OLTS-2021-T10-FT-VI-KVPY-CLASS-XII

FULL TEST – VI

PART – I
MATHEMATICS

1. For any real x the expression $2(k-x)\left[x+\sqrt{x^2+k^2}\right]$ can not exceed

(A) k^2 (B) $2k^2$
(C) $3k^2$ (D) none of these

Ans. B

Sol. Let $y=2(k-x)\left\{x+\sqrt{(x^2+k^2)}\right\}$

or $x+\sqrt{(x^2+k^2)}=\dfrac{y}{2(k-x)}$ ....(i)

$$\frac{\left\{x+\sqrt{(x^2+k^2)}\right\}\left\{\sqrt{(x^2+k^2)}-x\right\}}{\sqrt{(x^2+k^2)}-x}=\frac{y}{2(k-x)}$$

or $\dfrac{k^2}{\sqrt{(x^2+k^2)}-x}=\dfrac{y}{2(k-x)}$

$\Rightarrow$ $\sqrt{(x^2+k^2)}-x=\dfrac{2k^2(k-x)^2}{y}$ ....(ii)

Subtracting equation (ii) from equation (i),

$2x=\dfrac{y}{2(k-x)}-\dfrac{2k^2(k-x)}{y}$

$\Rightarrow$ $4(y-k^2)x^2+4k(2k^2-y)x+y^2-4k^4=0$

But x is real, $\therefore$ $D\ge 0$

Then, we get $y^2(y-2k^2)\le 0$

$\therefore$ $y\le 2k^2$ $(\because y^2\ge 0)$

Hence, maximum value of y is $2k^2$

2. If $g(x)=\begin{cases}[f(x)], & x\in\left(0,\dfrac{\pi}{2}\right)\cup\left(\dfrac{\pi}{2},\pi\right)\\ 3\ , & x=\dfrac{\pi}{2}\end{cases}$ where [x] denotes the greatest integer function

and $f(x)=\dfrac{2(\sin x-\sin^n x)+|\sin x-\sin^n x|}{2(\sin x-\sin^n x)-|\sin x-\sin^n x|}$, $n\in R$, then

(A) g(x) is continuous and differentiable at $x = \frac{\pi}{2}$, when $0 < n < 1$

(B) $g(x)$ is continuous and differentiable at $x = \frac{\pi}{2}$, when $n > 1$

(C) $g(x)$ is continuous but not differentiable at $x = \frac{\pi}{2}$, when $0 < n < 1$

(D) $g(x)$ is continuous but not differentiable, at $x = \frac{\pi}{2}$, when $n > 1$

Ans. B

Sol. For $0 < n < 1$, $\sin x < \sin^n x$ and for $n > 1$, $\sin x > \sin^n x$

Now, for $0 < n < 1$,

$$f(x) = \frac{2(\sin x - \sin^n x) - (\sin x - \sin^n x)}{2(\sin x - \sin^n x) + (\sin x - \sin^n x)} = \frac{1}{3} \text{ and for } n > 1,$$

$$f(x) = \frac{2(\sin x - \sin^n x) + (\sin x - \sin^n x)}{2(\sin x - \sin^n x) - (\sin x - \sin^n x)} = 3$$

For $n > 1$, $g(x) = 3$, $x \in (0, \pi)$

$\therefore g(x)$ is continuous and differentiable at: $x = \frac{\pi}{2}$, and for $0 < n < 1$,

$$g(x) = \begin{cases} \left[\frac{1}{3}\right] = 0, & x \in \left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right) \\ 3, & x = \frac{\pi}{2} \end{cases}$$

$\therefore g(x)$ is not continuous at $x = \frac{\pi}{2}$. Hence, $g(x)$ is also not differentiable at $x = \frac{\pi}{2}$

3. Let f(x) is a polynomial of degree 4 such that $f(x) = x$ has no real roots, then minimum possible number of real roots of $f(f(f(x))) + x^2 + x + 2016 = 0$ is

(A) 2 (B) 1

(C) 6 (D) 0

Ans. D

Sol. Let $f(x) - x > 0 \forall x$ .....(i)

$\Rightarrow f(f(x)) - f(x) > 0$ .....(ii)

$\Rightarrow f(f(f(x))) - f(f(x)) > 0$ ....(iii)

From (i), (ii) & (iii)

$f(f(f(x))) - x > 0$

4. Let $a_n = \sum_{k=1}^{n} \frac{1}{k(n+1-k)}$, then for $n \geq 2$

(A) $a_{2016} > a_{2015}$
(B) $a_{2017} < a_{2018}$
(C) $a_{2015} = a_{2016}$
(D) $a_{2017} < a_{2016}$

Ans. D

Sol. We have $a_n = \sum_{k=1}^{n} \left(\frac{1}{k} + \frac{1}{n+1-k}\right)$

$$= \frac{2}{n+1} \sum_{k=1}^{n} \left(\frac{1}{k} + \frac{1}{n+1-k}\right)$$

$$= \frac{2}{n+1} \sum_{k=1}^{n} \frac{1}{k}$$

For $n \geq 2$

$$\frac{1}{2}(a_n - a_{n+1}) = \frac{1}{n+1} \sum_{k=1}^{n} \frac{1}{k} - \frac{1}{n+2} \sum_{k=1}^{n+1} \frac{1}{k}$$

$$= \left(\frac{1}{n+1} - \frac{1}{n+2}\right) \sum_{k=1}^{n} \frac{1}{k} - \frac{1}{(n+1)(n+2)}$$

$$= \frac{1}{(n+1)(n+2)} \sum_{k=2}^{n} \frac{1}{k} > 0$$

$\Rightarrow a_n > a_{n+1}$

5. Let $C_1 : x^2 + y^2 - 2x + 2y + 1 = 0$ and $C_2 : x^2 + y^2 - 2x + 2y = 0$ are two given circles. From a moving point on C_2 tangents are drawn to C_1 at A and B. The locus of orthocenter of ΔPAB is

(A) $x^2 + y^2 - 2x + 2y + 1 = 0$
(B) $x^2 + y^2 - 2x + 2y = 0$
(C) $x^2 + y^2 + 2x - 2y = 0$
(D) $x^2 + y^2 - 2x - 2y + 1 = 0$

Ans. B

Sol. $C_1 : (x-1)^2 + (y+1)^2 = 1$ and $C_2 : (x-1)^2 + (y+1)^2 = \sqrt{2}$

C_2 is director circle of C_1

Since ΔPAB is right angled triangle and $\angle APB = 90^o$. So, orthocentre of ΔPAB is P

6. Compute the value of $E = \cot^2 \frac{x}{4} - 6\sqrt{10}$ where

$4\tan^3 x - 3\tan^2 x + 4\tan x - 3 = 0,\ x \in (0, \pi)$

(A) 9
(B) 19
(C) 18
(D) 12

Ans. B

Sol. Given $4\tan x\left(1+\tan^2 x\right)-3\left(1+\tan^2 x\right)=0$

$\therefore \left(1+\tan^2 x\right)\left(4\tan x-3\right)=0$

$\therefore \tan x=\frac{3}{4}>0$

Hence $x\in\left(0,\frac{\pi}{2}\right)$

$\therefore \sin x=\frac{3}{5}$ and $\cos x=\frac{4}{5}$

$\therefore \tan\frac{x}{2}=\frac{\sin x}{1+\cos x}=\frac{\frac{3}{5}}{1+\frac{4}{5}}=\frac{1}{3}$

$\therefore \sin\frac{x}{2}=\frac{1}{\sqrt{10}}$ and $\cos\frac{x}{2}=\frac{3}{\sqrt{10}}$

$\therefore \tan\frac{x}{4}=\frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}}=\frac{\frac{1}{\sqrt{10}}}{1+\frac{3}{\sqrt{10}}}=\frac{1}{3+\sqrt{10}}$

$\therefore \cot\frac{x}{4}=3+\sqrt{10}$

$\therefore \cot^2\frac{x}{4}=19+6\sqrt{10}\Rightarrow E=19$

7. Thirty two players ranked 1 to 32 are playing in a knockout tournament. Assume that in every match between any two players, the better – ranked player wins, the probability that ranked 1 and ranked 2 players are winner and runner up, respectively, is

(A) $\frac{16}{31}$ (B) $\frac{1}{2}$

(C) $\frac{17}{31}$ (D) None of these

Ans. A

Sol. For ranked 1 and 2 players to be winners and runner up, respectively, they should not be paired with each other in any round. Therefore, the required probability is $\frac{30}{31}\times\frac{14}{15}\times\frac{6}{7}\times\frac{2}{3}=\frac{16}{31}$.

8. If $n\in N>1$, then the sum of real parts of the roots of $z^n=\left(z+1\right)^n$ is equal to

(A) $\frac{n}{2}$ (B) $\frac{n-1}{2}$

(C) $-\frac{n}{2}$ (D) $\frac{1-n}{2}$

Ans. D

Sol. The equation $z^n = (z+1)^n$ will have exactly n – 1 roots.

$|z+1| = |z|$

Therefore, z lies on the right bisector of the segment connecting the points (0, 0) and (–1, 0). Thus, $\text{Re}(z) = -\frac{1}{2}$. Hence, the roots are collinear and have their real parts equal to $-\frac{1}{2}$. Hence, sum of the real parts of the roots is $\left(-\frac{1}{2}\right)(n-1)$.

