**JEE Main- 27-08-2021-Evening**
**PHYSICS**
**Section-A**

**Q1.** A player kicks a football with an initial speed of 25 $ms^{-1}$ at an angle of 45º from the ground. What are the maximum height and the time taken by the football to reach at the highest point during motion ? (Take g = 10 $ms^{-2}$)
(A) $h_{max} = 15.625\,m$, $T = 3.54$ s
(B) $h_{max} = 10\,m$, $T = 2.5$ s
(C) $h_{max} = 15.625\,m$, $T = 1.77$ s
(D) $h_{max} = 3.54\,m$, $T = 0.125$ s

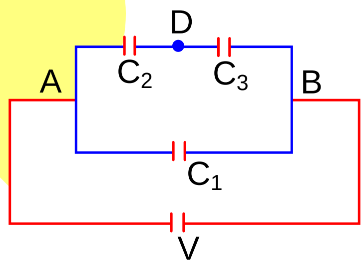
**Q2.** A mass of 50 kg is placed at the centre of a uniform spherical shell of mass 100 kg and radius 50 m. If the gravitational potential at a point, 25 m from the centre is V kg/m. The value of V is :
(A) + 2 G
(B) – 4 G
(C) – 20 G
(D) – 60 G

**Q3.** Match List I with List II

| List I | | List II | |
|---|---|---|---|
| (a) | $R_H$ (Rydberg constant) | (i) | kg $m^{-1}$ $s^{-1}$ |
| (b) | h (Planck's constant) | (ii) | kg $m^2$ $s^{-1}$ |
| (c) | $\mu_B$ (Magnetic field energy density) | (iii) | $m^{-1}$ |
| (d) | $\eta$ (coefficient of viscocity) | (iv) | kg $m^{-1}$ $s^{-2}$ |

(A) (a) – (ii), (b) – (iii), (c) – (iv), (d) – (i)
(B) (a) – (iii), (b) – (ii), (c) – (i), (d) – (iv)
(C) (a) – (iii), (b) – (ii), (c) – (iv), (d) – (i)
(D) (a) – (iv), (b) – (ii), (c) – (i), (d) – (iii)

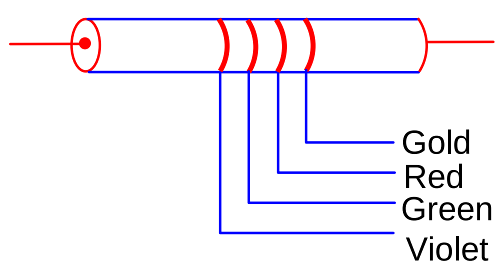
**Q4.** Three capacitors $C_1 = 2\mu F$, $C_2 = 12\mu F$ and $C_3 = 12\mu F$ are connected as shown in figure. Find the ratio of the charges on capacitors $C_1$, $C_2$ and $C_3$ respectively:




(A) 2:1:1
(B) 3:4:4
(C) 2:3:3
(D) 1:2:2

**Q5.** An antenna is mounted on a 400 m tall building. What will be the wavelength of signal that can be radiated effectively by the transmission tower upto a range of 44 km?
(A) 75.6 m
(B) 605 m
(C) 302 m
(D) 37.8 m

**Q6.** The colour coding on a carbon resistor is shown in the given figure. The resistance value of the given resistor is :



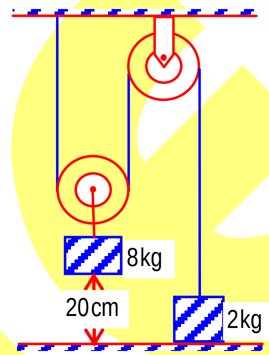

(A) $(7500 \pm 375)\Omega$
(B) $(5700 \pm 285)\Omega$
(C) $(7500 \pm 750)\Omega$
(D) $(5700 \pm 375)\Omega$

**Q7.** A coaxial cable consists of an inner wire of radius 'a' surrounded by an outer shell of inner and outer radii 'b' and 'c' respectively. The inner wire carries an electric current $i_0$, which is distributed uniformly across cross-sectional area. The outer shell carries an equal current in opposite direction and distributed uniformly. What will be the ratio of the magnetic field at a distance x from the axis when (i) x < a and (ii) a < x < b ?

(A) $\frac{x^2}{a^2}$ (B) $\frac{b^2-a^2}{x^2}$

(C) $\frac{a^2}{x^2}$ (D) $\frac{x^2}{b^2-a^2}$

**Q8.** The boxes of masses 2 kg and 8 kg are connected by a massless string passing over smooth pulleys. Calculate the time taken by box of mass 8 kg to strike the ground starting from rest. (use g = 10 m/s²)




(A) 0.25 s
(B) 0.4 s
(C) 0.34 s
(D) 0.2 s

**Q9.** For full scale deflection of total 50 divisions, 50 mV voltage is required in galvanometer. The resistance of galvanometer if its current sensitivity is 2 div/mA will be :

(A) 2Ω (B) 4Ω

(C) 1Ω (D) 5Ω

**Q10.** For a transistor $\alpha$ and $\beta$ are given as $\alpha = \frac{I_C}{I_E}$ and $\beta = \frac{I_C}{I_B}$. Then the correct relation between $\alpha$ and $\beta$ will be:

(A) $\beta = \frac{\alpha}{1-\alpha}$ (B) $\alpha\beta = 1$

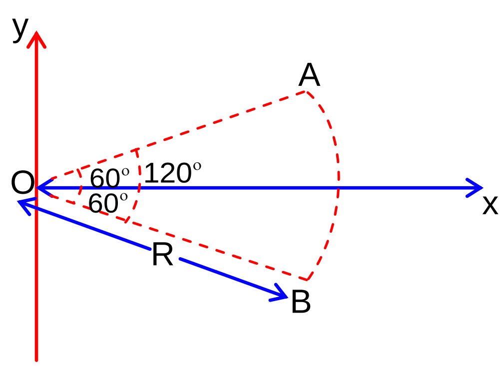
(C) $\alpha = \frac{\beta}{1-\beta}$ (D) $\alpha = \frac{1-\beta}{\beta}$

**Q11.** A monochromatic neon lamp with wavelength of 670.5 nm illuminates a photo-sensitive material which has a stopping voltage of 0.48 V. What will be the stopping voltage if the source light is changed with another source of wavelength of 474.6 nm?

(A) 1.5 V (B) 0.24 V

(C) 0.96 V (D) 1.25 V

**Q12.** Figure shows a rod AB, which is bent in a 120º circular arc of radius R. A charge (–Q) is uniformly distributed over rod AB. What is the electric field $\vec{E}$ at the centre of curvature O ?



