FIITJEE

NSEP – 2012-13

SOLUTIONS

PART-A

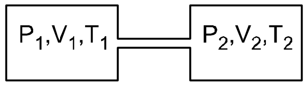
SUB-PART-A1

1. Two thermally insulated compartments 1 and 2 are filled with a perfect gas and are connected by a short tube having a valve which is closed. The pressure, volumes and absolute temperature of the two compartments are respectively (p_1, V_1, T_1) and (p_2, V_2, T_2). After opening the valve, the temperature and the pressure of both the compartments respectively are :

(a) $\dfrac{T_1T_2(p_1V_1+p_2V_2)}{(p_1V_1T_1+p_2V_2T_2)}, \dfrac{p_1V_1+p_2V_2}{V_1+V_2}$ (b) $\sqrt{T_1T_2}, \dfrac{p_1V_1+p_2V_2}{V_1+V_2}$

(c) $\dfrac{T_1T_2(p_1V_1+p_2V_2)}{(p_1V_1T_1+p_2V_2T_2)}, \dfrac{p_1V_1T_1+p_2V_2T_2}{V_1T_1+V_2T_2}$ (d) $\dfrac{T_1+T_2}{2}, \dfrac{p_1V_1+p_2V_2}{V_1+V_2}$

Ans. None of the option is correct



Sol. $n_1 = \dfrac{P_1V_1}{RT_1}, \quad n_2 = \dfrac{P_2V_2}{RT_2}$

No. of moles remain conserved

$$\left[\frac{PV_2}{RT}+\frac{PV_1}{RT}\right] = \frac{P_1V_1}{RT_1}+\frac{P_2V_2}{RT_2} \qquad ....(1)$$

Heat lost = Heat gained

so $n_1c(T - T_1) = n_2c(T_2 - T)$

$n_1T - n_1T_1 = n_2T_2 - n_2T$

$T(n_1 + n_2) = n_2T_2 + n_1T_1$

From (2) $T = \dfrac{n_2T_2+n_1T_1}{n_1+n_2}$

$$T = \frac{\frac{P_2V_2}{R}+\frac{P_1V_1}{R}}{\frac{P_1V_1}{RT_1}+\frac{P_2V_2}{RT_2}} = \frac{(P_1V_1+P_2V_2)T_1T_2}{P_1V_1T_2+P_2V_2T_1}$$

From (1) $P = \dfrac{P_1V_1+P_2V_2}{V_1+V_2}$

2. An inductance coil is connected to an ac source through a 60 ohm resistance in series. The source voltage, voltage across the coil and voltage across the resistance are found to be 33 V 27 V and 12 respectively. Therefore, the resistance of the coil is :

(a) 30 ohm (b) 45 ohm (c) 105 ohm (d) 75 ohm

Ans. **(d)**

Sol. $|I \times X_L\hat{j} + IR + I(60)| = 33 \qquad ....(1)$

$|I \times X_L\hat{j} + IR| = 27 \qquad ....(2)$

$IR = 12$

$I = \dfrac{1}{5} \qquad ....(3)$

Using (1), (2) & (3) we get R = 75 Ω

3. An ideal inductance coil is connected to a parallel plate capacitor. Electrical oscillations with energy W are set up in this circuit. The capacitor plates are slowly drawn apart till the frequency of oscillations is doubled. The work done in this process will be :

(a) W (b) 2 W (c) 3 W (d) 4 W

Ans. **(c)**

Sol.

$$\omega = \frac{1}{\sqrt{LC}}$$

$$\omega' = \omega\frac{1}{\sqrt{LC/4}}$$

So capacitance should be made $\frac{C}{4}$ i.e. $\frac{1}{4}$th of initial.

$$E_{final} = \frac{q_0^2}{2C'} = \frac{q_0^2}{2\times\frac{C}{4}} = 4\frac{q_0^2}{2C} \qquad \text{...........(1)}$$

$$E_{initial} = \frac{q_0^2}{2c} \qquad \text{...........(2)}$$

$$E_{final} = 4E_{initial}$$

$$\Delta E = \text{Work} = 3W$$

4. Two equals masses are connected by a spring satisfying Hooke's law and are placed on a frictionless table. The spring is elongated a little and allowed to go. Let the angular frequency of oscillations be w. Now one of the masses is stopped. The square of the new angular frequency is :

(a) ω^2 (b) $\frac{\omega^2}{2}$ (c) $\frac{\omega^2}{3}$ (d) $2\omega^2$

Ans. **(b)**

Sol. By reduced mass concept

$$\mu = \frac{m_1m_2}{m_1+m_2} = \frac{m}{2}$$

$$T' = 2\pi\sqrt{\frac{m}{K}} = \frac{2\pi}{\omega'}$$

$$T = 2\pi\sqrt{\frac{m}{2K}} = \frac{2\pi}{\omega}$$

$$\frac{T}{T'} = \frac{\omega'}{\omega} = \sqrt{\frac{1}{2}}$$

$$\omega' = \frac{1}{2}\omega^2$$

5. When a particle oscillates in simple harmonic motion, both its potential energy and kinetic energy vary sinusoidally with time, If v be the frequency of the motion of the particle, the frequency associated with the kinetic energy is :

(a) 4v (b) 2v (c) v (d) $\frac{v}{2}$

Ans. **(b)**

Sol. $v = A\omega \cos \omega t$

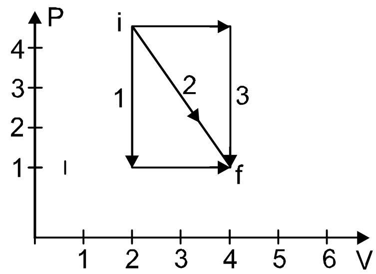
$KE = \frac{1}{2}mv^2$

$KE = \frac{1}{2}m(A\omega)^2 \cos^2 \omega t$

Here frequency of KE is 2ν

6. A gas expands form i to f along the three paths indicated. The work down along the three paths denoted by W_1, W_2 and W_3 have the relationship.

