

KVPY PAPER – 2012

STREAM SX (CLASS-XII)

PART-I

ONE MARK QUESTIONS

MATHEMATICS

1. Three children, each accompanied by a guardian, seek admission in a school. The principal wants to interview all the 6 persons one after the other subject to the condition that no child is interviewed before its guardian. In how many ways can this be done ?

(A) 60 (B) 90 (C) 120 (D) 180

Sol. (B)

$$\text{No. of ways} = {}^6C_2 \times {}^4C_2 \times {}^2C_2 = 90$$

2. In the real number system, the equation $\sqrt{x+3-4\sqrt{x-1}} + \sqrt{x+8-6\sqrt{x-1}} = 1$ has :

(A) No solution (B) Exactly two distinct solutions
(C) Exactly four distinct solutions (D) Infinitely many solutions

Sol. (D)

$$\sqrt{(x-1)+4-4\sqrt{x-1}} = 1 - \sqrt{(x-1)+9-6\sqrt{x-1}}$$

Put $\sqrt{x-1} = t$

$$\sqrt{t^2+4-4t} = 1 - \sqrt{t^2+9-6t}$$

Squaring we get infinitely many solutions.

3. The maximum value M of $3^x + 5^x - 9^x + 15^x - 25^x$, as x varies over reals, satisfies :

(A) $3 < M < 5$ (B) $0 < M < 2$ (C) $9 < M < 25$ (D) $5 < M < 9$

Sol. (B)

Max. value = 1

4. Suppose two perpendicular tangents can be drawn from the origin to the circle $x^2 + y^2 - 6x - 2py + 17 = 0$, for some real p. Then |p| is equal to :

(A) 0 (B) 3 (C) 5 (D) 17

Sol. (C)

Director circle passes through origin so

Radius of director circle = $\sqrt{2}$ radius of circle

$$\sqrt{3^2+p^2} = \sqrt{2}\sqrt{3^2+p^2-17}$$

$$3^2+p^2 = 2(3^2+p^2-17)$$

$$0 = 3^2+p^2-34$$

$$|P| = 5$$

5. Let a, b, c, d be numbers in the set {1, 2, 3, 4, 5, 6} such that the curves $y = 2x^3 + ax + b$ and $y = 2x^3 + cx + d$ have no point in common. The maximum possible value of $(a-c)^2 + b-d$ is :

(A) 0 (B) 5 (C) 30 (D) 36

Sol. (B)

$$ax + b = cx + d$$

So far no solution $a = c$

$$\text{so } (a-c)^2 + b-d = (a)^2 + 6 - 1 = 5$$

6. Consider the conic $ex^2 + \pi y^2 - 2e^2x - 2\pi^2y + e^3 + \pi^3 = \pi e$. Suppose P is any point on the conic and S_1, S_2 are the foci of the conic, then the maximum value of $(PS_1 + PS_2)$ is :

(A) πe (B) $\sqrt{\pi e}$ (C) $2\sqrt{\pi}$ (D) $2\sqrt{e}$

Sol. (C)

$$ex^2 + \pi y^2 - 2e^2x - 2\pi^2y + e^3 + \pi^3 = \pi e$$

$$\frac{(x-e)^2}{\pi} + \frac{(y-\pi)^2}{e} = 1$$

So $PS_1 + PS_2 = \text{Length of major axis} = 2\sqrt{\pi}$

7. Let $f(x) = \frac{\sin(x-a) + \sin(x+a)}{\cos(x-a) - \cos(x+a)}$, then :
- (A) $f(x+2\pi) = f(x)$ but $f(x+\alpha) \neq f(x)$ for any $0 < \alpha < 2\pi$
 (B) f is a strictly increasing function
 (C) f is strictly decreasing function
 (D) f is constant function

Sol. (D)

$$\begin{aligned} f(x) &= \frac{\sin(x-a) + \sin(x+a)}{\cos(x-a) - \cos(x+a)} \\ &= \frac{2\sin x \cdot \cos a}{2\sin x \cdot \sin a} = \tan a \end{aligned}$$

So constant function

8. The value of $\tan 81^\circ - \tan 63^\circ - \tan 27^\circ + \tan 9^\circ$ is :

(A) 0 (B) 2 (C) 3 (D) 4

Sol. (D)

$$\begin{aligned} &\tan 81^\circ - \tan 63^\circ - \tan 27^\circ + \tan 9^\circ \\ &= \frac{\sin 81^\circ}{\cos 81^\circ} + \frac{\sin 9^\circ}{\cos 9^\circ} - \left[\frac{\sin 63^\circ}{\cos 63^\circ} + \frac{\sin 27^\circ}{\cos 27^\circ} \right] \\ &= \frac{\sin 90^\circ}{\cos 81^\circ \cdot \cos 9^\circ} - \left[\frac{\sin 90^\circ}{\cos 63^\circ \cdot \cos 27^\circ} \right] \\ &= \frac{2}{\sin 18^\circ} - \frac{2}{\sin 54^\circ} \\ &= 2 \left[\frac{1}{\sin 18^\circ} - \frac{1}{\sin 54^\circ} \right] \\ &= 2 \left[\frac{\sin 54^\circ - \sin 18^\circ}{\sin 18^\circ \sin 54^\circ} \right] = 2 \left[\frac{2 \cos 36^\circ \cdot \sin 18^\circ}{\sin 18^\circ \cdot \sin 54^\circ} \right] = 4 \end{aligned}$$

9. The mid-point of the domain of the function $f(x) = \sqrt{4 - \sqrt{2x+5}}$ for real x is :

(A) $\frac{1}{4}$ (B) $\frac{3}{2}$ (C) $\frac{2}{3}$ (D) $-\frac{2}{5}$

Sol. (B)

$$\begin{aligned} f(x) &= \sqrt{4 - \sqrt{2x+5}} \\ x &\geq -\frac{5}{2} \\ 2x+5 &< 16 \\ x &\leq \frac{11}{2} \\ -\frac{5}{2} &\leq x \leq \frac{11}{2} \end{aligned}$$

\Rightarrow Mid-point of domain = $\frac{3}{2}$

10. Let n be a natural number and let a be a real number. The number of zeros of $x^{2n+1} - (2n+1)x + a = 0$ in the interval $[-1, 1]$ is :

(A) 2 if $a > 0$ (B) 2 if $a < 0$
 (C) At most one for every value of a (D) At least three for every value of a

Sol. (C)

Polynomial function is monotonic.

11. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be the function
 $f(x) = (x - a_1)(x - a_2) + (x - a_2)(x - a_3) + (x - a_3)(x - a_1)$
 with $a_1, a_2, a_3 \in \mathbb{R}$. Then $f(x) \geq 0$ if and only if :
 (A) at least two of a_1, a_2, a_3 are equal
 (B) $a_1 = a_2 = a_3$
 (C) a_1, a_2, a_3 are all distinct
 (D) a_1, a_2, a_3 are all positive and distinct

Sol. (B)
 $3x^2 - 2(a_1 + a_2 + a_3)x + a_1a_2 + a_2a_3 + a_3a_1$
 $4[a_1 + a_2 + a_3]^2 - 4[3(a_1a_2 + a_2a_3 + a_3a_1)] \leq 0$
 $(a_1 + a_2 + a_3)^2 - 3(a_1a_2) \leq 0$
 $a_1^2 + a_2^2 + a_3^2 - a_1a_2 - a_2a_3 - a_3a_1 \leq 0$
 $2[(a_1 - a_2)^2 + (a_2 - a_3)^2 + (a_3 - a_1)^2] \leq 0$

12. The value $\frac{\int_0^{\pi/2} (\sin x)^{\sqrt{2}+1} dx}{\int_0^{\pi/2} (\sin x)^{\sqrt{2}-1} dx}$ is :
 (A) $\frac{\sqrt{2}+1}{\sqrt{2}-1}$ (B) $\frac{\sqrt{2}-1}{\sqrt{2}+1}$ (C) $\frac{\sqrt{2}+1}{\sqrt{2}}$ (D) $2 - \sqrt{2}$

Sol. (D)

13. The value $\int_{-2012}^{2012} (\sin(x^3) + x^5 + 1) dx$ is :
 (A) 2012 (B) 2013 (C) 0 (D) 4024

Sol. (D)
 $\int_{-2012}^{2012} [\sin(x^3) + x^5 + 1] dx$
 $= \int_{-2012}^{2012} (\sin(x^3) + x^5) + \int_{-2012}^{2012} (1) dx$
 $= 0 + 4024$

14. Let $[x]$ and $\{x\}$ be the integer part and fractional part of a real number x respectively. The value of the integral $\int_0^5 [x]\{x\} dx$ is :

- (A) $5/2$ (B) 5 (C) 34.5 (D) 35.5

Sol. (B)
 $\int_0^5 [x]\{x\} dx = 1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{2} + 3 \cdot \frac{1}{2} + 4 \cdot \frac{1}{2} = 5$

15. Let $S_n = \sum_{k=1}^n k$ denote the sum of the first n positive integers. The numbers $S_1, S_2, S_3, \dots, S_{99}$ are written on 99 cards. The probability of drawing a card with an even number written on it is :

- (A) $\frac{1}{2}$ (B) $\frac{49}{100}$ (C) $\frac{49}{99}$ (D) $\frac{48}{99}$

Sol. (C)
 $S_1 \rightarrow \text{odd}$
 $S_2 \rightarrow \text{odd}$
 $S_3 \rightarrow \text{even}$
 $S_4 \rightarrow \text{even}$
 $S_5 \rightarrow \text{odd}$
 $S_6 \rightarrow \text{odd}$
 $S_7 \rightarrow \text{even}$
 So 49 cards will be even
 So probability = $\frac{49}{99}$

16. A purse contains 4 copper coins and 3 silver coins. A second purse contains 6 copper coins and 4 silver coins. A purse is chosen randomly and a coin is taken out of it. What is the probability that it is a copper coin ?
- (A) $\frac{41}{70}$ (B) $\frac{31}{70}$ (C) $\frac{27}{70}$ (D) $\frac{1}{3}$

Sol. (A)

$$\begin{aligned} \text{Required probability} &= \frac{1}{2} \left[\frac{4}{7} + \frac{6}{10} \right] \\ &= \frac{1}{2} \left[\frac{4}{7} + \frac{3}{5} \right] \\ &= \frac{1}{2} \left[\frac{41}{35} \right] = \frac{41}{70} \end{aligned}$$

17. Let H be the orthocenter of an acute-angled triangle ABC and O be its circumcentre. Then

$$\vec{HA} + \vec{HB} + \vec{HC}$$

- (A) Is equal to \vec{HO} (B) Is equal to $3\vec{HO}$
 (C) Is equal to $2\vec{HO}$ (D) Is not a scalar multiple of \vec{HO} in general

Sol. (C)

18. The number of ordered pairs (m, n), where $m, n \in \{1, 2, 3, \dots, 50\}$, such that $6^m + 9^n$ is a multiple of 5 is :
- (A) 1250 (B) 2500 (C) 625 (D) 500

Sol. (B)

We can choose m from (1, 2, ..., 50)
 but we can choose n from (1, 3, 5, 7, ..., 49)
 So that $6^m + 9^n$ is multiple of 5
 So total number of pairs = $50 \times 25 = 1250$

19. Suppose $a_1, a_2, a_3, \dots, a_{2012}$ are integers arranged on a circle. Each number is equal to the average of its two adjacent numbers. If the sum of all even indexed numbers is 3018, what is the sum of all numbers.

