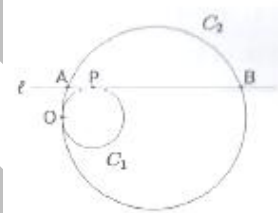


**KVPY – XII CLASS - STREAM – SB/SX**  
(Held on 1<sup>st</sup> November 2015)

**PART – I**  
**MATHEMATICS**

1. The number of ordered pairs  $(x, y)$  of real numbers that satisfy the simultaneous equations  
(A) 0 (B) 1  
(C) 2 (D) 4
2. If  $z$  is a complex number satisfying  $|z^3 + z^{-3}| \leq 2$ , then the maximum possible value of  $|z + z^{-1}|$  is  
(A) 2 (B)  $\sqrt[3]{2}$   
(C)  $2\sqrt{2}$  (D) 1
3. The largest perfect square that divides  $2014^3 - 2013^3 + 2012^3 - 2011^3 + \dots + 2^3 - 1^3$  is  
(A)  $1^2$  (B)  $2^2$   
(C)  $1007^2$  (D)  $2014^2$
4. Suppose OABC is a rectangle in the  $xy$ -plane where O is the origin and A, B lie on the parabola  $y = x^2$ . Then C must lie on the curve  
(A)  $y = x^2 + 2$  (B)  $y = 2x^2 + 1$   
(C)  $y = -x^2 + 2$  (D)  $y = -2x^2 + 1$
5. Circle  $C_1$  and  $C_2$  of radii  $r$  and  $R$  respectively, touch each other as shown in figure. The line  $\ell$ , which is parallel to the line joining the centres of  $C_1$  and  $C_2$  is tangent to  $C_1$  at P and intersects  $C_2$  at A, B. If  $R^2 = 2r^3$ , then  $\angle AOB$  equals



- (A)  $22\frac{1}{2}^\circ$  (B)  $45^\circ$   
(C)  $60^\circ$  (D)  $67\frac{1}{2}^\circ$
6. The shortest distance from the origin to a variable point on the sphere  $(x - 2)^2 + (y - 3)^2 + (z - 6)^2 = 1$  is  
(A) 5 (B) 6  
(C) 7 (D) 8
  7. The number of real numbers  $\lambda$  for which the equality  

$$\frac{\sin(\lambda\alpha)}{\sin\alpha} - \frac{\cos(\lambda\alpha)}{\cos\alpha} = \lambda - 1$$
 Holds for all real  $\alpha$  which are not integral multiples of  $\pi/2$  is  
(A) 1 (B) 2  
(C) 3 (D) Infinite
  8. Suppose ABCDEF is a hexagon such that  $AB = BC = CD = 1$  and  $DE = EF = FA = 2$ . If the vertices A, B, C, D, E, F are concyclic the radius of the circle passing through them is  
(A)  $\sqrt{\frac{5}{2}}$  (B)  $\sqrt{\frac{7}{3}}$

$$(C) \sqrt{\frac{11}{5}}$$

$$(D) \sqrt{2}$$

9. Let  $p(x)$  be a polynomial such that  $p(x) - p'(x) = x^n$ , where  $n$  is a positive integer. The  $p(0)$  equals

$$(A) n!$$

$$(B) (n-1)!$$

$$(C) \frac{1}{n!}$$

$$(D) \frac{1}{(n-1)!}$$

10. The value of the limit

$$\lim_{x \rightarrow 0} \left( \frac{x}{\sin x} \right)^{6/x^2} \text{ is}$$

$$(A) e$$

$$(B) e^{-1}$$

$$(C) e^{-1/6}$$

$$(D) e^6$$

11. Among all sectors of a fixed perimeter, choose the one with maximum area. Then the angle at the centre of this sector (i.e. the angle between the bounding radii) is

$$(A) \frac{\pi}{3}$$

$$(B) \frac{3}{2}$$

$$(C) \sqrt{3}$$

$$(D) 2$$

12. Define a function  $f: \mathbb{R} \rightarrow \mathbb{R}$  by  $f(x) = \max \{|x|, |x-1|, \dots, |x-2n|\}$ ,

where  $n$  is a fixed natural number, Then  $\int_0^{2n} f(x) dx$  is

$$(A) n$$

$$(B) n^2$$

$$(C) 3n$$

$$(D) 3n^2$$

13. If  $p(x)$  is a cubic polynomial with  $p(1) = 3$ ,  $p(0) = 2$  and  $p(-1) = 4$ , then  $\int_{-1}^1 p(x) dx$  is

$$(A) 2$$

$$(B) 3$$

$$(C) 4$$

$$(D) 5$$

14. Let  $x > 0$  be a fixed real number. Then the integral  $\int_0^x e^{-t} |x-t| dt$  is equal to

$$(A) x + 2e^{-x} - 1$$

$$(B) x - 2e^{-x} + 1$$

$$(C) x + 2e^{-x} + 1$$

$$(D) -x - 2e^{-x} + 1$$

15. An urn contains marbles of four colours: red, white, blue and green. When four marbles are drawn without replacement, the following events are equally likely

(1) the selection of four red marbles

(2) the selection of one white and three red marbles

(3) the selection of one white, one blue and two red marbles

(4) the selection of one marble of each colour

The smallest total number of marbles satisfying the given condition is

$$(A) 19$$

$$(B) 21$$

$$(C) 46$$

$$(D) 69$$

16. There are 6 boxes labeled  $B_1, B_2, \dots, B_6$ . In each trial, two fair dice  $D_1, D_2$  are thrown. If  $D_1$  shows  $j$  and  $D_2$  shows  $k$ , then  $j$  balls are put into the box  $B_k$ . After  $n$  trials, what is the probability that  $B_1$  contains the most one ball?

