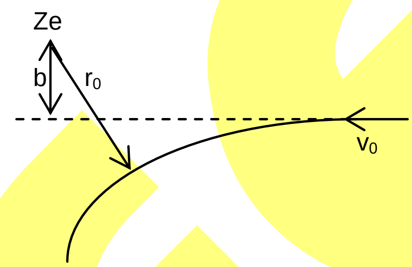



(A) $\frac{m^2g^2H^2}{L}$ (B) $\frac{mgH^2}{L}$
(C) mg(H+L) (D) mgH

Ans. D

Sol. $W_{agent} - mgH = 0$
$W_{agent} = mgH$

22. An $\alpha$ - particle is projected with velocity $v_0$ towards a very heavy nucleus of charge +Ze (where Z is the atomic number) from a very large distance as shown in the figure. If the distance of nearest approach is $r_0$ then the velocity at this point is




(A) $\frac{v_0 b}{r_0}$ (B) $\frac{v_0 r_0}{b}$

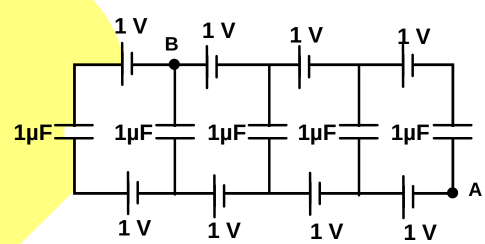
(C) $\frac{4v_0}{4 + A}$ (D) $\frac{4v_0}{4 - A}$

Ans. A

Sol. $mv_0 b = mvr_0$

$v = \frac{v_0 b}{r_0}$

23. The potential difference $V_B - V_A$ for the circuit shown in the figure is $\frac{24}{k}$ V. Then find the value of k.

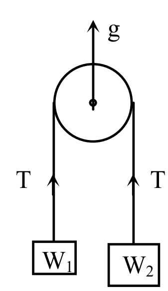



(A) 6
(B) 4
(C) 8
(D) 12

Ans. C

Sol. Apply KVL & symmetry charge on the capacitor 1μF is zero.

24. $W_1$ and $W_2$ is weights of blocks. If the pulley is taken up with an acceleration g then tension T will be



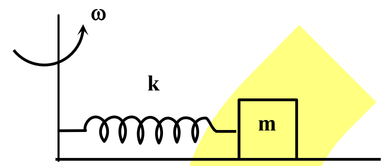

(A) $\frac{2W_1W_2}{W_1 + W_2}$ (B) $\frac{W_1W_2}{W_1 + W_2}$

(C) $\frac{4W_1W_2}{W_1 + W_2}$ (D) $\frac{4W_1W_2}{W_1 - W_2}$

Ans. C

Sol. w.r.t. pulley adding pseudo force also, effective weight of each block will become TWICE.

25. The extension in the spring will be (assume $\ell$ = natural length of spring)




(A) $\frac{m\omega^2 \ell_0}{k+\omega^2 m}$ (B) $\frac{m\omega^2 \ell_0}{k-\omega^2 m}$

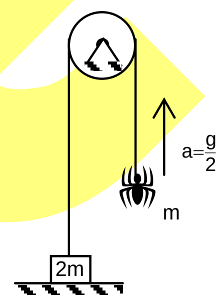
(C) zero (D) $\frac{m\omega^2 \ell_0}{k}$

Ans. B

Sol. F = M a for circular motion, gives

$K x = m(\ell + x) \times \omega^2$

26. An insect of mass m crawls along the hanging thread with an acceleration $a = \frac{g}{2}$. The reaction offered by ground on the block of mass 2 m is:




(A) $\frac{3mg}{2}$ (B) mg

(C) 2mg (D) $\frac{mg}{2}$

Ans. D

Sol. $T - mg = m\frac{g}{2}$

$N = 2\,mg - T = \frac{mg}{2}$

27. A stone is projected with a kinetic energy K at an angle of $30^0$ with the horizontal. Its kinetic energy at the highest point of its trajectory will be

(A) K (B) $\frac{K}{4}$

(C) $\frac{K}{2}$ (D) $3\frac{K}{4}$

Ans. D

Sol. $\frac{1}{2}m\left(u\cos 30^0\right)^2$

28. The threshold frequency for a metallic surface corresponds to an energy of 6.2 eV, and the stopping potential for a radiation incident on this surface 5 V. The incident radiation lies in

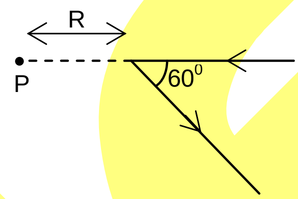
(A) X-ray region (B) ultra-violet region

(C) infra-red region (D) visible region

Ans. B

Sol. $\lambda = \dfrac{1240\,\text{eV nm}}{11.2} \approx 1100\,\text{Å}$. Ultraviolet region

29. A long straight wire, carrying a current I is bent at its mid point to form an angle of $60^0$. AT a point P, distance R from the point of bending the magnetic field is



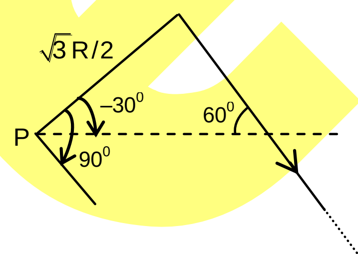

(A) $\dfrac{(\sqrt{2}-1)\mu_0 i}{4\pi R}$ (B) $\dfrac{(\sqrt{2}+1)\mu_0 i}{4\pi R}$

(C) $\dfrac{\mu_0 i}{4\sqrt{3}\pi R}$ (D) $\dfrac{\mu_0 i}{8R}$

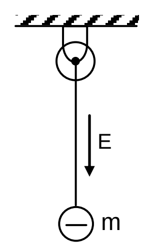
Ans. C

Sol. $B = \dfrac{\mu_o I}{4\pi\sqrt{3}\,R/2}\left[\sin 90^o + \sin(-30^o)\right]$

$= \dfrac{\mu_o I}{4\sqrt{3}\,R}$




30. A simple pendulum charged negatively to q coulomb oscillates with a time period T in a downward electric field. If the electric field is with-drawn, the new time period is:



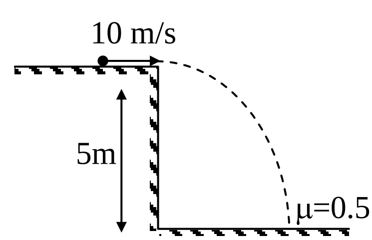

(A) = T (B) > T

(C) < T (D) $T\dfrac{mg}{qE}$

Ans. B

Sol. $T = 2\pi\sqrt{\dfrac{\ell}{g_{ef}}}$, $g_{ef} = \left(\dfrac{mg+qE}{m}\right)$

31. A small ball moving with a velocity 10 m/s, horizontally (as shown in figure) strikes a rough horizontal surface having $\mu = 0.5$. If the coefficient of restitution is e = 0.4. Horizontal component of velocity of ball after first impact will be (g = 10 m/s$^2$)




(A) 10 m/s (B) 8 m/s

(C) 3 m/s (D) 4 m/s

Ans. C

Sol. $\int N dt = mv_y - mu_y$, $e = \dfrac{v_y}{u_y} \Rightarrow v_y = 0.4 \times 10 = 4$ m/s