(a) $W_1 < W_2 < W_3$ (b) $W_2 < W_1 = W_3$
(c) $W_2 < W_1 < W_3$ (d) $W_1 > W_2 > W_3$



Ans. **(a)**

Sol. Work done = Area under P – V graph

So $w_3 > w_2 > w_1$

7. An ideal gas at 30°C enclosed in cylinder with perfectly non conducting side and a piston moving without friction in it. The base of the cylinder is perfectly conducting. Cylinder is first placed on a heat source till the gas is heated to 100°C and the piston raised by 20 cm and the atmospheric pressure is 100 kPa. The piston is then held in final position and cylinder is placed on the heat sink to cool the gas to 30°C. Denoting ΔQ_1 as the heat supplied during heating and ΔQ_2 as the heat lost during the cooling, then $[\Delta Q_1 \sim \Delta Q_2]$ would be equal to :

(a) 436 J (b) 336 J (c) 236 J (d) 136 J

Sol. Insufficient information

8. Equals amounts liquid helium and water at their respective boiling points are boiled by supplying the heat from identical heaters in time t_{He} and t_w. The latent heats of vaporization of He and Water are 2.09×10^4 J / Kg and 540 kcal / kg, then t_{He} is :

(a) about 0.1 t_w (b) about 0.05 t_w
(c) just greater than 0.01 t_w (d) just less than 0.01 t_w

Ans. **(d)**

Sol.
$$\frac{t_{He}}{t_\omega} = \frac{L_{He}}{L_\omega}$$
$$= \frac{20.9 \times 10^3}{540 \times 4.2 \times 10^3}$$
$$= \frac{20.9}{540 \times 4.2} = 0.00921$$

i.e. Just less than $t_{He} \approx 0.01\, t_\omega$

9. A 5 litre vessel contains 2 mole of oxygen gas at a pressure of 8 atm. The average translational kinetic energy of an oxygen molecule under this condition is :

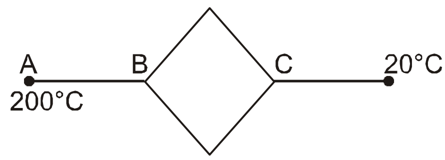
(a) 8.4×10^{-14} J (b) 4.98×10^{-21} J (c) 7.4×10^{-16}J (d) 4.2×10^{-21} J

Ans. **(b)**

Sol.
$$PV = nRT$$
$$(8) \times 5 = (2) \times R \times T$$
$$20 = RT$$
$$T = \frac{20}{0.0821} = 243.60\text{K}$$
$$KE = \frac{3}{2}KT = \frac{3}{2} \times 243.60 \times 1.38 \times 10^{-23}$$
$$= 4.98 \times 10^{-21}\text{J}$$

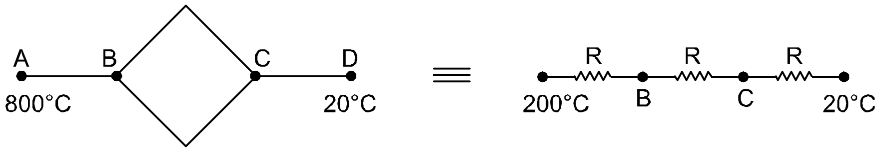
10. Six identical conducting rods are joined as shown. The ends A and D are maintained at 200° C and 20° C respectively. No heat is lost to surroundings. The temperature of the junction C will be :

(a) 60°C (b) 80°C
(c) 100°C (d) 120°C



Ans. **(b)**

Sol.



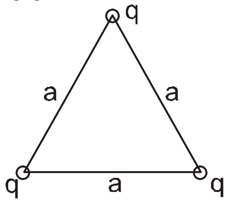
Equal temperature is lost at each section so T_C = 80°C

11. Three corners of an equilateral triangle of side a are occupied by three charges of magnitude q. If the charges are transferred to infinity, their kinetic energy will be $\frac{1}{4\pi\varepsilon_0}$ times.

(a) $\frac{2q}{a}$ (b) $\frac{3q^2}{a}$ (c) $\frac{q^2}{3a}$ (d) $\frac{3q}{a}$

Ans. (b)

Sol.



PE of system is $3\frac{kq^2}{a}$

At infinity $KE = \Delta PE$

$$\frac{1}{2}(2m)u^2 = 3\frac{kq^2}{a}$$

$$= \frac{3q^2}{q} \times \frac{1}{4\pi\varepsilon_0}$$

12. An LDR (light dependent resistance) is connected to an appropriate voltage source and a current measuring meter in series (Assuming that the LDR current is proportional to the intensity of the incident light). The LDR is illuminated with light from a distant metal filament bulb. The filament voltage V, the distance d of LDR from the bulb and the LDR current I are noted. If both V and are doubled, the LDR current is :

(a) 61 I (b) 4 I (c) I (d) less than I

Ans. (c)

Sol. Due to increment of voltage of filament V, energy produced by filament in 4E

$$I = \frac{KE}{4\pi d^2}$$

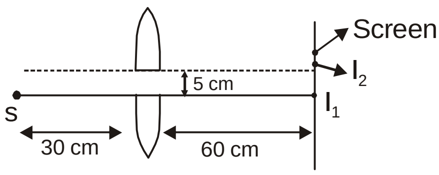
$$I' = \frac{K4E}{4\pi(2d)^2} = I$$

13. A point source is placed at a distance of 30 cm from a convex lens of focal length f on its axis and the image is formed on a screen at a distance of 60 cm from the lens. Now the lens is split into two halves. One half is moved perpendicular to the lens axis through a distance of 5 cm. It is found that the two halves of the lens form two images on the screen and the images are separated by a distance d. The values of f and d respectively, are :

(a) 20 cm, 15 cm (b) 20 cm, 10 cm (c) 30 cm, 10 cm (d) 30 cm, 5 cm

Ans. (a)

Sol.



By lens formula

$$\frac{1}{60} - \frac{1}{(-30)} = \frac{1}{f} \Rightarrow f = 20 \text{ cm}$$

$$M = \frac{-v}{u} = \frac{-60}{30} = -2$$

So, Distance I_1, I_2 = 15 cm

14. The angle of refraction of a very thin prism is 1°. A light ray is incident normally on one of the refracting surfaces. The ray that ultimately emerges from the first surface after suffering reflection from the second surface, makes an angle of 3.32° with the normal. The deviation of the ray emerging from the second surface and the refractive index of the material of the prism respectively, are :

(a) 0.66°, 1.66 (b) 1.66°, 1.5 (c) 1.5°, 1.66 (d) 0.66°, 1.5

Ans. **(a)**

(a) $\mu = \frac{\sin 3.32°}{\sin 2°} = \frac{3.32}{2°}$

$\mu = 1.66$

(b) $\delta = r - i \Rightarrow \mu(1°) - 1° = 1.66 - 1° = 0.66°$

15. A beam of light from a distant axial point source is incident on the plane surface of a thin planoconvex lens; a real image is formed at a distance of 40 cm. Now if the curved surface is silvered, the real image is formed at a distance of 7.5 cm. The radius of curvature of the curved surface of the lens and the refractive index of the material of the lens respectively, are :

(a) 40 cm, 1.5 (b) 24 cm, 1.6 (c) 20 cm, 1.6 (d) 7.5 cm, 1.5

Ans. **(b)**

Focal length of plane convex lens.