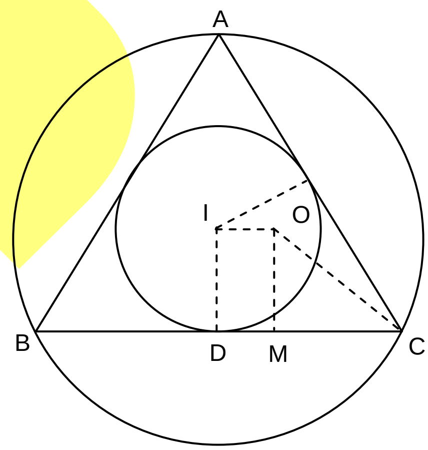
9. In a triangle the line joining the circumcentre to the incentre is parallel to BC, then $\cos B + \cos C$ is equal to

(A) $\frac{3}{2}$ (B) 1

(C) $\frac{3}{4}$ (D) $\frac{1}{2}$

Ans. B

Sol. M is the mid point of BC



$\therefore$ $MC = \frac{a}{2}$

$OC = R$

$OM = ID = r$

$\therefore \Delta OMC$

$R^2 = r^2 + \frac{a^2}{4}$

$R^2 = r^2 + \frac{(2R\sin A)^2}{4}$

$\Rightarrow$ $R^2 \cos^2 A = r^2$

$\Rightarrow$ $\cos A = \frac{r}{R} \Rightarrow r = R\cos A$

$\because$ $\cos A + \cos B + \cos C$

$= 1 + 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

$= 1 + \frac{r}{R} = 1 + \cos A$

$\therefore$ $\cos B + \cos C = 1$

10. 'A' is a set containing n elements. A subset 'P' of 'A' is chosen. The set 'A' is reconstructed by replacing the elements of 'P', A subset 'Q' of 'A' is again chosen. The number of ways of choosing 'P' and 'Q', so that $P \cap Q$ contains exactly two elements is

(A) $9\ {}^{n}C_2$ (B) $3^n - {}^{n}C_2$

(C) $2\ {}^{n}C_n$ (D) none of these

Ans. D

Sol. $A = \{a_1, a_2, a_3, ..., a_n\}$

$(i) a_i \in p,\ a_i \in Q$

$(ii) a_i \notin P,\ a_i \notin Q$

$(iii) a_i \notin P,\ a_i \in Q$

$(iv) a_i \notin P,\ a_i \notin Q$

$P \cap Q$ contains exactly two elements, taking 2 elements in (i) and (n – 2) elements in (ii) or (iii) or (iv)

$\therefore$ Number of ways $= {}^{n}C_2 \times 3^{n-2}$

11. If α, β are non real numbers satisfying $x^3 - 1 = 0$, then the value of $\begin{vmatrix} \lambda+1 & \alpha & \beta \\ \alpha & \lambda+\beta & 1 \\ \beta & 1 & \lambda+\alpha \end{vmatrix}$ is equal to

(A) 0 (B) λ^3

(C) $\lambda^3 + 1$ (D) $\lambda^3 - 1$

Ans. B

Sol. $x^3 - 1 = 0 \therefore x = 1, \omega, \omega^2$

Here, $\alpha = \omega, \beta = \omega^2$

$\begin{vmatrix} \lambda+1 & \omega & \omega^2 \\ \omega & \lambda+\omega^2 & 1 \\ \omega^2 & 1 & \lambda+\omega \end{vmatrix}$ Applying $C_1 \to C_2 + C_2 + C_3$,

then $\begin{vmatrix} \lambda & \omega & \omega^2 \\ \lambda & \lambda+\omega^2 & 1 \\ \lambda & 1 & \lambda+\omega \end{vmatrix}$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, then we get $\begin{vmatrix} \lambda & \omega & \omega^2 \\ 0 & \lambda+\omega^2-\omega & 1-\omega^2 \\ 0 & 1-\omega & \lambda+\omega-\omega^2 \end{vmatrix}$

$= \lambda\{(\lambda + \omega^2 - \omega)(\lambda + \omega - \omega^2) - (1-\omega)(1-\omega^2)\}$

$= \lambda(\lambda^2) = \lambda^3$

12. The number of real solutions of the equation $\left(\frac{9}{10}\right)^x = -3 + x - x^2$ is

(A) none
(B) one
(C) two
(D) more than two

Ans. A

Sol. $\left(\frac{9}{10}\right)^x = -3 + x - x^2$

$\Rightarrow \left(\frac{9}{10}\right)^x = -\left\{\left(x - \frac{1}{2}\right)^2 + \frac{11}{4}\right\}$

$\because$ LHS is always positive and RHS is always negative for all x.
Hence, no solution.

13. The consecutive odd integers whose sum is $45^2 - 21^2$ are
(A) 43, 45,……75
(B) 43, 45,……..79
(C) 43,45,……..85
(D) 43, 45,……..89

Ans. D

Sol. Let n consecutive odd integers are $2m+1, 2m+3, 2m+5, \ldots\ldots, 2m+2n-1$
Given that, $(2m+1)+(2m+3)+(2m+5)+\ldots.+(2m+2n-1) = 45^2 - 21^2$

$\Rightarrow \quad 2mn + (1+3+5\ldots\ldots+2n-1) = 45^2 - 21^2$

$\Rightarrow \quad 2mn + \frac{n}{2}(1+2n-1) = 45^2 - 21^2$

$\Rightarrow \quad 2mn + n^2 = 45^2 - 21^2$

$\Rightarrow \quad (n+m)^2 - m^2 = 45^2 - 21^2$

On comparing $n+m = 45,\ m = 21$

$\therefore \quad n = 24$

$\therefore$ Consecutive odd integers are 43, 45, 47,…….89.

14. If a, b, c are in AP, then the sum of the coefficients of $\left\{1+\left(ax^2-2bx+c\right)^2\right\}^{1973}$ is

(A) –2
(B) –1
(C) 0
(D) 1

Ans. D

Sol. $\because a, b, c$ are in AP

$\Rightarrow \quad 2b = a + c$

$\Rightarrow \quad a - 2b + c = 0$

Putting $x = 1$

Required sum $= (1+a-2b+c)^{1973} = (1+0)^{1973} = 1$

15. A ray of light coming along the line $3x+4y-5=0$ gets reflected from the line $ax+by-1=0$ and goes along the line $5x-12y-10=0$, then

(A) $a=\frac{64}{115}, b=\frac{112}{15}$

(B) $a=-\frac{64}{115}, b=\frac{8}{115}$

(C) $a=\frac{64}{115}, b=-\frac{8}{115}$

(D) $a=-\frac{64}{115}, b=-\frac{8}{115}$

Ans. C

Sol. Equation of bisectors of the given lines are $\left(\frac{3x+4y-5}{\sqrt{3^2+4^2}}\right)=\pm\left(\frac{5x-12y-10}{\sqrt{5^2+(-12)^2}}\right)$

$\therefore (39x+52y-65)=\pm(25x-60y-50)$

or $\frac{14}{15}x+\frac{112}{15}y-1=0$

or $\frac{64}{115}x-\frac{8}{115}y-1=0$

$\therefore$ $a=\frac{14}{15}, b=\frac{112}{15}$ or $a=\frac{64}{115}, b=-\frac{8}{115}$

16. If the tangent at P on $y^2=4ax$ meets the tangent at the vertex in Q and S is the focus of the parabola, then $\angle SQP$ is equal to:

(A) $\frac{\pi}{3}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{2}$

(D) $\frac{2\pi}{3}$

Ans. C

Sol. Let $P=(at^2, 2at)$,

$\therefore$ Tangent at P is, $ty=x+at^2$ .....(i)

tangent at vertex is $x=0$ .....(ii)

Solving equation (i) and (ii), we get $Q=(0, at)$ and $S=(a, 0)$

$\because$ Slope of QP is $\frac{2at-at}{at^2-0}=\frac{1}{t}=m_1$ (say) and slope of QS is $\frac{0-at}{a-0}=-t=m_2$ (say)

$\because$ $m_1m_2=-1$

$\therefore$ $\angle SQP=\frac{\pi}{2}$

17. Values of x and y satisfying the equation $\sin^7 y=|x^3-x^2-9x+9|+|x^3-x^2-4x+4|+\sec^2 2y+\cos^4 y$ are

(A) $x=1, y=n\pi, n\in I$

(B) $x=1, y=2n\pi+\frac{\pi}{2}, n\in I$

(C) $x=1, y=2n\pi, n\in I$

(D) none of the above

Ans. B

Sol. $\sin^7 y = |x^3 - x^2 - 9x + 9| + |x^3 - x^2 - 4x + 4| + \sec^2 2y + \cos^4 y$

for $x = 1$

$\sin^7 y = \sec^2 2y + \cos^4 y$

$\Rightarrow \sin^7 y \cos^2 2y = 1 + \cos^4 y \cos^2 2y$

Since, $\text{LHS} \le 1$ and $\text{RHS} \ge 1$

which is possible only when

$\therefore \quad \sin^7 y \cos^2 2y = 1$

$\Rightarrow \quad \sin^7 y = 1$ and $\cos^2 2y = 1$

$y = \frac{\pi}{2}$

General value of y is $2n\pi + \frac{\pi}{2}$

Hence, $x = 1$ and $y = 2n\pi + \frac{\pi}{2}, n \in I$

18. Let r, s and t be the roots of the equation, $8x^3 + 1001x + 2008 = 0$. The value of $(r+s)^3 + (s+t)^3 + (t+r)^3$ is:

(A) 251
(B) 751
(C) 735
(D) 753

Ans. D

Sol. $8x^3 + 1001x + 2008 = 0$

$r + s + t = 0, \; rst = -\frac{2008}{8} = -251$

So, $(r+s)^3 + (s+t)^3 + (t+r)^3$

$= -(t^3 + s^3 + r^3) = -3rst = -3(-251)$

= 753

As $r + s + t = 0$, so $r^3 + s^3 + t^3 = 3rst$

19. Let A, B, C, D be (not necessarily square) real matrices such that $A^T = BCD : B^T = CDA; C^T = DAB$, and $D^T = ABC$ for the matrix S = ABCD; consider the two statements :

I : $S^3 = S$ II : $S^2 = S^4$

(A) II is true but not I
(B) I is true but not II
(C) both I and II are true
(D) Both I and II are false

Ans. C

Sol. $S = ABCD = A(BCD) = AA^T$ ....(i)

$S^3 = (ABCD)(ABCD)(ABCD)$

$= (ABC)(DAB)(CDA)(BCD)$
$= D^T C^T B^T A^T$
$= (ABCD)^T = (BCD)^T A^T$
$= AA^T$ ....(ii)
From equation (i) and (ii)
$S = S^3$
$S^2 = S^4$