$\int N dt = m \times 4 - (-10m) = 14$ m

$$-\int \mu N dt = mv_x - mu_x$$
$$-0.5 \times 14m = mv_x - m \times 10 \Rightarrow v_x = 3\,m/s$$

32. A person sitting firmly over a rotating stool has his arms folded with two identical balls. If he stretched his arms along with balls and then the work done by him
(A) zero
(B) positive
(C) negative
(D) any of these

Ans. C

Sol. From conservation of angular momentum
$$I\omega = I_o\omega_o$$
$$\therefore \quad \frac{1}{2}I\omega^2 = \frac{1}{2}\times I\frac{I_o^2\omega_o^2}{I^2} = \frac{1}{2}I_o\omega_o^2\cdot\left(\frac{I_o}{I}\right)$$
$(K.E.)_{final} < (K.E.)_{initial}$ ; $I > I_o$
$\therefore \quad W_{man} = -ve.$

33. In a one dimensional collision between two identical particles. A and B, B is stationary and A has momentum p before impact. During impact, B gives impulse J to A.
(A) The total momentum of the 'A plus B ' system is p before and after the impact, and (p-J) during the impact
(B) During the impact, A gives impulse J to B
(C) The coefficient of restitution is $\frac{J}{p} - 1$
(D) The coefficient of restitution is $\frac{J}{p} + 1$

Ans. B

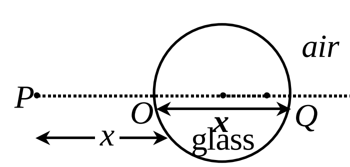
Sol. Factual

34. A spherical surface of radius of curvature R separates air (refractive index 1.0) from glass (refractive index 1.5). The centre of curvature is in the glass. A point object P placed in air is found to have a real image Q in the glass. The line PQ cuts the surface at a point O, and PO =OQ. The distance PO is equal to
(A) 5R
(B) 3R
(C) 2R
(D) 1.5R

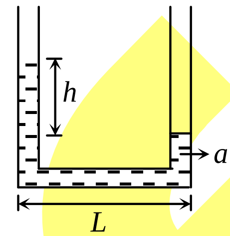
Ans. A

Sol. $\frac{1.5}{x} + \frac{1}{x} = \frac{1.5-1}{R}$, $\qquad \frac{2.5}{x} = \frac{0.5}{R}$

$x = 5R$



35. At rest, a liquid stands at the same level in the tubes. As the system is given an acceleration a towards the right, a height difference h occurs as shown in the figure. The value of h is:

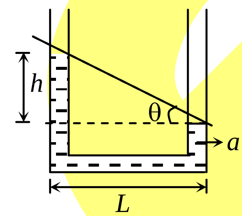


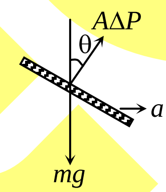

(A) $\frac{aL}{2g}$ (B) $\frac{gL}{2a}$

(C) $\frac{gL}{a}$ (D) $\frac{aL}{g}$

Ans. D

Sol. Newton's equations are :




$A\Delta P \sin\theta = ma$ ...(i)

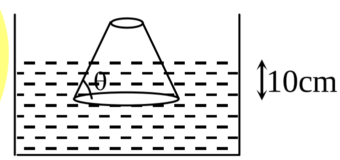
and $A\Delta P \cos\theta = mg$ ...(ii)

By (i) and (ii)

$\tan\theta = \frac{a}{g} = \frac{h}{L}$

or $h = \frac{aL}{g}$

36. A conical flask of mass 10 kg and base area $10^3$ cm$^2$ is floating in liquid of specific gravity 1.2 as shown in the figure. The force that liquid exerts on curved surface of conical flask is (g = 10 m/s$^2$)




(A) 20 N in downward direction
(B) 40 N in downward direction
(C) 20 N in upward direction
(D) 40 N in upwards direction

Ans. A

Sol. F = p × A

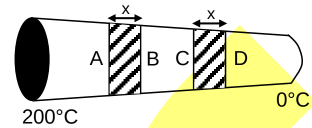
37. Imagine a Young's double slit interference experiment performed with wave associated with fast moving electrons produced from an electron gun. The distance between successive maxima will decrease maximum if
(A) the accelerating potential in the electron gun is decreased.
(B) the accelerating potential is increased and the distance of screen from slit is decreased.
(C) the distance of the screen from the slit is increased.
(D) the distance between the slits is decreased.

Ans. B

Sol. $\Delta x = \frac{\lambda D}{d} = \left(\frac{h}{mV}\right)\left(\frac{D}{d}\right)$, upon increasing ΔV, V increases.

Since, $e\Delta V = \frac{1}{2}mV^2$

38. Two ends of a conducting rod of varying cross-section are maintained at 200°C and 0°C respectively. In steady state

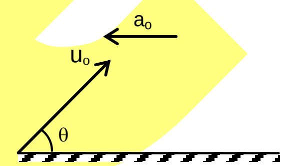



(A) temperature difference across AB and CD are equal.
(B) temperature difference across AB is greater than that of across CD.
(C) temperature difference across AB is less than that of across CD.
(D) temperature difference may be equal or different depending on the thermal conductivity of the rod.

Ans. C

Sol. $\frac{dq}{dt} = kA\left(\frac{dT}{dx}\right)$ = constant throughout rod.

39. From a point on ground, a projective is thrown with velocity $u_o$ at an angle "θ" with horizontal ground.
It is given that due to air drag, a constant acceleration $a_o$ is imparted on projectile, if projectile follows a straight line path, then




(A) $a_o = g\sin\theta$
(B) $a_o = g\cos\theta$
(C) $a_o = g\cot\theta$
(D) $a_o = g\tan\theta$

Ans. C

Sol. For projectile to follow straight line, net acceleration $(\vec{a} + \vec{g})$ must be along line of projection.

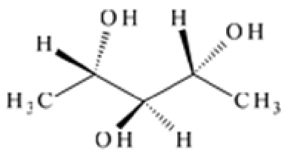
40. A photosensitive metallic surface has work function $h\nu_0$. If photons of energy $2h\nu_0$ falls on this surface, the electrons come out with a maximum velocity of $4 \times 10^6$ m/s. When the photon energy is increased to $5h\nu_0$, then maximum velocity of photoelectrons will be:
(A) $2 \times 10^6$ m/s
(B) $2 \times 10^7$ m/s
(C) $8 \times 10^7$ m/s
(D) $8 \times 10^6$ m/s

Ans. D

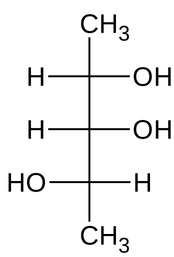
Sol. $K_1 = E_1 - W = 2h\nu_0 - h\nu_0 = h\nu_0$
$K_2 = E_2 - W = 5h\nu_0 - h\nu_0 = 4h\nu_0$

# CHEMISTRY

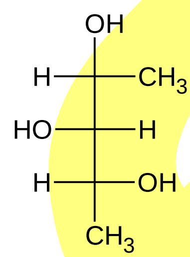
41 The fischer projection formula that represents the following compound is



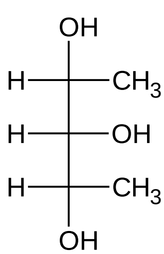
(A)



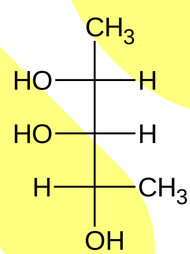
(B)



(C)

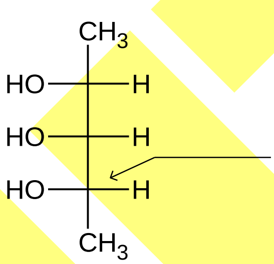


(D)



Ans. D

Sol.