$$\frac{1}{f} = (\mu - 1)\frac{1}{R}$$

$$\frac{1}{40} = \frac{(\mu - 1)}{R} \quad ....(1)$$

For mirror

So, $$\frac{1}{7.5} = \left(\frac{2}{R} + \frac{2}{40}\right)$$

$$\frac{1}{15} = \frac{1}{R} + \frac{1}{40}$$

$$R = \frac{40 \times 15}{25} = 24\text{cm}$$

Put R in eqn. (1)

$$\frac{1}{40} = \frac{(\mu - 1)}{24}$$

$$\frac{24}{40} = \mu - 1$$

$$\mu = 1 + \frac{24}{40} = 1 + \frac{3}{5} = \frac{8}{5} = 1.6$$

16. A convex lens forms the image of an axial point on a screen. A second lens with focal length f cm is placed between the screen and the first lens at a distance of 10 cm from the screen. To view the image the screen has to be shifted away from the lens by 5 cm. A third lens having focal length of the same magnitude f cm is used to replace the second lens at the same position. But this time to view the image the screen has to be shifted towards the lens by d cm. The values of f and d respectively, are :

(a) 30 cm, 2.5 cm (b) 30 cm, 5 cm (c) 7.5 cm, 2.5 cm (d) 7.5 cm, 5 cm

Ans. **(a)**

Sol. for 2^{nd} lens

u = +10 cm V = +15 cm

$$\frac{1}{V} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{f} = \frac{1}{15} - \frac{1}{10} = \frac{10 - 15}{150} = \frac{-5}{150}$$

f = –30 cm

Now

$$f' = +30$$

$$\frac{1}{V} - \frac{1}{10} = \frac{1}{30}$$

$$V = 7.5 \text{ cm}$$

So, $d = 2.5$ cm

17. Cerenkov effect: If the speed of an electron in a medium is greater than the speed of light in that medium then the electron emits light. An electron beam in a medium is accelerated by a voltage V. The light that is emitted just suffers total internal reflection at the boundary of the medium placed in air when the angle of incidence is 45°. The value of the voltage is :

(a) 63.19 kV (b) 255.64 kV (c) 200.34 kV (d) 127.82 kV

Ans. **(d)**

Sol.

$$ev = \frac{1}{2}m_e\left(\frac{3\times10^8}{\sqrt{2}}\right)^2$$

$$1.6 \times 10^{19} V = \frac{1}{2}\times9.1\times10^{-31}\times\frac{9\times10^{16}}{2}$$

$$V = 127.82 \text{ KV}$$

18. In a electrolytic process certain amount of charge liberates 0.8 gram of oxygen. Then the amount of silver liberated by the same amount of charge is :

(a) 10.8 gram (b) 1.08 gram (c) 0.9 gram (d) 9, 0 gram

Ans. **(a)**

Sol. moles of Oxygen is $\frac{0.8}{16} = \frac{0.1}{2}$ mole

Electrons flown = $\left(\frac{0.1}{2}\times 2\right) = 0.1$ mole

So, These no of electron liberate .1 mole of Ag.

i.e. $0.1 \times 108 = 10.8$ gm

19. The energy state of doubly ionized lithium having the same energy as that of the first excited state of hydrogen is :

(a) 4 (b) 6 (c) 3 (d) 2

Ans. **(b)**

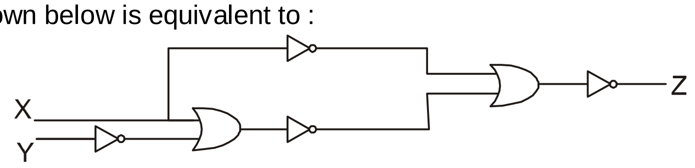
Sol.

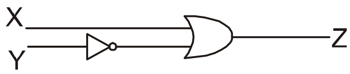
$$-13.6\times\frac{1^2}{2^2} = -13.6\times\frac{3^2}{n^2}$$

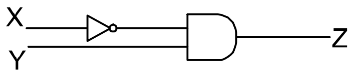
$$\frac{1}{4} = \frac{9}{n^2}$$

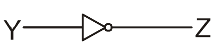
$$n = 6$$

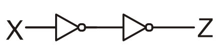
20. The logic circuit shown below is equivalent to :



(a) 

(b) 

(c) 

(d) 

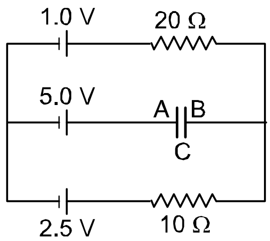
Ans. **(d)**

Sol. Truth table of circuit

X	Y	Output
0	0	0
0	1	0
1	0	1
1	1	1

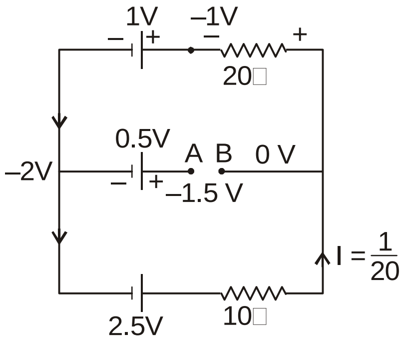
21. In the circuit shown below, the potential of a with respect to B of the capacitor C is :

(a) 2.00 volt (b) –2.00 volt
(c) –1.50 volt (d) +1.50 volt



Ans. **(c)**

Sol. $V_A - V_B = -1.5$ V



22. Two pendulums differ in lengths by 22 cm. They oscillate at the same place so that one of them makes 30 oscillations and the other makes 36 oscillations during the same time. The lengths (in cm) of the pendulums are :

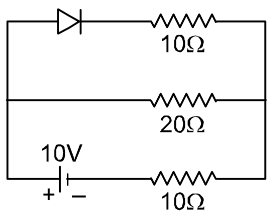
(a) 72 and 50 (b) 60 and 38 (c) 50 and 28 (d) 80 and 58

Ans. **(a)**

Sol.
$$\frac{T_1}{T_2} = \frac{36}{30} = \frac{6}{5} = \sqrt{\frac{L_1}{L_2}}$$
$$\frac{L_1}{L_2} = \frac{36}{25} \quad \text{and} \quad L_1 - L_2 = 22 \text{ cm}$$
$$\left(\frac{36}{25} - 1\right)L_2 = 22\ ; \quad L_2 = \frac{22 \times 25}{11} = 50\,\text{cm}$$
$$L_1 = 72 \text{ cm}$$