20. If $\vec{x}$ and $\vec{y}$ be unit vectors and $|\vec{z}| = \frac{2}{\sqrt{7}}$ such that $\vec{z} + (\vec{z} \times \vec{x}) = \vec{y}$ and θ is the angle between $\vec{x}$ and $\vec{z}$, then the value of $\sin\theta$ is:

(A) $\frac{1}{2}$
(B) 1
(C) $\frac{\sqrt{3}}{2}$
(D) $\frac{\sqrt{3}-1}{2\sqrt{2}}$

Ans. C

Sol. $|\vec{z}| = \frac{2}{\sqrt{7}}; \vec{z} + (\vec{z} \times \vec{x}) = \vec{y}$

$\vec{y}.\vec{y} = (\vec{z} + (\vec{z} \times \vec{x})).(\vec{z} + (\vec{z} \times \vec{x}))$

$= |\vec{z}|^2 + |\vec{z} + \vec{x}|^2 + 2\vec{z}.(\vec{z} \times \vec{x})$

$1 = \frac{4}{7} + |\vec{z}|^2 |\vec{x}|^2 \sin^2\theta$

$1 = \frac{4}{7} + \frac{4}{7}\sin^2\theta; \sin^2\theta = \frac{3}{4}$

$\sin\theta = \frac{\sqrt{3}}{2}; \theta = \frac{\pi}{3}$

PHYSICS

21. The mass of a block is 87.2 g and its volume is 25 cm^3. Its density upto correct significant figures is

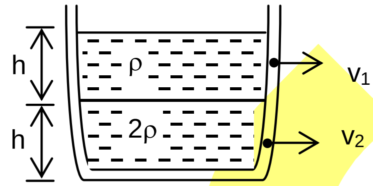
(A) 3.55 g/cc
(B) 3.5 g/cc
(C) 3.65 g/cc
(D) 3.7 g/cc

Ans. B

Sol. Density = $\frac{m}{V} = \frac{87.2}{25}$ = 3.488

= 3.5 g/cc as per significant figures rule for multiplication / division.

22. Equal volumes of two immiscible liquids of densities ρ and 2ρ are filled in a vessel as shown in figure. Two small holes are punched at depth h/2 and 3h/2 from the upper surface of lighter liquid. If v_1 and v_2 are the velocities of efflux at these two holes, then v_1/v_2 is



(A) $\frac{1}{2\sqrt{2}}$
(B) $\frac{1}{2}$
(C) $\frac{1}{4}$
(D) $\frac{1}{\sqrt{2}}$

Ans. D

Sol. $V_1 = \sqrt{2g\left(\frac{h}{2}\right)} = \sqrt{gh}$

$V_2 = \sqrt{2gh_{eff}} = \sqrt{2gh}$

$\frac{V_1}{V_2} = \frac{1}{\sqrt{2}}$

23. A thin prism P_1 with angle 4^o and made from glass ($\mu = 1.54$) is combined with another prism P_2 made of another glass of $\mu = 1.72$ to produce dispersion without deviation. The angle of prism P_2 is

(A) 3.3°
(B) 4°
(C) 3°
(D) 2.6°

Ans. C

Sol. For no deviation

$(\mu_1 - 1)A_1 = (\mu_2 - 1)A_2$

$4^o(1.54 - 1) = (1.72 - 1)A_2$

$A_2 = \frac{4 \times 0.54}{0.72} = 3°.$

24. If ν_1, ν_2 and ν_3 are the fundamental frequencies of three segments of a stretched string of sonometer wire, then the fundamental frequency of the string (when whole string is used for vibration) is

(A) $\nu_1 + \nu_2 + \nu_3$
(B) $\left[\frac{1}{\nu_1} + \frac{1}{\nu_2} + \frac{1}{\nu_3}\right]^{-1}$
(C) $\nu_1\nu_2\nu_3$
(D) $[\nu_1\nu_2\nu_3]^{1/3}$

Ans. B

Sol. For string fixed at both ends:

$f = \frac{V}{2\ell}$ (fundamental mode)

$\Rightarrow \quad f \propto \frac{1}{\ell}$

$\ell = \ell_1 + \ell_2 + \ell_3$

$\Rightarrow \quad \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$

25. Imagine a Young's double slit interference experiment performed with wave associated with fast moving electrons produced from an electron gun. The distance between successive maxima will decrease maximum if

(A) the accelerating potential in the electron gun is decreased.

(B) the accelerating potential is increased and the distance of screen from slit is decreased.

(C) the distance of the screen fro the slit is increased.

(D) the distance between the slits is decreased.

Ans. B

Sol. $\Delta x = \frac{\lambda D}{d} = \left(\frac{h}{mV}\right)\left(\frac{D}{d}\right)$, upon increasing ΔV, V increases.

Since, $e\Delta V = \frac{1}{2}mV^2$

26. A bulb of 100 W is connected in parallel with an ideal inductance of 1 H. This arrangement is connected to a 90 V D.C. battery through a switch. On Pressing the switch, the

(A) bulb does not glow

(B) bulb glows

(C) bulb glows after a short time and then continues to glow

(D) bulb glows for a short time and then stops glowing

Ans. D

Sol. Initially, inductor will withdraw nearly zero current hence resulting in large current through bulb. At steady state, inducter will withdraw whole curretn making current throgh bulb zero.

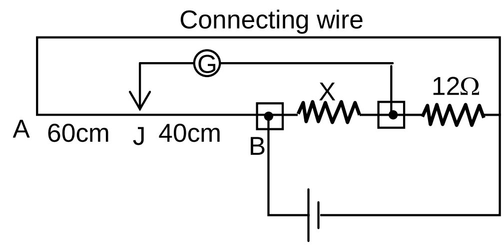
27. If reading of galvanometer is 0 then value of unknown resistance x is

(A) 8 Ω

(B) 18 Ω

(C) 200 Ω

(D) 12 Ω



Ans. A

Sol. At balance:

$$\frac{40}{x} = \frac{60}{12}$$

$\Rightarrow x = 8\Omega$

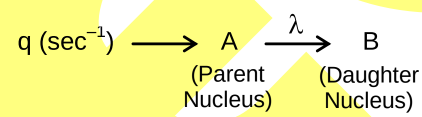
28. Hot wire instruments (ammeter or voltmeter) are used for measuring:

(A) dc only
(B) ac only
(C) Both ac and dc
(D) Neither ac nor dc

Ans. C

Sol. Hot wire instruments are based on heating effect of current.

29. In a radioactive reaction an unstable nucleus A dis-integrates into a stable nucleus B. But A is generated at a constant rate of q nucleus per second. Then at steady state number of nucleus of A will be



(A) $q\lambda$
(B) $\frac{q}{\lambda}$
(C) $q - \lambda$
(D) $\frac{2q}{\lambda}$

Ans. B

Sol. At steady state.
Rate of generation of A = Rate of decay of A.
$q = \lambda N_A$

$\Rightarrow N_A = \frac{q}{\lambda}$

30. A body of mass m is suspended vertically from rubber cord with force constant K. The maximum distance over which the body can be pulled down for the body's oscillation to remain harmonic is

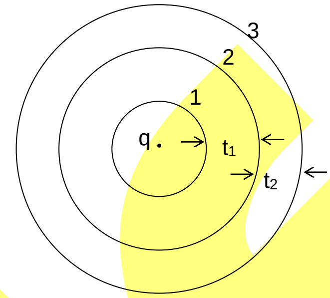
(A) $\frac{2mg}{K}$
(B) $\frac{mg}{K}$
(C) $\frac{4mg}{K}$
(D) $\frac{mg}{2K}$

Ans. B

Sol. Extension in cord due to mass = $\frac{mg}{k}$. Since rubber cord cannot compress (it slacks). Hence, block should not move up beyond a point which corresponds to a compression in the cord.

$\Rightarrow$ Answer = $\frac{mg}{K}$

31. Figure show three spherical equipotential surface 1,2 and 3 round a point charge q. The potential difference $V_1 - V_2 = V_2 - V_3$. If t_1 and t_2 be the distance between them. Them



(A) $t_1 = t_2$ (B) $t_1 > t_2$
(C) $t_1 < t_2$ (D) $t_1 \le t_2$

Ans. C

Sol. $\because$ E is continuously decreasing along radially outward direction.

$\therefore \quad \frac{V_1 - V_2}{t_1} > \frac{V_2 - V_3}{t_2}$; $\quad \therefore \quad t_1 < t_2$

32. A quantity X is given by $\varepsilon_o L \frac{\Delta V}{\Delta t}$ where ε_o is the permittivity of free space, L is a length, ΔV is a potential difference and Δt is a time interval. The dimensional formula (i.e. unit) of X is the same as that of

(A) resistance (B) charge
(C) voltage (D) current

Ans. D

Sol. Units and dimensions.

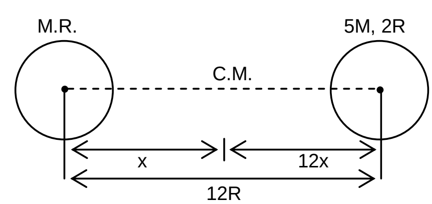
33. Two spherical bodies of masses M and 5M and radii R and 2R respectively are released in free space with initial separation between their centres equal to 12R. If they attract each other due to gravitational force only, then the distance covered by smaller body just before collision is

(A) 8 R (B) 9 R
(C) 10 R (D) 7.5 R

Ans. D

Sol. Initially

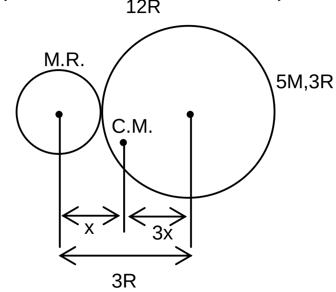
$Mx = 5M(12 - x) \Rightarrow x = 10R$



Finally

$Mx = 5M(3R - x)$

$x = 2.5R$



Distance travelled by smaller block = 10R – 2.5R
= 7.5 R

34. The pans of a physical balance (one pan on each side) are in equilibrium. Air is blown below the right hand pan. Then the right hand pan will

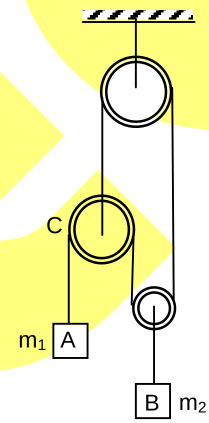
(A) move down (B) move up
(C) move sideways (D) no effect.