- (A) 0 (B) 1509 (C) 3018 (D) 6036

Sol. (D)

$a_1, a_2, a_3, \dots, a_{2012}$ must be in an A.P.
 and $a_{2012} + a_2 = a_1$
 So difference of these terms = 0
 All numbers are equal.
 So sum of all number = $2 \times 3018 = 6036$.

20. Let $S = \{1, 2, 3, \dots, n\}$ and $A = \{(a, b) \mid 1 \leq a, b \leq n\} = S \times S$. A subset B of A is said to be a good subset if $(x, x) \in B$ for every $x \in S$. Then the number of good subsets of A is :

- (A) 1 (B) 2^n (C) $2^{n(n-1)}$ (D) 2^{n^2}

Sol. (C)

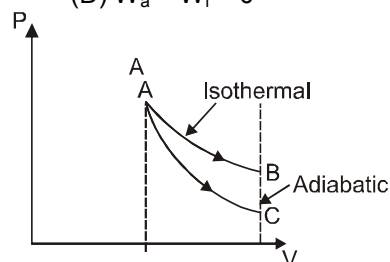
PHYSICS

21. An ideal monatomic gas expands to twice its volume. If the process is isothermal, the magnitude of work done by the gas is W_i . If the process is adiabatic, the magnitude of work done by the gas is W_a . Which of the following is true ?

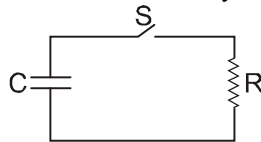
- (A) $W_i = W_a > 0$ (B) $W_i > W_a = 0$ (C) $W_i > W_a > 0$ (D) $W_a > W_i = 0$

Sol. (B)

Area under the curve P vs V gives work done by gas and expansion work done by gas is positive.
 $W_i > W_a > 0$



22. The capacitor of capacitance C in the circuit shown is fully charged initially. Resistance is R.



After the switch S is closed, the time taken to reduce the stored energy in the capacitor to half its initial value is :

- (A) RC_2 (B) RC (C) $\ln 2 \ 2RC \ \ln 2$ (D) $\frac{RC \ln 2}{2}$

Sol. (D)

Let initial charge on capacitor be Q_0 .

$$Q = Q_0 e^{-t/CR}$$

Given , $U = \frac{U_i}{2}$

$$\Rightarrow \frac{Q^2}{2c} = \frac{1}{2} \frac{Q_0^2}{2c}$$

$$\Rightarrow Q = \frac{Q_0}{\sqrt{2}}$$

$$\Rightarrow e^{-t/CR} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow t = \frac{CR \ln 2}{2}$$

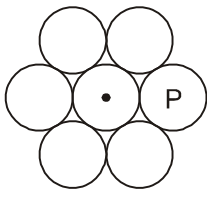
23. A liquid drop placed on a horizontal plane has a near spherical shape (slightly flattened due to gravity). Let R be the radius of its largest horizontal section. A small disturbance cause the drop to vibrate with frequency ν about its equilibrium shape. By dimensional analysis the ratio. $\frac{\nu}{\sqrt{\sigma/\rho R^3}}$ can be (Here σ is density, g is acceleration due to gravity, and k is an arbitrary dimensionless constant.)

- (A) $k\rho g R^2/\sigma$ (B) $k\rho R^3/g\sigma$ (C) $k\rho R^2/g\sigma$ (D) $k\rho/g\sigma$

Sol. (A)

$\frac{U}{\sqrt{\frac{6}{\delta R^3}}}$ is a dimensionless quality and the $\frac{K\delta g R^2}{6}$ is also a dimensionally quantity.

24. Seven identical coins are rigidly arranged on a flat table in the pattern shown below so that each coin touches its neighbors. Each coin is a thin disc of mass m and radius r. Note that the moment of inertia of an individual coin about an axis passing through centre and perpendicular to the plane of the coin is $mr^2/2$.



The moment of inertia of the system of seven coins about an axis that passes through the point P (the centre of the coin positioned directly to the right of the central coin) and perpendicular to the plane of the coins is :

- (A) $\frac{55}{2}mr^2$ (B) $\frac{127}{2}mr^2$
 (C) $\frac{111}{2}mr^2$ (D) $55 mr^2$

Sol. (C)

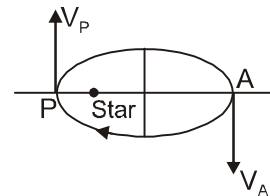
$$I_{cm} = \left(\frac{mr^2}{2} + 4mr^2 \right) \times 6 + \frac{mr^2}{2}$$

$$= \frac{55mr^2}{2}$$

using parallel axis theorem

$$I_p = \frac{55mr^2}{2} + 28mr^2 = \frac{111mr^2}{2}$$

25. A planet orbits in an elliptical path of eccentricity e round a massive star considered fixed at one of the foci. The point in space where it is closest to the star is denoted by P and the point where it is farthest is denoted by A. Let v_p and v_A be the respective speeds at P and A. Then



- (A) $\frac{V_P}{V_A} = \frac{1+e}{1-e}$ (B) $\frac{V_P}{V_A} = 1$ (C) $\frac{V_P}{V_A} = \frac{1+e^2}{1-e}$

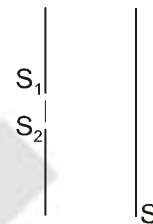
Sol. (A)

From conservation of Angular momentum about centre of star.

$$mV_P(a - ae) = mV_A(a + ae)$$

$$\Rightarrow \frac{V_P}{V_A} = \frac{1+e}{1-e}$$

26. In a young's double slit experiment the intensity of light at each slit is I_0 . Interference pattern is observed along a direction parallel to the line $S_1 S_2$ on screen S.



The minimum, maximum, and the intensity averaged over the entire screen are respectively.

- (A) 0, $4I_0$, $2I_0$ (B) I_0 , $2I_0$, $3I_0/2$
 (C) 0, $4I_0I_0$ (D) 0, $2I_0$, I_0

Sol. (A)

$$I_r = I_0 + I_0 + 2\sqrt{I_0^2} \cos \phi$$

$$= 4I_0 \cos^2 \frac{\phi}{2}$$

$$I_{\min} = 0$$

$$I_{\max} = 4I_0$$

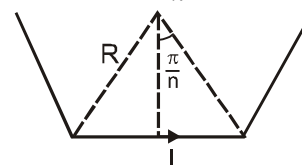
$$\text{Average Intensity} = 2I_0$$

27. A loop carrying current i has the shape of a regular polygon of n sides. If R is the distance from the centre to any vertex, then the magnitude of the magnetic induction vector \vec{B} at the centre of the loop is :

- (A) $n \frac{\mu_0 i}{2\pi R} \tan \frac{\pi}{n}$ (B) $n \frac{\mu_0 i}{2\pi R} \tan \frac{2\pi}{n}$ (C) $\frac{\mu_0 i}{2R}$ (D) $\frac{\mu_0 i}{\pi R} \tan \frac{\pi}{n}$

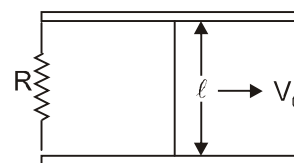
Sol. (A)

$$B = \frac{n\mu_0 i}{4\pi R \cos(\pi/n)} \cdot 2 \sin(\pi/n) = \frac{n\mu_0 i \tan(\pi/n)}{2\pi R}$$



28. A conduction rod of mass m and length ℓ is free to move without friction on two parallel long conducting rails, as shown below. There is a resistance R across the rails. In the entire space around, there is a uniform magnetic field B normal to the plane of the rod and rails. The rod is given an impulsive velocity V_0 .

\otimes
 B



Finally, the initial energy $\frac{1}{2}mv_0^2$

- (A) Will be converted fully into heat energy in the resistor
 (B) Will enable rod to continue to move with velocity v_0 since the rails are frictionless
 (C) Will be converted fully into magnetic energy due to induced current
 (D) Will be converted into the work done against the magnetic field

Sol. (A)

Initial kinetic energy will be fully converted into heat energy in the resistor.