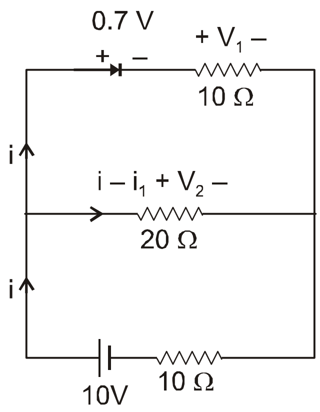
23. The voltage drop across a forward biased diode is 0.7 volt. In the following circuit, the voltage across the 10 ohm resistance in series with the diode and 20 ohm resistance are :

(a) 0.70 V, 4.28 V (b) 3.58 V, 4.28 V
(c) 5.35 V, 2.14 V (d) 3.58 V, 9.3 V



Ans. **(b)**

Sol.
$$0.7 + 10i_1 + 10i - 10 = 0$$
$$10i + 10i_1 = 9.3 \qquad ....(1)$$
$$20(i - i_1) + 10i - 10 = 0$$
$$3i - 2i_1 = 1 \qquad ....(2)$$
$$i^1 = \frac{27.9 - 10}{50} = \frac{17.5}{50} = 0.358$$
$$V_1 = 10i_1 = 3.58$$
$$3i - 0.716 = 1$$
$$i = \frac{1.716}{3} = 0.578$$
$$V_2 = 20\,(i - i_1) = 4.28 \text{ V}$$



24. The magnetic flux φ through a stationary loop of wire having a resistance R varies with time as $\varphi = at^2 + bt$ (a and b are positive constants). The average emf and the total charge flowing in the loop in the time interval t = 0 to t = τ respectively are :

(a) $a\tau + b, \frac{a\tau^2 + b\tau}{R}$ (b) $a\tau + b, \frac{a\tau^2 + b\tau}{2R}$ (c) $\frac{a\tau + b}{2}, \frac{a\tau^2 + b\tau}{R}$ (d) $2(a\tau + b), \frac{a\tau^2 + b\tau}{2R}$

Ans. **(a)**

Sol.
$$|\varepsilon| = \left|-\frac{dQ}{dt}\right| = \left|-(2at + b)\right| = 2at + b$$

$$\text{average emf} = \frac{1}{\tau}\int_0^{\tau}(2at + b)bt$$

$$= \frac{1}{\tau}(2a\frac{\tau^2}{2} + bc) = a\tau + b$$

$$Q = \frac{\Delta\phi}{R} = \frac{(a\tau^2 + bc) - 0}{R} = \frac{a\tau^2 + b\tau}{R}$$

25. Three waves of the same amplitude have frequencies (n – 1), n and (n + 1) Hz. They superpose on one another to produce beats. The number of beats produced per second is :

(a) n (b) 2 (c) 1 (d) 3n

Ans. **(b)**

Sol. Δf = no of beat

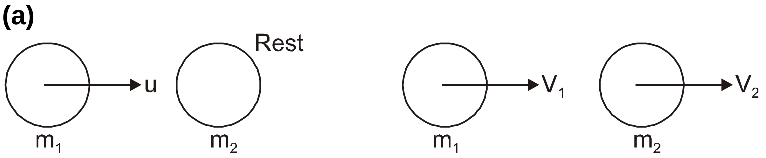
Δf = 2

26. A spherical ball of mass m_1 collides head on with another ball of mass m_2 at rest. The collision is elastic. The fraction of kinetic energy lost by m_1 is :

(a) $\frac{4m_1m_2}{(m_1 + m_2)^2}$ (b) $\frac{m_1}{m_1 + m_2}$ (c) $\frac{m_2}{m_1 + m_2}$ (d) $\frac{m_1m_2}{(m_1 + m_2)^2}$

Ans. **(a)**

Sol.



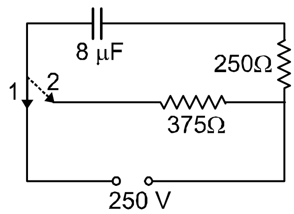
$$m_1u = m_1V_1 + m_2V_2$$
$$u = -V_1 + V_2$$
$$V_1 = \frac{m_1 - m_2}{m_1 + m_2}u$$

$$\text{fraction of K.E. loss} = \frac{\frac{1}{2}m_1(u^2 - V_1^2)}{\frac{1}{2}m_1u^2}$$

$$= \frac{u^2 - \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 u^2}{u^2} = \frac{4m_1m_2}{(m_1 + m_2)^2}$$

27. In the circuit shown below, the switch is in position 1 for a long time. At some moment after that the switch is thrown in position 2. The quantity of heat generated in the resistance of 375 ohm after the switch is changed to position 2 is :

(a) 0.15 J (b) 0.25 J
(c) 0.50 J (d) 0.10 J



Ans. **(a)**

Sol. When switch in position 1

$$\text{Stored energy} = \frac{1}{2} \times 8 \times 10^{-6} \times (250)^2 = H$$

Heat loss in 375 Ω

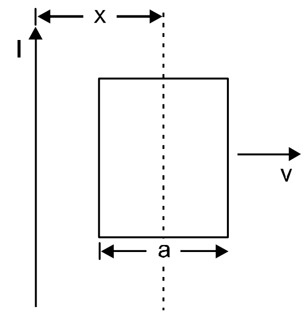
Resistor $\quad H_1 = \dfrac{375}{(375+250)} \times H$

$$= \frac{3}{5}H$$

$$H_1 = \frac{3}{5} \times \frac{1}{2} \times 8 \times 10^{-6} \times \frac{10^6}{16}$$

$$= \frac{3}{10 \times 2} = 0.15\,J$$

28. A conducting square frame of side a and a long straight wire carrying current I are located in the same plane as shown in the figure. The frame moves to the right with a constant velocity v. The emf induced in the frame will be proportional to :

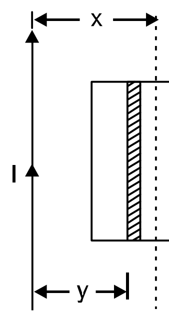


(a) $\dfrac{1}{x^2}$ (b) $\dfrac{1}{(2x-a)^2}$

(c) $\dfrac{1}{(2x+a)^2}$ (d) $\dfrac{1}{(2x-a)(2x+a)}$

Ans. **(d)**

Sol.