Ans. A

Sol. Blowing air below righ pan will reduce pressure below pan. (Bernaulli's equaiton) equilibrium will get disturbed and this pan will move down.

35. In the arrangement shown in figure neglect the masses of the pulley and string and also friction. The accelerations of blocks A and B are

(A) g, g/2 (B) g/2, g
(C) 3g/2, 3g/4 (D) g, g



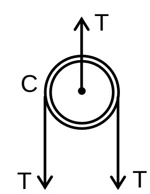
Ans. D

Sol. Since tension in ideal string is uniform, hence for pulley C, we will have:

$2T - T = 0$

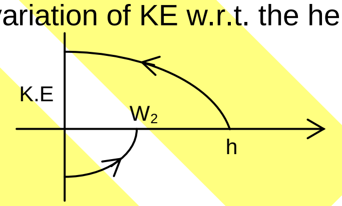
$\Rightarrow \quad T = 0$

In such condition, both blocks fall with acceleration 'g'.

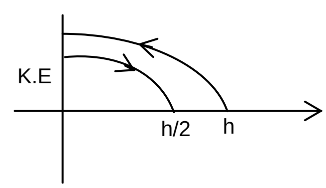


36. A ball is dropped from a height h on the ground and rebounds to a height h/2. What is the variation of KE w.r.t. the height?

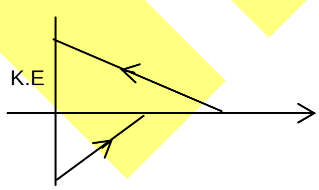
(A)



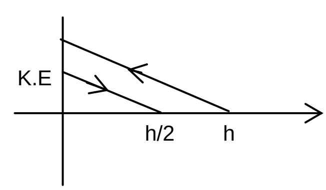
(B)



(C)



(D)



Ans. D

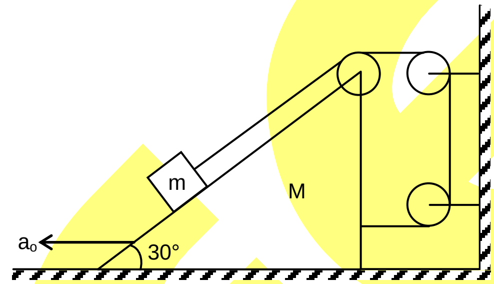
Sol. C.O.E.

$$mgh_o = mgh + \frac{1}{2}mV^2$$

$$K.E. = \frac{1}{2}mV^2 = mgh_o - mgh$$

$\Rightarrow$ Straight line graph. Also, kinetic energy will always be positive.

37. As shown in the figure, the wedge is being pulled towards left with a acceleration "a_o". In this condition acceleration of block of mass "m" will be



(A) $2a_o$

(B) $\sqrt{3}a_o$

(C) $\left(\sqrt{5-2\sqrt{3}}\right)a_o$

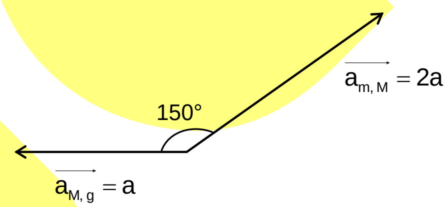
(D) $5a_o$

Ans. C

Sol. $\overrightarrow{a_{m,g}} = \overrightarrow{a_{m,M}} + \overrightarrow{a_{M,g}}$

$a_{m,g} = \sqrt{a^2 + (2a)^2 + 2(a)(2a)\cos(150^o)}$

$= \left(\sqrt{5-2\sqrt{3}}\right)a_o$



38. The power radiated by a black body is P, and it radiates maximum energy around the wavelength λ_0. If the temperature of black body is now changed so that it radiates maximum energy around a wavelength $\frac{3\lambda_0}{4}$, the new power radiated by it will be

(A) $\frac{4}{3}P$ (B) $\frac{16}{9}P$

(C) $\frac{64}{27}P$ (D) $\frac{256}{81}.P$

Ans. D

Sol. Wein's displacement law:

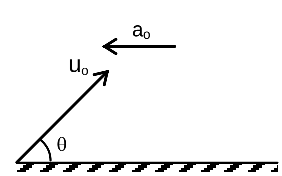
$\lambda_m = \frac{b}{T}$

Stefan's law:

$U = e\sigma AT^4$

39. From a point on ground, a projective is thrown with velocity u_o at an angle "θ" with horizontal ground.

It is given that due to air drag, a constant acceleration a_o is imparted on projectile, if projectile follows a straight line path, then



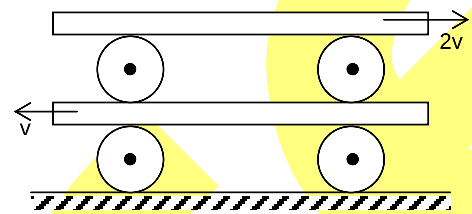
(A) $a_o = g\sin\theta$ (B) $a_o = g\cos\theta$

(C) $a_o = g\cot\theta$ (D) $a_o = g\tan\theta$

Ans. C

Sol. For projectile to follow straight line, net acceleration $(\vec{a}+\vec{g})$ must be along line of projection.

40. All cylinders are identical and no slipping at any contact. The ratio of angular speeds of upper cylinders to lower cylinders is



(A) 1/3
(B) 3
(C) 1
(D) none

Ans. B

Sol. Upper cylinders:

$$\omega = \frac{2V+V}{2R} = \frac{3V}{2R}$$

Lower cylinders:

$$\omega = \frac{V+0}{2R} = \frac{V}{2R}$$

CHEMISTRY

41. Which of the following complex has strongest M – C bond (metal-carbon bond)?
(A) $[Cr(CO)_6]$
(B) $[Fe(CO)_5]$
(C) $[Fe(CO)_6]^{2+}$
(D) $[Mn(CO)_6]^+$

Ans. B

Sol. Metal in lower oxidation state forms strong back bond.

42. Among the following complexes
(i) $[Ru(bipyridyl)_3]^+$
(ii) $[Cr(EDTA)]^-$
(iii) trans-$[CrCl_2(ox)_2]^{3-}$
(iv) cis-$[CrCl_2(ox)_2]^{3-}$
The ones that show chirality are
(A) i, ii, iv
(B) i, ii, iii
(C) ii, iii, iv
(D) i, iii, iv

Ans. A

Sol. They do not have symmetry elements.

43. For a homonuclear diatomic molecule, the bonding molecular orbital is
(A) σ_u of lowest energy
(B) σ_u of second lowest energy
(C) π_g of lowest energy
(D) π_u of lowest energy

Ans. D

Sol. Bonding 'σ' orbitals are gerade, bonding 'π' orbitals are gerade.

44. The ionisation potential of 'H' atom is 13.6 eV. The first ionisation potential of a sodium atom, assuming that the energy of its outer electron can be represented by a H-atom like model. (with shielding provided by core electron is 9.16)

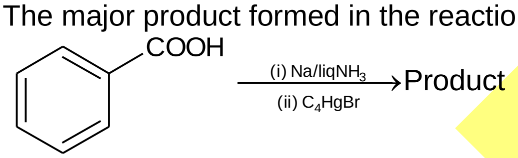
(A) 46.0 eV (B) 11.5 eV
(C) 5.1 eV (D) 2.9 eV

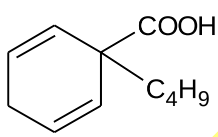
Ans. D

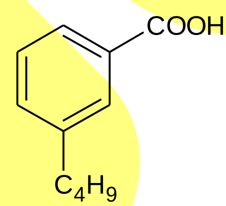
Sol. $E_i = 13.6\frac{Z_{eff}^2}{n^2}$

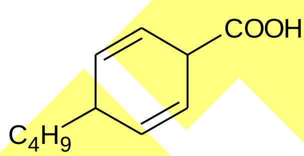
$Z_{eff} = 11 - 9.16$

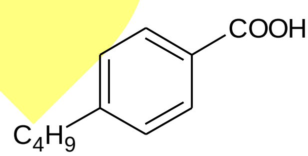
45. The major product formed in the reaction is



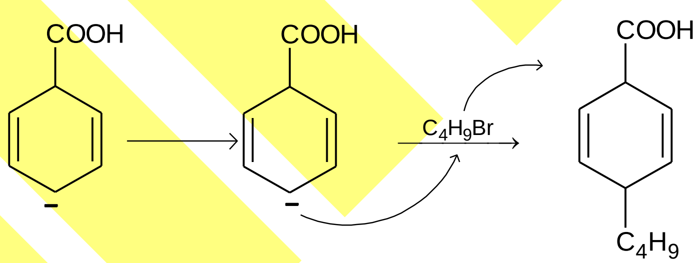
(A) 

(B) 

(C) 

(D) 

Ans. C

Sol. 

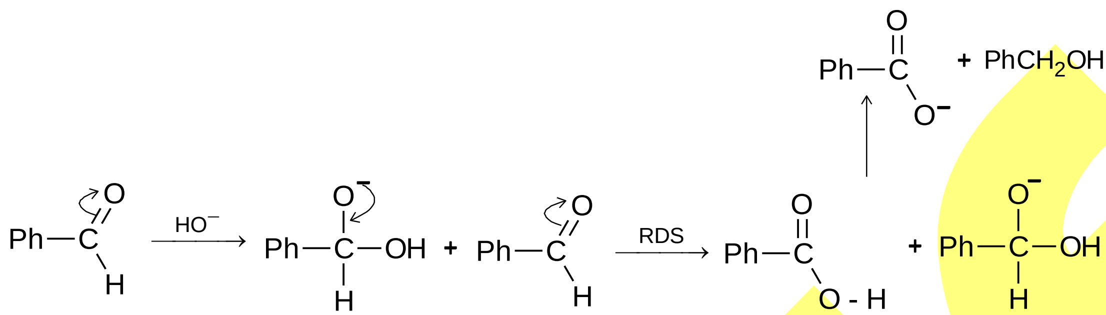
46. In the Cannizaro's reaction given below

$2Ph - CHO \xrightarrow{OH^-} PhCH_2OH + PhCOO^-$, the slowest step is

(A) the attack of OH^- at the carboxyl group
(B) the transfer of hydride to the carbonyl group
(C) the abstraction of proton from the carboxylic acid
(D) the deprotenation of Ph – CH_2OH

Ans. B

Sol.