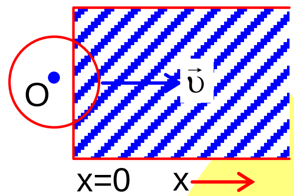

(A) $\frac{3\sqrt{3}Q}{8\pi^2\varepsilon_0R^2}(-\hat{i})$

(B) $\frac{3\sqrt{3}Q}{8\pi^2\varepsilon_0R^2}(\hat{i})$

(C) $\frac{3\sqrt{3}Q}{8\pi\varepsilon_0 R^2}(\hat{i})$

(D) $\frac{3\sqrt{3}Q}{16\pi^2\varepsilon_0 R^2}(\hat{i})$

**Q13.** A constant magnetic field of 1 T is applied in the x > 0 region. A metallic circular ring of radius 1m is moving with a constant velocity of 1 m/s along the x-axis. At t = 0, the centre of O of the ring is at x = – 1m. What will be the value of the induced emf in the ring at t = 1s? (Assume the velocity of the ring does not change.)




(A) 0 V
(B) $2\pi$ V
(C) 2 V
(D) 1 V

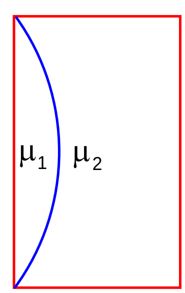
**Q14.** The height of victoria falls is 63 m. What is the difference in temperature of water at the top and at the bottom of fall ?
[Given 1 cal = 4.2 J and specific heat of water $= 1 \text{cal} g^{-1}\,{}^{\circ}C^{-1}$]

(A) 0.147 $^{\circ}$C
(B) 14.76 $^{\circ}$C
(C) 1.476 $^{\circ}$C
(D) 0.014 $^{\circ}$C

**Q15.** Two discs have moments of intertia $I_1$ and $I_2$ about their respective axes perpendicular to the plane and passing through the centre. They are rotating with angular speeds, $\omega_1$ and $\omega_2$ respectively and are brought into contact face to face with their axes of rotation coaxial. The loss in kinetic energy of the system in the process is given by:

(A) $\frac{(I_1 - I_2)^2 \omega_1\omega_2}{2(I_1 + I_2)}$
(B) $\frac{I_1 I_2}{(I_1 + I_2)}(\omega_1 - \omega_2)^2$
(C) $\frac{I_1 I_2}{2(I_1 + I_2)}(\omega_1 - \omega_2)^2$
(D) $\frac{(\omega_1 - \omega_2)^2}{2(I_1 + I_2)}$

**Q16.** Curved surfaces of a plano-convex lens of refractive index $\mu_1$ and a plano-concave lens of refractive index $\mu_2$ have equal radius of curvature as shown in figure. Find the ratio of radius of curvature to the focal length of the combined lenses.




(A) $\frac{1}{\mu_2 - \mu_1}$
(B) $\mu_2 - \mu_1$
(C) $\frac{1}{\mu_1 - \mu_2}$
(D) $\mu_1 - \mu_2$

**Q17.** The light waves from two coherent sources have same intensity $I_1 = I_2 = I_0$. In interference pattern the intensity of light at minima is zero. What will be the intensity of light at maxima ?

(A) $I_0$
(B) 2 $I_0$

(C) 5 $I_0$ (D) 4 $I_0$

**Q18.** If force (F), length (L) and time (T) are taken as the fundamental quantities. Then what will be the dimension of density :

(A) $\left[FL^{-3}T^{2}\right]$ (B) $\left[FL^{-5}T^{2}\right]$

(C) $\left[FL^{-4}T^{2}\right]$ (D) $\left[FL^{-3}T^{3}\right]$

**Q19.** If the rms speed of oxygen molecules at 0°C is 160 m/s, find the rms speed of hydrogen molecules at 0°C.

(A) 332 m/s (B) 640 m/s

(C) 40 m/s (D) 80 m/s

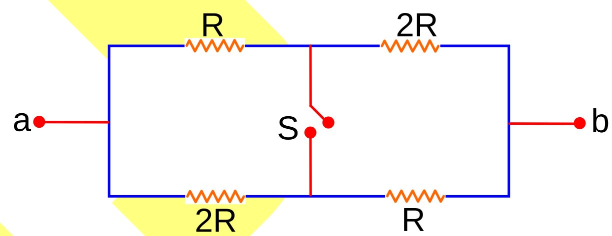
**Q20.** Water drops are falling from a nozzle of a shower onto the floor, from a height of 9.8 m. The drops fall at a regular interval of time. When the first drop strikes the floor, at that instant, the third drop begins to fall. Locate the position of second drop from the floor when the first drop strikes the floor.

(A) 7.35 m (B) 2.45 m

(C) 2.94 m (D) 4.18 m

## Section-B

**Q1.** The ratio of the equivalent resistance of the network (shown in figure) between the points a and b when switch is open and switch is closed is x : 8. The value of x is _______.



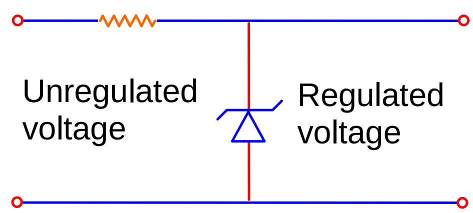

**Q2.** A heat engine operates between a cold reservoir at temperature $T_2$ = 400 K and a hot reservoir at temperature $T_1$. It takes 300 J of heat from the hot reservoir and delivers 240 J of heat to the cold reservoir in a cycle. The minimum temperature of the hot reservoir has to be _________K.

**Q3.** A plane electromagnetic wave with frequency of 30 MHz travels in free space. At particular point in space and time, electric field is 6 V/m. The magnetic field at this point will be $x \times 10^{-8}$ T. The value of x is _____.

**Q4.** A tuning fork is vibrating at 250 Hz. The length of the shortest closed organ pipe that will resonate with the tuning fork will be _____ cm. (Take speed of sound in air as 340 $ms^{-1}$)

**Q5.** X different wavelengths may be observed in the spectrum from a hydrogen sample if the atoms are excited to states with principal quantum number n = 6 ? The value of X is _______.

**Q6.** A zener diode of power rating 2W is to be used as a voltage regulator. If the zener diode has a breakdown of 10 V and it has to regulate voltage fluctuated between 6 V and 14 V, the value of $R_s$ for safe operation should be _______ Ω.