$$(A) \left( \frac{5^{n-1}}{6^{n-1}} \right) + \left( \frac{5^n}{6^n} \right) \left( \frac{1}{6} \right)$$

$$(B) \left( \frac{5^n}{6^n} \right) + \left( \frac{5^{n-1}}{6^{n-1}} \right) \left( \frac{1}{6} \right)$$

$$(C) \left(\frac{5^n}{6^n}\right) + n\left(\frac{5^{n-1}}{6^{n-1}}\right)\left(\frac{1}{6}\right)$$

$$(D) \left(\frac{5^n}{6^n}\right) + n\left(\frac{5^{n-1}}{6^{n-1}}\right)\left(\frac{1}{6^2}\right)$$

17. Let  $\vec{a} = 6\vec{i} - 3\vec{j} - 6\vec{k}$  and  $\vec{d} = \vec{i} + \vec{j} + \vec{k}$ . Suppose that  $\vec{a} = \vec{b} + \vec{c}$  where  $\vec{b}$  and  $\vec{c}$  is perpendicular to  $\vec{d}$ . Then  $\vec{c}$  is

$$(A) 5\vec{i} - 4\vec{j} - \vec{k}$$

$$(B) 7\vec{i} - 2\vec{j} - 5\vec{k}$$

$$(C) 4\vec{i} - 5\vec{j} + \vec{k}$$

$$(D) 3\vec{i} + 6\vec{j} - 9\vec{k}$$

18. If  $\log_{(3x-1)}(x-2) = \log_{(9x^2-6x+1)}(2x^2-10x-2)$ , then x equals

$$(A) 9 - \sqrt{15}$$

$$(B) 3 + \sqrt{15}$$

$$(C) 2 + \sqrt{5}$$

$$(D) 6 - \sqrt{5}$$

19. Suppose a, b, c are positive integers such that  $2^a + 4^b + 8^c = 328$ . Then  $\frac{a+2b+3c}{abc}$  is equal to

$$(A) \frac{1}{2}$$

$$(B) \frac{5}{8}$$

$$(C) \frac{17}{24}$$

$$(D) \frac{5}{6}$$

20. The sides of a right – angled triangle are integers. The length of one of the sides is 12. The largest possible radius of the incircle of such a triangle is

$$(A) 2$$

$$(B) 3$$

$$(C) 4$$

$$(D) 5$$

### PHYSICS

21. A small box resting on one edge of the table is stuck in such a way that it slides off the other edge, 1 m away, after 2 seconds. The coefficient of kinetic friction between the box and the table

(A) must be less than 0.05

(B) must be exactly zero

(C) must be more than 0.05

(D) must be exactly 0.05

22. Carbon – 11 decays to boron – 11 according to the following formula.



Assume that positrons ( $e^{+}$ ) produced in the decay combine with free electrons in the atmosphere and annihilate each other almost immediately. Also assume that the neutrinos ( $\nu_e$ ) are massless and do not interact with the environment. At  $t = 0$  we have  $1 \mu\text{g}$  of  ${}_{6}^{11}\text{C}$ . If the half – life of the decay process is  $t_0$ , then net energy produced between time  $t = 0$  and  $t = 2t_0$  will be nearly

$$(A) 8 \times 10^{18} \text{ MeV}$$

$$(B) 8 \times 10^{16} \text{ MeV}$$

$$(C) 4 \times 10^9 \text{ MeV}$$

$$(D) 4 \times 10^{16} \text{ MeV}$$

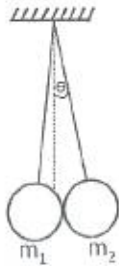
23. Two uniform plates of the same thickness and area but of different materials, one shaped like an isosceles triangle and the other shaped like a rectangle are joined together to form a composite body as shown in the figure. If the centre of mass of the composite body is located at the mid point of their common side, the ratio between masses of the triangle to that of the rectangle is



- (A) 1 : 1  
(C) 3 : 4

- (B) 4 : 3  
(D) 2 : 1

24. Two spherical objects each of radii  $R$  and masses  $m_1$  and  $m_2$  are suspended using two strings of equal length  $L$  as shown in the figure ( $R \ll L$ ). The angle,  $\theta$  which mass  $m_2$  makes with the vertical is approximately



(A)  $\frac{m_1 R}{(m_1 + m_2)L}$

(B)  $\frac{2m_1 R}{(m_1 + m_2)L}$

(C)  $\frac{2m_2 R}{(m_1 + m_2)L}$

(D)  $\frac{m_2 R}{(m_1 + m_2)L}$

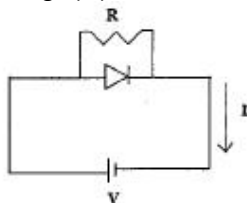
25. A horizontal disk of moment of inertia  $4.25 \text{ kg} - \text{m}^2$  with respect to its axis of symmetry is spinning counter clockwise at 15 revolutions per seconds about its axis, as viewed from above. A second disk of moment of inertia  $1.80 \text{ kg} - \text{m}^2$  with respect to its axis of symmetry is spinning clockwise at 25 revolutions per second as viewed from above about the same axis and is dropped on top of the first disk. The two disks stick together and rotate as one about their axis of symmetry. The new angular velocity of the system as viewed from above is close to

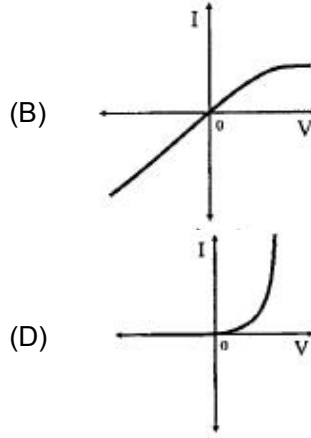
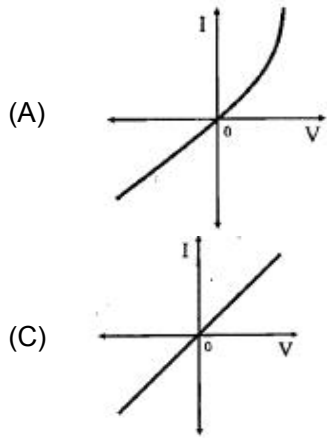
- (A) 18 revolutions/second and clockwise  
(B) 18 revolutions/second and counter clockwise  
(C) 3 revolutions/second and clockwise.  
(D) 3 revolutions/second and counter clockwise