$$d\phi = \left(\frac{\mu_0 I}{2\pi y}\right) a\,dy$$

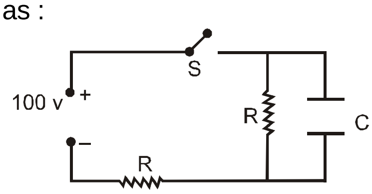
$$\phi = \frac{\mu_0 Ia}{2\pi} \ln\left(\frac{x+a/2}{x-a/2}\right)$$

$$= \frac{\mu_0 Ia}{2\pi} \ln\left(\frac{2x+a}{2x-a}\right)$$

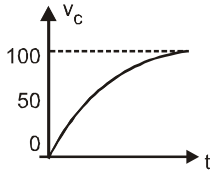
$$\varepsilon = \left|\frac{-d\phi}{dt}\right| = \left|\frac{\mu_0 Ia}{2\pi}\left(\frac{2x-a}{2x+a}\right) \cdot \left[\frac{2(2x-a)-2(2x+a)}{(2x-a)^2}\right] v\right|$$

$$= \frac{\mu_0 Ia}{2\pi} \times \frac{2[2a] \times V}{(2x-a)(2x+a)}$$

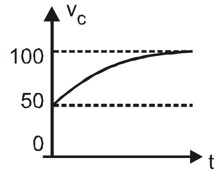
29. In the circuit shown below, the switch S is closed at the moment t = 0. As a result the voltage across the capacitor C will change with time as :



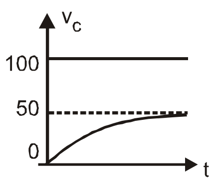
(a)



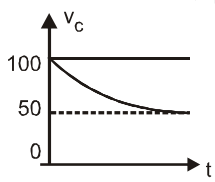
(b)



(c)



(d)



Ans. **(c)**

Sol. At steady state condition potential across capacitance = 50 V

At initial potential across capacitance = 0 V

30. The ratio of the rational kinetic energy to the total kinetic energy of one mole of a gas of rigid diatomic molecules is ?

(a) $\frac{2}{3}$ (b) $\frac{2}{5}$ (c) $\frac{3}{5}$ (d) $\frac{5}{2}$

Ans. **(b)**

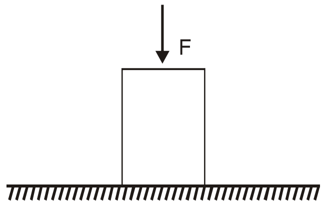
Sol. **For diatomic gas :**

3 degree of freedom for translation motion
2 degree of freedom for rotation motion.

So Rotational K.E. = $2\times\frac{1}{2}KT$

Total K.E. = $5\times\frac{1}{2}KT$

31. A metal cylinder of length L is subjected to a uniform compressive force F as shown in the figure. The material of the cylinder has Young's modulus Y and Poisson's ratio μ. The change in volume of the cylinder is :



(a) $\frac{\mu FL}{Y}$ (b) $\frac{(l-\mu)FL}{Y}$ (c) $\frac{(1+2\mu)FL}{Y}$ (d) $\frac{(1-2\mu)FL}{Y}$

Ans. **(d)**

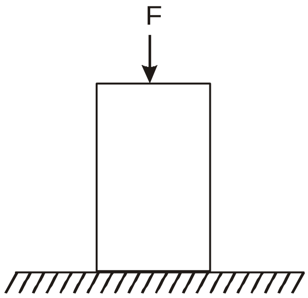
Sol.

$$\mu = \frac{\frac{-\Delta r}{r}}{\frac{\Delta L}{L}}$$

$$\Rightarrow \quad \frac{\Delta r}{r} = -\frac{\mu \Delta L}{L} \quad ....(1)$$

$$Y = \frac{F/A}{\Delta L/L} \quad ...(2)$$

$$\frac{\Delta v}{v} = \frac{\Delta A}{A} + \frac{\Delta L}{L} \quad ...(3)$$



From (1), (2) and (3)

$$\Delta v = \frac{(1-2\mu)FL}{Y}$$

32. Three person A, B, and C not the time taken by their train to cover the distance between two successive station by observing the digital clock on the platforms of two stations. The clocks display time in hours and minutes. The three persons find the intervals 3, 5 and 4 minutes respectively. Assume the maximum discrepancy of 2 seconds in actual starting and stopping of the train and the observations by A,. B and C. Then, ?

(a) All A, B and C can be correct
(b) Only A and B or B and C can be correct
(c) Only one of A, B and C can be correct
(d) C is correct since it is equal to the average of the three observations

Ans. **(c)**

33. When two drops of water coalesce – (I) Total surface area decreases. (II) There is some rise in temperature.

Which of the following is correct?

(a) Both (I) and (II) are wrong statements
(b) Statement (I) is true but (II) is not true.
(c) Both (I) and (II) are true and the two statements are independent of each other.
(d) Both (I) and (II) are true and (I) is the cause of (II).

Ans. **(d)**

Sol. When two drops of water coalesce, the total surface area decrease. The loss of surface energy will increase temperature of drop :

34. Two capacitors 0.5 μF and 1.0 μF in series are connected to a dc source of 30 V. The voltages across the capacitors respectively are :

(a) 10 V, 20 volt (b) 15 V, 15 V (c) 20 V, 10 V (d) 30 V, 30 V

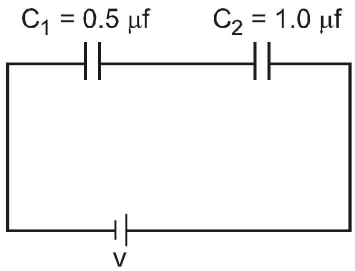
Ans. **(c)**

Sol.

$$C_{eq} = \frac{C_1C_2}{C_1+C_2} = \frac{0.5}{1.5} = \frac{1}{3}$$

$$Q = C_{eq}V = \frac{1}{3}\times 30 = 10,$$

$$V_1 = \frac{Q}{C_1} = \frac{10}{0.5} = 20, \quad V_2 = \frac{Q}{C_2} = \frac{10}{1} = 10$$



35. The Th_{90}^{232} atom has successive alpha and beta decays to the end product Pb_{82}^{208}. The numbers of alpha and beta particles emitted in the process respectively are :

(a) 4, 6 (b) 4, 4 (c) 6, 2 (d) 6, 4

Ans. **(d)**

Sol. Parent atom — Th_{90}^{232}

End product — Pb_{82}^{208}

No. of alpha particle = $\frac{232-208}{4} = 6$

No. of beta particle = 4

36. If the breakdown field of air is 2.0 × 10^6 V/ m, the maximum charge that can be given to a sphere of diameter 10 cm is :

(a) 2.0 × 10^{-4} (b) 5.6 × 10^{-7}C (c) 5.6 × 10^{-5} C (d) 2.0 × 10^2 C

Ans. **(b)**

Sol. $E_b = 2 \times 10^6$ v/m

$$\frac{Q}{4\pi \in_0 R^2} = 2\times 10^6$$

$$\Rightarrow \quad Q = \frac{2\times 10^6 \times 5\times 5}{9\times 10^9 \times 100\times 100}$$

$$= \frac{50}{9}\times 10^{-7} = 5.6 \times 10^{-7}\text{ C}$$

37. Density of ocean water varies with depth. This is due to :

(a) Elasticity (b) Viscosity (c) surface tension (d) All the three

Ans. **(a)**

Density of ocean water depends on elasticity of water.