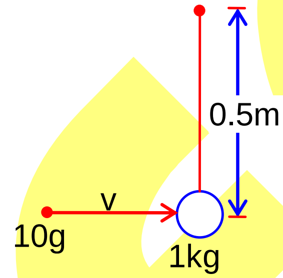


**Q7.** Two simple harmonic motion, are represented by the equations $y_1 = 10\sin\left(3\pi t + \frac{\pi}{3}\right)$

$y_2 = 5\left(\sin 3\pi t + \sqrt{3}\cos 3\pi t\right)$

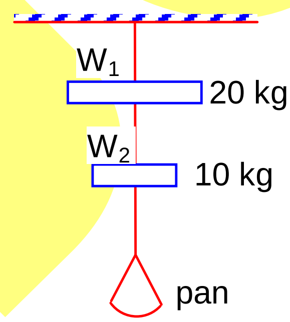
Ratio of amplitude of $y_1$ to $y_2$ = x:1. The value of x is ____

**Q8.** A bullet of 10 g, moving with velocity v, collides head-on with the stationary bob of a pendulum and recoils with velocity 100 m/s. The length of the pendulum is 0.5 m and mass of the bob is 1 kg. The minimum value of v = _____ m/s so that the pendulum describes a circle. (Assume the string to be inextensible and g = 10 m/s$^2$)




**Q9.** An ac circuit has an inductor and a resistor of resistance R in series, such that $X_L = 3R$. Now, a capacitor is added in series such that $X_C = 2R$. The ratio of new power factor with the old power factor of the circuit is $\sqrt{5} : x$ The value of x is _____.

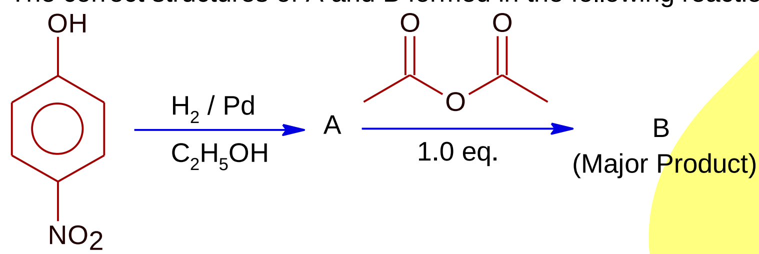
**Q10.** Wires $W_1$ and $W_2$ are made of same material having the breaking stress of $1.25 \times 10^9\ N/m^2$ . $W_1$ and $W_2$ have cross-sectional area of $8 \times 10^{-7}\ m^2$ and $4 \times 10^{-7}\ m^2$, respectively. Masses of 20 kg and 10 kg hang from them as shown in the figure. The maximum mass that can be placed in the pan without breaking the wires is ____ kg. (Use g = 10 m/s$^2$)

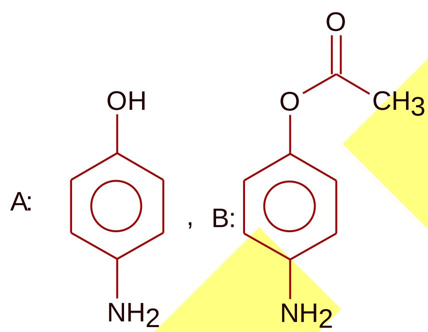


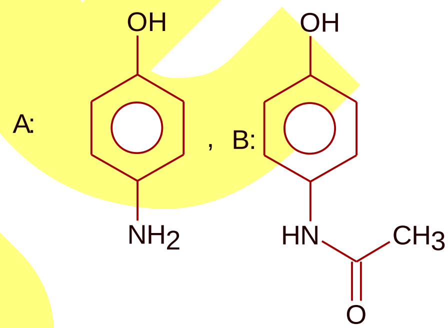
**CHEMISTRY**
**Section-A**

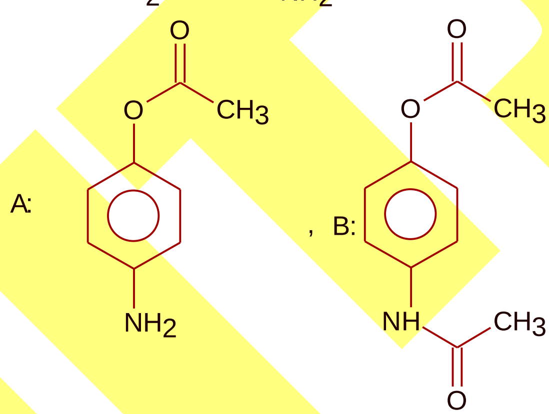
**Q1.** The oxide that gives $H_2O_2$ most readily on treatment with $H_2O$ is :
(A) $Na_2O_2$
(B) $BaO_2 \cdot 8H_2O$
(C) $PbO_2$
(D) $SnO_2$

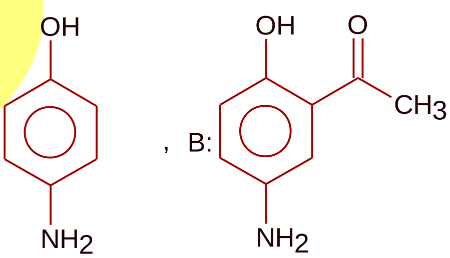
**Q2.** The correct structures of A and B formed in the following reactions are:




(A)



(B)



(C)



(D)


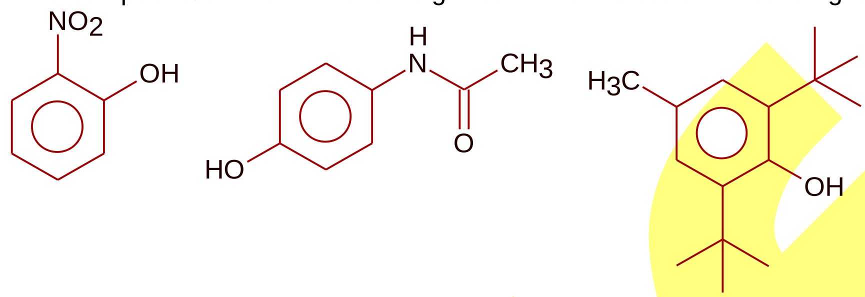

**Q3.** Potassium permanganate on heating at 513 K gives a product which is :
(A) paramagnetic and colourless
(B) diamagnetic and colourless
(C) diamagnetic and green
(D) paramagnetic and green

**Q4.** In stratosphere most of the ozone formation is assisted by :
(A) $\gamma$–rays.
(B) cosmic rays.
(C) visible radiations
(D) ultraviolet radiation.