29. A steady current I flows through a wire of radius r , length L and resistivity ρ . The current produces heat in the wire. The rate of heat loss in a wire is proportional to its surface area. The steady temperature of the wire is independent of :

- (A) L (B) r (C) I (D) ρ

Sol. (A)

$i^2 R = \text{Rate of heat loss through radiation}$

$$\Rightarrow i^2 \frac{\rho l}{\pi r^2} \propto \pi r^2 l (\theta - \theta_0)$$

$$\Rightarrow \theta - \theta_0 \propto \frac{i^2 \rho}{\pi^2 r^4}$$

30. The ratio of the speed of sound to the average speed of an air molecule at 300K and 1 atmospheric pressure is close to :

- (A) 1 (B) $\sqrt{300}$ (C) $\sqrt{1/300}$ (D) 300

Sol. (A)

$$v_s = \sqrt{\frac{\gamma RT}{m_0}}$$

$$\langle v \rangle = \sqrt{\frac{8 RT}{\pi m_0}}$$

$$\therefore \frac{v_s}{\langle v \rangle} = \sqrt{\frac{\gamma \pi}{8}} = \sqrt{0.55}$$

Nearest option is A

31. In one model of the electron, the electron of mass m_e is thought to be a uniformly charged shell of radius R and total charge e , whose electrostatic energy E is equivalent to its mass m_e via Einstein's mass energy relation $E = m_e c^2$. In this model, R is approximately ($m_e = 9.1 \times 10^{-31}$ kg, $c = 3 \times 10^8$ m.s $^{-1}$, $\frac{1}{4} \pi \epsilon_0 = 9 \times 10^9$ Farads.m $^{-1}$, magnitude of the electron charge = 1.6×10^{-19} C)

- (A) 1.4×10^{-15} m (B) 2×10^{-13} m (C) 5.3×10^{-11} m (D) 2.8×10^{-35} m

Sol. (A)

$$\frac{Ke^2}{2R} = m_e c^2$$

$$\Rightarrow R = \frac{Ke^2}{2m_e c^2} = 1.407 \times 10^{-15} \text{ m}$$

32. A body is executing simple harmonic motion of amplitude a and period T about the equilibrium position $x = 0$. Large numbers of snapshots are taken at random of this body in motion. The probability of the body being found in a very small interval x to $x + |dx|$ is highest at :

- (A) $x = \pm a$ (B) $x = 0$ (C) $x = \pm a/2$ (D) $x = \pm a/\sqrt{2}$

Sol. (A)

Probability of finding body will be max. where speed is minimum.

33. Two identical bodies are made of a material for which the heat capacity increases with temperature. One of these is held at a temperature of 100°C while the other one is kept at 0°C. If the two are brought into contact, then, assuming no heat loss to the environment, the final temperature that they will reach is :

- (A) 50°C (B) more than 50°C (C) less than 50°C (D) 0°C

Sol. (B)

$$\text{Temp} \propto \frac{1}{\text{Heat capacity}}$$

With no change in heat capacity eq. temp. would be $\theta = 50^\circ\text{C}$.

As heat capacity increase with temperature $\theta > 50^\circ$

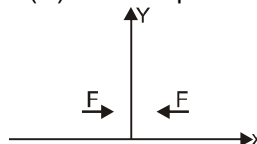
34. A particle is acted upon by a force given by $F = -\alpha x^3 - \beta x^4$ where α and β are positive constants. At the point $x = 0$, the particle is :

- (A) in stable equilibrium (B) in unstable equilibrium
(C) in neutral equilibrium (D) not in equilibrium

Sol. (A)

For $x > 0$: $F < 0$
 $x < 0$: $F > 0$

\therefore stable equilibrium



35. The potential energy of a point particle is given by the expression $V(x) = \alpha x + \beta \sin(x / \gamma)$. A dimensionless combination of the constants α , β and γ is :

(A) $\alpha/\beta\gamma$ (B) $\alpha^2/\beta\gamma$ (C) $\gamma/\alpha\beta$ (D) $\alpha\gamma/\beta$

Sol. (D)

$$[\alpha] = \frac{[V]}{[x]}$$

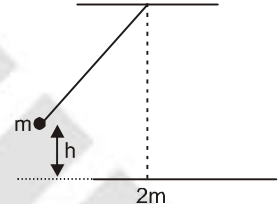
$$[\beta] = [V]$$

$$[\gamma] = [x]$$

$$\left[\frac{\alpha\gamma}{\beta} \right] = \frac{[V]}{[x]} \cdot \frac{1}{[V]} \cdot [x] = 1$$

36. A ball of mass m suspended from a rigid support by an inextensible massless string is released from a height h above its lowest point. At its lowest point it collides elastically with a block of mass $2m$ at rest on a frictionless surface. Neglect the dimensions of the ball and the block. After the collision the ball rises to a maximum height of :

(A) $h/3$ (B) $h/2$
(C) $h/8$ (D) $h/9$



Sol. (D)

Velocity of ball after collision

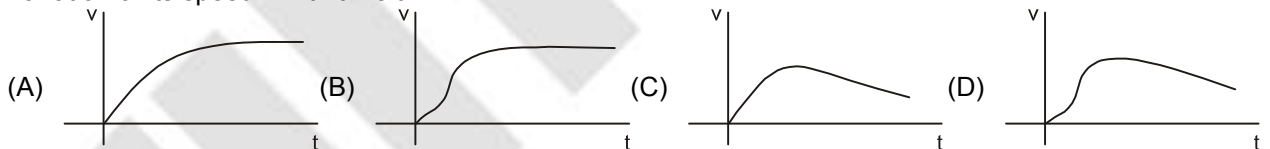
$$v_1 = \frac{m - 2m}{m + 2m} \sqrt{2gh}$$

$$= -\frac{\sqrt{2gh}}{3}$$

\therefore Maximum height reached is given by

$$h_{\max} = \frac{v_1^2}{2g} = \frac{h}{9}$$

37. A particle released from rest is falling through a thick fluid under gravity. The fluid exerts a resistive force on the particle proportional to the square of its speed. Which one of the following graphs best depicts the variation of its speed v with time t ?



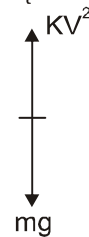
Sol. (A)

Force acting on particle.

$$a = \frac{dv}{dt} = mg - kv^2$$

Hence speed of particle will increase and becomes constant

$$\text{when } kv^2 = mg \Rightarrow v = \sqrt{\frac{mg}{k}}$$



38. A cylindrical steel rod of length 0.10 m and thermal conductivity $50 \text{ W.m}^{-1}.\text{K}^{-1}$ is welded end to end to copper rod of thermal conductivity $400 \text{ W.m}^{-1}.\text{K}^{-1}$ and of the same area of cross section but 0.20 m long. The free end of the steel rod is maintained at 100°C and that of the copper rod at 0°C . Assuming that the rods are perfectly insulated from the surrounding, the temperature at the junction of the two rods is :

(A) 20° (B) 30° (C) 40° (D) 50°

Sol. (A)

$$100^\circ\text{C} \quad \begin{array}{|c|c|} \hline \text{Steel} & \text{Copper} \\ \hline \end{array} \quad 0^\circ\text{C}$$

$$\frac{T - 100}{R_S} = \frac{T - 0}{R_C}$$

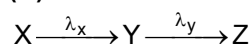
$$\Rightarrow \frac{T - 100}{T} = -\frac{R_S}{R_C} = -4$$

$$\therefore T = 20^\circ\text{C}$$

39. A parent nucleus X is decaying into daughter nucleus Y which in turn decays to Z. The half lives of X and Y are 40000 years and 20 years respectively. In a certain sample, it is found that the number of Y nuclei hardly changes with time. If the number of X nuclei in the sample is 4×10^{20} , the number of Y nuclei present in it is :

(A) 2×10^{17} (B) 2×10^{20} (C) 4×10^{23} (D) 4×10^{20}

Sol. (A)



$$\lambda_x N_x = \lambda_y N_y$$

\Rightarrow

$$N_y = \frac{\lambda_x}{\lambda_y} N_x$$

$$= \frac{(t_{1/2})_y}{(t_{1/2})_x} \cdot N_x$$

$$= \frac{20}{40000} \times 4 \times 10^{20}$$

$$= 2 \times 10^{17}$$

40. An unpolarized beam of light of intensity I_0 passes through two linear polarizers making an angle of 30° with respect to each other. The emergent beam will have an intensity :

(A) $\frac{3I_0}{4}$ (B) $\frac{\sqrt{3}I_0}{4}$ (C) $\frac{3I_0}{8}$ (D) $\frac{I_0}{8}$

Sol. (A)

Malus's law

$$I = I_0 \cos^2 \theta$$

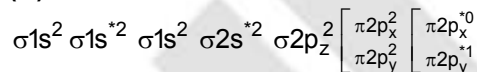
$$I = \frac{3I_0}{4}$$

CHEMISTRY

41. Among the following, the species with the highest bond order is :

(A) O_2 (B) F_2 (C) O_2^+ (D) F_2^-

Sol. (C)

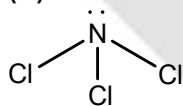


$$BO = \frac{n - n^*}{2} = \frac{10 - 5}{2} = \frac{5}{2} = 2.5$$

42. The molecule with non-zero dipole moment is :

(A) BCl_3 (B) $BeCl_2$ (C) CCl_4 (D) NCl_3

Sol. (D)

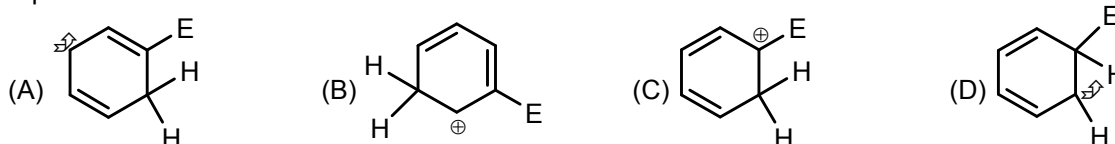


43. For a one-electron atom, the set of allowed quantum numbers is :

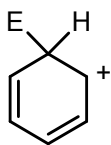
(A) $n = 1, \ell = 0, m_\ell = 0, m_s = +1/2$ (B) $n = 1, \ell = 1, m_\ell = 0, m_s = +1/2$
 (C) $n = 1, \ell = 0, m_\ell = -1, m_s = -1/2$ (D) $n = 1, \ell = 1, m_\ell = 1, m_s = -1/2$

Sol. (A)

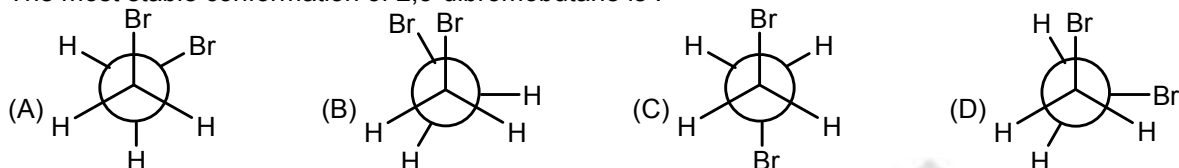
44. In the reaction of benzene with an electrophile E^+ , the structure of the intermediate σ -complex can be represented as :



Sol. (D)



45. The most stable conformation of 2,3-dibromobutane is :



Sol. (C)

Staggered is most stable.