26. A boy is standing on top of a tower of height 85 m and throws a ball in the vertically upward direction with a certain speed. If 5.25 seconds later he hears the ball hitting the ground, then the speed with which the boy threw the ball is (take  $g = 10 \text{ m/s}^2$ , speed of sound in air = 340 m/s)

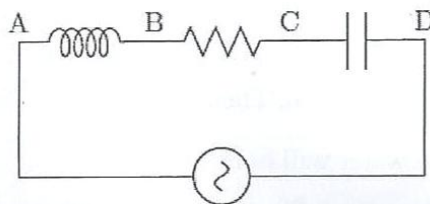
- (A) 6 m/s  
(C) 10 m/s
- (B) 8 m/s  
(D) 12 m/s

27. For a diode connected in parallel with a resistor, which is the most likely current(I) – voltage(V) characteristic?





28. A beam of monoenergetic electrons, which have been accelerated from rest by a potential  $U$ , is used to form an interference pattern in a Young's Double Slit experiment. The electrons are now accelerated by potential  $4U$ . Then the fringe width  
 (A) remains the same  
 (B) is half the original fringe width.  
 (C) is twice the original fringe width.  
 (D) is one-fourth the original fringe width.
29. A point charge  $Q (= 3 \times 10^{-12} \text{ C})$  rotates uniformly in a vertical circle of radius  $R = 1 \text{ mm}$ . the axis of the circle is aligned along the magnetic axis of the earth. At what value of the angular speed  $\omega$ , the effective magnetic field at the center of the circle will be reduced to zero? (Horizontal component of Earth's magnetic field is  $30 \text{ micro Tesla}$ )  
 (A)  $10^{11} \text{ rad/s}$   
 (B)  $10^9 \text{ rad/s}$   
 (C)  $10^{13} \text{ rad/s}$   
 (D)  $10^7 \text{ rad/s}$
30. A closed bottle containing water at  $30^\circ\text{C}$  is open on the surface of the moon. Then  
 (A) the water will boil  
 (B) the water will come out as a spherical ball.  
 (C) the water will freeze  
 (D) the water will decompose into hydrogen and oxygen
31. A simple pendulum of length  $\ell$  is made to oscillate with an amplitude of  $45^\circ$ . The acceleration due to gravity is  $g$ . Let  $T_0 = 2\pi\sqrt{\ell/g}$ . The time period of oscillation of this pendulum will be  
 (A)  $T_0$  irrespective of the amplitude  
 (B) slightly less than  $T_0$   
 (C) slightly more than  $T_0$   
 (D) dependent on whether it swings in a plane aligned with the north-south or east-west directions.
32. An ac voltmeter connected between points A and B in the circuit below reads  $36 \text{ V}$ . If it is connected between A and C, the reading is  $39 \text{ V}$ . The reading when it is connected between B and D is  $25 \text{ V}$ . What will the voltmeter read when it is connected between A and D? (Assume that the voltmeter reads true rms voltage values and that the source generates a pure ac)

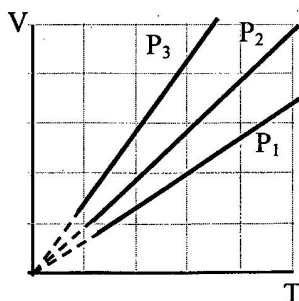


- (A)  $\sqrt{481}$  V (B) 31 V  
 (C) 61 V (D)  $\sqrt{3361}$  V

33. A donor atom in a semiconductor has a loosely bound electron. The orbit of this electron is considerably affected by the semiconductor material but behaves in many ways like an electron orbiting a hydrogen nucleus. Given that the electron has an effective mass of  $0.07 m_e$ , (where  $m_e$  is mass of the free electron) and the space in which it moves has a permittivity  $13\epsilon_0$ , then the radius of the electron's lowermost energy orbit will be close to (The Bohr radius of the hydrogen atom is  $0.53 \text{ \AA}$ )

- (A)  $0.53 \text{ \AA}$  (B)  $243 \text{ \AA}$   
 (C)  $10 \text{ \AA}$  (D)  $100 \text{ \AA}$

34. The state of an ideal gas was changed isobarically. The graph depicts three such isobaric lines. Which of the following is true about the pressures of the gas?



- (A)  $P_1 = P_2 = P_3$  (B)  $P_1 > P_2 > P_3$   
 (C)  $P_1 < P_2 < P_3$  (D)  $P_1/P_2 = P_3/P_1$

35. A metallic ring of radius  $a$  and resistance  $R$  is held fixed with its axis along a spatially uniform magnetic field whose magnitude is  $B_0 \sin(\omega t)$ . Neglect gravity. Then,

- (A) the current in the ring oscillates with a frequency of  $2\omega$ .  
 (B) the Joule heating loss in the ring is proportional to  $a^2$ .  
 (C) the force per unit length on the ring will be proportional to  $B_0^2$ .  
 (D) the net force on the ring is non-zero.