**Q5.** Lyophilic sols are more stable than lyophobic sols because :
(A) The colloidal particles have positive charge.
(B) There is a strong electrostatic repulsion between the negatively charged colloidal particles.
(C) The colloidal particles have no charge.
(D) The colloidal particles are solvated.

**Q6.** Choose the **correct** statement from the following :
(A) The standard enthalpy of formation for alkali metal bromides becomes less negative on descending the group.
(B) LiF has least negative standard enthalpy of formation among alkali metal fluorides.
(C) The low solubility of CsI in water is due to its high lattice enthalpy.
(D) Among the alkali metal halides, LiF is least soluble in water.

**Q7.** The compound/s which will show significant intermolecular H–bonding is/are :




(a) (b) (c)

(A) (c) only (B) (b) only
(C) (a), (b) and (c) (D) (a) and (b) only

**Q8.** Match list I with list II

| List I (Name of ore / mineral) | List II (Chemical formula) |
|---|---|
| (a) Calamine | (i) ZnS |
| (b) Malachite | (ii) $FeCO_3$ |
| (c) Siderite | (iii) $ZnCO_3$ |
| (d) Sphalerite | (iv) $CuCO_3{\cdot}Cu(OH)_2$ |

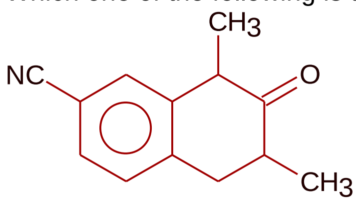
Choose the most appropriate answer from the options given below:
(A) (a) – (iv), (b) – (iii), (c) – (i), (d) – (ii) (B) (a) – (iii), (b) – (iv), (c) – (i), (d) – (ii)
(C) (a) – (iii), (b) – (ii), (c) – (iv), (d) – (i) (D) (a) – (iii), (b) – (iv), (c) – (ii), (d) – (i)

**Q9.** Which one of the following is formed (mainly) when red phosphorus is heated in a sealed tube at 803 K ?
(A) White phosphorus (B) Yellow phosphorus
(C) $\alpha$-Black phosphorus (D) $\beta$-Black phosphorus

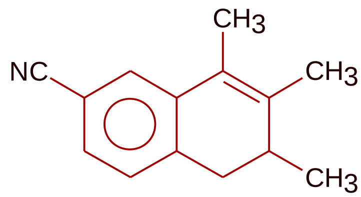
**Q10.** The correct order of ionic radii for the ions, $P^{3-}, S^{2-}, Ca^{2+}, K^{+}, Cl^{-}$ is:
(A) $K^{+} > Ca^{2+} > P^{3-} > S^{2-} > Cl^{-}$ (B) $P^{3-} > S^{2-} > Cl^{-} > Ca^{2+} > K^{+}$
(C) $Cl^{-} > S^{2-} > P^{3-} > Ca^{2+} > K^{+}$ (D) $P^{3-} > S^{2-} > Cl^{-} > K^{+} > Ca^{2+}$

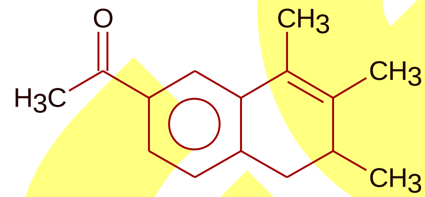
**Q11.** Given below are two statements:
Statement I: Ethyl pent –4–yn-oate on reaction with $CH_3MgBr$ gives a 3$^{o}$ –alcohol.
Statement II: In this reaction one mole of ethyl pent-4-yn-oate utilizes two moles of $CH_3MgBr$.
In the light of the above statements, choose the most appropriate answer from the options given below.
(A) Both **Statement I** and **Statement II** are false.
(B) Both **Statement I** and **Statement II** are true.
(C) **Statement I** is false but **Statement II** is true.
(D) **Statement I** is true but **Statement II** is false.

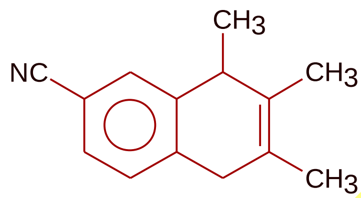
**Q12.** Which one of the following is the major product of the given reaction?

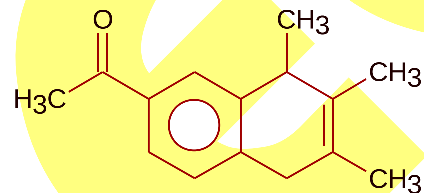



(i) $2CH_3MgBr$
(ii) $H_3O^+$
(iii) $H_2SO_4$, heat
→ Major Product

(A)


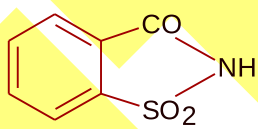

(B)


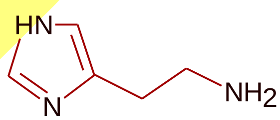

(C)


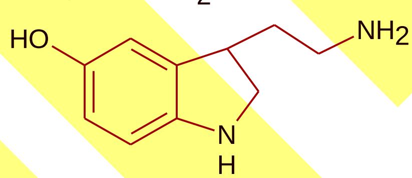

(D)


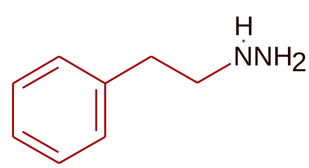

**Q13.** Hydrolysis of sucrose gives:
(A) $\alpha$ - D - (+) – Glucose and $\alpha$ - D – (–) – Fructose
(B) $\alpha$ - D - (–) – Glucose and $\alpha$ - D – (+) – Fructose
(C) $\alpha$ - D - (+) – Glucose and $\beta$ - D – (–) – Fructose
(D) $\alpha$ - D - (–) – Glucose and $\beta$ - D – (–) – Fructose

**Q14.** Which one of the following chemicals is responsible for the production of HCl in the stomach leading to irritation and pain?