46. Typical electronic energy gaps in molecules are about 1.0 eV. In terms of temperature, the gap is closest to :

- (A) 10^2 K (B) 10^4 K (C) 10^3 K (D) 10^5 K

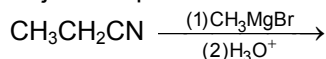
Sol. (C)

$k_B T \approx 0.025$ eV at 0 K

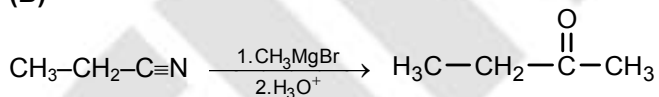
$$\text{So, } \frac{k_B T}{k_B T_2} = \frac{0.025}{1.00}$$

$$T_2 = 4 \times T_1 \\ = 4 \times 273 = 1092 \approx 10^3 \text{ K}$$

47. The major final product in the following reaction is :



Sol. (B)



48. A zero-order reaction, $A \rightarrow \text{Product}$, with an initial concentration $[A]_0$ has a half-life of 0.2 s. If one starts with the concentration $2[A]_0$, then the half-life is :

- (A) 0.1 s (B) 0.4 s (C) 0.2 s (D) 0.8 s

Sol. (B)

Half-life of zero order reaction is given by

$$t_{50} = \frac{C_0}{2k}, \quad t_{50} \propto C_0$$

Hence half life will get doubled.

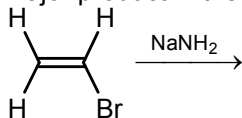
49. The isoelectronic pair of ions is :

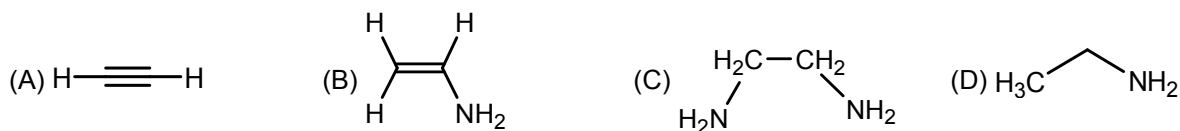
- (A) Sc^{2+} and V^{3+} (B) Mn^{3+} and Fe^{2+} (C) Mn^{2+} and Fe^{3+} (D) Ni^{3+} and Fe^{2+}

Sol. (C)

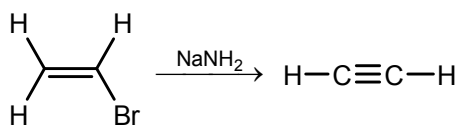
Both have 24 electrons.

50. The major product in the following reaction is :

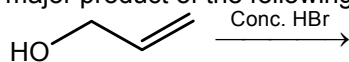




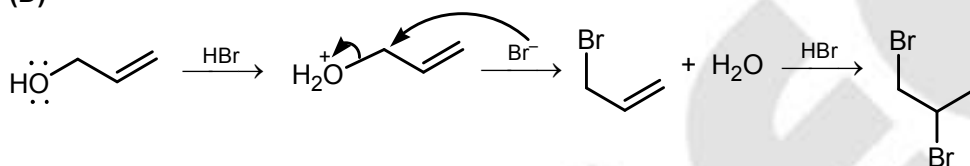
Sol. (A)



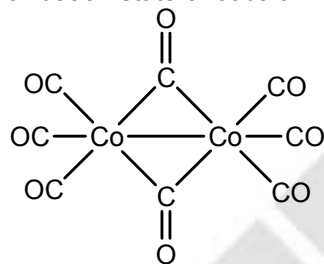
51. The major product of the following reaction is :



Sol. (D)



52. The oxidation state of cobalt in the following molecule is :



(A) 3 (B) 1 (C) 2 (D) 0

Sol. (D)

All ligands are neutral.

53. The pK_a of a weak acid is 5.85. The concentrations of the acid and its conjugate base are equal at a pH of:

(A) 6.85 (B) 5.85 (C) 4.85 (D) 7.85

Sol. (B)

$\text{pH} = \text{pK}_a = 5.85$

$K_a = \frac{[\text{H}^+][\text{A}^-]}{[\text{AA}]}$

$K_a = [\text{H}^+]$
 $\text{pK}_a = \text{pH}$

54. For a tetrahedral complex $[\text{MCl}_4]^{2-}$, the spin-only magnetic moment is 3.83 BM. The element M is :

(A) Co (B) Cu (C) Mn (D) Fe

Sol. (A)

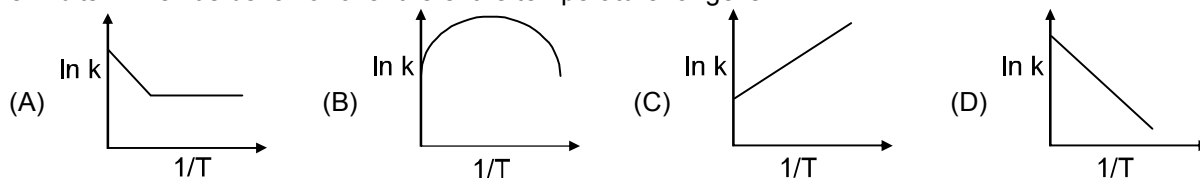
$3.83 \text{ BM} = \sqrt{n(n+2)} \text{ BM}$

$3.83 = \sqrt{n(n+2)}$

$n = 3$

So M^{2+} has three unpaired electrons.
Hence metal is Co.

55. Among the following graphs showing variation of rate (k) with temperature (T) for a reaction, the one that exhibits Arrhenius behavior over the entire temperature range is :



Sol.

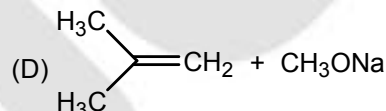
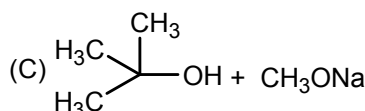
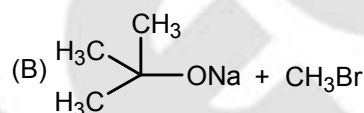
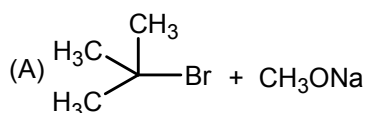
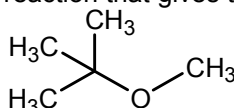
(D)

$$K = Ae^{-E/RT}$$

$$\ln K = \ln A - \frac{E}{RT}$$

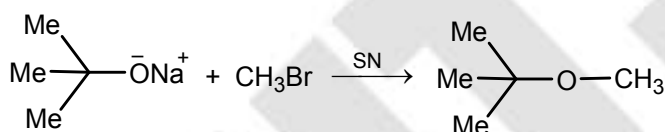
So the $\ln K$ vs $\frac{1}{T}$ is linear with -ve slope & +ve intercept.

56. The reaction that gives the following molecule as the major product is :



Sol.

(B)



57. The C-O bond length in CO, CO₂, and CO₃²⁻ follows the order :

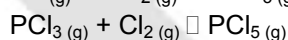
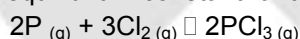
(A) CO < CO₂ < CO₃²⁻ (B) CO₂ < CO₃²⁻ < CO (C) CO > CO₂ > CO₃²⁻ (D) CO₃²⁻ < CO₂ < CO

Sol.

(A)

As decided by bond order
CO < CO₂ < CO₃²⁻

58. The equilibrium constant for the following reactions are K₁ and K₂, respectively.



Then the equilibrium constant for the reaction



(A) K₁K₂

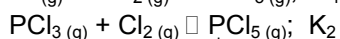
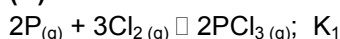
(B) K₁K₂²

(C) K₁²K₂²

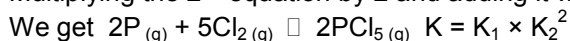
(D) K₁²K₂

Sol.

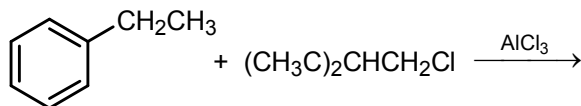
(B)

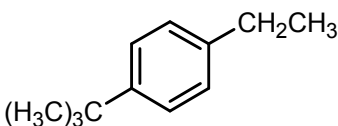
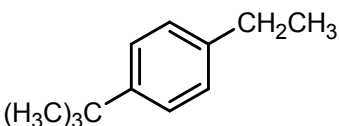
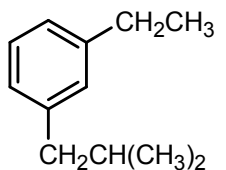
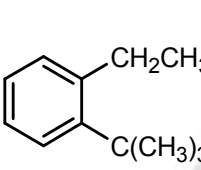


Multiplying the 2nd equation by 2 and adding it with 1st

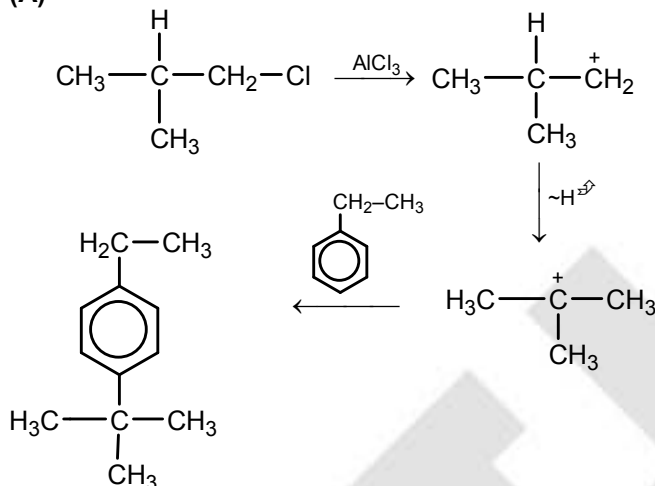


59. The major product of the following reaction is :



- (A)  (B) 
- (C)  (D) 

Sol. (A)



60. Doping silicon with boron produces a :

- (A) n-type semiconductor (B) metallic conductor (C) p-type semiconductor (D) insulator

Sol. (C)

Doping with trivalent impuring produces p-type semiconductor.