42. Metallic copper dissolves in

(A) dilute HCl
(B) concentrated HCl
(C) aqueous KCN
(D) pure ammonia

Ans. C

Sol. $Cu + KCN + O_2 + H_2O \longrightarrow [Cu(CN)_4]^{3-}$

43. A 50 mL solution of pH = 1 is mixed with a 50 mL solution of pH = 2. The pH of the mixture is

(A) 0.86
(B) 1.26
(C) 1.76
(D) 2.26

Ans. A

Sol. $pH = -\log[H_3O^+]$

44. With respect to halogens, four statements are given below

(I) The bond dissociation energies for halogens are in the order: $I_2 < F_2 < Br_2 < Cl_2$

(II) The only oxidation state is -1

(III) The amount of energy required for the excitation of electrons to first excited state decreases progressively as we move from $F_2$ to $I_2$

(IV) They form $HX_2^-$ species in their aqueous solutions (X = halogen)

The correct statements are

(A) I, II, IV
(B) I, III, IV
(C) II, III, IV
(D) I, III

Ans. D

Sol. $:\ddot{\underset{..}{F}}-\ddot{\underset{..}{F}}:$

L.P – L.P repulsion weakens the F – F bond strength.

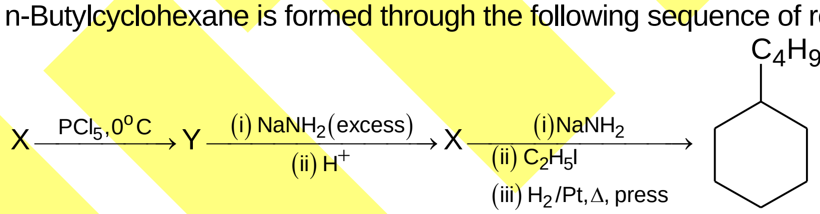
45. The vapor pressure of two pure isomeric liquids X and Y are 200 torr and 100 torr respectively at a given temperature. Assuming a solution of these components to obey Raoult's law, the mole fraction of component X in vapor phase in equilibrium with the solution containing equal amounts of X and Y, at the same temperature, is

(A) 0.33
(B) 0.50
(C) 0.66
(D) 0.80

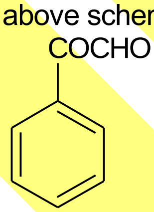
Ans. C

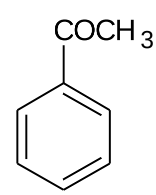
Sol. According to Raoult's law $P_x = P_x^0 \chi_x$.

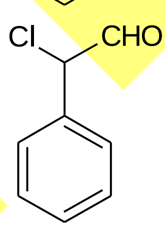
46. n-Butylcyclohexane is formed through the following sequence of reactions.

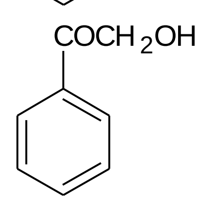



In the above scheme of reactions, "X" is

(A)


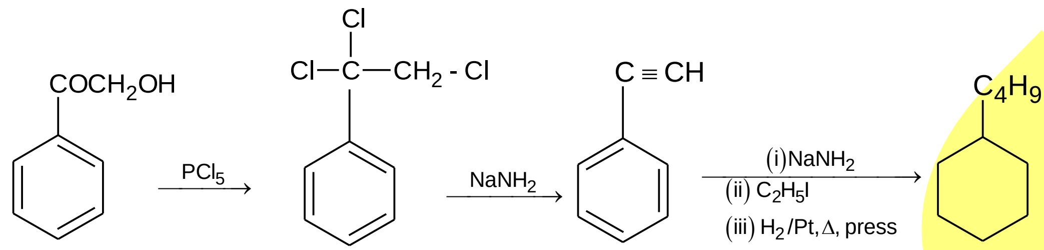

(B)



(C)



(D)



Ans. D

Sol.



47. Four statements for Cr and Mn are given below.
(I) $Cr^{2+}$ and $Mn^{3+}$ have the same electronic configuration.
(II) $Cr^{2+}$ is a reducing agent while $Mn^{3+}$ is an oxidizing agent.
(III) $Cr^{2+}$ is an oxidizing agent while $Mn^{3+}$ is a reducing agent.
(IV) both Cr and Mn are oxidizing agents.
The correct statements are
(A) I, III, IV (B) I, II
(C) I, II, IV (D) I, IV

Ans. B

Sol. $Co^{2+}$ configuration is $t_{2g}^{3}e_{g}^{1}$ hence it acts as a reducing agent.

48. Passing $H_2S$ gas into a mixture of $Mn^{2+}$, $Ni^{2+}$, $Cu^{2+}$, and $Hg^{2+}$ in an acidified aqueous solution precipitates
(A) CuS and HgS (B) MnS and CuS
(C) MnS and NiS (D) NiS and HgS

Ans. A

Sol. Cus and HgS have very low $K_{sp}$.

49. The complex that shows optical activity is
(A) trans-$[CoCl_2(en)_2]^+$ (B) cis-$[CoCl_2(en)_2]^+$
(C) trans-$[PtCl_2(NH_3)_2]$ (D) $[CoCl_2(NH_3)_2(en)]^+$

Ans. B

Sol. Compound has not plane of symmetry

50. The reaction that does not produce nitrogen is
(A) heating $(NH_4)_2Cr_2O_7$ (B) $NH_3$ + excess of $Cl_2$
(C) heating of $NaN_3$ (D) heating of $NH_4NO_3$

Ans. B

Sol. $NH_3 + \text{excess of } Cl_2 \longrightarrow NCl_3 + HCl$

51. The order of pπ-dπ interaction in the compounds containing bond between Si/P/S/Cl and oxygen is in the order
(A) P > Si > Cl > S (B) Si < P < S < Cl
(C) S < Cl < P < Si (D) Si > P > S > Cl

Ans. B

Sol. $\pi$-interaction is stronger between smaller atoms of comparable size.

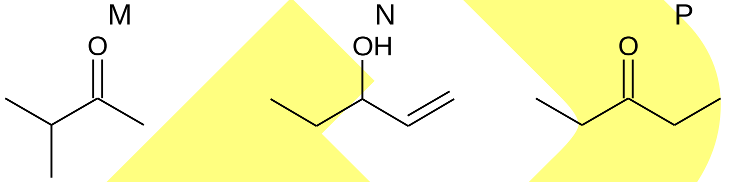
52. The solubility products ($K_{sp}$) of three salts MX, $MY_2$ and $MZ_3$ are $1 \times 10^{-8}$, $4 \times 10^{-9}$ and $27 \times 10^{-8}$, respectively. The correct order for solubilities of these salts is
(A) $MX > MY_2 > MZ_3$
(B) $MZ_3 > MY_2 > MX$
(C) $MZ_3 > MX > MY_2$
(D) $MY_2 > MX > MZ_3$

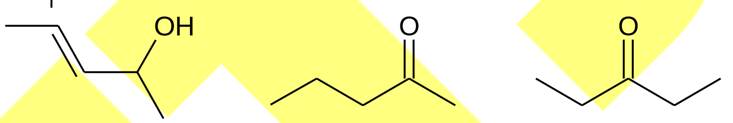
Ans. B

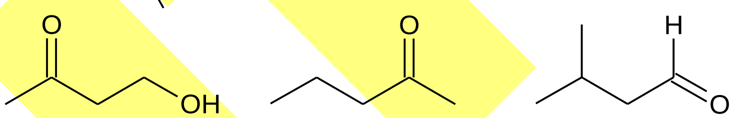
Sol. $K_{sp}$ of a $A_xB_y$ type salt is $K_{sp} = [A^{y+}]^x[B^{x-}]^y$.