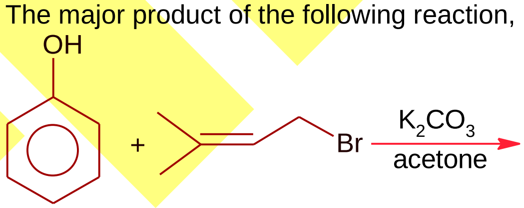
(A)



(B)


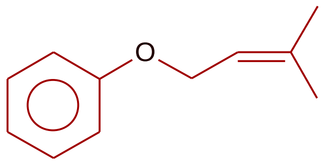

(C)



(D)


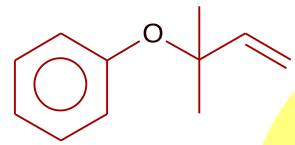

**Q15.** The major product of the following reaction, if it occur by $S_N2$ mechanism is:



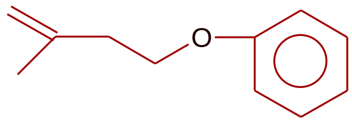
(A)



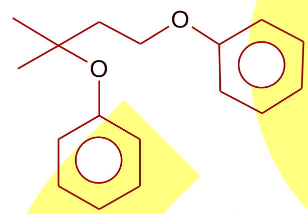
(B)



(C)



(D)



**Q16.** Which one of the following is used to remove most of plutonium from spent nuclear fuel?
(A) $BrO_3$
(B) $O_2F_2$
(C) $I_2O_5$
(D) $ClF_3$

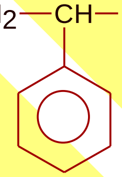
**Q17.** Which one of the following reactions will not yield propionic acid?
(A) $CH_3CH_2CH_2Br + Mg, CO_2$ dry ether / $H_3O^+$
(B) $CH_3CH_2CH_3 + KMnO_4\,(Heat), OH^- / H_3O^+$
(C) $CH_3CH_2CCl_3 + OH^- / H_3O^+$
(D) $CH_3CH_2COCH_3 + OI^- / H_3O^+$

**Q18.** Which one of the following tests used for the identification of functional groups in organic compounds does not use copper reagent ?
(A) Biuret test for peptide bond
(B) Benedict's test
(C) Barfoed's test
(D) Seliwanoff's test

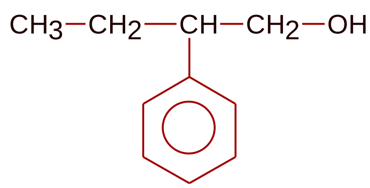
**Q19.** The addition of dilute NaOH to $Cr^{3+}$ salt solution will give:
(A) precipitate of $[Cr(OH)_6]^{3-}$
(B) precipitate of $Cr_2O_3(H_2O)_n$
(C) precipitate of $Cr(OH)_3$
(D) a solution of $[Cr(OH)_4]^-$

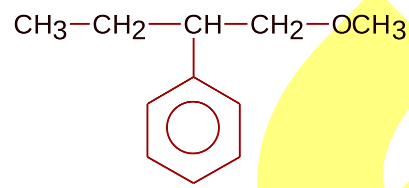
**Q20.** The major product (A) formed in the reaction given below is

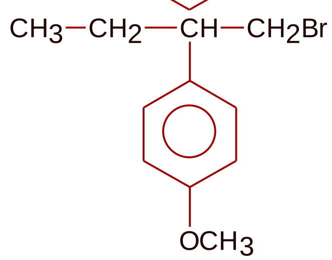
$CH_3-CH_2-CH-CH_2-Br$

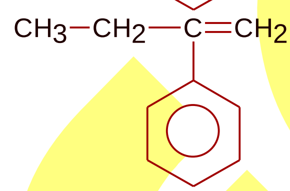


$+ CH_3O^{\ominus} \xrightarrow{CH_3OH}$ A
(Major Product)

(A) 

(B) 

(C) 

(D) 

## Section-B

**Q1.** The number of optical isomers possible for $\left[Cr\left(C_2O_4\right)_3\right]^{3-}$ is ____.

**Q2.** The first order rate constant for the decomposition of $CaCO_3$ at 700 K is $6.36 \times 10^{-3}$ $s^{-1}$ and activation energy is 209 kJ $mol^{-1}$. Its rate constant (in $s^{-1}$) at 600 K is $x \times 10^{-6}$. The value of x is ___. (nearest integer).
$\left[\text{Given } R = 8.31\ J\ K^{-1}\ mol^{-1}; \log 6.36 \times 10^{-3} = -2.19, 10^{-4.79} = 1.62 \times 10^{-5}\right]$

**Q3.** When 5.1 g of solid $NH_4HS$ is introduced into a two litre evacuated flask at 27°C, 20% of the solid decomposes into gaseous ammonia and hydrogen sulphide. The $K_p$ for the reaction at 27°C is $x \times 10^{-2}$. The value of x is _______. (Integer answer)
[Given R = 0.082 L atm $K^{-1}$ $mol^{-1}$]

**Q4.** 100 g of propane is completely reacted with 1000 g of oxygen. The mole fraction of carbon dioxide in the resulting mixture is $x \times 10^{-2}$. The value of x is _______. (Nearest integer)
[Atomic weight : H = 1.008; C = 12.00; O = 16.00]

**Q5.** Data given for the following reaction is as follows:
$FeO_{(s)} + C_{(graphite)} \longrightarrow Fe_{(s)} + CO_{(g)}$

| Substance | $\Delta_f H^0$ $\left(kJ\ mol^{-1}\right)$ | $\Delta S^0$ $\left(J\ mol^{-1}\ K^{-1}\right)$ |
|---|---|---|
| $FeO_{(s)}$ | –266.3 | 57.49 |
| $C_{(graphite)}$ | 0 | 5.74 |
| $Fe_{(s)}$ | 0 | 27.28 |
| $CO_{(g)}$ | –110.5 | 197.6 |

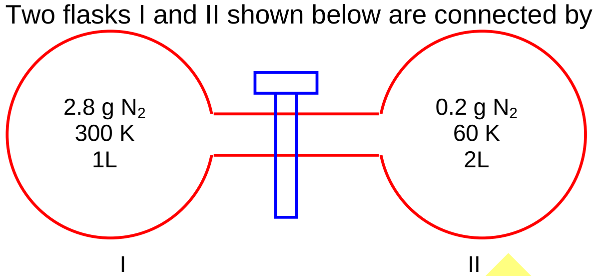
The minimum temperature in K at which the reaction becomes spontaneous is ___ (Integer answer)

**Q6.** The number of species having non – pyramidal shape among the following is ___.
(A) $SO_3$ (B) $NO_3^-$
(C) $PCl_3$ (D) $CO_3^{2-}$

**Q7.** The resistance of a conductivity cell with cell constant $1.14\ cm^{-1}$, containing 0.001 M KCl at 298 K is 1500 Ω. The molar conductivity of 0.001 M KCl solution at 298 K in $S\ cm^2\ mol^{-1}$ is___________. (Integer answer)