BIOLOGY

61. The disorders that arise when the immune system destroys 'self' cells are called autoimmune disorders. Which of the following would be classified under this ?

- (A) rheumatoid arthritis (B) asthma (C) rhinitis (D) eczema

Sol. (A)

Rheumatoid arthritis is an autoimmune disease while rhinitis, asthma and eczema are inflammatory disease of, mucous membrane inside nose, bronchioles and epidermis respectively.

62. Which of the following class of immunoglobulin can trigger the complement cascade ?

- (A) IgA (B) IgM (C) IgD (D) IgE

Sol. (B)

Classical complement pathway comprising of 21 different serum proteins primarily activated by antibody molecules of either IgG or IgM.

63. Diabetes insipidus is due to :

- (A) hypersecretion of vasopressin (B) hyposecretion of insulin
(C) hypersecretion of insulin (D) hyposecretion of vasopressin

Sol. (D)

Vasopressin or ADH hormone induce reabsorption of water in DCT and collecting duct of nephron. It's hyposecretion may cause frequent urination which is also called Diabetes insipidus.

64. Fossils are most often in which kind of rocks ?

- (A) meteorites (B) sedimentary rocks (C) igneous rocks (D) metamorphic rocks

- Sol. (B)**
Fossils are most often found in Sedimentary kind of rocks.
- 65.** Peptic ulcers are caused by :
(A) a fungus, *Candida albicans* (B) a virus, cytomegalovirus
(C) a parasite, *Trypanosoma brucei* (D) a bacterium *Helicobacter pylori*
- Sol. (D)**
Helicobacter pylori previously named *compylobacter pyloridis*, is a gram – negative bacteria found in stomach, causes peptic ulcers.
- 66.** Transfer RNA (tRNA)
(A) is present in the ribosomes and provides structural integrity
(B) usually has clover leaf-like structure
(C) carries genetic information from DNA to ribosomes
(D) codes for proteins
- Sol. (B)**
rRNA is present in the ribosomes and mRNA carries genetic information from DNA and codes for protein, while tRNA carries amino acids at site of protein synthesis and also have clover leaf-like structure
- 67.** Some animals excrete uric acid in urine (uricotelic) as it requires very little water loss. Which animals among the following are most likely to be uricotelic ?
(A) fishes (B) amphibians (C) birds (D) mammals
- Sol. (C)**
Birds are uricotelic.
- 68.** A ripe mango, kept with unripe mangoes causes their ripening. This is due to the release of a gaseous plant hormone.
(A) auxin (B) gibberlin (C) cytokinine (D) ethylene
- Sol. (D)**
Ethylene is only gaseous plant hormone that induce ripening of fruit.
- 69.** Human chromosomes undergo structural changes during the cell cycle. Chromosomal structure can be best visualized if a chromosome is isolated from a cell at :
(A) G1 phase (B) S phase (C) G2 phase (D) M phase
- Sol. (D)**
Chromosomal structure can be best visualized in metaphase of M Phase .
- 70.** By which of the following mechanisms is glucose reabsorbed from the glomerular filtrate by the kidney tubule ?
(A) osmosis (B) diffusion (C) active transport (D) passive transport
- Sol. (B)**
In kidney tubule PCT, 100% glucose from glomerular filtrate is reabsorbed by diffusion.
- 71.** In mammals, the hormones secreted by the pituitary, the master gland, is itself regulated by the pituitary, the master gland, is itself regulated by :
(A) hypothalamus (B) median cortex (C) pineal gland (D) cerebrum
- Sol. (A)**
Secretion of hormones by the pituitary gland is regulated by a part of brain “hypothalamus”.
- 72.** Which of the following is true TCA cycle in eukaryotes ?
(A) takes place in mitochondrion (B) produces no ATP
(C) takes place in Golgi complex (D) independent of electron transport chain
- Sol. (A)**
Kreb Cycle/TCA cycle in eukaryotes takes place in mitochondria.
- 73.** A hormone molecule binds to a specific protein on the plasma membrane inducing a signal. The protein it binds to is called :
(A) ligand (B) antibody (C) receptor (D) histone
- Sol. (C)**
Proteins on plasma membrane to which hormone molecules bind is called receptor.
- 74.** DNA mutations that do not cause any functional change in the protein product are known as :
(A) nonsense mutations (B) missense mutations (C) deletion mutations (D) silent mutations

- Sol. (D)**
Silent mutations are DNA mutations that do not result in a change to the amino acid sequence of a protein.
- 75.** Plant roots are usually devoid of chlorophyll and cannot perform photosynthesis. However, there are exceptions. Which of the following plant root can perform photosynthesis ?
(A) Arabidopsis (B) Tinospora (C) Rice (D) Hibiscus
- Sol. (B)**
Some plants develop green, chlorophyll containing roots along with colourless roots, these green roots perform photosynthesis
e.g. – Trapa and Tinospora
- 76.** Vitamin A deficiency leads to night-blindness. Which of the following is the reason for the disease ?
(A) rod cells are not converted to cone cells
(B) rhodopsin pigment of rod cells is defective
(C) melanin pigment is not synthesized in cone cells
(D) cornea of eye gets dried
- Sol. (B)**
Vitamin A is structural component of visual pigment. "Rhodopsin", vitamin A deficiency may lead to night blindness because rhodopsin pigment of rod cells become defective.
- 77.** In Dengue virus infection, patients often develop haemorrhagic fever due to internal bleeding. This happens due to the reduction of :
(A) platelets (B) RBCs (C) WBCs (D) lymphocytes
- Sol. (A)**
Haemorrhagic fever in dengue virus infection may cause internal bleeding due to the reduction of platelets.
- 78.** If the sequence of bases in sense strand of DNA is 5' – GTTCATCG-3', then the sequence of bases in its RNA transcript would be :
(A) 5'-GTTCATCG-3' (B) 5'-GUUCAUCG-3'
(C) 5'-CAAGTAGC-3' (D) 5'-CAAGUAGC-3'
- Sol. (B)**
Transcribed RNA is exactly like that of the sense strand of DNA
DNA → 5' - GTTCATCG - 3'
RNA → 5' - GUUCAUCG - 3'
- 79.** A reflex action is a quick involuntary response to stimulus. Which of the following is an example of BOTH, unconditioned and conditioned reflex ?
(A) knee Jerk reflex
(B) secretion of saliva in response to the aroma of food
(C) sneezing reflex
(D) contraction of pupil in response to bright light
- Sol. (B)**
Secretion of saliva in response to the aroma of food could be both unconditional and conditional reflex.
- 80.** In a food chain such as grass → deer → lion, the energy cost of respiration as a proportion of total assimilated energy at each level would be :
(A) 60% - 30% - 20% (B) 20% - 30% - 60% (C) 20%- 60%- 30% (D) 30%- 30% - 30%
- Sol. (D)**
According to 10% law at each trophic level 90% of assimilated energy is lost in respiration and only 10% is passed on to the next level so respiration energy loss would be constant at each trophic level.

PART-II

TWO MARKS QUESTIONS

MATHEMATICS

- 81.** Suppose a, b, c are real numbers, and each of the equations $x^2 + 2ax + b^2 = 0$ and $x^2 + 2bx + c^2 = 0$ has two distinct real roots. Then the equation $x^2 + 2cx + a^2 = 0$ has :
(A) two distinct positive real roots (B) two equal roots
(C) one positive and one negative root (D) no real roots

Sol. (D)

$$\begin{aligned}x^2 + 2ax + b^2 = 0 &\Rightarrow D_1 > 0 \Rightarrow 4a^2 - 4b^2 > 0 \Rightarrow a^2 - b^2 > 0 \Rightarrow a^2 > b^2 \\x^2 + 2bx + c^2 = 0 &\Rightarrow D_2 > 0 \Rightarrow 4b^2 - 4c^2 > 0 \Rightarrow b^2 > c^2 \Rightarrow a^2 > b^2 > c^2 \\&\& \quad x^2 + 2cx + a^2 = 0 \Rightarrow D_3 = 4(c^2 - a^2) < 0\end{aligned}$$

82. The coefficient of x^{2012} in $\frac{1+x}{(1+x^2)(1-x)}$ is :

- (A) 2010 (B) 2011 (C) 2012 (D) 2013

Sol. (Option are wrong) Answer is 1

$$\frac{(1+x)^2}{(1+x^2)(1-x^2)} = \frac{(1+x)^2}{1-x^4}$$

Coefficient of x^{2012} is 1.