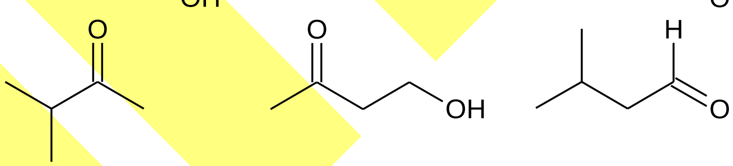
53. Three isomeric compounds M, N, and P ($C_5H_{10}O$) give the following tests:
(i) M and P react with sodium bisulfite to form an adduct
(ii) N consumes 1 mol of bromine and also gives turbidity with conc. HCl/anhydrous $ZnCl_2$ after prolong heating
(iii) M reacts with excess of iodine in alkaline solution to give yellow crystalline compound with a characteristic smell.
(iv) p-Rosaniline treated with sulphur dioxide develops pink colour on shaking with P
The structures of M, N, and P, respectively are

M N P

(A)



(B)



(C)



(D)



Ans. D

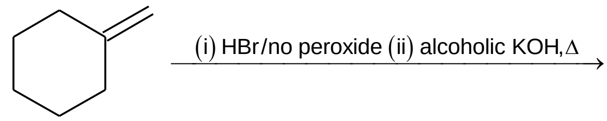
Sol. Carry our required reactions.

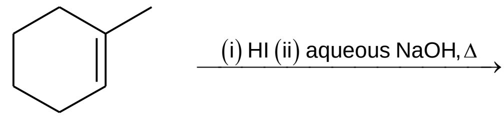
54. Certain combinations of cations and anions lead to the formation of colored salts in solid state even though each of these ions with other counter ions may produce colorless salts. This phenomenon is due to temporary charge transfer between the two ions. Out of the following, the salt that can exhibit this behavior is
(A) $SnCl_2$
(B) $SnCl_4$
(C) $SnBr_2$
(D) $SnI_4$

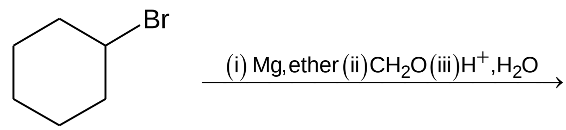
Ans. D

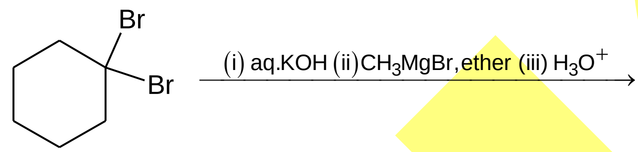
Sol. $SnI_4$ show charge transfer spectra because $I^-$ can easily get oxidized.

55. Four processes are indicated below:

(I) 


(II) 


(III) 

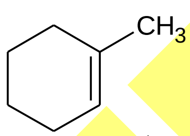

(IV) 

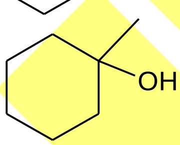

The processes that do not produce 1-methylcyclohexanol are

(A) II, IV
(B) I, II
(C) III, IV
(D) I, III

Ans. D

Sol.

(I) 


(III) 

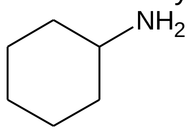

56. In an experiment, it was found that for a gas at constant temperature, PV = C. The value of C depends on

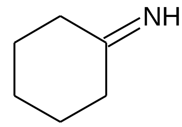
(A) atmospheric pressure
(B) quantity of gas
(C) molecular weight of gas
(D) volume of chamber

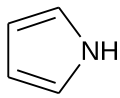
Ans. B

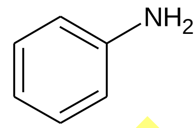
Sol. PV = const(K)
K depends on temp and mass of gas (i.e. number of moles)

57. The order of basicity of the following compounds is

(I) 

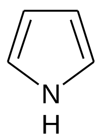
(II) 

(III) 

(IV) 

(A) I > II > IV > III
(B) IV > II > I > II
(C) III > II > I > IV
(D) I > II > III > IV

Ans. A

Sol. In pyrrole  lone pair of electron is permanently involved in resonance.

58. An ideal gas taken in an insulated chamber is released into interstellar space. The statement that is nearly true for this process is
(A) $Q = 0, W \neq 0$
(B) $W = 0, Q \neq 0$
(C) $\Delta U = 0, Q \neq 0$
(D) $Q = W = \Delta U = 0$

Ans. D

Sol. Fact based.

59. For $[FeF_6]^{3-}$ and $[CoF_6]^{3-}$, the statement that is correct is
(A) both are colored
(B) both are colorless
(C) $[FeF_6]^{3-}$ is colored and $[CoF_6]^{3-}$ is colorless
(D) $[FeF_6]^{3-}$ is colorless and $[CoF_6]^{3-}$ is colored

Ans. D

Sol. $[FeF_6]^{3-}$ $t_{2g}^3e_g^2$ CFSE is 'zero' colourless complex, $[CoF_6]^{3-}$ $t_{2g}^4e_g^2$ coloured. CFSE is not zero.

60. Benzene can not be iodinated with $I_2$ directly. How ever in presence of oxidants such as $HNO_3$, iodination is possible. The electrophile formed in this case is
(A) $[\,I^+\,]$
(B) $[\,I^{o}\,]$
(C) $\left[I^{\delta+}....OH_2^{\delta+}\right]^+$
(D) $\left[I^{\delta+}....OH_2^{-\delta}\right]$

Ans. C

Sol. $I_2 + HNO_3 \longrightarrow \left[I^+\right] + NO_2 + HI + H_2O$

## PART – II

## MATHEMATICS

61. Number of all (+)ve integers n that have exactly 16 (+)ve integral divisors $d_1, d_2, ..........., d_{16}$ such that

$1 = d_1 < d_2 < ...........d_{16} = n$

$d_6 = 18$ and $d_9 - d_8 = 17$

(A) 0 (B) 2
(C) 4 (D) 16

Ans. B

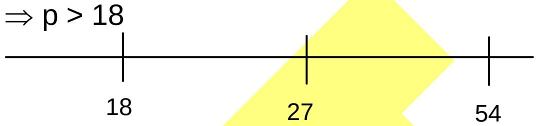
Sol. $d_6 = 18 = 2.3^2$ having divisors 1, 2, 3, 6, 9, 18 which can be taken as $d_1, d_2, d_3, d_4, d_5, d_6$ respectively

Now $16 = 2 \times 8 = 2 \times 2 \times 4 = 2 \times 2 \times 2 \times 2$
$= 4 \times 4$

In which only one case $2 \times 4 \times 2$ is accepted.