36. The dimensions of the area  $A$  of a black hole can be written in terms of the universal gravitational constant  $G$ , its mass  $M$  and the speed of light  $C$  as  $A = G^\alpha M^\beta c^\gamma$ . Here

- (A)  $\alpha = -2$ ,  $\beta = -2$ , and  $\gamma = 4$  (B)  $\alpha = 2$ ,  $\beta = 2$ , and  $\gamma = -4$   
 (C)  $\alpha = 3$ ,  $\beta = 3$ , and  $\gamma = -2$  (D)  $\alpha = -3$ ,  $\beta = -3$ , and  $\gamma = 2$

37. A 160 watt infrared source is radiating light of wavelength  $50000 \text{ \AA}$  uniformly in all directions. The photon flux at a distance of  $1.8 \text{ m}$  is of the order of

- (A)  $10 \text{ m}^{-2} \text{ s}^{-1}$  (B)  $10^{10} \text{ m}^{-2} \text{ s}^{-1}$   
 (C)  $10^{15} \text{ m}^{-2} \text{ s}^{-1}$  (D)  $10^{20} \text{ m}^{-2} \text{ s}^{-1}$

38. A wire bent in the shape of a regular  $n$  – polygonal loop carries a steady current  $I$ . Let  $l$  be the perpendicular distance of a given segment and  $R$  be the distance of a vertex both from the centre of the loop. The magnitude of the magnetic field at the centre of the loop is given by

- (A)  $\frac{n\mu_0 I}{2\pi l} \sin(\pi/n)$  (B)  $\frac{n\mu_0 I}{2\pi R} \sin(\pi/n)$   
 (C)  $\frac{n\mu_0 I}{2\pi l} \cos(\pi/n)$  (D)  $\frac{n\mu_0 I}{2\pi R} \cos(\pi/n)$

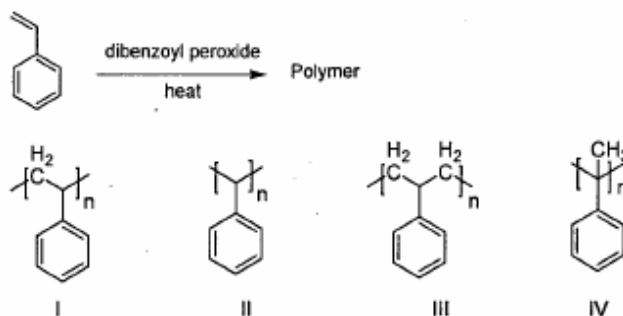
39. The intensity of sound during the festival season increased by 100 times. This could imply a decibel level rise from

- (A) 20 to 120 dB (B) 70 to 72 dB  
 (C) 100 to 10000 dB (D) 80 to 100 dB

40. One end of a slack wire (Young's modulus  $Y$ , length  $L$  and cross-sectional area  $A$ ) is clamped to a rigid wall and the other end to a block (mass  $m$ ) which rests on a smooth horizontal plane. The block is set in motion with a speed  $v$ . What is the maximum distance the block will travel after the wire becomes taut?
- (A)  $v\sqrt{\frac{mL}{Ay}}$  (B)  $v\sqrt{\frac{2mL}{AY}}$   
 (C)  $v\sqrt{\frac{mL}{2AY}}$  (D)  $L\sqrt{\frac{mv}{AY}}$

### CHEMISTRY

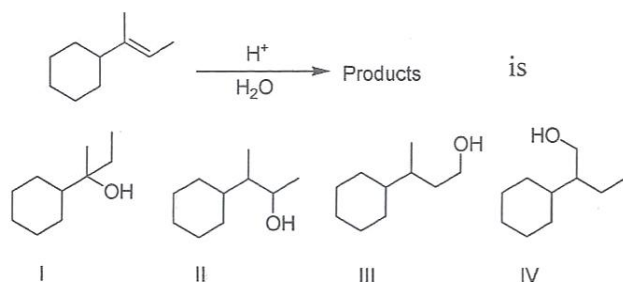
41. The Lewis acid strength of  $BBr_3$ ,  $BCl_3$  and  $BF_3$  is in the order  
 (A)  $BBr_3 < BCl_3 < BF_3$  (B)  $BCl_3 < BF_3 < BBr_3$   
 (C)  $BF_3 < BCl_3 < BBr_3$  (D)  $BBr_3 < BF_3 < BCl_3$
42.  $O^{2-}$  is isoelectronic with  
 (A)  $Zn^{2+}$  (B)  $Mg^{2+}$   
 (C)  $K^+$  (D)  $Ni^{2+}$
43. The  $H-C-H$ ,  $H-N-H$ , and  $H-O-H$  bond angles (in degrees) in methane, ammonia and water are respectively, closest to  
 (A) 109.5, 104.5, 107.1 (B) 109.5, 107.1, 104.5  
 (C) 104.5, 107.1, 109.5 (D) 107.1, 104.5, 109.5
44. In alkaline medium, the reaction of hydrogen peroxide with potassium permanganate produces a compound in which the oxidation state of Mn is  
 (A) 0 (B) +2  
 (C) +3 (D) +4
45. The rate constant of a chemical reaction at a very high temperature will approach  
 (A) Arrhenius frequency factor divided by the ideal gas constant  
 (B) activation energy  
 (C) Arrhenius frequency factor  
 (D) activation energy divided by the ideal gas constant
46. The standard reduction potentials (in V) of a few metal ion/metal electrodes are given below.  $Cr^{3+}/Cr = -0.75$ ;  $Cu^{2+}/Cu = +0.34$ ;  $Pb^{2+}/Pb = -0.13$ ;  $Ag^+/Ag = +0.8$ . The reducing strength of the metals follows the order.  
 (A)  $Ag > Cu > Pb > Cr$  (B)  $Cr > Pb > Cu > Ag$   
 (C)  $Pb > Cr > Ag > Cu$  (D)  $Cr > Ag > Cu > Pb$
47. Which of the following molecules can exhibit optical activity?  
 (A) 1 - bromopropane (B) 2 - bromobutane  
 (C) 3 - bromopentane (D) bromocyclohexane
48. The structure of the polymer obtained by the following reaction is