38. A spring of certain length and having spring constant k is cut into two pieces of length in a ratio 1 : 2. The spring constants of the two pieces are in a ratio :

(a) 1 : 1 (b) 1 : 4 (c) 1 : 2 (d) None of these

Ans. **(d)**

$$K \propto \frac{1}{l}, \quad \frac{K_1}{K_2} = \frac{l_2}{l_1}$$

$$\Rightarrow \quad \frac{l_1}{l_2} = \frac{1}{2} \Rightarrow \frac{K_1}{K_2} = \frac{2}{1}$$

39. When a metal surface is illuminated with light of wavelength λ the stopping potential is V_0. When the same surface is illuminated with light of wavelength 2 λ, the stopping potential is $\frac{V_0}{4}$. If the velocity of light in air is c, the threshold frequency of photoelectric emission is :

(a) $\frac{c}{6\lambda}$ (b) $\frac{c}{3\lambda}$ (c) $\frac{2c}{3\lambda}$ (d) $\frac{4c}{3\lambda}$

Ans. **(b)**

Sol.
$$eV_0 = \frac{hc}{\lambda} - \phi \quad ....(1)$$
$$\frac{eV_0}{4} = \frac{hc}{2\lambda} - \lambda \quad ....(2)$$

From (1) and (2)
$$4 = \frac{\frac{hc}{\lambda} - \phi}{\frac{hc}{2\lambda} - \phi} \Rightarrow \frac{2hc}{\lambda} - 4\phi = \frac{hc}{\lambda} - \phi$$
$$\Rightarrow \quad 3\phi = \frac{hc}{\lambda} \Rightarrow \phi = \frac{hc}{3\lambda} = h\nu_0 \Rightarrow \nu_0 = \frac{c}{3\lambda}$$

40. Two elastic waves move along the same direction in the same medium. The pressure amplitudes of both the waves are equal, but the wavelength of the first wave is three times that of the second. If the average power transmitted through unit area by the first wave is W_1 and that by the second is W_2, then :

(a) $W_1 = W_2$ (b) $W_1 = 3\,W_2$ (c) $W_2 = 3W_1$ (d) $W_1 = 9\,W_2$

Ans. **(a)**

Sol. Given $(\Delta P_0)_1 = (\Delta P_0)_2$ ....(1)
$$\lambda_1 = 3\lambda_2 \quad ....(2)$$
$$W_1 = \frac{(\Delta P_0)_1^2}{2\rho v}$$
$$W_2 = \frac{(\Delta P_0)_2^2}{2\rho v}$$

From eq. (1) and (2)
$$W_1 = W_2$$

SUB-PART A – 2

In question 41 to 50 any number of options (1 or 2 or 3 r all 4) may be correct. You are to identify all of them correctly to get 6 marks. Even if one answer identified is incorrect or one correct answer is missed, you get zero

41. A cube floats both in water and in a liquid of specific gravity 0.8. Therefore,
(a) Apparent weight of the cube is the same in water and in the liquid
(b) The cube has displaced equal volume of water and the liquid while floating.
(c) The cube has displaced equal weight of water and the liquid while floating.
(d) If some weights are placed on the top surface of the cube to make it just sink, the load is case of water will be 0.8 times of that to be used in case of the liquid

Ans. **(a, c)**

Sol. Apparent weight of Cube in water and in liquid is zero. Buoyant force is equal to weight of liquid displaced.

42. On the basis of the kinetic theory of gases one compares 1 gram of hydrogen with 1 gram of argon both at 0°C. Then,
(a) The same temperature implies that the average kinetic energy of the molecules is the same in both the cases
(b) The same temperature implies that the average potential energy of the molecules is the same in both the cases
(c) Internal energies in both the cases are equal
(d) When both the samples are heated through 1°C, the total energy added to both of them is not the same.

Ans. **(b, d)**

Although the change in temperature is same, total heat added to both of them will be different as degree of freedom is different.

43. While explaining the action of heat engine, one can say that :
(a) Heat cannot be fully converted into mechanical work.
(b) The first law of thermodynamics is necessary but not sufficient.
(c) Heat under no circumstances can flow from lower to higher temperature
(d) A body can not be cooled to absolute zero

Ans. **(a, b, d)**
Heat cant't be fully converted into mechanical work.

44. The rate of change of angular momentum of a system of particles about the centre of mass is equal to the sum of external torques about the centre of mass when the centre of mass is :
(a) Fixed with respect to an inertial frame.
(b) In linear acceleration
(c) In rotational motion
(d) Is in a translation motion

Ans. **(a, b, c, d)**
From inertial frame

$$\vec{\tau}_{ex} = \frac{d\vec{L}}{dt}$$

From non-inertial frame, torque of pseudo force is also included apart from external force. Torque of pseudo force about centre of mass is zero.

So $\frac{d\vec{L}_{cm}}{dt} = \vec{\tau}_{ex}$ is valid for all type of frame.

45. Light is traveling in vacuum along the Z axis. The sets of possible electric and magnetic fields could be :
(a) $\vec{E} = \hat{i}\vec{E}_0 \sin(\omega t - kz), \vec{B} = \hat{j}B_0 \sin(\omega t - kz)$
(b) $\vec{E} = \hat{i}\vec{E}_0 \sin(\omega t - kz), \vec{B} = \hat{j}B_0 \cos(\omega t - kz)$
(c) $\vec{E} = \hat{j}\vec{E}_0 \sin(\omega t - kz), \vec{B} = -\hat{i}B_0 \sin(\omega t - kz)$
(d) $\vec{E} = \hat{i}\vec{E}_0 \sin(\omega t - kz), \vec{B} = \hat{j}B_0 \sin(\omega t - kz + \delta)$

Ans. **(a,c)**
Sol. Electric & magnetic field oscillates perpendicular to each other and also perpendicular to direction of propagation of light.

46. In case of photoelectric effect,
(a) Since photons are absorbed as a single unit, there is no significant time delay in the emission of photoelectrons.
(b) Einstein's analysis gives a critical frequency $v_0 = \frac{e\varphi}{h}$, where φ is the work function and the light of this frequency ejects electrons with maximum kinetic energy.
(c) Only a small fraction of the incident photons succeed in ejecting photoelectrons while most of them are absorbed by the system as a whole and generate thermal energy
(d) The maximum kinetic energy of the electrons is dependent on the intensity of radiation.