$\therefore\ n = 2.3^3.P$, P – Prime

$\Rightarrow$ p > 18




Case I $18 < p < 27$

$d_1 = p,\ d_8 = 27,\ d_9 = 2p \Rightarrow d_9 - d_8 = 17$

$\Rightarrow p = 22$ Not possible

Case II $27 < p < 54$

$d_7 = 27,\ d_8 = p,\ d_9 = 54 \Rightarrow d_9 - d_8 = 17 \Rightarrow p = 37$

$\therefore\ n = 2.3^3.37$

Case III $p > 54$ $d_7 = 27,\ d_8 = 54,\ d_9 = p$

$\therefore$ p = 71

62. Given three cubes with integer edge lengths, if the sum of their surface areas is 564 $cm^2$, then the sum of their volumes is

(A) 764 $cm^3$ or 586 $cm^3$ (B) 764 $cm^3$
(C) 586 $cm^3$ or 564 $cm^3$ (D) 586 $cm^3$

Ans. A

Sol. Denote the edge lengths of the three cubes as a, b and c, respectively. Then we have $6\left(a^2 + b^2 + c^2\right) = 564$, 5 i.e. $a^2 + b^2 + c^2 = 94$. We may assume that $1 \le a \le b \le c < 10$

Then $3c^2 \ge a^2 + b^2 + c^2 = 94$

It follows that $c^2 > 31$. So $6 \le c < 10$, and this means that c can only be 9, 8, 7 or 6.

If c = 9, then $a^2 + b^2 = 94 - 9^2 = 13$.

It is easy to see that a = 2, b = 3. So we get the solution (a, b, c) = (2, 3, 9).
If c = 8, then $a^2 + b^2 = 94 - 8^2 = 30$
This means that $b \geq 4$ and $2b^2 \geq 30$; it follows that $b = 4$ or 5.
So $a^2 = 5$ or 14; in both cases a has no integer solution.
If c = 7, then $a^2 + b^2 = 94 - 7^2 = 45$
It is easy to see that a = 3, b = 6 is the only solution.
If c = 6, then $a^2 + b^2 = 94 - 6^2 = 58$.
So $2b^2 \geq 58$, or $b^2 \geq 29$. This means that $b \geq 6$, but $b \leq c = 6$, so b = 6. Then $a^2 = 22$ and a cannot be an integer.
In summary, there are two solutions: (a, b, c) = (2, 3, 9) and (a, b, c) = (3, 6, 7). Then the possible volumes are $V_1 = 2^3 + 3^3 + 9^3 = 764\ \text{cm}^3$.

$$V_2 = 3^3 + 6^3 + 7^3 = 586\ \text{cm}^3$$

63. Given the line $L: x + y - 9 = 0$ and the circle $M: 2x^2 + 2y^2 - 8x - 8y - 1 = 0$, point A is on L and points B, C are on M; $\angle BAC = 45^\circ$ and the line AB is through the center of M. Then the maximum value of the x coordinate of point A is ____________.

(A) 3
(B) 6
(C) 9
(D) 12

Ans. B

Sol. Suppose that A (a, 9 –a). Then the distance from the centre of M to the line AC is

$d = |AM| \times \sin\angle BAC$

$= \sqrt{(a-2)^2 + (9-a-2)^2} \times \sin 45^\circ$

$= \sqrt{2a^2 - 18a + 53} \times \dfrac{\sqrt{2}}{2}$.

On the other hand, since the line AC intercepts M, it follows that $d \leq$ the radius of $M = \sqrt{\dfrac{17}{2}}$, i.e. $\sqrt{2a^2 - 18a + 53} \times \dfrac{\sqrt{2}}{2} \leq \sqrt{\dfrac{17}{2}}$

The solution is $3 \leq a \leq 6$.

64. Consider the set P defined as $P = \{S = 0$, where 'S' represents any conic with directrix $x + ky + 16 = 0, k \in R\}$ and consider an ellipse $E: 9x^2 + 16y^2 = 144$
If each member of P intersects the ellipse such that common chord is of maximum length. Then the set of values of k if P contains exactly 1 or 2 elements if member of P are parabolas only.

(A) $\left[-\sqrt{15}, \sqrt{15}\right] - \{0\}$
(B) $\left[-\sqrt{15}, \infty\right) - \{0\}$
(C) $\left(-\infty, \sqrt{15}\right]$
(D) $(-5, 5)$

Ans. A

Sol. Case I

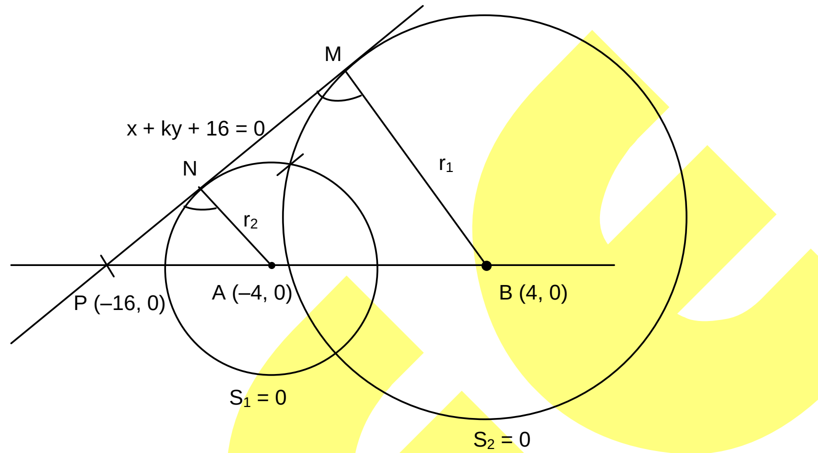
For exactly one parabola

$c_1c_2 = r_1 + r_2$

Case II

For exactly two parabolas

$|r_1 - r_2| < c_1c_2 < r_1 + r_2$




65. The minimum value of $f(x) = 8^x + 8^{-x} - 4(4^x + 4^{-x})\ \forall x \in R$.

(A) –1
(B) –2
(C) –3
(D) 1

Ans. C

Sol. Let $u = 2^x + 2^{-x}$

$4^x + 4^{-x} = 4^u - 2$

$8^x + 8^{-x} = u^3 - 3u$

$\Rightarrow f(x) = u^3 - 3u - 4(u^2 - 2) = u^3 - 4u^2 - 3u + 8$

Let $g(u) = u^3 - 4u^2 - 3u + 8;\ u \geq 2$

$g'(u) = (3u+1)(u-3) \Rightarrow u = 3$

$g''(u) = 6u - 8 \quad \Rightarrow g''(3) > 0 \quad \Rightarrow u = 3$ is point of minimum

$g(3) = 27 - 36 - 9 + 8 = -10$

66. The number of solutions of the equation $\sin^3 x + 1 = 2\sqrt[3]{2\sin x - 1}$ on the interval $\left[0, \frac{\pi}{2}\right]$

(A) 0
(B) 1
(C) 2
(D) 3

Ans. C

Sol. Use If $f(x) = f^{-1}(x) \Rightarrow f(x) = x$

67. Consider a set S = {1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60} and

four natural numbers $\alpha, \beta, \gamma, \delta$. Then total number of (+) ve integral solution of the equation

$\alpha\beta\gamma\delta = N$, where $N \in S$

(A) 325
(B) 375
(C) 126
(D) 125

Ans. B

Sol. Put $\alpha\beta\delta x = 60$, where x is a dummy variable.