47. The increasing order of the rate of HCN addition to compound is

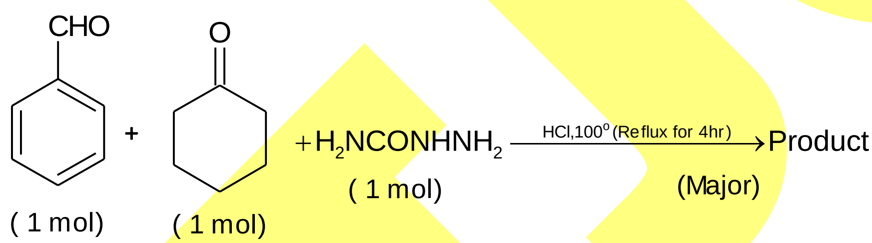
(i) HCHO
(ii) CH_3COCH_3
(iii) $PhCOCH_3$
(iv) PhCOPh

(A) (i) < (ii) < (iii) < (iv)
(B) (iv) < (ii) < (iii) < (i)
(C) (iv) < (iii) < (ii) < (i)
(D) (iii) < (iv) < (ii) < (i)

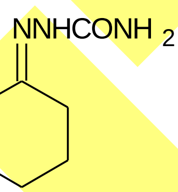
Ans. C

Sol. Less hindered carbonyl compounds are more reactive towards nucleophilic addition.

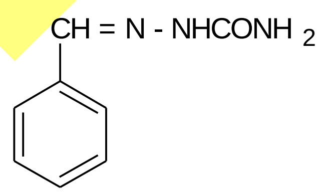
48.



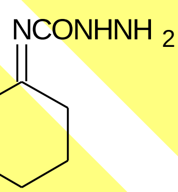
(A)



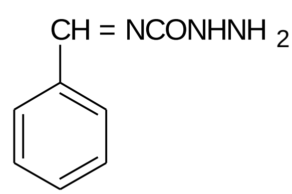
(B)



(C)



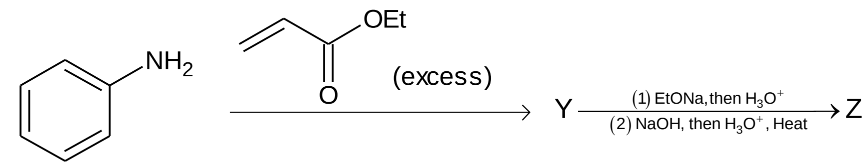
(D)



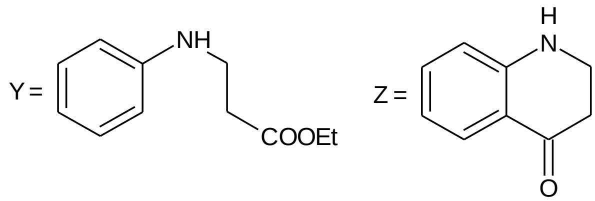
Ans. B

Sol. Reaction is thermodynamic control, stable product will be formed.

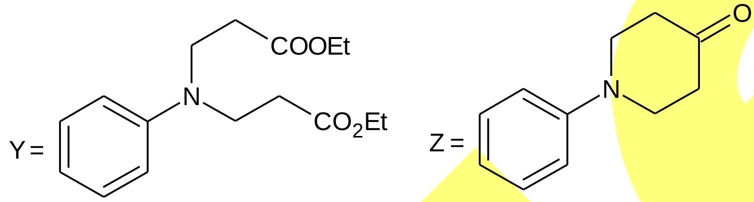
49.



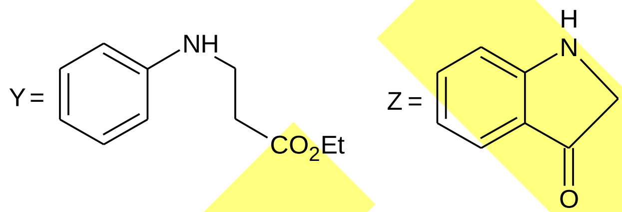
(A)



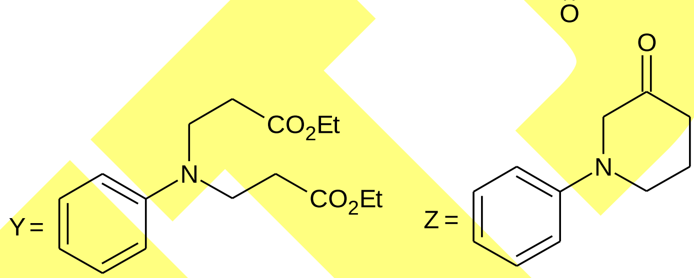
(B)



(C)

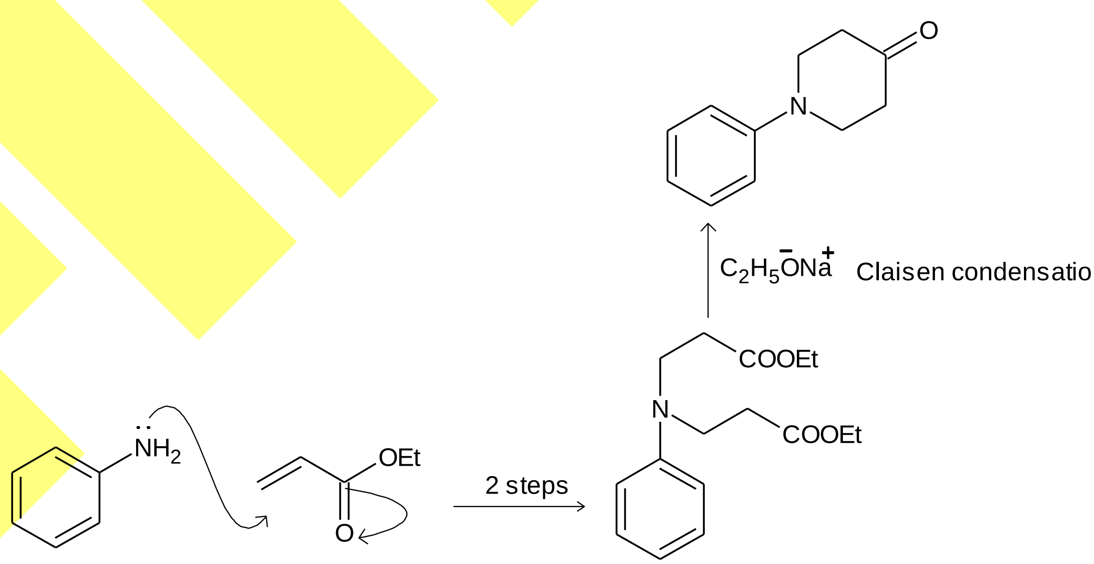


(D)



Ans. B

Sol.



50. The average value of C – C bond order in graphite is

(A) 1 (B) $\frac{3}{2}$

(C) $\frac{3}{4}$ (D) $\frac{4}{3}$

Ans. D

Sol. Fact based.

51. Which of the following does not contain acidic hydrogen?

(A) NaH_2PO_4 (B) NaH_2PO_3

(C) Na_2HPO_4 (D) Na_2HPO_3

Ans. D

Sol. The proton of Na_2HPO_3 is not acidic because H_3PO_3 is a dibasic acid.

52. Which of the following is least soluble in water?

(A) $Mg(OH)_2$ (B) $Ca(OH)_2$

(C) $Sr(OH)_2$ (D) $Ba(OH)_2$

Ans. A

Sol. The solubility of hydroxides of gr-2 elements increases on moving down the group.

53. Which of the following gases is evolved when calcium nitride is dissolved in water?

(A) NO_2 (B) NH_3

(C) NO (D) N_2

Ans. B

Sol. $Ca_3N_2 + 6H_2O \longrightarrow 3Ca(OH)_2 + 2NH_3$

54. A radioactive isotope having a half life of 3 days was received after 12 days. It was found that there were 3 gm of the isotope in the container. The initial weight of the isotope when it was packed: (antilog 1.203 = 16)

(A) 12 gm (B) 24 gm

(C) 36 gm (D) 48 gm

Ans. D

Sol. $\frac{0.693}{3} = \frac{2.303}{12}\log\left(\frac{A_0}{3}\right) \quad \Rightarrow \frac{A_0}{3}$ = antilog (1.203)

$A_0 = 16 \times 3 = 48$ gm

55. The ratio of the difference in energy between the first and second Bohr orbit to that between the second and third Bohr orbit is:

(A) $\frac{1}{2}$
(B) $\frac{1}{2}$
(C) $\frac{4}{9}$
(D) $\frac{27}{5}$

Ans. D

Sol. $\frac{E_2 - E_1}{E_3 - E_2} = \frac{\frac{3}{4}E_1}{\frac{E_1}{9} - \frac{E_1}{4}} = \frac{3}{4} \times \frac{36}{5} = \frac{27}{5}$

56. Proteins are polymers of

(A) β-keto acids
(B) α-amino acids
(C) β-keto esters
(D) α-chloro esters

Ans. B

Sol. Proteins contain $R-\underset{}{CH}(NH_2)-COOH$ unit.