- (A) I  
(C) III
- (B) II  
(D) IV
49. The major product of the reaction between  $\text{CH}_3\text{CH}_2\text{ONa}$  and  $(\text{CH}_3)_3\text{CCl}$  in ethanol is  
 (A)  $\text{CH}_3\text{CH}_2\text{OC}(\text{CH}_3)_3$   
 (B)  $\text{CH}_2 = \text{C}(\text{CH}_3)_2$   
 (C)  $\text{CH}_3\text{CH}_2\text{C}(\text{CH}_3)_3$   
 (D)  $\text{CH}_3\text{CH} = \text{CHCH}_3$
50. When  $\text{H}_2\text{S}$  gas is passed through a hot acidic aqueous solution containing  $\text{Al}^{3+}$ ,  $\text{Cu}^{2+}$ ,  $\text{Pb}^{2+}$  and  $\text{Ni}^{2+}$ , a precipitate is formed which consists of  
 (A)  $\text{CuS}$  and  $\text{Al}_2\text{S}_3$   
 (B)  $\text{PuS}$  and  $\text{NiS}$   
 (C)  $\text{CuS}$  and  $\text{NiS}$   
 (D)  $\text{PbS}$  and  $\text{CuS}$
51. The electronic configuration of an element with the largest difference between the 1<sup>st</sup> and 2<sup>nd</sup> ionization energies is  
 (A)  $1s^2 2s^2 2p^6$   
 (B)  $1s^2 2s^2 2p^6 3s^1$   
 (C)  $1s^2 2s^2 2p^6 3s^2$   
 (D)  $1s^2 2s^2 2p^1$
52. The order of electronegativity of carbon is  $sp$ ,  $sp^2$  and  $sp^3$  hybridized states follows  
 (A)  $sp > sp^2 > sp^3$   
 (B)  $sp^3 > sp^2 > sp$   
 (C)  $sp > sp^3 > sp^2$   
 (D)  $sp^2 > sp > sp^3$
53. The most abundant transition metal in human body is  
 (A) copper  
 (B) iron  
 (C) zinc  
 (D) manganese
54. The molar conductivities of  $\text{HCl}$ ,  $\text{NaCl}$ ,  $\text{CH}_3\text{COOH}$ , and  $\text{CH}_3\text{COONa}$  at infinite dilution follow the order  
 (A)  $\text{HCl} > \text{CH}_3\text{COOH} > \text{NaCl} > \text{CH}_3\text{COONa}$   
 (B)  $\text{CH}_3\text{COONa} > \text{HCl} > \text{NaCl} > \text{CH}_3\text{COOH}$   
 (C)  $\text{HCl} > \text{NaCl} > \text{CH}_3\text{COOH} > \text{CH}_3\text{COONa}$   
 (D)  $\text{CH}_3\text{COOH} > \text{CH}_3\text{COONa} > \text{HCl} > \text{NaCl}$
55. The spin only magnetic moment of  $[\text{ZCl}_4]^{2-}$  is 3.87 BM where Z is  
 (A) Mn  
 (B) Ni  
 (C) Co  
 (D) Cu
56. If  $\alpha$  - D - Glucose is dissolved in water and kept for a few hours, the major constituent(s) present in the solution is (are)  
 (A)  $\alpha$  - D - glucose  
 (B) mixture of  $\beta$  - D - glucose and open chain D - glucose  
 (C) open chain D - glucose  
 (D) mixture of  $\alpha$  - D - glucose and  $\beta$  - D - glucose
57. The pH of 1M aqueous solutions of  $\text{HCl}$ ,  $\text{CH}_3\text{COOH}$  and  $\text{HCOOH}$  follows the order  
 (A)  $\text{HCl} > \text{HCOOH} > \text{CH}_3\text{COOH}$   
 (B)  $\text{HCl} = \text{HCOOH} > \text{CH}_3\text{COOH}$   
 (C)  $\text{CH}_3\text{COOH} > \text{HCOOH} > \text{HCl}$   
 (D)  $\text{CH}_3\text{COOH} = \text{HCOOH} > \text{HCl}$

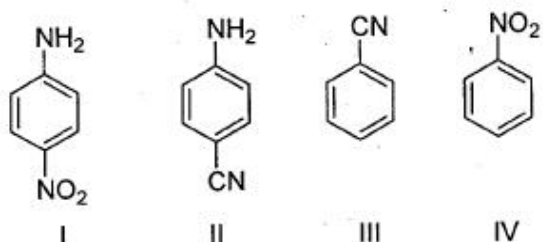


58. The major product of the reaction



- (A) I (B) II  
(C) III (D) IV

59. Reaction of aniline with  $\text{NaNO}_2 + \text{dil. HCl}$  at  $0^\circ\text{C}$  followed by reaction with  $\text{CuCN}$  yields



- (A) I (B) II  
(C) III (D) IV

60. Schottky defect in a crystal arises due to  
(A) creation of equal number of cation and anion vacancies  
(B) creation of unequal number of cation and anion vacancies  
(C) migration of cations to interstitial voids  
(D) migration of anions to interstitial voids

## PART – II MATHEMATICS

81. Let  $c = (\sqrt{50} + 7)^{1/3} - (\sqrt{50} - 7)^{1/3}$ . Then

- (A)  $x = 2$   
(B)  $x = 3$   
(C)  $x$  is a rational number, but not an integer  
(D)  $x$  is an irrational number

82. Let  $(1 + x + x^2)^{2014} = a_0 + a_1x + a_2x^2 + a_3x^3 + \dots + a_{4028}x^{4028}$ , and let

$$A = a_0 - a_3 + a_6 - \dots + a_{4026}$$

$$B = a_1 - a_4 + a_7 - \dots - a_{4027}$$

$$C = a_2 - a_5 + a_8 - \dots + a_{4028}$$

- (A)  $|A| = |B| > |C|$  (B)  $|A| = |B| < |C|$   
(C)  $|A| = |C| > |B|$  (D)  $|A| = |C| < |B|$

83. A mirror in the first quadrant is in the shape of a hyperbola whose equation is  $xy = 1$ . A light source in the second quadrant emits a beam of light that hits the mirror at the point  $(2, 1/2)$ . If the reflected ray is parallel to the  $y$ -axis, the slope of the incident beam is