68. If $f(x) = ax^2 + bx + c$ be such that $|f(0)| \le 1$, $|f(1)| \le 1$ and $|f(-1)| \le 1$, then for $x \in [-1, \ 1]$, $|f(x)|$ cannot have the value

(A) $\frac{1}{4}$ (B) 1

(C) $\frac{5}{4}$ (D) $\frac{7}{4}$

Ans. D

Sol. Given $f(x) = ax^2 + bx + c$ ...(i)

$\therefore$ $f(0) = c$ ...(ii)

$f(1) = a + b + c$ ...(iii)

and $f(-1) = a - b + c$ ...(iv)

Solving (ii), (iii), (iv), we get

$a = \frac{f(-1) + f(1) - 2f(0)}{2}$,

$b = \frac{f(1) - f(-1)}{2}$, $c = f(0)$

Substituting the values of *a*, *b* and *c* in (i), we have

$$f(x) = \left\{\frac{f(-1) + f(1) - 2f(0)}{2}\right\} x^2 + \left\{\frac{f(1) - f(-1)}{2}\right\} x + f(0)$$

$$\Rightarrow \quad 2f(x) = (x^2 - x) f(-1) + (x^2 + x) f(1) + 2(1 - x^2) f(0)$$

$$\therefore \quad |2f(x)| = |(x^2 - x) \ f(-1) + (x^2 + x) f(1) + 2(1 - x^2) f(0)|$$

$$\le |x^2 - x| \, |f(-1)| + |x^2 + x| \, |f(1)| + 2|(1 - x^2)| \, |f(0)|$$

$$\le |x^2 - x| + |x^2 + x| + 2|1 - x^2|$$

$$= |x| \, |x - 1| + |x| \, |x + 1| + 2|1 - x| \, |1 + x|$$

$$\therefore \ 2|f(x)| \le -|x|(x - 1) + |x|(x + 1) + 2(1 - x^2)$$

$$= 2|x| + 2 - 2x^2$$

or $|f(x)| \le |x| + 1 - x^2$

$$= -(x^2 - |x| - 1) = -(|x|^2 - |x| - 1) = -\left\{\left(|x| - \frac{1}{2}\right)^2 - \frac{1}{4} - 1\right\} = \frac{5}{4} - \left(|x| - \frac{1}{2}\right)^2 \le \frac{5}{4}$$

Hence, $|f(x)| \le \frac{5}{4}$.

69. The sum of the series

$$^nC_1 - \left(1+\frac{1}{2}\right)\ ^nC_2 + \left(1+\frac{1}{2}+\frac{1}{3}\right)\ ^nC_3 - ... + (-1)^{n-1}\left(1+\frac{1}{2}+...+\frac{1}{n}\right)\ ^nC_n$$ is equal to

(A) 0
(B) 1
(C) $\frac{1}{n}$
(D) $\frac{1}{n^2}$

Ans. C

Sol. $$^nC_1 - \left(1+\frac{1}{2}\right)\ ^nC_2 + \left(1+\frac{1}{2}+\frac{1}{3}\right)\ ^nC_3 - ... + (-1)^{n-1}\left(1+\frac{1}{2}+...+\frac{1}{n}\right)\ ^nC_n$$

$$= (C_1 - C_2 + C_3 - ...) - \frac{1}{2}(C_2 - C_3 + C_4 - ...) + \frac{1}{3}(C_3 - C_4 + C_5 - ...) - ...$$

$$= C_0 + \frac{1}{2}(C_0 - C_1) + \frac{1}{3}(C_0 - C_1 + C_2) + ...$$

$$= {}^{n-1}C_0 + \frac{1}{2}\ {}^{n-1}C_1(-1)^1 + \frac{1}{3}\ {}^{n-1}C_2(-1)^2 + \frac{1}{4}\ {}^{n-1}C_3(-1)^3 + ... = \frac{1}{n}$$

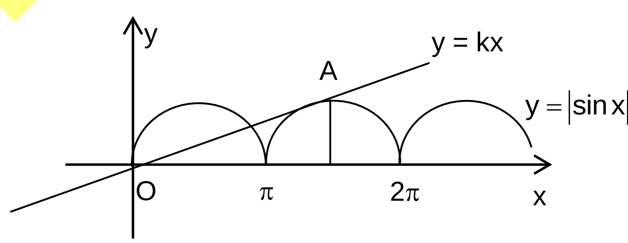
(consider $(1-x)^n \times (1-x)^{-1}$)

70. It is known that the curve $f(x) = |\sin x|$ intercepts the line $y = kx(k > 0)$ at exactly three points, the maximum x coordinate of these points being $\alpha$. Then $\frac{\cos\alpha}{\sin\alpha + \sin 3\alpha} =$

(A) $\frac{2+\alpha^2}{\alpha}$
(B) $\frac{1+\alpha^2}{\alpha}$
(C) $\frac{1+\alpha^2}{4\alpha}$
(D) $\alpha$

Ans. C

Sol. The image of the three intercepting point of $f(x)$ and $y = kx$ is shown in the figure. It is easy to see that the curve and the line are tangent to each other at point $A(\alpha, -\sin\alpha)$, and $\alpha \in \left(\pi, \frac{3\pi}{2}\right)$.




As $f'(x) = -\cos x$ for $x \in \left(\pi, \frac{3\pi}{2}\right)$,

we have $-\cos\alpha = -\frac{\sin\alpha}{\alpha}$, i.e. $\alpha = \tan\alpha$. Then

$$\frac{\cos\alpha}{\sin\alpha + \sin 3\alpha} = \frac{\cos\alpha}{2\sin 2\alpha\cos\alpha} = \frac{1}{4\sin\alpha\cos\alpha}$$

$$= \frac{\cos^2\alpha + \sin^2\alpha}{4\sin\alpha\cos\alpha} = \frac{1+\tan^2\alpha}{4\tan\alpha}$$

$$= \frac{1+\alpha^2}{4\alpha}.$$

# PHYSICS

71. Two blocks A and B of mass 2m and m respectively are connected to a massless spring of spring constant K. If A and B moving on the horizontal frictionless surface with velocity v to right. If A collides with C of mass m elastically and head on, then the maximum compressions of the spring will be

(A) $\sqrt{\frac{3}{2}\frac{m}{k}}V$

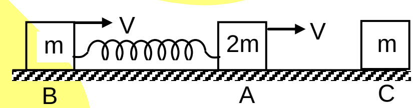
(B) $\sqrt{\frac{27\,m}{8\,K}}V$

(C) $\sqrt{\frac{9\,m}{8\,K}}V$

(D) $\sqrt{\frac{8\,m}{27\,K}}V$

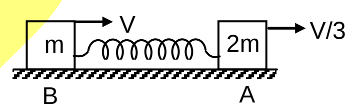
Ans. D

Sol. Velocity of A after the collision with C, $V_1 = \frac{V}{3}$




$\therefore$ Velocity of each block A and B at maximum compression

$V_2 = \frac{5}{9}V$

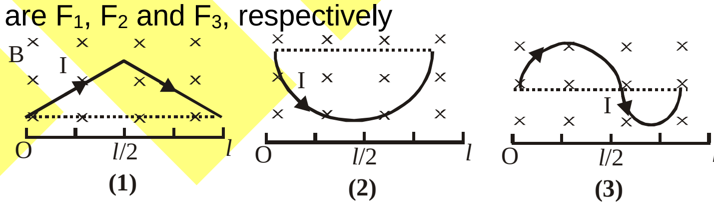



$$\therefore \quad \frac{1}{2}k\,x^2 + \frac{1}{2}3m\left(\frac{5}{9}V\right)^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}2m\left(\frac{V}{3}\right)^2$$

$$x = \sqrt{\frac{8m}{27K}}V$$

72. Three conductors 1, 2, and 3 each carrying the same current I are placed in a uniform magnetic field B, as shown in figure. The forces experienced by conductors 1, 2 and 3 are $F_1$, $F_2$ and $F_3$, respectively




(A) $F_3 > F_2 > F_1$
(B) $F_1 \neq 0$; $F_1 \neq 0$ ; $F_3 = 0$
(C) $F_1$ acts upwards, $F_2$ acts downwards ; $F_3 = 0$
(D) All experience the same force in same direction

Ans. D

Sol. F is same for all.