**Q8.** The number of photons emitted by a monochromatic (single frequency) infrared range finder of power 1 mW and wavelength of 1000 nm, in 0.1 second is $x \times 10^{13}$. The value of x is _____. (Nearest integer)
($h = 6.63 \times 10^{-34}$ Js, $c = 3.00 \times 10^{8}\ ms^{-1}$)

**Q9.** Two flasks I and II shown below are connected by a valve of negligible volume.




When the valve is opened, the final pressure of the system in bar is $x \times 10^{-2}$. The value of x is ______. (Integer answer)
[Assume–Ideal gas; 1 bar = $10^5$ Pa; Molar mass of $N_2$ = 28.0 g $mol^{-1}$ ; R = 8.31 $Jmol^{-1}K^{-1}$]

**Q10.** 40 g of glucose (Molar mass = 180) is mixed with 200 mL of water. The freezing point of solution is _____ K. (Nearest integer)
[Given : $K_f$ = 1.86 K kg $mol^{-1}$ ; Density of water = 1.00 g $cm^{-3}$; Freezing point of water = 273.15 K].

## MATHEMATICS
### Section-A

**Q1.** The Boolean expression $(p \wedge q) \Rightarrow ((r \wedge q) \wedge p)$ is equivalent to:

(A) $(p \wedge r) \Rightarrow (p \wedge q)$
(B) $(q \wedge r) \Rightarrow (p \wedge q)$
(C) $(p \wedge q) \Rightarrow (r \wedge q)$
(D) $(p \wedge q) \Rightarrow (r \vee q)$

**Q2.** Let M and m respectively be the maximum and minimum values of the function $f(x) = \tan^{-1}(\sin x + \cos x)$ in $\left[0, \frac{\pi}{2}\right]$. Then the value of $\tan(M - m)$ is equal to

(A) $3 - 2\sqrt{2}$
(B) $2 + \sqrt{3}$
(C) $3 + 2\sqrt{2}$
(D) $2 - \sqrt{3}$

**Q3.** The angle between the straight lines, whose direction cosines are given by the equations $2\ell + 2m - n = 0$ and $mn + nl + lm = 0$, is:

(A) $\frac{\pi}{3}$
(B) $\pi - \cos^{-1}\left(\frac{4}{9}\right)$
(C) $\cos^{-1}\left(\frac{8}{9}\right)$
(D) $\frac{\pi}{2}$

**Q4.** The value of the integer $\int_0^1 \frac{\sqrt{x}\,dx}{(1+x)(1+3x)(3+x)}$ is:

(A) $\frac{\pi}{4}\left(1 - \frac{\sqrt{3}}{6}\right)$
(B) $\frac{\pi}{8}\left(1 - \frac{\sqrt{3}}{2}\right)$
(C) $\frac{\pi}{4}\left(1 - \frac{\sqrt{3}}{2}\right)$
(D) $\frac{\pi}{8}\left(1 - \frac{\sqrt{3}}{6}\right)$

**Q5.** If $\lim_{x \to \infty}\left(\sqrt{x^2 - x + 1} - ax\right) = b$, then the ordered pair (a, b) is :

(A) $\left(-1, \frac{1}{2}\right)$
(B) $\left(-1, -\frac{1}{2}\right)$
(C) $\left(1, -\frac{1}{2}\right)$
(D) $\left(1, \frac{1}{2}\right)$

**Q6.** The area of the region bounded by the parabola $(y-2)^2 = (x-1)$, the tangent to it at the point whose ordinate is 3 and the x – axis is:

(A) 4
(B) 9
(C) 6
(D) 10

**Q7.** A differential equation representing the family of parabolas with axis parallel to y – axis and whose length of latus rectum is the distance of the point (2, –3) form the line 3x + 4y = 5, is given by :

(A) $10\frac{d^2x}{dy^2} = 11$
(B) $11\frac{d^2y}{dx^2} = 10$
(C) $11\frac{d^2x}{dy^2} = 10$
(D) $10\frac{d^2y}{dx^2} = 11$

**Q8.** The equation of the plane passing through the line of intersection of the planes $\vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right)=1$ and $\vec{r}.\left(2\hat{i}+3\hat{j}-\hat{k}\right)+4=0$ and parallel to the x – axis is

(A) $\vec{r}.\left(\hat{j}-3\hat{k}\right)-6=0$
(B) $\vec{r}.\left(\hat{i}+3\hat{k}\right)+6=0$
(C) $\vec{r}.\left(\hat{j}-3\hat{k}\right)+6=0$
(D) $\vec{r}.\left(\hat{i}-3\hat{k}\right)+6=0$

**Q9.** The set of all values of k > –1, for which the equation $\left(3x^2+4x+3\right)^2-(k+1)\left(3x^2+4x+3\right)\left(3x^2+4x+2\right)+k\left(3x^2+4x+2\right)^2=0$ has real roots is:

(A) $\left(\frac{1}{2},\frac{3}{2}\right]-\{-1\}$
(B) $\left(1,\frac{5}{2}\right]$
(C) $\left[-\frac{1}{2},1\right)$
(D) $[2,3)$

**Q10.** Let $A=\left(\begin{bmatrix}[x+1] & [x+2] & [x+3]\\ [x] & [x+3] & [x+3]\\ [x] & [x+2] & [x+4]\end{bmatrix}\right)$ where [t] denotes the greatest integer less than or equal to t. If det(A) = 192, then the set of values of x is the interval.

(A) $[60,61)$
(B) $[68,69)$
(C) $[62,63)$
(D) $[65,66)$

**Q11.** If $y(x)=\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right), x\in\left(\frac{\pi}{2},\pi\right)$, then $\frac{dy}{dx}$ at $x=\frac{5\pi}{6}$ is:

(A) –1
(B) 0
(C) $-\frac{1}{2}$
(D) $\frac{1}{2}$

**Q12.** It two tangents from a point P to the parabola $y^2=16(x-3)$ are at right angles, then the locus of point P is:

(A) x + 4 = 0
(B) x + 2 = 0
(C) x + 3 = 0
(D) x + 1 = 0

**Q13.** Let A(a, 0), B(b, 2b + 1) and C(0, b), b ≠ 0, $|b|\neq 1$, be points such that the area of triangle ABC is 1 sq. unit, then the sum of all possible values of a is:

(A) $\frac{-2b^2}{b+1}$
(B) $\frac{2b}{b+1}$
(C) $\frac{2b^2}{b+1}$
(D) $\frac{-2b}{b+1}$