83. Let (x, y) be a variable point on the curve $4x^2 + 9y^2 - 8x - 36y + 15 = 0$. Then $\min(x^2 - 2x + y^2 - 4y + 5) + \max(x^2 - 2x + y^2 - 4y + 5)$ is :

- (A) $\frac{325}{36}$ (B) $\frac{36}{325}$ (C) $\frac{13}{25}$ (D) $\frac{25}{13}$

Sol. (A)

$$\begin{aligned}4x^2 + 9y^2 - 8x - 36y + 15 &= 0 \\4(x^2 - 2x + 1) - 4 + 9(y^2 - 4y + 4) - 36 + 15 &= 0 \\4(x-1)^2 + 9(y-2)^2 &= 40 - 15 = 25\end{aligned}$$

$$\frac{(x-1)^2}{(5/2)^2} + \frac{(y-2)^2}{(5/3)^2} = 1$$

$$x-1 = \frac{5}{2} \cos \theta$$

$$y-2 = \frac{5}{3} \sin \theta$$

$$\text{Min.}((x-1)^2 + (y-2)^2) + \text{Max.}((x-1)^2 + (y-2)^2)$$

$$\begin{aligned}\text{Min.} \left(25 \frac{\cos^2 \theta}{4} + \frac{25 \sin^2 \theta}{9} \right) + \text{Max.} \left(25 \frac{\cos^2 \theta}{4} + 25 \frac{\sin^2 \theta}{9} \right) \\= \frac{25}{9} + \frac{25}{4} = \frac{325}{36}\end{aligned}$$

84. The sum of all $x \in [0, \pi]$ which satisfy the equation $\sin x + \frac{1}{2} \cos x = \sin^2 \left(x + \frac{\pi}{4} \right)$ is :

- (A) $\frac{\pi}{6}$ (B) $\frac{5\pi}{6}$ (C) π (D) 2π

Sol. (C)

$$\sin x + \frac{1}{2} \cos x = \sin^2 \left(x + \frac{\pi}{4} \right)$$

$$2 \sin x + \cos x = 2 \sin^2 \left(x + \frac{\pi}{4} \right)$$

$$2 \sin x + \cos x = 1 + \sin 2x$$

$$2 \sin x + \cos x - 1 - 2 \sin x \cos x = 0$$

$$2 \sin x (1 - \cos x) - 1(1 - \cos x) = 0$$

$$(2 \sin x - 1)(1 - \cos x) = 0$$

$$\sin x = \frac{1}{2}, \cos x = 1$$

$$x = 30^\circ, 150^\circ, x = 0$$

85. A polynomial $P(x)$ with real coefficients has the property that $P''(x) \neq 0$ for all x . Suppose $P(0) = 1$ and $P'(0) = -1$. What can you say about $P(1)$?

- (A) $P(1) \geq 0$ (B) $P(1) \leq 0$ (C) $P(1) \neq 0$ (D) $-1/2 < P(1) < 1/2$

Sol. (Question is wrong)

86. Define a sequence $\langle a_n \rangle$ by $a_1 = 5$, $a_n = a_1 a_2 \dots a_{n-1} + 4$ for $n > 1$. Then $\lim_{n \rightarrow \infty} \frac{\sqrt{a_n}}{a_{n-1}}$
- (A) equals $\frac{1}{2}$ (B) equal 1 (C) equals $\frac{2}{5}$ (D) does not exist

Sol. (C)

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{\sqrt{a_n}}{a_{n-1}} &= \lim_{n \rightarrow \infty} \sqrt{\frac{a_1 a_2 \dots a_{n-1} + 4}{(a_{n-1})^2}} \\ &= \lim_{n \rightarrow \infty} \sqrt{\frac{a_1 a_2 \dots a_{n-2} + \frac{4}{a_{n-1}}}{(a_{n-1})^2}} \\ &= \lim_{n \rightarrow \infty} \sqrt{\frac{1}{1 + \frac{4}{a_1 a_2 \dots a_{n-2}}} + \frac{4}{(a_{n-1})^2}} = 1 \end{aligned}$$

87. The value of the integral $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx$, where $a > 0$, is :

- (A) π (B) $a\pi$ (C) $\frac{\pi}{2}$ (D) 2π

Sol. (C)

$$\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx = \int_0^{\pi} \cos^2 x dx = \frac{\pi}{2}$$

88. Consider
 $L = \sqrt[3]{2012} + \sqrt[3]{2013} + \dots + \sqrt[3]{3011}$
 $R = \sqrt[3]{2013} + \sqrt[3]{2014} + \dots + \sqrt[3]{3012}$
and $I = \int_{2012}^{3012} \sqrt[3]{x} dx$.

- Then
(A) $L + R < 2I$ (B) $L + R = 2I$ (C) $L + R > 2I$ (D) $\sqrt{LR} = I$

Sol. (A)

$$\frac{L+R}{2} < I \Rightarrow L+R < 2I$$

89. A man tosses a coin 10 times, scoring 1 point for each head and 2 points for each tail. Let $P(K)$ be the probability of scoring at least K points. The largest value of K such that $P(K) > \frac{1}{2}$ is :

- (A) 14 (B) 15 (C) 16 (D) 17

Sol. (B)

90. Let $f(x) = \frac{x+1}{x-1}$ for all $x \neq 1$. Let.
 $f^1(x) = f(x), f^2(x) = f(f(x))$ and generally
 $f^n(x) = f(f^{n-1}(x))$ for $n > 1$.

$$\text{Let } P = f^1(2)f^2(3)f^3(4)f^4(5)$$

Which of the following is a multiple of P

- (A) 125 (B) 375 (C) 250 (D) 147

Sol. (B)

$$f(2) = 3, f^2(3) = -3, f^3(4) = -5/3, f^4(5) = 5$$

$$P = 75$$

PHYSICS

91. The total energy of a black body radiation source is collected for five minutes and used to heat water. The temperature of the water increases from 10.0°C to 11.0°C . The absolute temperature of the black body is doubled and its surface area halved and the experiment repeated for the same time. Which of the following statements would be most nearly correct ?

- (A) The temperature of the water would increase from 10.0°C to a final temperature of 12°C
 (B) The temperature of the water would increase from 10.0°C to a final temperature of 18°C
 (C) The temperature of the water would increase from 10.0°C to a final temperature of 14°C
 (D) The temperature of the water would increase from 10.0°C to a final temperature of 11°C

Sol. (B)

$$H = ms\Delta T$$

$$\Rightarrow \sigma AT_0^4 = ms \times 1 \quad \dots\dots\dots(i)$$

$$\sigma \frac{A}{2} (2T_0)^2 = ms \Delta T \quad \dots\dots\dots(ii)$$

from (i) & (ii)

$$\Delta T = 8^\circ\text{C}$$

92. A small asteroid is orbiting around the sun in a circular orbit of radius r_0 with speed V_0 . A rocket is launched from the asteroid with speed $V = \alpha V_0$, where V is the speed relative to the sun. The highest value of α for which the rocket will remain bound to the solar system (ignoring gravity due to the asteroid and effects of other planets).

- (A) $\sqrt{2}$ (B) 2 (C) $\sqrt{3}$ (D) 1

Sol. (A)

$$V_0 = \sqrt{\frac{GM}{r_0}} \quad \dots\dots\dots(i)$$

For rocket to remain bounded with solar system, mechanical energy < 0

$$\Rightarrow \frac{1}{2}mv^2 - \frac{GMm}{r_0} \leq 0 \quad \dots\dots\dots(ii)$$

$$\Rightarrow \alpha \leq \sqrt{2}$$

$$\alpha_{\max} = \sqrt{2}$$

93. A radioactive nucleus A has a single decay mode with half life τ_A . Another radioactive nucleus B has two decay modes 1 and 2. If decay mode 2 were absent, the half life of B would have been $\tau_A/2$. If decay mode 1 were absent, the half life of B would have been $2\tau_A$. If the actual half life of B is τ_B . then the ratio τ_B/τ_A is :

- (A) 3/7 (B) 7/2 (C) 7/3 (D) 1

Sol. (A)

$$\tau_B = \frac{\ln 2}{\lambda_{\text{eff.}}}$$

$$\lambda_{\text{eff.}} = \lambda_1 + \lambda_2$$

$$= \frac{2\ln 2}{\tau_A} + \frac{\ln 2}{3\tau_A}$$

$$\Rightarrow \tau_B = \frac{\ln 2}{\frac{2\ln 2}{\tau_A} + \frac{\ln 2}{3\tau_A}}$$

$$= \tau_A \left(\frac{3}{7} \right) \Rightarrow \frac{\tau_B}{\tau_A} = \frac{3}{7}$$

94. A stream of photons having energy 3 eV each impinges on a potassium surface. The work function of potassium is 2.3 eV. The emerging photo-electrons are slowed down by a copper plate placed 5 mm away. If the potential difference between the two metal plates is 1 V, the maximum distance the electrons can move away from the potassium surface before being turned back is :

- (A) 3.5 mm (B) 1.5 mm (C) 2.5 mm (D) 5.0 mm

Sol. (A)

$$K_{\max} = 3\text{eV} - 2.3\text{eV}$$

$$= 0.7 \text{ eV}$$

using work energy theorem

$$0.7 = 200 \ell$$

$$\ell = \frac{0.7}{200} = 3.5 \text{ mm.}$$

95. Consider three concentric metallic spheres A, B and C of radii a, b, c respectively where $a < b < c$. A and B are connected whereas C is grounded. The potential of the middle sphere B is raised to V then the charge on the sphere C is :

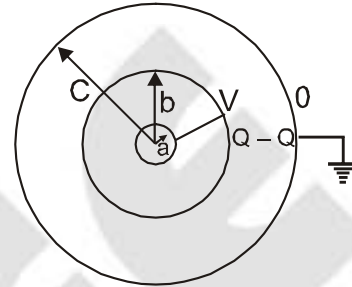
- (A) $-4\pi\epsilon_0 V \frac{bc}{c-b}$ (B) $+4\pi\epsilon_0 V \frac{bc}{c-b}$ (C) $-4\pi\epsilon_0 V \frac{ac}{c-a}$ (D) Zero

Sol. (A)

$$\Delta V_{BC} = Q \left(\frac{1}{4\pi\epsilon_0 b} - \frac{1}{4\pi\epsilon_0 c} \right)$$

$$Q = \frac{4\pi\epsilon_0 Vbc}{c-b}$$

$$\text{Change on sphere } c = -Q = \frac{-4\pi\epsilon_0 Vbc}{c-b}$$



96. On a bright sunny day a diver of height h stands at the bottom of a lake of depth H . Looking upwards, he can see objects outside the lake in a circular region of radius R . Beyond this circle he sees the images of objects lying on the floor of the lake. If refractive index of water is $4/3$, then the value of R is :

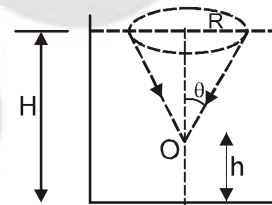
- (A) $3(H-h)/\sqrt{7}$ (B) $3h\sqrt{7}$ (C) $(H-h)/\sqrt{7/3}$ (D) $(H-h)/\sqrt{5/3}$

Sol. (A)

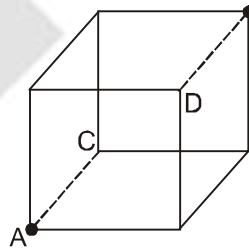
$$\sin \theta = \frac{1}{\mu} = \frac{1}{4} \times 3 = \frac{3}{4}$$

$$\tan \theta = \frac{3}{\sqrt{7}} = \frac{R}{H-h}$$

$$\Rightarrow R = \frac{(H-h)3}{\sqrt{7}}$$



97. As shown in the figure below, a cube is formed with ten identical resistances R (thick lines) and two shorting wires (dotted lines) along the arms AC and BD .