Use $F = I(\vec{\ell} \times \vec{B})$

73. A uniform solid hemisphere of radius r is joined to uniform solid right circular cone of base of radius r. Both have same density. The centre of mass of the composite solid lies on the common face. The height (h) of the cone is

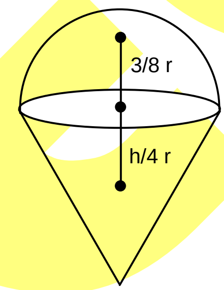
(A) 2r (B) $\sqrt{3}\,r$

(C) 3r (D) $r\sqrt{6}$

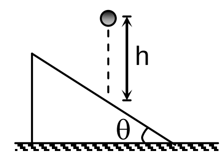
Ans. B

Sol. $\frac{3}{8}r \cdot \rho \frac{2}{3}\pi r^3 = \frac{h}{4} \cdot \rho \frac{1}{3}\pi r^2 h$

$\Rightarrow \quad h = \sqrt{3}\,r$




74. A ball after falling through a distance h collides with an inclined plane of inclination θ as shown. It moves horizontally after the impact. The co-efficient of restitution between inclined plane and ball is (inclined surface is friction less)




(A) 1 (B) $\tan^2\theta$

(C) $\cot^2\theta$ (D) $\sin^2\theta$

Ans. B

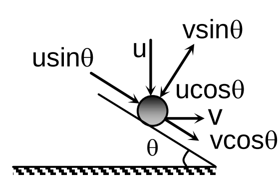
Sol. Impact takes place along the normal to the inclined plane

$\therefore \quad u\sin\theta = v\cos\theta$

$v = u\tan\theta \qquad \ldots(i)$

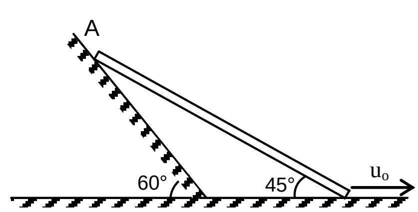
$e = \frac{v\sin\theta}{u\cos\theta} = \frac{u\tan\theta \cdot \sin\theta}{u\cos\theta}$

$e = \tan^2\theta$




75. A rod of length $\ell$ placed between two surfaces is being moved by pulling the end B with speed $u_o$ along horizontal surface as shown.

The angular velocity (ω) of the rod at this instant will be



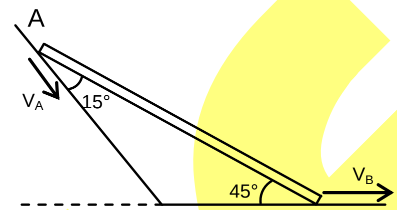

(A) $\frac{u_o}{\ell}$ (B) $\frac{u_o}{\sqrt{2}\,\ell}$

(C) $\frac{\sqrt{2}\,u_o}{\ell}(1 + \tan 15°)$ (D) $\frac{u_o}{\sqrt{2}\,\ell}(1 + \tan 15°)$

Ans. D

Sol. $V_B \cos 45° = V_A \cos (15°)$

$$\omega = \frac{V_B \sin 45^o + V_A \sin(15^o)}{\ell}$$

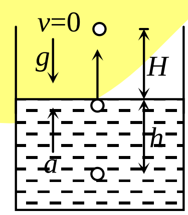



76. A ball of mass m and density ρ is immersed in a liquid of density 3 ρ at a depth h and released. To what height will the ball jump up above the surface of liquid? (neglect the resistance of water and air, radius of ball << h).

(A) h (B) h/2
(C) 2h (D) 3h

Ans. C

Sol. Volume of ball $V = \frac{m}{\rho}$




Acceleration of ball inside the liquid

$$a = \frac{F_{net}}{m} = \frac{\text{upthrust} - \text{weight}}{m}$$

or $$a = \frac{\left(\frac{m}{\rho}\right)(3\rho)(g) - mg}{m} = 2g \text{ (upwards)}$$

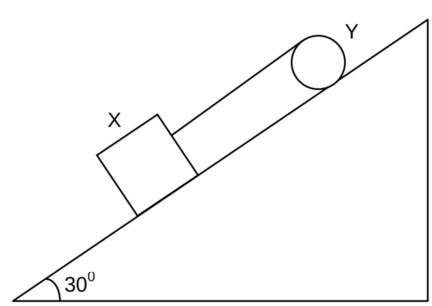
∴ Velocity of ball while reaching at surface

$$v = \sqrt{2ah} = \sqrt{4gh}$$

∴ The ball will jump to a height

$$H = \frac{v^2}{2g} = \frac{4gh}{2g} = 2h$$

77. A block X of mass 0.5 kg is held by a long massless string on a frictionless inclined plane of inclination $30^0$ to the horizontal. The string is wound on a uniform solid cylindrical drum Y of mass 1.5 kg and of radius 0.2 m as shown in figure. The drum is given an initial velocity such that the block X starts moving up the plane. At a certain instant of time when the magnitude of the angular velocity of Y is 10 rad/$s^{-1}$ calculate the distance traveled by X (in m)from the instant of time until it comes to rest ($g = 10$ m/$s^2$)




(A) 1 (B) 2
(C) 3 (D) 4

Ans. A

Sol. $mg\sin\theta - T = ma$

$TR = \frac{1}{2}MR^2\alpha$ , $a = R\alpha$

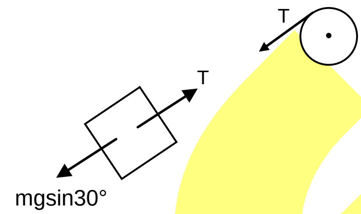
$m = 0.5$ kg ; $\theta = 30°$ ; $M = 1.5$ kg ; $R = 0.2$ m

Solving we get, $a = 2\ m/s^2$

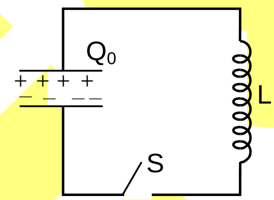
$u = 2$ m/s $(u = R\omega)$

$v^2 = u^2 + 2as$

$\Rightarrow \quad x = 1m.$




78. A capacitor of capacitance $C = \frac{18}{\pi}$ mF having initial charge $Q_0$ connected to an inductor of inductance $L = \frac{18}{\pi}$ mH at $t = 0$. Find the time (in milli second) after energy stored in electric field is three times energy stored in magnetic field.