57. Glucose is added to 1L water to such an extent that $\Delta T_f / k_f$ becomes equal to 10^{-3}, the wt. of glucose ($C_6H_{12}O_6$) added is:

(A) 180gm
(B) 18 gm
(C) 1.8 gm
(D) 0.18 gm

Ans. D

Sol. $\Delta T_f = K_f . m$

$\frac{\Delta T_f}{K_f} = \frac{w}{180} \times \frac{1000}{1000} = 10^{-3} = w = 0.18$ gm

58. For the reaction $C(s) + CO_2(g) \rightleftharpoons 2CO(g)$, the partial pressure of CO_2 and CO are 4 and 8 atm. respectively. K_p for the reaction is:

(A) 16
(B) 2
(C) 0.5
(D) 4

Ans. A

Sol. $K_P = \frac{(p_{CO})^2}{p_{CO_2}} = \frac{8 \times 8}{4} = 16$

59. When equal volumes of the following solution are mixed, precipitation of AgCl ($K_{sp} = 2.8 \times 10^{-10}$) will occur only with:

(A) 10^{-4} M(Ag^+) and 10^{-4} M (Cl^-)
(B) 10^{-4} M(Ag^+) and 10^{-5} M (Cl^-)
(C) 10^{-5} M(Ag^+) and 10^{-5} M (Cl^-)
(D) in all cases

Ans. A

Sol. When mixing of equal volume

$[Ag^+] = \frac{1}{2} \times 10^{-4}$ and $[Cl^-] = \frac{1}{2} \times 10^{-4}$

$[Ag^+]\,[Cl^-] = \frac{1}{4} \times 10^{-8} = 2.5 \times 10^{-9} > K_{SP}$

60. Which of the following two are isostructural?

(A) XeF_2, IF_2^-
(B) NH_3, BF_3
(C) CO_3^{2-}, SO_3^{2-}
(D) PCl_5, ICl_5

Ans. A

Sol. XeF_2, IF_2^- both are linear.

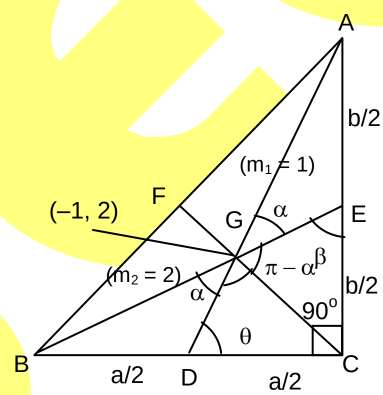
PART – II

MATHEMATICS

61. The triangle ABC, right angled at C, has median AD, BE and CF, AD lies along the line $y = x + 3$, BE lies along the line $y = 2x + 4$. If the length of the hypotenuse is 60, then the area of the triangle ABC (in sq. units)

(A) 225 (B) 400
(C) 250 (D) 200

Ans. B



Sol. Area $= \frac{1}{2}ab$

$AD: y = x + 3$

$BE: y = 2x + 4$

solve $G(-1, 2)$

acute angle α between the median is $\tan\alpha = \frac{m_1 - m_2}{1 + m_1 m_2}$

$\tan\alpha = \frac{2-1}{1+2} \Rightarrow \tan\alpha = \frac{1}{3}$

now $(180 - \alpha) + 90^o + \theta + \beta = 360^o$

$\Rightarrow \quad \alpha = \theta + \beta - 90^o$

$\cot\alpha = -\tan(\theta + \beta)$

$-3 = \frac{\tan\theta + \tan\beta}{1 - \tan\theta\tan\beta}$ or $-3 = \frac{\frac{2b}{a} + \frac{2a}{b}}{1 - \frac{2b}{a}.\frac{2a}{b}} \Rightarrow 9 = \frac{2(a^2 + b^2)}{ab}$

$9ab = 2 \times 360 \qquad \Rightarrow \frac{1}{2}ab = 400$

$\therefore$ Area = 400 sq. units

62. A series of ellipses $E_1, E_2, ..., E_n$ are drawn such that E_n touches E_{n-1} at the extremities of the major axis of E_{n-1} and the foci of E_n coincide with the extremities of the minor axis of E_{n-1}. If the eccentricity of the ellipses is independent of n, then the value of eccentricity is

(A) $\frac{\sqrt{5}}{3}$ (B) $\frac{\sqrt{5}+1}{4}$
(C) $\frac{\sqrt{5}-1}{2}$ (D) $\frac{\sqrt{5}-1}{3}$

Ans. C

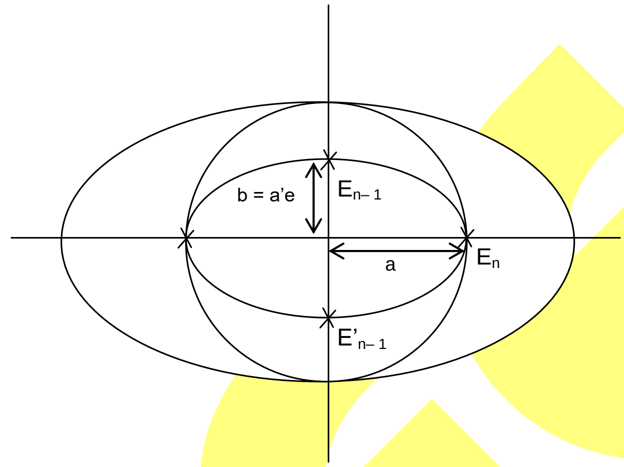
Sol. $b' = a,\ a'e = b$

$b'^2 = a'^2 - a'^2 e^2$

$a^2 = \frac{b^2}{e^2} - b^2$

$e^2 = \frac{b^2}{a^2}\left(1-e^2\right) = \left(1-e^2\right)^2$

$1 - e^2 = e$



63. The arithmetic mean of a numbers of pair wise distinct primes is 27. The biggest prime among them is

(A) 31
(B) 129
(C) 139
(D) 97

Ans. C

Sol. Let the primes be $p_1 < p_2 < .... < p_n$. We have

$\frac{p_1 + p_2 + ... + p_n}{n} = 27 \Rightarrow p_1 + p_2 + ... + p_n = 27n$

The primes less than 27 are 2, 3, 5, 7, 11, 13, 17, 19, 23.

$p_n = 27n - \left(p_1 + p_2 + ... + p_{n-1}\right)$

$= 27 + \left(27 - p_1\right) + \left(27 - p_2\right) + ... + \left(27 - 23\right)$

$\leq 27 + \left(27 - 2\right) + \left(27 - 3\right) + ... + \left(27 - 23\right)$

= 145

Largest prime less than 145 is 139, we have $p_n = 139$

Also, A.M. of 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31 and 139 is 27. Thus the largest prime is 139.

64. A monic quadratic trinomial P(x) is such that $P(x) = 0$ and $P\left(P\left(P(x)\right)\right) = 0$ have a common root, then (monic polynomial has its leading coefficient equal to 1)

(A) $P(0).P(1) > 0$
(B) $P(0).P(1) < 0$
(C) $P(0).P(1) = 0$
(D) none

Ans. C

Sol. Let $x = \alpha$ be root of both $P(x) = 0$ as well as of $P\left(P\left(P(x)\right)\right) = 0$

$\Rightarrow P(\alpha) = 0$

$\Rightarrow P\left(P\left(P(\alpha)\right)\right) = 0 \Rightarrow P\left(P(0)\right) = 0$

If $P(x)=x^2+ax+b$, then $P(0)=b \Rightarrow P(P(0))=P(b) \Rightarrow P(b)=0$

$\Rightarrow b^2+ab+b=0 \Rightarrow b(1+a+b)=0 \Rightarrow P(0)P(1)=O$

65. The equation of the curve for which the square of the ordinate is twice the rectangle contained by the abscissa and the intercept of the normal on x – axis and passing through (2, 1) is

(A) $x^2+y^2-x=0$ (B) $4x^2+2y^2-9y=0$

(C) $2x^2+4y^2-9x=0$ (D) $4x^2+2y^2-9x=0$

Ans. D

Sol. $\because$ Equation of normal at (x, y) is $Y-y=-\frac{dx}{dy}(X-x)$

Put $Y=0$

Then, $X=x+y\frac{dy}{dx}$

Given, $y^2=2x\ X$

$$\Rightarrow \quad y^2=2x\left(x+y\frac{dy}{dx}\right)$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{y^2-2x^2}{2xy}=\frac{\left(\frac{y}{x}\right)^2-2}{2\left(\frac{y}{x}\right)}$$

Put $y=vx$

$$\therefore \quad \frac{dy}{dx}=v+x\frac{dv}{dx}$$

Then, $v+x\frac{dv}{dx}=-\frac{2-v^2}{2v}$

or $x\frac{dv}{dx}=-\frac{(2+v^2)}{2v}$

or $\frac{2vdv}{(2+v^2)}+\frac{dx}{x}=0$

Integrating, we get

$$\ln(2+v^2)+\ln|x|=\ln c$$

$$\Rightarrow \quad \ln(|x|(2+v^2))=\ln c$$

or $|x|\left(2+\frac{y^2}{x^2}\right)=c$

$\because$ It passes through (2, 1), then $2\left(2+\frac{1}{4}\right)=c$

$$\Rightarrow \quad c=\frac{9}{2}$$

Then, $|x|\left(2+\frac{y^2}{x^2}\right)=\frac{9}{2}$

or, $2x^2+y^2=\frac{9}{2}|x|$

or $4x^2+2y^2=9|x|$

66. $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{k^{1/\alpha}\left\{n^{a-\frac{1}{a}}+k^{a-\frac{1}{a}}\right\}}{n^{a+1}}$ is equal to

(A) 1 (B) 2

(C) 3 (D) none of these

Ans. A

Sol. $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{k^{1/\alpha}\left\{n^{a-\frac{1}{a}}+k^{a-\frac{1}{a}}\right\}}{n^{a+1}}$

$=\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{n}.\left\{\left(\frac{k}{n}\right)^{\frac{1}{a}}+\left(\frac{k}{n}\right)^{a}\right\}$

$=\int_0^1\left(x^{1/a}+x^a\right)dx$

$=\left\{\frac{x^{(1/a)+1}}{\frac{1}{a}+1}+\frac{x^{a+1}}{a+1}\right\}_0^1$

$=\frac{a}{a+1}+\frac{1}{a+1}$

= 1

67. Let n be a natural number and p be a real number. The number of zeros of the equation $x^{2n+1}-(2n+1)x+p=0\,(p\in R)$ in the interval $[-1,1]$ is:

(A) 2 if p > 0 (B) 2 if p <0

(C) at most one for every real value of p (D) at least three for every real value of p

Ans. C

Sol. $x^{2n+1}-(2n+1)x+p=0;$

Let $f(x)=x^{2n+1}-(2n+1)x+p$

$f'(x)=(2n+1)x^{2n}-(2n+1)$

$=(2n+1)(x^{2n}-1)$

$f'(x)=0;\ x^{2n}=1;\ x=\pm1$

But for $x \in [-1, 1]$; $x^2 \in [0, 1]$

$0 \le x^{2n} \le 1$; $-1 \le x^{2n} - 1 \le 0$

$f'(x) \le 0$; thus function is decreasing in the interval $[-1, 1]$

68. If $\tan\left(\sum_{r=1}^{n} \tan^{-1}\left(\frac{2r-1}{(r^2+r+1)(r^2-r+1)-2r^3}\right)\right) = 961$, then the value of n is equal to:

(A) 31
(B) 30
(C) 60
(D) 61

Ans. A

Sol. Let $y = \frac{(2r-1)}{(r^2+r+1)(r^2-r+1)-2r^3} = \frac{(2r-1)}{(r^2+1)^2 - r^2 - 2r^3}$

$= \frac{(2r-1)}{1+r^4+r^2-2r^3} = \frac{(2r-1)}{1+r^2(r^2-2r+1)}$

$= \frac{(2r-1)}{1+r^2(r-1)^2}$

$y = \frac{r^2-(r-1)^2}{1+r^2(r-1)^2}$

Thus, $y = \tan^{-1}(r^2) - \tan^{-1}(r-1)^2$

Thus, $t_1 = \tan^{-1}1 - \tan^{-1}0$

$t_2 = \tan^{-1}2^2 - \tan^{-1}1$

$\vdots \qquad \vdots$

$t_n = \tan^{-1}n^2 - \tan^{-1}(n-1)^2$

sum $= \tan^{-1}n^2$;

$\tan(\tan^{-1}n^2) = 961$

$n^2 = 961$

$n = 31$

69. If $\int_0^1 e^{x^2}(x-\alpha)\,dx = 0$, then

(A) $1 < \alpha < 2$
(B) $\alpha < 0$
(C) $0 < \alpha < 1$
(D) $\alpha = 0$

Ans. C

Sol. We have $\int_0^1 e^{x^2}(x-\alpha)\,dx = 0$

$\Rightarrow \quad \int_0^1 xe^{x^2}\,dx = \int_0^1 \alpha e^{x^2}\,dx$

Put $x^2 = t$ in first integral

$$\therefore \quad \frac{1}{2}\int_0^1 e^t\,dt = \alpha \int_0^1 e^{x^2}\,dx$$

$$\frac{1}{2}(e-1) = \alpha\int_0^1 e^{x^2}\,dx \qquad ...(i)$$

But $(1-0)e^0 \le \int_0^1 e^{x^2}\,dx \le e^1(1-0)$

$$\Rightarrow \quad 1 \le \int_0^1 e^{x^2}dx \le e \qquad ...(ii)$$

$$\frac{1}{e} \le \frac{1}{\int_0^1 e^{x^2}\,dx} \le 1$$

$$\Rightarrow \quad \frac{(e-1)}{2e} \le \frac{2}{\int_0^1 e^{x^2}dx} \le \left(\frac{e-1}{2}\right)$$

$$\Rightarrow \quad \left(\frac{e-1}{2e}\right) \le \alpha \le \left(\frac{e-1}{2}\right)$$ [from equation (i)]

70. If the vectors $\vec{c}, \vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$ and $\vec{b} = \hat{j}$ are such that $\vec{a}, \vec{c}$ and $\vec{b}$ from a right handed system, then $\vec{c}$ is

(A) $z\hat{i} - x\hat{k}$
(B) $\vec{0}$
(C) $y\hat{j}$
(D) $-z\hat{i} + x\hat{k}$

Ans. A

Sol. $\because \vec{a}, \vec{c}, \vec{b}$ form a right handed system

$\therefore \vec{c} = \vec{b} \times \vec{a} = \hat{j} \times \left(x\hat{i} + y\hat{j} + z\hat{k}\right)$

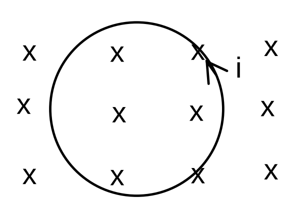
$= x\left(\hat{j} \times \hat{i}\right) + y\left(\hat{j} \times \hat{j}\right) + z\left(\hat{j} + \hat{k}\right)$

$= -\hat{k}x + 0 + \hat{i}\,z$

$= z\hat{i} - x\hat{k}$

PHYSICS

71. Figure shows a wire forming a circular loop of radius a carrying a current i. The force of compression in the body of the wire is



(A) 2iaB
(B) iaB
(C) 2πiaB
(D) none of these

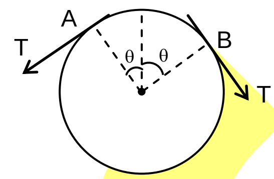
Ans. B

Sol. For arc AB:

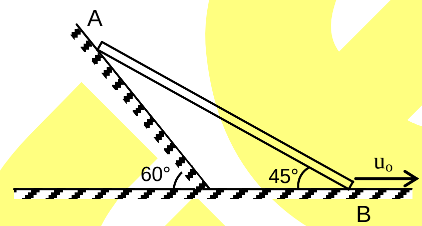
$2T \sin\theta = F_m$

$= IB\,(2R \sin\theta)$

$\Rightarrow \quad T = IBR$



72. A rod of length ℓ placed between two surfaces is being moved by pulling the end B with speed u_o along horizontal surface as shown.
The angular velocity (ω) of the rod at this instant will be



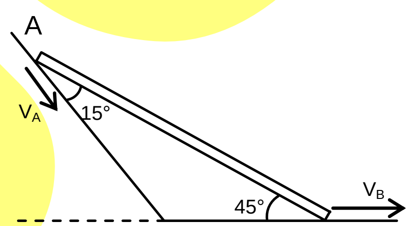
(A) $\frac{u_o}{\ell}$ (B) $\frac{u_o}{\sqrt{2}\,\ell}$

(C) $\frac{\sqrt{2}\,u_o}{\ell}(1 + \tan 15°)$ (D) $\frac{u_o}{\sqrt{2}\,\ell}(1 + \tan 15°)$

Ans. D

Sol. $V_B \cos 45° = V_A \cos (15°)$

$$\omega = \frac{V_B \sin 45^o + V_A \sin(15^o)}{\ell}$$



73. The moment of inertia of a rectangular lamina of mass 'm', length 'ℓ' and width 'b' about an axis passing through its centre of mass, perpendicular to its diagonal and lies in the plane.

(A) $m\left(\frac{\ell^2 + b^2}{12}\right)$ (B) $\frac{m}{12}\left[\frac{\ell^4 + b^4}{\ell^2 + b^2}\right]$

(C) $\frac{m}{6}\left[\frac{\ell^4 + b^4}{\ell^2 + b^2}\right]$ (D) $\frac{m(\ell^2 + b^2)}{24}$

Ans. B

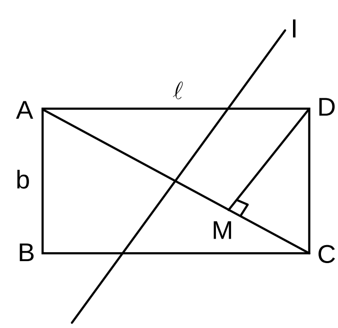
Sol. $DM = \frac{\ell b}{\sqrt{\ell^2 + b^2}}$

$$I_{A_C} = \frac{1}{6}m(DM)^2 = \frac{1}{6}\frac{m\ell^2 b^2}{(\ell^2 + b^2)}$$

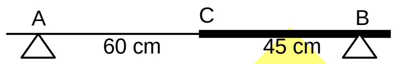
$$I_z = \frac{1}{12}m(\ell^2 + b^2)$$

$$\therefore \quad I = \frac{1}{12}m\,(\ell^2 + b^2) - \frac{1}{6}m\frac{\ell^2 b^2}{(\ell^2 + b^2)}$$

$$= \frac{m}{12(\ell^2 + b^2)}\left[(\ell^2 + b^2)^2 - 2\ell^2 b^2\right] = \frac{m(\ell^4 + b^4)}{12(\ell^2 + b^2)}$$



74. A steel wire of length 60 cm and area of cross section $10^{-6} m^2$ is joined with an aluminium wire of length 45 cm and area of cross section $3 \times 10^{-6} m^2$. The composite string is stretched by a tension of 80 N. Density of steel is 7800 kg m^{-3} and that of aluminium is 2600 kg m^{-3}. The minimum frequency of turning fork, which can produce standing wave in it with node at joint is



(A) 357.3 Hz
(B) 375.3 Hz
(C) 337.5 Hz
(D) 325.3 Hz

Ans. C

Sol. Mass per unit length, $\mu = \frac{m}{\ell} = \frac{\rho A \ell}{\ell} = \rho A$

$\mu_s = \mu_{A\ell} = 78 \times 10^{-4}$ kg/m

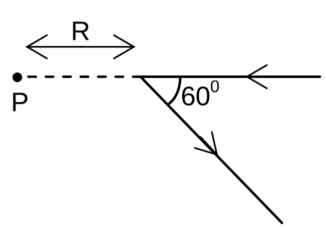
$\therefore$ Speed of wave is same in both wire

$$V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{80 \times 10^4}{78}} = \frac{2 \times 10^2}{\sqrt{3.9}}$$

$$\nu_{min} = \frac{V}{\lambda_{max}} = \frac{200}{\sqrt{3.9} \times 0.3} \left[\frac{\lambda_{max}}{2} = 15 \text{ cm for C as a node}\right]$$