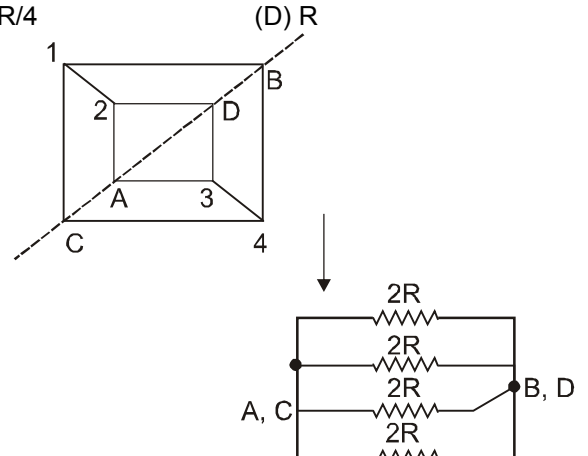
Resistance between point A and B is :

- (A) $R/2$ (B) $5R/6$ (C) $3R/4$ (D) R

Sol. (A)

Transforming the circuit

$$\therefore \text{Effective resistance} = \frac{R}{2}$$



98. A standing wave in a pipe with a length $L = 1.2$ m is described by

$$y(x,t) = y_0 \sin[(2\pi/L)x] \sin[(2\pi/L)x + \pi/4]$$

Based on above information, which one of the following statement is **incorrect**.

(Speed of sound in air is 300 m.s.^{-1})

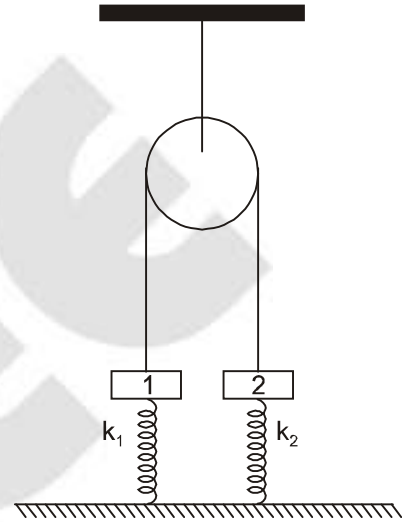
- (A) The pipe is closed at both ends
 (B) The wavelength of the wave could be 1.2 m
 (C) There could be a node at $x = 0$ and antinode at $x = L/2$
 (D) The frequency of the fundamental mode of vibrations is 137.5 Hz

Sol. (Wrong DATA)

99. Two blocks (1 and 2) of equals mass m are connected by an ideal string (see figure below) over a frictionless pulley. The blocks are attached to the ground by springs having spring constants k_1 and k_2 such that $k_1 > k_2$.

Initial, both springs are unscratched. The block 1 is slowly pulled down a distance x and released. Just after the release the possible values of magnitudes of the accelerations of the blocks a_1 and a_2 can be :

- (A) either $\left(a_1 = a_2 = \frac{(k_1 + k_2)x}{2m} \right)$ or $\left(a_1 = \frac{k_1x}{m} - g \text{ and } a_2 = \frac{k_2x}{m} + g \right)$
 (B) $\left(a_1 = a_2 = \frac{(k_1 + k_2)x}{2m} \right)$ only
 (C) $\left(a_1 = a_2 = \frac{(k_1 - k_2)x}{2m} \right)$ only
 (D) either $\left(a_1 = a_2 = \frac{(k_1 - k_2)x}{2m} \right)$ or $\left(a_1 = a_2 = \frac{(k_1 k_2)x}{(k_1 + k_2)m} - g \right)$



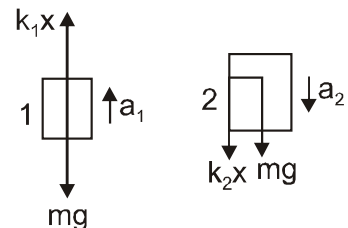
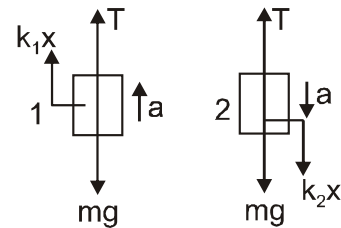
- Sol. (A)
 (i) If $k_1x > mg$

$$\therefore a = \frac{(k_1 + k_2)x}{2m}$$

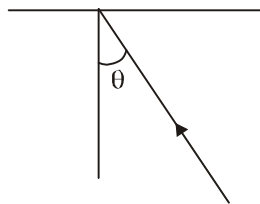
- (ii) If $k_1x > mg$

$$a_1 = \frac{k_1x}{m} - g$$

$$a_2 = \frac{k_2x}{m} + g$$



100. A simple pendulum is released from rest at the horizontally stretched position. When the string makes an angle θ with the vertical, the angle ϕ which the acceleration vector of the bob makes with the string is given by :



- (A) $\phi = 0$ (B) $\phi = \tan^{-1}\left(\frac{\tan\theta}{2}\right)$ (C) $\phi = \tan^{-1}(2\tan\theta)$ (D) $\phi = \pi/2$

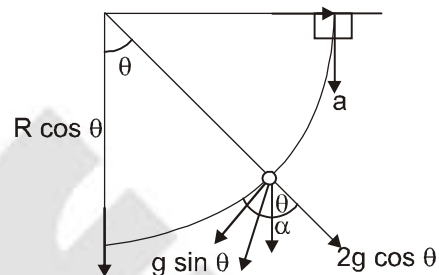
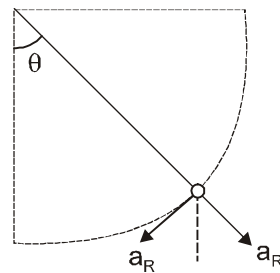
Sol. (B)

$$a_T = g \sin \theta$$

$$a_R = \frac{v^2}{R} = \frac{2gR \cos \theta}{R} = 2g \cos \theta$$

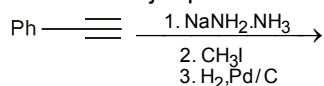
$$\therefore \tan \varphi = \frac{g \sin \theta}{2g \cos \theta}$$

$$= \frac{\tan \theta}{2}$$

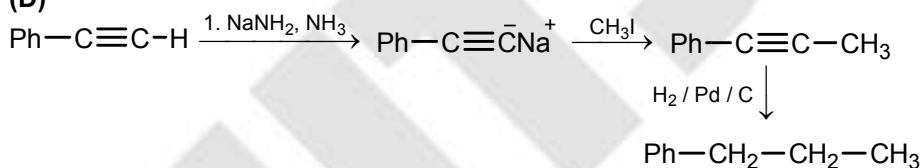


CHEMISTRY

101. The final major product obtained in the following sequence of reaction is :



Sol. (D)



102. In the DNA of E. Coli the mole ratio of adenine to cytosine is 0.7. If the number of moles of adenine in the DNA is 350000, the number of moles of guanine is equal to :

- (A) 350000 (B) 500000 (C) 225000 (D) 700000

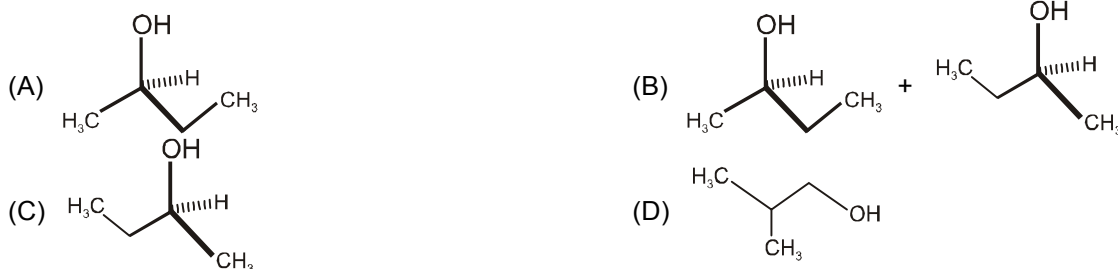
Sol. (B)

$$\frac{A}{C} = 0.7$$

$$C = \frac{A}{0.7} = \frac{350000}{0.7} = 50,000$$

$$A = G$$

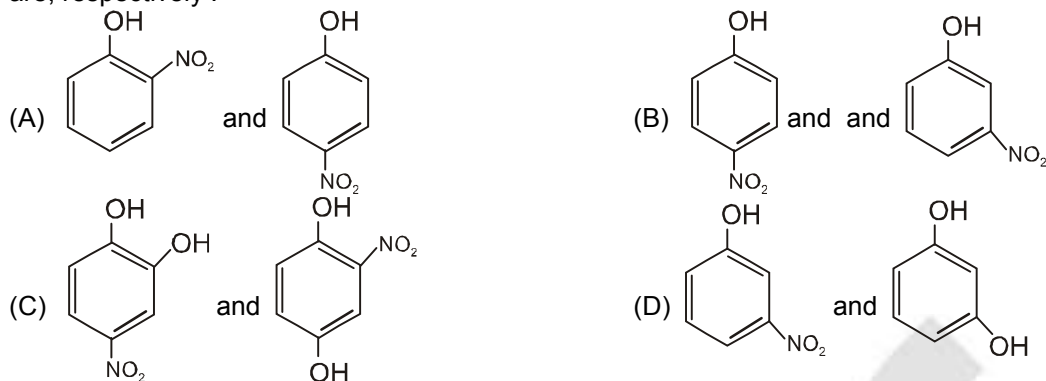
103. (R)-2-bromobutane upon treatment with aq. NaOH gives :



Sol. (C)

(R)-2-bromobutane upon treatment with aq. NaOH will give (S)-2-butanol with S_N2 (inversion).