**Q14.** Let [λ] be the greatest integer less than or equal to λ. The set of all values of λ for which the system of linear equations $x+y+z=4, 3x+2y+5z=3, 9x+4y+\left(28+[\lambda]\right)z=[\lambda]$ has a solution is:

(A) $(-\infty,-9)\cup(-9,\infty)$
(B) $[-9,-8)$
(C) R
(D) $(-\infty,-9)\cup[-8,\infty)$

**Q15.** Each of the persons A and B independently tosses three fair coins. The probability that both of them get the same number of heads is:

(A) $\frac{5}{16}$ (B) 1

(C) $\frac{1}{8}$ (D) $\frac{5}{8}$

**Q16.** Two poles AB of length a meter and CD of length a + b (b ≠ a) metres are erected at the same horizontal level with bases at B and D. If BD = x and $\tan\angle ACB = \frac{1}{2}$, then:

(A) $x^2 + 2(a+2b)x - b(a+b) = 0$

(B) $x^2 - 2ax + a(a+b) = 0$

(C) $x^2 + 2(a+2b)x + a(a+b) = 0$

(D) $x^2 - 2ax + b(a+b) = 0$

**Q17.** A box open from top is made from a rectangular sheet of dimension a x b by cutting squares each of side x from each of the four corners and folding up the flaps. If the volume of the box is maximum, then x is equal to:

(A) $\frac{a+b-\sqrt{a^2+b^2-ab}}{12}$ (B) $\frac{a+b-\sqrt{a^2+b^2-ab}}{6}$

(C) $\frac{a+b+\sqrt{a^2+b^2-ab}}{6}$ (D) $\frac{a+b-\sqrt{a^2+b^2+ab}}{6}$

**Q18.** Let Z, be the set of all integers,

A = $\{(x,y) \in Z \times Z : (x-2)^2 + y^2 \le 4\}$

B = $\{(x,y) \in Z \times Z : x^2 + y^2 \le 4\}$

C = $\{(x,y) \in Z \times Z : (x-2)^2 + (y-2)^2 \le 4\}$

If the total number of relations from $A \cap B$ to $A \cap C$ is $2^p$, then the value of p is:

(A) 49 (B) 9

(C) 16 (D) 25

**Q19.** If the solution curve of the differential equation $(2x - 10y^3)dy + ydx = 0$, passes through the points (0, 1) and (2, β), then β is a root of the equation:

(A) $y^5 - y^2 - 1 = 0$ (B) $y^5 - 2y - 2 = 0$

(C) $2y^5 - 2y - 1 = 0$ (D) $2y^5 - y^2 - 2 = 0$

**Q20.** If $0 < x < 1$ and $y = \frac{1}{2}x^2 + \frac{2}{3}x^3 + \frac{3}{4}x^4 + \ldots$, then the value of $e^{1+y}$ at x = $\frac{1}{2}$ is:

(A) $2e^2$ (B) $\frac{1}{2}e^2$

(C) 2e (D) $\frac{1}{2}\sqrt{e}$

## Section-B

**Q1.** Let S be the sum of all solutions (in radians) of the equation $\sin^4\theta + \cos^4\theta - \sin\theta\cos\theta = 0$ in $[0, 4\pi]$. Then $\frac{8S}{\pi}$ is equal to ____.

**Q2.** Let S = [1, 2, 3, 4, 5, 6, 9]. Then the number of elements in the set $T = \{A \subseteq S : A \neq \phi$ and the sum of all the elements of A is not a multiple of 3} is ___

**Q3.** Two circles each of the radius 5 units touch each other at the point (1, 2). If the equation of their common tangent is 4x + 3y = 10, and $C_1$ ($\alpha$, $\beta$) and $C_2$ ($\gamma$, $\delta$), $C_1 \neq C_2$ are their centres, then $|(\alpha+\beta)(\gamma+\delta)|$ is equal to ___

**Q4.** $3\times7^{22} + 2\times10^{22} - 44$ when divided by 18 leaves the remainder ___

**Q5.** Let S be the mirror image of the point Q(1, 3, 4) with respect to the plane $2x - y + z + 3 = 0$ and let R (3, 5, $\gamma$) be a point of this plane. Then the square of the length of the line segment SR is ___.

**Q6.** The probability distribution of random variable X is given by:

| X | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| P(X) | K | 2K | 2K | 3K | K |

Let $p = P(1 < X < 4 \mid X < 3)$. If 5p = $\lambda$.k then $\lambda$ is equal to ___

**Q7.** Let $z_1$ and $z_2$ be two complex numbers such that $\arg(z_1 - z_2) = \frac{\pi}{4}$ and $z_1, z_2$ satisfy the equation $|z - 3|$ = Re(z). Then the imaginary part of $z_1 + z_2$ is equal to ___

**Q8.** An online exam is attempted by 50 candidates out of which 20 are boys. The average marks obtained by boys is 12 with a variance 2. The variance of marks obtained by 30 girls is also 2. The average marks of all 50 candidates is 15. If $\mu$ is the average marks of girls and $\sigma^2$ is the variance of marks of 50 candidates, then $\mu + \sigma^2$ is equal to ___

**Q9.** Let A (sec $\theta$, 2tan$\theta$) and $\beta$ (sec$\phi$, 2tan$\phi$), where $\theta + \phi = \pi/2$ be two points on the hyperbola $2x^2 - y^2 = 2$. If ($\alpha$, $\beta$) is the point of the intersection of the normals to the hyperbola at A and B, then $(2\beta)^2$ is equal to ___

**Q10.** If $\int \frac{2e^x + 3e^{-x}}{4e^x + 7e^{-x}}dx = \frac{1}{14}\left(ux + v\log_e\left(4e^x + 7e^{-x}\right)\right) + C$ where C is a constant of integration, then u + v is equal to ___.