$= 337.5$ Hz

75. A long straight wire, carrying a current I is bent at its mid point to form an angle of 60^0. AT a point P, distance R from the point of bending the magnetic field is

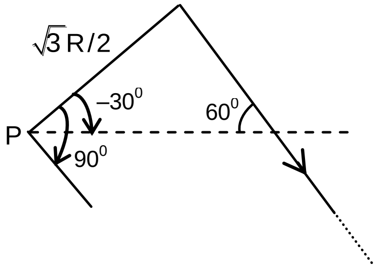


(A) $\frac{(\sqrt{2}-1)\mu_0 i}{4\pi R}$
(B) $\frac{(\sqrt{2}+1)\mu_0 i}{4\pi R}$
(C) $\frac{\mu_0 i}{4\sqrt{3}\pi R}$
(D) $\frac{\mu_0 i}{8R}$

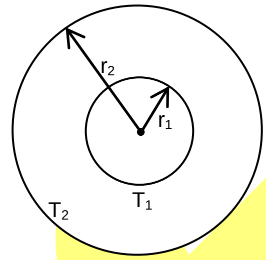
Ans. C

Sol. $B = \frac{\mu_o I}{4\pi\sqrt{3}\,R/2}\left[\sin 90^o + \sin(-30^o)\right]$

$= \frac{\mu_o I}{4\sqrt{3}\,R}$



76. The figure shows a hollow conducting sphere radius (r_2) with cavity radius (r_1) kept at temperatures T_1 and T_2 respectively. The radial rate of flow of heat in a substance between the two concentric spheres in proportional to



(A) $\frac{r_1 r_2}{r_2 - r_1}$
(B) $(r_2 - r_1)$
(C) $\frac{(r_2 - r_1)}{r_1 r_2}$
(D) $\log_e\left(\frac{r_2}{r_1}\right)$

Ans. A

Sol. $H = -k4\pi r^2 \frac{dT}{dr} \Rightarrow H \propto \frac{r_1 r_2}{r_2 - r_1}$

77. The minimum force required to punch a hole of diameter 'd' in a plate of thickness 't' when the ultimate shear strength of steel = S is given by

(A) π d s t
(B) $\pi\left(\frac{d}{2}\right)^2$ s t
(C) 2π d s t
(D) $\pi \frac{d}{2}$ s t

Ans. A

Sol. $S = \frac{F_{tangntial}}{A}$

Force = S × (Slant surface area)

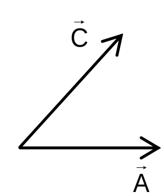
= $S(\pi dt)$

78. If $\bar{A} \times \bar{B} = \bar{C} + \bar{D}$, then the correct alternative is

(A) $\bar{B}$ is parallel to ($\bar{C} + \bar{D}$)
(B) $\bar{A}$ is perpendicular to $\bar{C}$
(C) component of $\bar{C}$ along $\bar{A}$ = component of $\bar{D}$ along $\bar{A}$
(D) component of $\bar{C}$ along $\bar{A}$ = - component of $\bar{D}$ along $\bar{A}$

Ans. D

Sol. Component of $\vec{C}$ along $\vec{A}$



$= (C\cos\theta)\,(\hat{A})$

$= C\left(\frac{\vec{C}\cdot\vec{A}}{C\,A}\right)\hat{A}$

$= \frac{\vec{C}\cdot\vec{A}}{A}(\hat{A})$

Component of $\vec{D}$ along $\vec{A} = \frac{\vec{D}\cdot\vec{A}}{A}(\hat{A})$

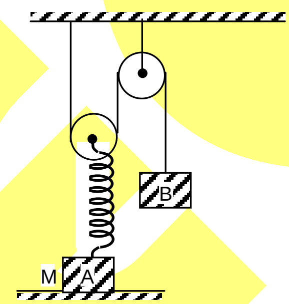
$$\vec{A}\times\vec{B} = \vec{C}+\vec{D}$$

$$\Rightarrow \vec{A}\cdot(\vec{A}\times\vec{B}) = \vec{A}\cdot(\vec{C}+\vec{D})$$

$$0 = \vec{A}\cdot\vec{C}+\vec{A}\cdot\vec{D}$$

$$\Rightarrow \vec{D}.\vec{A} = -\vec{C}\cdot\vec{A}$$

79. Find minimum mass of block B so that A leaves the surface when B is released from rest with spring unelongated



(A) M / 4
(B) M / 2
(C) M
(D) 2M

Ans. A

Sol. For block A to lift

$$Kx_0 = Mg$$

$$x_0 = \frac{Mg}{k}$$

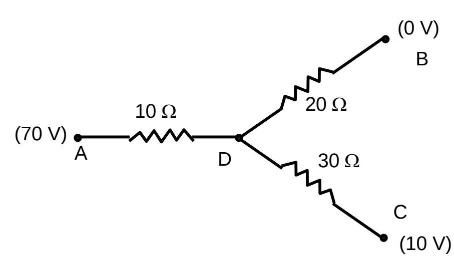
Block B will descend by $2x_0$

C.O.E.

$$mg(2x_0) = \frac{1}{2}k(x_0)^2$$

$$\Rightarrow \quad m = \frac{M}{4} \quad (m \to \text{mass of B})$$

80. In the network shown points A, B, C are at potentials of 70 V, zero and 10 V respectively:



(a) point D is at a potential of 40 V
(b) the currents in the sections AD, DB, DC are in the ratio 3 : 2 : 1
(c) the currents in the sections AD, DB, DC are in the ratio 1 : 2 : 3
(d) the network draws a total power of 200 W

(A) only (a) is correct
(B) only (b) is correct
(C) all are correct
(D) only (a), (b), (d) are correct

Ans. D

Sol. Let potential of junction = V

Junction law:

$$\frac{70-V}{10} = \frac{V-0}{20} + \frac{V-10}{30}$$

$$\Rightarrow \quad V = 40\ V$$

$$I_{AD} = \frac{30}{10} = 3A$$

$$I_{DB} = \frac{40}{20} = 2A$$

$$I_{DC} = \frac{30}{30} = 1A$$

$P_{Total} = P_{AD} + P_{BD} + P_{DC}$

CHEMISTRY

81. Which of the following is a colourless complex?

(A) $[Ti(H_2O)_6]Cl_4$ (B) $[Ti(H_2O)_6]Cl_3$

(C) $[Ti(H_2O)_6]Cl_2$ (D) $[Ti(H_2O)_6]Cl$

Ans. A

Sol. Ti^{4+} ion contains no d-electron. Hence it is coloureless.

82. How many Faradays of electricity is needed to oxidise one mole of H_2O completely to dioxygen gas?

(A) 1 (B) 2

(C) 4 (D) 1.5

Ans. B

Sol. One mole H_2O contains one mole O^{2-}

$$O^{2-} - 2e \longrightarrow O = \frac{1}{2}O_2$$

For two moles of electrons, two Faraday's charge is required.

83. Which of the following complex displays ionization isomerism?

(A) $[Cr(NH_3)_4(Br)_2]Br$ (B) $[Zn(NH_3)_4]Br_2$

(C) $[Co(NH_3)_3(Br)_3]$ (D) $[Mn(NH_3)_4(Br)_2]Cl$

Ans. D

Sol. In (D) the Br^- and Cl^- ligands exchange their positions.

84. Which of the following solution boils at the highest temperature?

(A) 0.1 m KCl (B) 0.4 m NaCl

(C) 0.3 m $AlCl_3$ (D) 0.1 m $C_6H_{12}O_6$

Ans. C

Sol. $\Delta T_b = i \times K_b \times m$

For(C), $\Delta T_b = 4 \times K_b \times 0.3 = 1.2\ K_b$. It is the highest as compared to other solutions.

85. What is the coordination number of the following complex?
$[Cr(en)_2(H_2O)_2]Cl_3$

(A) 7 (B) 4
(C) 6 (D) 9

Ans. C

Sol. Coordination number = No. of dative bonds formed by ligands with the metal ion.
Two en($NH_2CH_2CH_2NH_2$) = 2 + 2 = 4 dative bonds
Two H_2O = 2 dative bond
Total no. of dative bond = 6

86. Which of the following decreases on dilution?

(A) Molar conductance of CH_3COOH (B) Specific conductance of NaCl
(C) Equivalent conductance of $Al_2(SO_4)_3$ (D) Equivalent conductance of $Ca(OH)_2$

Ans. B

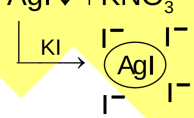
Sol. Specific conductance decreases on dilution.

87. What type of colloid is formed if $AgNO_3$ is added to excess of KI?

(A) Positive sol (B) Negative sol
(C) Neutral sol (D) KNO_3 sol

Ans. B

Sol. $AgNO_3 + KI \longrightarrow AgI\downarrow + KNO_3$



88. How many moles of CH_3I is consumed by one mole of $C_2H_5NH_2$?

(A) 2 (B) 1
(C) 3 (D) 4

Ans. C

Sol. $C_2H_5NH_2 \xrightarrow[-HI]{CH_3I} C_2H_5NHCH_3 \xrightarrow[-HI]{CH_3I} C_2H_5N(CH_3)_2 \xrightarrow{CH_3I} I^-\overset{+}{\left[C_2H_5N(CH_3)_3\right]}$

89. $CH_3-CH=CH-CH_2OCH_2CH_2CH=CH_2$
How many maximum no. of moles of HI can be consumed by one mole of the above compound?

(A) 2 (B) 3
(C) 4 (D) 5

Ans. C

Sol. Two double bond – Two moles
One ether – 1 mole, one alcohol produced – one mole

90. The pyknometric density of sodium chloride crystal is 2.165×10^3 kg m^{-3}, while its x-ray density is 2.178×10^3 kg m^{-3}. The fraction of the unoccupied sites in sodium chloride crystal is

(A) 5.96
(B) 5.96×10^{-2}
(C) 5.96×10^{-1}
(D) 5.96×10^{-3}

Ans. D

Sol. $\left(\frac{2.178 - 2.165}{2.178}\right) = 5.96 \times 10^{-3}$