- (A)  $13/8$  (B)  $7/4$   
(C)  $15/8$  (D)  $2$

84. Let  $C(\theta) = \sum_{n=0}^{\infty} \frac{\cos(n\theta)}{n!}$  which of the following statements is FALSE?
- (A)  $C(0) \cdot C(\pi) = 1$  (B)  $C(0) + C(\pi) > 2$   
 (C)  $C(\theta) > 0$  for all  $\theta \in \mathbb{R}$  (D)  $C'(\theta) \neq 0$  for all  $\theta \in \mathbb{R}$

85. Let  $a > 0$  be a real number. Then the limit

$$\lim_{x \rightarrow 2} \frac{a^x + a^{3-x} - (a^2 + a)}{a^{3-x} - a^{x/2}}$$
 is

- (A)  $2 \log a$  (B)  $-\frac{4}{3}a$   
 (C)  $\frac{a^2 + a}{2}$  (D)  $\frac{2}{3}(1-a)$

86. Let  $f(x) = ax^2 - 2 + \frac{1}{x}$  where  $\alpha$  is a real constant. The smallest  $\alpha$  for which  $f(x) \geq 0$  for all  $x > 0$

- (A)  $\frac{2^2}{3^3}$  (B)  $\frac{2^3}{3^3}$   
 (C)  $\frac{2^4}{3^3}$  (D)  $\frac{2^5}{3^3}$

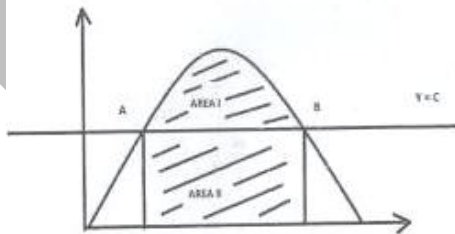
87. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a continuous function satisfying

$$f(x) + \int_0^x f(t) dt + x^2 = 0$$

For all  $x \in \mathbb{R}$ . Then

- (A)  $\lim_{x \rightarrow \infty} f(x) = 2$   
 (B)  $\lim_{x \rightarrow \infty} f(x) = -2$   
 (C)  $f(x)$  has more than one point in common with the  $x$ -axis  
 (D)  $f(x)$  is an odd function

88. The figure shows a portion of the graph  $y = 2x - 4x^3$ . The line  $y = c$  is such that the areas of the regions marked I and II are equal. If  $a, b$  are the  $x$ -coordinates of A, B respectively, then  $a + b$  equals



- (A)  $\frac{2}{\sqrt{7}}$  (B)  $\frac{3}{\sqrt{7}}$   
 (C)  $\frac{4}{\sqrt{7}}$  (D)  $\frac{5}{\sqrt{7}}$

89. Let  $X_n = \{1, 2, 3, \dots, n\}$  and let a subset  $A$  of  $X_n$  be chosen so that every pair of elements of  $A$  differ by at least 3. (For example, if  $n = 5$ ,  $A$  can be  $\emptyset$ ,  $\{2\}$  or  $\{1, 5\}$  among others). When  $n = 10$ , let the probability that  $1 \in A$  be  $p$  and let the probability that  $2 \in A$  be  $q$ . Then

- (A)  $p > q$  and  $p - q = \frac{1}{6}$  (B)  $p < q$  and  $p - q = \frac{1}{6}$

$$(C) p > q \text{ and } p - q = \frac{1}{10}$$

$$(D) p < q \text{ and } q - p = \frac{1}{10}$$

90. The remainder when the determinant

$$\begin{vmatrix} 2014^{2014} & 2015^{2015} & 2016^{2016} \\ 2017^{2017} & 2018^{2018} & 2019^{2019} \\ 2020^{2020} & 2021^{2021} & 2022^{2022} \end{vmatrix}$$

is divided by 5 is

(A) 1

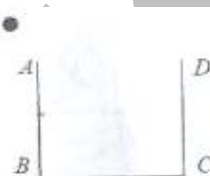
(B) 2

(C) 3

(D) 4

### PHYSICS

91. A cubical vessel has opaque walls. An observer (dark circle is figure below) is located such that she can see only the wall CD but not the bottom. Nearly to what height should water be poured so that she can see an object placed at the bottom at a distance of 10 cm from the corner C? Refractive index of water is 1.33.



(A) 10 cm  
(C) 27 cm

(B) 16 cm  
(D) 45 cm

92. The moments of inertia of a non-uniform circular disc (of mass M and radius R) about four mutually perpendicular tangents AB, BC, CD, DA are  $I_1, I_2, I_3$  and  $I_4$ , respectively (the square ABCD circumscribes the circle). The distance of the center of mass of the disc from its geometrical center is given by

(A)  $\frac{1}{4MR} \sqrt{(I_1 - I_3)^2 + (I_2 - I_4)^2}$

(B)  $\frac{1}{12MR} \sqrt{(I_1 - I_3)^2 + (I_2 - I_4)^2}$

(C)  $\frac{1}{3MR} \sqrt{(I_1 - I_2)^2 + (I_3 - I_4)^2}$

(D)  $\frac{1}{2MR} \sqrt{(I_1 - I_3)^2 + (I_2 + I_4)^2}$

93. A horizontal steel railroad track has a length of 100 m when the temperature is  $25^\circ\text{C}$ . The track is constrained from expanding or bending. The stress on the track on a hot summer day, when the temperature is  $40^\circ\text{C}$ , is (Note: the linear coefficient of thermal expansion for steel is  $1.1 \times 10^{-5}/^\circ\text{C}$  and the Young's modulus of steel is  $2 \times 10^{11}$  Pa)