Ans. **(a,c)**
Sol. Efficiency of photons are usually very less.

47. A parallel combination of an inductor coil and a resistance of 60 ohm is connected to an ac source. The current in coil, current in the resistance and the source current are respectively 3A, 2.5 A and 4.5 A respectively. Therefore,
(a) Kirchhoff's currect law is NOT applicable to ac circuits
(b) impendance of the coil is 50 ohm
(c) electric power dissipated in the coil is 150 watt.
(d) impedance of the circuit is 33.3 ohm.

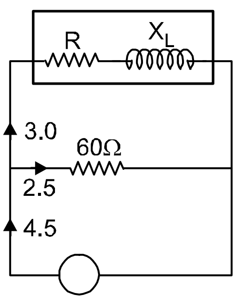
Ans. **(b,c,d)**
Sol.

$$3\sqrt{R^2 + X_L^2} = 2.5 \times 60$$

$$\sqrt{R^2 + X_L^2} = 50 \qquad ....(1)$$

$$Z_{eq} = \frac{(R + \hat{j}X_L)(60)}{60 + R + \hat{j}X_L}$$

$$|Z_{eq}| = \frac{60\sqrt{R^2 + X_L^2}}{\sqrt{(60+R)^2 + X_L^2}} = \frac{60 \times 50}{\sqrt{3600 + 2500 + 120R}}$$



$$\frac{4.5\times60\times50}{\sqrt{6100+120R}} = 2.5 \times 60$$

$$6100 + 120\,R = (90)^2 = 8100$$

$$120\,R = 2000;\;\; R = \frac{50}{3}\Omega$$

$$|\,Z_{eq}\,| = \frac{60\times50}{90} = \frac{100}{3}\Omega$$

$$P_R = (3)^2\times\frac{50}{3} = 150\,\text{watt}$$

48. The nuclear forces
(a) are stronger being roughly hundred times that of electromagnetic forces.
(b) have a short range dominant over a distance of about a few Fermi.
(c) are central forces independent of the spin of the nucleons.
(d) are independent of the nuclear charge.

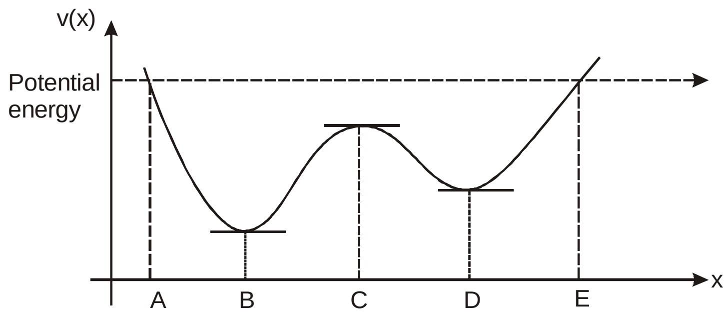
Ans. **(a,b,d)**

49. Consider a mole of a sample of hydrogen gas at NTP
(a) The volume of the gas is exactly 2.24×10^{-2} m^3
(b) The volume of the gas is approximately 2.24×10^{-2} m^3
(c) The gas will be thermal equilibrium with 1 mole of oxygen gas at NTP
(d) The gas will be in thermodynamic equilibrium with 1 mole of oxygen at NTP

Ans. **(b,c,d)**

Sol. Two gases will be in thermal equilibrium if temperature of the two gases are same. Two gases are said to be in thermodynamic equilibrium if both temperature & pressure of two gases are same.

50. A particle moves in one dimension in a conservative force field. The potential energy is depicted in the graph below:



If the particle starts to move from rest from the point A, then
(a) The speed is zero at the points A and E
(b) The acceleration vanishes at the A, B, C, D E
(c) The acceleration vanishes at the points B, C, D
(d) The speed is maximum at the point D

50. **(a,c)**

Sol. As particle is moving in conservative force field U + K = constant.

$$F = \frac{\partial U}{\partial x}$$

Hence force will be zero at B, C and D.

PART B

1. (a) A conductor having resistance R (independent of temperature) and thermal capacity C is initially at temperature T_0 same as that of the surrounding. At time t = 0 it is connected to a source with constant voltage V. The thermal power dissipated by the conductor to the surrounding varies as $q = k(T-T_0)$. Determine the temperature T of the conductor at any time t and at the time $t = \frac{C}{k}$.

(b) A particle moves rectilinearly in an electric field $E = E_0 - ax$ where a is a positive constant and x is the distance from the point where the particle is initially at rest. Let the particle have a specific charge $\frac{q}{m}$. Find (I) the distance covered by the particle till the moment at which it once again comes to rest, and (II) acceleration of the particle at this moment

Sol. **(a)** Let temperature of conductor changes from T to T + dT in time 't' to 't + dt'.

$\therefore \quad CdT = i^2Rdt - K(T - T_0)\,dt$

$\Rightarrow \int_{T_0}^{T} \frac{dT}{i^2R + KT_0 - KT} = \int_0^t \frac{1}{c}dt$

$\Rightarrow \quad T = \frac{i^2R}{K}\left(1 - e^{-\frac{K}{a}t}\right) + T_0$ Temperature after time t

At $t = \frac{c}{K}$

$T = \frac{i^2R}{K}\left(1 - \frac{1}{e}\right) + T_0$

(b) Acceleration of particle

$f = \frac{q}{m}(E_0 - ax)$ ....(i)

$\Rightarrow \quad v\frac{dv}{dx} = \frac{q}{m}(E_0 - ax)\ \frac{q}{m}$

$\Rightarrow \quad \frac{v^2}{2} = \frac{q}{m}\left(E_0x - \frac{ax^2}{2}\right)$

$v = 0 \Rightarrow x = 0$ or $x = \frac{2E_0}{a}$

$\therefore$ Required distance travelled = $\frac{2E_0}{a}$

From (i)

Required acceleration = $\frac{q}{m}\left\{E_0 - a\left(\frac{2E_0}{a}\right)\right\} = \frac{q}{m}E_0$

2. One mole of an ideal gas ($\gamma = 1.4$) with initial pressure of 2 atm and temperature of 57°C is taken to twice its volume through different processes that include isothermal, isobaric and adiabatic processes. Determine the processes where maximum work is done and the amount of work in this case. By what percentage is this work larger than the work done in a processes in which it is the least?