(A) 1 (B) 2
(C) 3 (D) 4

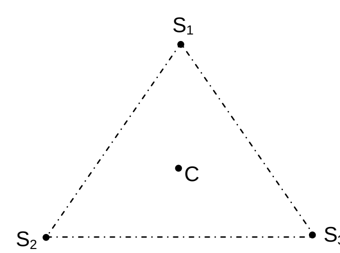
Ans. C

Sol. $U_C + U_L = \frac{Q_0^2}{2C}$

$\frac{4}{3}U_C = \frac{Q_0}{2C} \Rightarrow \frac{4}{3}\frac{Q^2}{2C} = \frac{Q_0^2}{2C}$, $Q = \frac{\sqrt{3}}{2}Q_0$ and $Q = Q_0\cos\omega t$

$t = \frac{\pi}{6\omega} = \frac{\pi}{6}\sqrt{LC} = \frac{\pi}{6}\sqrt{\left(\frac{18}{\pi}\right)\left(\frac{18}{\pi}\right)\times 10^{-6}} = 3$ ms

79. Three identical sources $S_1$, $S_2$ and $S_3$ are placed at the vertices of an equilateral triangle. If they have intensity $I_0$ each at centroid c of triangle. The resulting intensity of sound at c will be

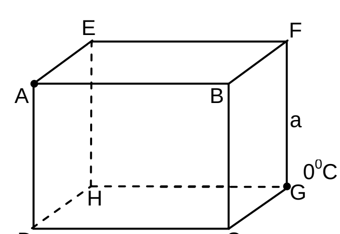



(A) $3I_0$ (B) $6I_0$
(C) zero (D) $9I_0$

Ans. D

Sol. $I_r = \left(\sqrt{I_o} + \sqrt{I_o} + \sqrt{I_o}\right)^2 = 9I_o$

80. A cubical frame is made by connecting 12 identical uniform conducting rods as shown in the figure. In the steady state the temperature of junction A is $100^0$ C while that of the G is $0^0C$. Then,




(A) B will be Hotter than H
(B) Temperature of F is $40^0C$
(C) Temperature of D is $66.67^0C$
(D) Temperature of E is $50^0C$

Ans. B

Sol. $100°C - Ir - \frac{I}{2}r - Ir = 0°C$

$\Rightarrow$ $Ir = 40°C$ ; $\therefore$ $t_F = 0° + Ir = 40°C$

## CHEMISTRY

81. A diatomic molecule has a dipole moment of 1.92 D and bong length of 2.0 Å. What is the percentage ionic character of the molecules?

(A) 33% (B) 20%
(C) 70% (D) 50%

Ans. B

Sol. Calculated $\mu = 4.8 \times 10^{-10} \times 2 \times 10^{-8}$ esu-cm = 9.6 D

$\therefore$ % ionic character $= \frac{\mu_{observed}}{\mu_{calculated}} \times 100 = \frac{1.92 \times 100}{9.0} = 20\%$

82. Which of the following is a strongest base?

(A) $CN^-$ (B) $Br^-$
(C) $OH^-$ (D) $C_6H_5O^-$

Ans. C

Sol. Because conjugate acid of $OH^-$(i.e. $H_2O$) is a weakest acid.

83. Which among the following alkenes will be oxidized by $SeO_2$?

(A) $CH_2 = CH_2$

(B)
```
CH3 - CH - CH = CH2
      |
      CH3
```

(C)
```
       CH3
       |
CH3 -  C - CH = CH2
       |
       CH3
```

(D)
```
      CH3              CH3
      |                |
CH3 - C - CH = CH  -   C - CH3
      |                |
      CH3              CH3
```

Ans. B

Sol. Only (B) has got allylic hydrogen.

84. Which of the following species is most stable?

(A) $p - NO_2C_6H_4 - \overset{\oplus}{C}H_2$ (B) $C_6H_5\overset{\oplus}{C}H_2$

(C) $p - Cl - C_6H_4 - \overset{\oplus}{C}H_2$ (D) $p - H_3CO - C_6H_4 - \overset{\oplus}{C}H_2$

Ans. D

Sol. Greater the electron donating mesomeric effect, greater will be stability.

85. In the following groups

| –OAc | –OMe | $–OSO_2Me$ | $–OSO_2CF_3$ |
|---|---|---|---|
| I | II | III | IV |

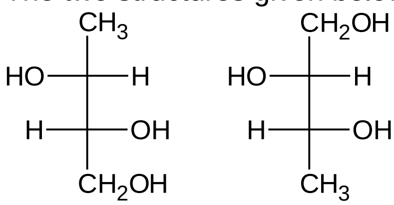
The order of leaving group ability is

(A) I > II > III > IV
(B) IV > III > I > II
(C) III > II > I > IV
(D) II > III > IV > I

Ans. B

Sol. A weaker base is a better leaving group or greater the acidity of the conjugate acid of the base, greater will be the leaving group ability.

86. The two structures given below represent:




(A) pair of diastereomers
(B) pair of enantiomers
(C) same molecule
(D) both are optically inactive

Ans. C

Sol. One is obtained on rotating the other by 180º in the plane of paper.

87. The compound(s) formed upon combustion of sodium metal in excess air is(are)

| $Na_2O_2$ | $Na_2O$ | $NaO_2$ | NaOH |
|---|---|---|---|
| I | II | III | IV |

(A) I only
(B) I & II
(C) II & III
(D) IV only

Ans. B

Sol. $Na + O_2\,(\text{limited}) \xrightarrow{\Delta} Na_2O$

$Na + O_2\,(\text{excess}) \xrightarrow{\Delta} Na_2O_2$

88. Sodium oxide can be obtained by heating

(A) sodium carbonate alone
(B) sodium nitrate alone
(C) Sodium nitrite with sodium
(D) None of these

Ans. C

Sol. $2NaNO_2 + 6Na \longrightarrow 4Na_2O + N_2$

89. A colourless solid(X) on heating evolved $CO_2$ and also gave a white residue, soluble in water. Residue also gave $CO_2$ when treated with dilute acid. X is

(A) $Na_2CO_3$
(B) $CaCO_3$
(C) $NaHCO_3$
(D) $Ca(HCO_3)_2$

Ans. C

Sol. $$2NaHCO_3 \xrightarrow{\Delta} Na_2CO_3 + H_2O + CO_2$$

$$Na_2CO_3 \xrightarrow{\text{Acid}} CO_2$$

90. $$NH_4Cl + (A) \longrightarrow \text{Microcosmic salt}$$

$$\text{Microcosmic salt} \xrightarrow{\text{Heat}} (B) \xrightarrow[\text{MnO}]{\text{Heat}} \underset{\text{Violet bead}}{(C)}$$

(A), (B) and (C) are

(A) $Na_3PO_4$, $NaPO_3$, $Mn_3(PO_4)_2$
(B) $Na_2HPO_4$, $Na_3PO_4$, $Mn_3(PO_4)_2$
(C) $Na_2HPO_4$, $NaPO_3$, $Mn(PO_3)_2$
(D) $Na_2HPO_4$, $NaPO_3$, $NaMnPO_4$

Ans. D

Sol. A = $Na_2HPO_4$, B = $NaPO_3$ & C = $NaMnPO_4$