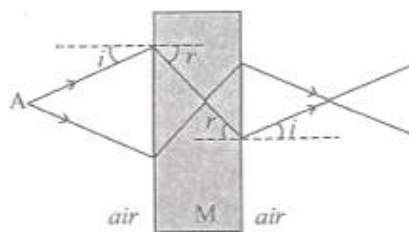
(A)  $6.6 \times 10^7$  Pa

(B)  $8.8 \times 10^7$  Pa

(C)  $3.3 \times 10^7$  Pa

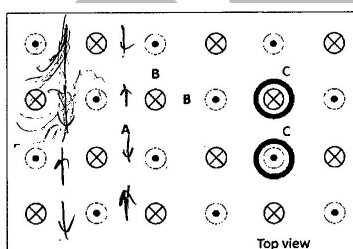
(D)  $5.5 \times 10^7$  Pa

94. Electromagnetic waves emanating from a point A (in air) are incident on a rectangular block of material M and emerge from the other side as shown. The angles  $i$  and  $r$  are angles of incidence and refraction when the wave travels from air to the medium. Such paths of the rays are possible



- (A) if the material has a refractive index very nearly equal to zero.  
 (B) only with gamma rays with a wavelength smaller than the atomic nuclei of the material  
 (C) if the material has a refractive index less than zero.  
 (D) only if the wave travels in M with a speed faster than the speed of light in vacuum.

95. Two small metal balls of different mass  $m_1$  and  $m_2$  are connected by strings of equal length to a fixed point. When the balls are given equal charges, the angles that the two strings make with the vertical are  $30^\circ$  and  $60^\circ$ , respectively. The ratio  $m_1/m_2$  is close to  
 (A) 1.7 (B) 3.0  
 (C) 0.58 (D) 2.0
96. Consider the regular array of vertical identical current carrying wires (with direction of current flow as indicated in the figure below) protruding through a horizontal table. If we scatter some diamagnetic particles on the table, they are likely to accumulate



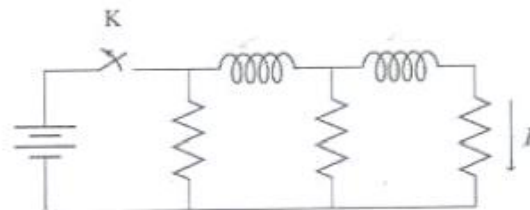
- (A) around regions such as A.  
 (B) around regions such as B.  
 (C) in circular regions around individual wires such as C.  
 (D) uniformly everywhere.
97. The distance between the vertex and the centre of mass of a uniform solid planar circular segment of angular size  $\theta$  and radius  $R$  is given by

- (A)  $\frac{4}{3}R \frac{\sin(\theta/2)}{\theta}$  (B)  $R \frac{\sin(\theta/2)}{\theta}$   
 (C)  $\frac{4}{3}R \cos\left(\frac{\theta}{2}\right)$  (D)  $\frac{2}{3}R \cos(\theta)$

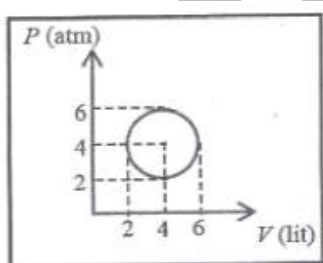
98. An object is propelled vertically to a maximum height of  $4R$  from the surface of a planet of radius  $R$  and mass  $M$ . The speed of object when it returns to the surface of the plane is

- (A)  $2\sqrt{\frac{2GM}{5R}}$  (B)  $\sqrt{\frac{GM}{2R}}$   
 (C)  $\sqrt{\frac{3GM}{2R}}$  (D)  $\sqrt{\frac{GM}{5R}}$

99. In the circuit shown below, all the inductors (assumed ideal) and resistors are identical. The current through the resistance on the right is  $I$  after the key  $K$  has been switched on for a long time. The currents through the three resistors (in order, from left to right) immediately after the key is switched off are



- (A)  $2I$  upwards,  $I$  downwards and  $I$  downwards  
 (B)  $2I$  downwards,  $I$  downwards and  $I$  downwards  
 (C)  $I$  downwards,  $I$  downwards and  $I$  downwards  
 (D)  $0$ ,  $I$  downwards and  $I$  downwards.
100. An ideal gas undergoes a circular cycle centered at  $4 \text{ atm}$ ,  $4 \text{ lit}$  as shown in the diagram. The maximum temperature attained in this process is close to



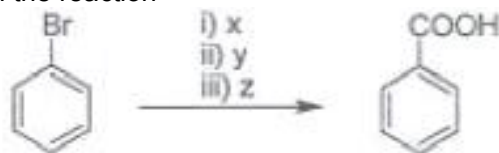
- (A)  $\frac{30}{R}$  (B)  $\frac{36}{R}$   
 (C)  $\frac{24}{R}$  (D)  $\frac{16}{R}$

### CHEMISTRY

101. For the reaction  $\text{N}_2 + 3\text{X}_2 \rightarrow 2\text{NX}$ , where  $\text{X} = \text{F}, \text{Cl}$  (the average bond energies are  $\text{F} - \text{F} = 155 \text{ kJ mol}^{-1}$ ,  $\text{N} - \text{F} = 272 \text{ kJ mol}^{-1}$ ,  $\text{Cl} - \text{Cl} = 242 \text{ kJ mol}^{-1}$ ,  $\text{N} - \text{Cl} = 200 \text{ kJ mol}^{-1}$  and  $\text{N} \equiv \text{N} = 941 \text{ kJ mol}^{-1}$ ), the heats of formation of  $\text{NF}_3$  and  $\text{NCl}_3$  in  $\text{kJ mol}^{-1}$ , respectively, are closest to
- (A)  $-226$  and  $+467$  (B)  $+226$  and  $-467$   
 (C)  $-151$  and  $+311$  (D)  $+151$  and  $-311$
102. The equilibrium constants for the reaction  $\text{X} = 2\text{Y}$  and  $\text{Z} = \text{P} + \text{Q}$  are  $K_1$  and  $K_2$ , respectively. If the initial concentrations and the degree of dissociation of  $\text{X}$  and  $\text{Z}$  are the same, the ratio  $\frac{K_1}{K_2}$  is:
- (A) 4 (B) 1  
 (C) 0.5 (D) 2
103. The geometry and the number of unpaired electron(s) of  $[\text{MnBr}_4]^{2-}$ , respectively, are
- (A) tetrahedral and 1 (B) square planar and 1  
 (C) tetrahedral and 5 (D) square planar and 5
104. The standard cell potential for  $\text{Zn}|\text{Zn}^{2+}||\text{Cu}^{2+}|\text{Cu}$  is  $1.10\text{V}$ . When the cell is completely discharged,  $\frac{\log[\text{Zn}^{2+}]}{[\text{Cu}^{2+}]}$  is closest to