Sol. **Work done in isothermal process**

$W_T = nRT\ \ln\left(\frac{V_2}{V_1}\right)$

$= 1 \times 8.314 \times 330 \times \ln 2$

$= 1901.33$ J

Work done in isobaric process :

$W_P = P(V_2 - V_1)$

$= PV_0$

$= nRT$

$\therefore \quad W_P = 1 \times 8.314 \times 330$

$= 2743.62$ J

Work done in adiabatic process :

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

$\Rightarrow$ $$T = 330\left(\frac{V_0}{2V_0}\right)^{(1.4-1)}$$

$\therefore$ $$T = \frac{330}{2^{(0.4)}}K$$

$$W_A = \frac{-nR(T_2 - T_1)}{\gamma - 1}$$

$$= \frac{8.314\left(330 - \frac{330}{2^{(0.4)}}\right)}{0.4} = 1662.8 \text{ J}$$

Obviously $W_P > W_T > W_A$

Thus work done in isobaric process is maximum and

Thus work done in adiabatic process is least.

Now required percentage = $\frac{W_P - W_A}{W_P} \times 100$ = 39.39%

3. A railway carriage of mass M_c filled with sand of mass M_s moves along the rails. The carriage is given an impulse and its starts with a velocity v_0. At the same time it is observed that the sand starts leaking through a hole at the bottom of the carriage at a constant mass rate λ. Find the distance at which the carriage becomes empty and the velocity attained by the carriage at the time. (Neglect the friction along the rails)

Sol. Force of thrust

$$\vec{F}_{Th} = \vec{V}_{rel.}\frac{dm}{dt} = 0 \quad \because \vec{V}_{rel.} = 0$$

$\therefore$ Velocity of railway carriage will remain same

$\therefore$ Velocity of railway carriage when it becomes empty = V_0

Rate at which mass of carriage is decreasing

$$\frac{dm}{dt} = -\lambda \Rightarrow \Delta m = \lambda \Delta t \quad (\because \lambda \text{ is constant})$$

$\therefore$ $\Delta t = \frac{M_s}{\lambda}$

4. Show that, for any angle of incidence on a prism

$$\frac{\sin\frac{1}{2}(A+\delta)}{\sin\frac{1}{2}(A)} = \mu\frac{\cos\frac{1}{2}(r_1 - r_2)}{\cos\frac{1}{2}(i-e)}$$

(symbols have usual meanings)

and that the right-hand side reduces to at minimum deviation.

Sol. $\sin i = \mu \sin r_1$ ....(1)

$\sin e = \mu \sin r_2$ ....(2)

$A = r_1 + r_2$ ....(3)

$\delta = i + e - A$ ....(4)

From (1) & (2)

$$\frac{\sin i}{\sin e} = \frac{\sin r_1}{\sin r_2}$$

$$\frac{\sin i + \sin e}{\sin e} = \frac{\sin r_1 + \sin r_2}{\sin r_2}$$

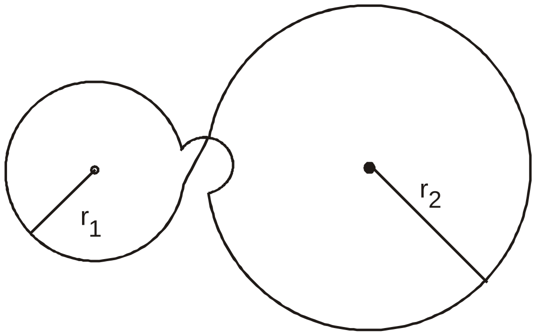
$$\frac{2\sin\left(\frac{i+e}{2}\right)\cos\left(\frac{i-e}{2}\right)}{\sin e} = \frac{2\sin\left(\frac{r_1+r_2}{2}\right)\cos\left(\frac{r_1-r_2}{2}\right)}{\sin r_2}$$

$$\frac{\sin\left(\frac{A+\delta}{2}\right)}{\sin(A/2)} = \frac{\sin e}{\sin r_2}\cdot\frac{\cos\frac{1}{2}(r_1-r_2)}{\cos\frac{1}{2}(i-e)}$$

$$\frac{\sin\left(\frac{A+\delta}{2}\right)}{\sin(A/2)} = \mu\frac{\cos\frac{1}{2}(r_1-r_2)}{\cos\frac{1}{2}(i-e)}$$

5. (a) A small amount of solution containing Na^{24} nuclides with activity 20500 disintegrations per second was injected in the blood stream of a person. The activity of 1 ml of blood sample taken after 5 hours later, was found to be 20 disintegration per minute. The half life of the radioactive nuclides is 15 hours. Find the total volume of the blood of this person.

(b) The wire loop shown in the figure lies in uniform magnetic induction $B = B_0\cos\omega t$ perpendicular to its plane. (Given $r_1 = 10$ cm and $r_2 = 20$ cm, $B_0 = 20$ m T and $\omega = 100\pi$). Find the amplitude of the current induced in the loop if its resistance is 0.1 Ω/m



Sol. (a) Activity of blood after 5 yrs.

$A_1 = \frac{A_0}{2^{(t/t_{1/2})}} \Rightarrow A_0 = \frac{20}{60}\times 2^{1/3} = \frac{1}{3}(2^{1/3})$ dps

$\therefore$ Volume of blood = $\frac{20500}{\frac{1}{3}\times 2^{1/3}}$ ml = 48809.52 ml

(b) $\phi = B\pi(r_2^2 - r_1^2) = B_0\cos\omega t\, \pi(r_2^2 - r_1^2)$

$\therefore$ $E_{ind} = -\frac{d\phi}{dt} = B_0\omega\sin\omega t\,\pi(r_2^2 - r_1^2)$

$= B_0\pi\omega(r_2^2 - r_1^2)\sin\omega t$

$\therefore$ $i = \frac{B_0\pi\omega(r_2^2 - r_1^2)}{R}\sin\omega t = \frac{10}{\pi}\sin(100\pi t)$

$\therefore$ Current amplitude = $\frac{10}{\pi}$ Amp.

* * * * *

Physics constant you may need :

1. Charge on electron $e = 1.6 \times 10^{-19}$C
2. Mass of electron $m_e = 9.1 \times 10^{-31}$ kg
3. Universal gravitational constant $G = 6.67 \times 10^{11}$ Nm^2/kg^2
4. Permittivity of free space $\varepsilon_0 = 8.85\times 10^{-12}\ C^2/N\ m^2$
5. Gas constant R = 8.31 J/K mol
6. Planck constant $h = 6.62 \times 10^{-34}$ Js
7. Stefan constant $\sigma = 5.67 \times 10^{-8}$ $W/m^2\ K^4$
8. Boltzman constant $k = 1.38 \times 10^{-23}$ J/K