104. Phenol on treatment with dil. HNO_3 gives two products P and Q. P is steam volatile but Q is not. P and Q are, respectively :



Sol. (A)
P is steam volatile due to intramolecular H-bonding.

105. A metal is irradiated with light of wavelength 660 nm. Given that the work function of the metal is 1.0 eV, the de Broglie wavelength of the ejected electron is close to :

(A) 6.6×10^{-7} m (B) 8.9×10^{-11} m (C) 1.3×10^{-9} m (D) 6.6×10^{-13} m

Sol. (C)

$$\frac{hc}{\lambda} = \frac{12400}{600} = 1.88 \text{ eV}$$

$$h\nu = \phi + \text{KE}$$

$$\text{KE} = (1.88 - 1) \text{ eV} = 0.88 \text{ eV}$$

$$l = \frac{\sqrt{150e}}{\sqrt{E(\text{eV})}}$$

$$= \frac{\sqrt{150}}{\sqrt{0.88}} = \sqrt{170.45} \text{ \AA}$$

$$= 13 \text{ \AA} = 1.3 \times 10^{-9} \text{ m}$$

106. The inter-planar spacing between the (2 2 1) planes of a cubic lattice of length 450 pm is :

(A) 50 pm (B) 150 pm (C) 300 pm (D) 450 pm

Sol. (B)

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}} = \frac{450}{\sqrt{2^2 + 2^2 + 1^2}} = \frac{450}{\sqrt{9}} = 150 \text{ pm}$$

107. The ΔH for vaporization of a liquid is 20 kJ/mol. Assuming ideal behaviour, the change in internal energy for the vaporization of 1 mol of the liquid at 60°C and 1 bar is close to :

(A) 13.2 kJ/mol (B) 17.2 kJ/mol (C) 19.5 kJ/mol (D) 20.0 kJ/mol

Sol. (B)

$$H = E + PV$$

$$\Delta H = \Delta E + \Delta(PV)$$

$$\Delta H = \Delta E + P\Delta V \text{ (as } P = \text{constant)}$$

$$\Delta E = \Delta H - P\Delta V$$

$$= \Delta H - n_g RT$$

$$= \left(20 - \frac{1 \times 8.314 \times 333}{1000} \right) \text{ kJ/mol}$$

$$= 20 - 2.7687 \text{ kJ/mol}$$

$$= 17.2 \text{ kJ/mol}$$

108. Among the following, the species that is both tetrahedral and diamagnetic is :

(A) $[\text{NiCl}_4]^{2-}$ (B) $[\text{Ni}(\text{CN})_4]^{2-}$ (C) $\text{Ni}(\text{CO})_4$ (D) $[\text{Ni}(\text{H}_2\text{O})_6]^{2+}$

Sol. (C)

$\text{Ni}(\text{CO})_4$ oxidation state of Ni is zero so hybridization is sp^3 without any unpaired electron.

109. Three moles of an ideal gas expands reversibly under isothermal condition from 2 L to 20 L at 300 K. The amount of heat-change (in kJ/mol) in the process is :

- (A) 0 (B) 7.2 (C) 10.2 (D) 17.2

Sol. (D)

$$\begin{aligned} \Delta E &= Q + w \\ \Delta E &= 0 \text{ under given condition} \\ Q &= -w \\ &= -\left(-nRT \ln \frac{V_2}{V_1}\right) \\ &= 3 \times 8.314 \times 300 \times 2.303 \log \frac{20}{2} \\ &= 17.232 \text{ kJ/mol} \end{aligned}$$

110. The following data are obtained for a reaction, $X + Y \rightarrow \text{Products}$.

Expt.	$[X_0]/\text{mol}$	$[Y_0]/\text{mol}$	rate/mol L ⁻¹ s ⁻¹
1.	0.25	0.25	1.0×10^{-6}
2.	0.50	0.25	4.0×10^{-6}
3.	0.25	0.50	8.0×10^{-6}

The overall order of the reaction is :

- (A) 2 (B) 4 (C) 3 (D) 5

Sol. (C)

As rate of reaction gets 8 times by doubling the conc. of y while keeping conc. of x constant so order of reaction with respect to y is 3. Similarly, keeping Y constant rate gets quadrupled on doubling conc. of X. Hence order of reaction with respect to X is 2. So over all order of reaction is 5.

BIOLOGY

111. When hydrogen peroxide is applied on the wound as a disinfectant, there is frothing at the site of injury, which is due to the presence of an enzyme in the skin that uses hydrogen peroxide as a substrate to produce :

- (A) hydrogen (B) carbon Dioxide (C) water (D) oxygen

Sol. (D)

Peroxidase enzyme use hydrogen peroxide as a substrate to produce oxygen that cause frothing at the site of injury.

112. Persons suffering from hypertension (high blood pressure) are advised a low-salt diet because :

- (A) more salt is absorbed in the body of a patient with hypertension
 (B) high salt leads to water retention in the blood that further increases the blood pressure
 (C) high salt increases nerve conduction and increases
 (D) high salt causes adrenaline release that increases blood pressure

Sol. (B)

High salt leads to water retention in the blood that increases blood volume and further increases the blood pressure.

113. Insectivorous plants that mostly grow on swampy soil use insects as a source of :

- (A) carbon (B) nitrogen (C) phosphorous (D) magnesium

Sol. (B)

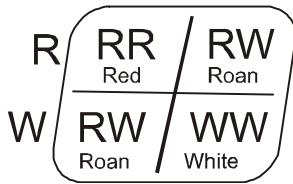
Insectivorous plants use insects as source of nitrogen.

114. In cattle, the coat colour red and white are two dominant traits, which express equally in F1 to produce roan (red and white colour in equal proportion). If F1 progeny are self-bred, the resulting progeny in F2 will have phenotypic ratio (red : roan : white) is :

- (A) 1 : 1 : 1 (B) 3 : 9 : 3 (C) 1 : 2 : 1 (D) 3 : 9 : 4

Sol. (C)

- Phenotypic Ratio (Red : Roan : White) is 1 : 2 : 1
- As red and white colour trait are codominant on selfing of F₂ generation – RW



115. The restriction endonuclease EcoR-I recognises and cleaves DNA sequence as shown below :



What is the probable number of cleavage sites that can occur in a 10 kb long random DNA sequence ?

- (A) 10 (B) 2 (C) 100 (D) 50

Sol. (I)

116. Which one of the following is true about enzyme catalysis ?

- (A) the enzyme changes at the end of the reaction
 (B) the activation barrier of the process is lower in the presence of an enzyme
 (C) the rate of the reaction is retarded in the presence of an enzyme
 (D) the rate of the reaction is independent of substrate concentration

Sol. (B)

Enzyme serve as biocatalyst, it increases rate of a biological reaction by decreasing activation energy.

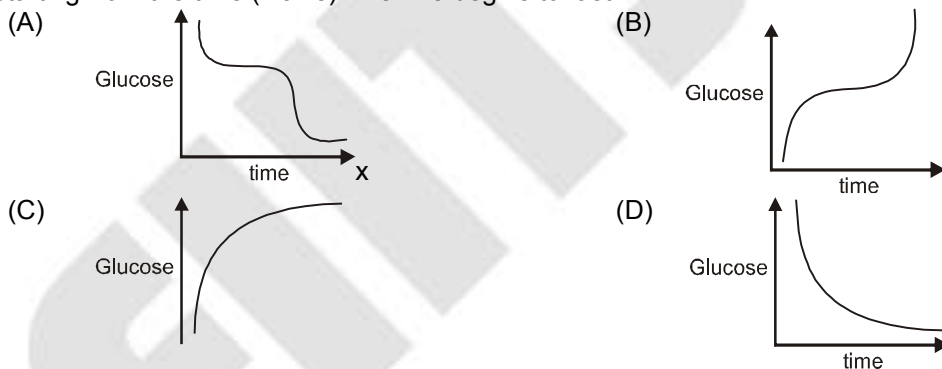
117. Vibrio cholerae causes cholera in humans. Ganga water was once used successfully to combat the infection. The possible reason could be :

- (A) high salt content of Ganga water (B) low salt content of Ganga water
 (C) presence of bacteriophages in Ganga water (D) presence of antibiotics in Ganga water

Sol. (D)

Ganga water may possibly contain antibiotic that may kill vibrio cholerae bacteria certainly.

118. When a person begins to fast, after some time glycogen stored in the liver is mobilized as a source of glucose. Which of the following graphs best represents the change of glucose level (y axis) in his blood, starting from the time (x axis) when he begins to fast ?



Sol. (A) (B) (C) (D)

On starting fast glucose level decreased and then remain constant as glucose began to mobilize from stored glycogen in liver and then further decreased.

119. The following sequence contains the open reading frame of a polypeptide. How many amino acids will the polypeptide consist of ?



- (A) 4 (B) 2 (C) 10 (D) 7

Sol. (D)

An open reading frame (ORF) contains both initiating and stop codons, given ORF may possibly encode 7 amino acids.

- 120.** Insects constitute the largest animals group on earth. About 25-30% of the insect species are known to be herbivores. In spite of such huge herbivore pressure, globally, green plants have persisted. One possible reason for this persistence is :
- (A) food presence of insects has tended to change with time
 - (B) herbivore insects have become inefficient feeders of green plants
 - (C) herbivore population has been kept in control by predators
 - (D) decline in reproduction of herbivores with time

Sol. (C)
Herbivore population has been kept in control by predators.

* * * * *

FIITJEE