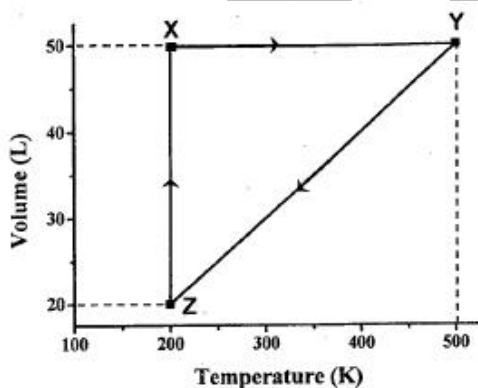
- (A) 37.3  
(C) 18.7
- (B) 0.026  
(D) 0.052

105. In the reaction



x, y and z are

- (A) x = Mg, dry ether; y = CH<sub>3</sub>Cl; z = H<sub>2</sub>O  
(C) x = Mg, dry ether; y = CO<sub>2</sub>; z = dil.HCl
- (B) x = Mg, dry methanol; y = CO<sub>2</sub>; z = dil.HCl  
(D) x = Mg, dry methanol; y = CH<sub>3</sub>Cl; z = H<sub>2</sub>O
106. An organic compound having molecular formula C<sub>2</sub>H<sub>6</sub>O undergoes oxidation with K<sub>2</sub>Cr<sub>2</sub>O<sub>7</sub>/H<sub>2</sub>SO<sub>4</sub> to produce X which contains 40% carbon, 6.7% hydrogen and 53.3% oxygen. The molecular formula of the compound X is
- (A) CH<sub>2</sub>O  
(C) C<sub>2</sub>H<sub>4</sub>O
- (B) C<sub>2</sub>H<sub>4</sub>O<sub>2</sub>  
(D) C<sub>2</sub>H<sub>6</sub>O<sub>2</sub>
107. The maximum number of cyclic isomers (positional and optical) of a compound having molecular formula C<sub>3</sub>H<sub>2</sub>Cl<sub>2</sub> is
- (A) 2  
(C) 4
- (B) 3  
(D) 5
108. The volume vs. temperature graph of 1 mole of an ideal gas is given below



The pressure of the gas (in atm) at X, Y and Z, respectively are

- (A) 0.328, 0.820, 0.820  
(C) 0.238, 0.280, 0.280
- (B) 3.28, 8.20, 3.28  
(D) 32.8, 0.280, 82.0
109. MnO<sub>2</sub> when fused with KOH and oxidized in air gives a dark green compound X. In acidic solution, X undergoes disproportionation to give an intense purple compound Y and MnO<sub>2</sub>. The compound X and Y, respectively, are
- (A) K<sub>2</sub>MnO<sub>4</sub> and KMnO<sub>4</sub>  
(C) K<sub>2</sub>MnO<sub>4</sub> and Mn<sub>2</sub>O<sub>7</sub>
- (B) Mn<sub>2</sub>O<sub>7</sub> and KMnO<sub>4</sub>  
(D) KMnO<sub>4</sub> and K<sub>2</sub>MnO<sub>4</sub>
110. A metal (X) dissolves both in dilute HCl and dilute NaOH to liberate H<sub>2</sub>. Addition of NH<sub>4</sub>Cl and excess NH<sub>4</sub>O to an HCl solution of X produces Y as a precipitate. Y is also produced by adding NH<sub>4</sub>Cl to the NaOH solution of X. The species X and Y, respectively, are
- (A) Zn and Zn(OH)<sub>2</sub>  
(C) Zn and Na<sub>2</sub>ZnO<sub>2</sub>
- (B) Al and Al(OH)<sub>3</sub>  
(D) Al and NaAlO<sub>2</sub>

## ANSWERS

1. **D**                      2. **A**                      3. **D**
4. No Option matches (Answer is  $y^2 = x^2y - 2x^2$ )
5. **B**                      6. **B**                      7. **B**                      8. **B**
9. **A**                      10. **A**                      11. **D**                      12. **D**
13. **D**                      14. **A**                      15. **B**                      16. **D**
17. **B**                      18. **B**                      19. **C**                      20. **C**
21. **A**                      22. **D**                      23. **C**                      24. **D**
25. **D**                      26. **B**                      27. **A**                      28. **D**
29. **A**                      30. **A**                      31. **C**                      32. **A**
33. **D**                      34. **B**                      35. **C**                      36. **B**
37. **D**                      38. **A**                      39. **D**                      40. **A**
- 
41. **C**                      42. **B**                      43. **B**                      44. **D**
45. **C**                      46. **B**                      47. **B**                      48. **A**
49. **B**                      50. **D**                      51. **B**                      52. **A**
53. **B**                      54. **A**                      55. **C**                      56. **D**
57. **C**                      58. **A**                      59. **C**                      60. **A**
- 
81. **A**                      82. **C**                      83. **C**                      84. **D**
85. **D**                      86. **D**                      87. **B**                      88. **A**
89. **C**                      90. **D**                      91. **C**                      92. **A**
93. **C**                      94. **C**                      95. **A**                      96. **A**
97. **A**                      98. **A**                      99. **A**                      100. **A**
101. **C** (Probably question was to be asked for heat of reaction. In that case(A) could have been the correct option otherwise C is the closest option but not correct)
102. **A**                      103. **C**                      104. **A**                      105. **C**
106. **B**                      107. **C**                      108. **A**                      109. **A**
110. **B**