## ANSWER- KEY

### ANSWER: JEE Main- 27-08-2021-Evening

| PHYSICS | CHEMISTRY | MATHEMATICS |
|---|---|---|
| **Section-A** | **Section-A** | **Section-A** |
| **Ans1.** C | **Ans1.** A | **Ans1.** C |
| **Ans2.** B | **Ans2.** B | **Ans2.** A |
| **Ans3.** C | **Ans3.** D | **Ans3.** D |
| **Ans4.** D | **Ans4.** D | **Ans4.** B |
| **Ans5.** B | **Ans5.** D | **Ans5.** C |
| **Ans6.** A | **Ans6.** D | **Ans6.** B |
| **Ans7.** A | **Ans7.** B | **Ans7.** B |
| **Ans8.** B | **Ans8.** D | **Ans8.** C |
| **Ans9.** A | **Ans9.** C | **Ans9.** B |
| **Ans10.** A | **Ans10.** D | **Ans10.** C |
| **Ans11.** D | **Ans11.** D | **Ans11.** C |
| **Ans12.** B | **Ans12.** B | **Ans12.** D |
| **Ans13.** C | **Ans13.** C | **Ans13.** A |
| **Ans14.** A | **Ans14.** B | **Ans14.** C |
| **Ans15.** C | **Ans15.** A | **Ans15.** A |
| **Ans16.** D | **Ans16.** B | **Ans16.** D |
| **Ans17.** D | **Ans17.** A | **Ans17.** B |
| **Ans18.** C | **Ans18.** D | **Ans18.** D |
| **Ans19.** B | **Ans19.** B | **Ans19.** A |
| **Ans20.** A | **Ans20.** D | **Ans20.** B |
| **Section-B** | **Section-B** | **Section-B** |
| **Ans1.** 9 | **Ans1.** 2 | **Ans1.** 56 |
| **Ans2.** 500 | **Ans2.** 16 | **Ans2.** 80 |
| **Ans3.** 2 | **Ans3.** 6 | **Ans3.** 40 |
| **Ans4.** 34 | **Ans4.** 19 | **Ans4.** 15 |
| **Ans5.** 15 | **Ans5.** 964 | **Ans5.** 72 |
| **Ans6.** 20 | **Ans6.** 3 | **Ans6.** 30 |
| **Ans7.** 1 | **Ans7.** 760 | **Ans7.** 6 |
| **Ans8.** 400 | **Ans8.** 50 | **Ans8.** 25 |
| **Ans9.** 1 | **Ans9.** 84 | **Ans9.** Not Correct(As per NTA 36) |
| **Ans10.** 40 | **Ans10.** 271 | **Ans10.** 7 |

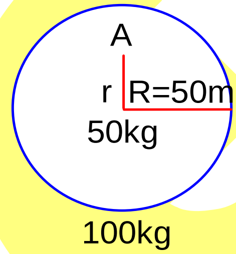
## SOLUTION: JEE Main- 27-08-2021-Evening

## PHYSICS
### Section-A

**Sol1.** $H_{max} = \frac{u^2 \sin^2\theta}{2g} = \frac{(25\times\sin 45^o)^2}{2\times 10} = 15.625\,m$

$T = \frac{u\sin\theta}{g} = \frac{25\times\sin 45}{10} = 1.77s$

**Sol2.** $V_A = -\frac{G M_1}{r} - \frac{G M_2}{R} = -\frac{50G}{25} - \frac{100G}{50} = -4G$




**Sol3.** $R_H : m^{-1}$

$h : kgm^2s^{-1}$

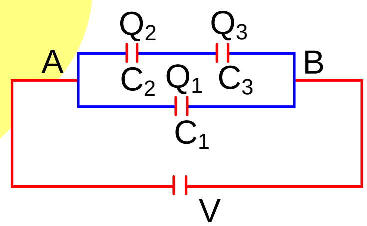
$\mu_B : kgm^{-1}s^{-2}$

$\eta : kgm^{-1}s^{-1}$

**Sol4.** $Q_1 = C_1V = 2V\,\mu C$

$Q_2 = Q_3 = \frac{C_2C_3}{C_2+C_3}V = \frac{6\times 12}{6+12}V = 4V\,\mu C$

$Q_1 : Q_2 : Q_3 = 1:2:2$




**Sol5.** $\lambda > h \Rightarrow \lambda > 400m$

**Sol6.** $R = 75\times 10^2 \pm 5\%$

$\Rightarrow R = 7500 \pm 375\ \Omega$

**Sol7.** For $x < a$,

$B_1 = \frac{\mu_0 i_0 x}{2\pi a^2}$

For $a < x < b$,

$B_2 = \frac{\mu_0 i_0}{2\pi x}$

$\frac{B_1}{B_2} = \frac{x^2}{a^2}$

**Sol8.** For 8kg block,

$8g - 2T = 8a \quad ...(i)$

For 2kg block,

$T - 2g = 2 \times 2a \quad ...(ii)$

Solving equations (i) and (ii),

$$a = \frac{g}{4} = 2.5\,m/s^2$$

$$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 0.2}{2.5}} = 0.4s$$

**Sol9.** Current at full deflection,

$$I_{max} = \frac{50}{2} = 25\,mA$$

$$R = \frac{V}{I_{max}} = \frac{50}{25} = 2\Omega$$

**Sol10.** $\beta = \frac{I_C}{I_E} = \frac{I_C}{I_E - I_C} = \frac{I_C / I_E}{1 - I_C / I_E} = \frac{\alpha}{1-\alpha}$

**Sol11.** $eV_{S2} = \frac{hc}{\lambda_1} - \phi$

$$eV_{S2} = \frac{hc}{\lambda_2} - \phi$$

$$eV_{S2} - eV_{S1} = \frac{hc}{\lambda_2} - \frac{hc}{\lambda_1}$$

$$\Rightarrow V_{S2} = V_{S1} + \frac{hc}{e}\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right)$$

$$\Rightarrow V_{S2} = 0.48 + 12.43 \times 10^{-7} \times \left(\frac{1}{474.6} - \frac{1}{670.5}\right) \times 10^{-9}$$

$$= 1.25V$$

**Sol12.** $\lambda = \frac{-Q}{R \times 2\pi/3}$

$$\vec{E} = \frac{2k\lambda}{R}\sin\left(\frac{\theta}{2}\right)\left(-\hat{i}\right)$$

$$\Rightarrow \vec{E} = \frac{2k}{R} \times \frac{-3Q}{2\pi R}\sin 60^{o}\left(-\hat{i}\right)$$

$$\Rightarrow \vec{E} = \frac{3\sqrt{3}Q}{8\pi^2\varepsilon_0 R^2}\left(\hat{i}\right)$$

**Sol13.** $\varepsilon = Bv(2R) = 1 \times 1 \times 2 \times 1 = 2V$

**Sol14.** $mgh = ms\Delta T$

$$\Rightarrow \Delta T = \frac{gh}{s} = \frac{10 \times 63}{4200} = 0.147^{o}C$$

**Sol15.** From Conservation of Angular Momentum,

$$\Rightarrow I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega$$

$$\Rightarrow \omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$$

$$KE_i = \frac{1}{2} I_1 \omega_1^2 + \frac{1}{2} I_2 \omega_2^2$$

$$KE_f = \frac{1}{2}(I_1 + I_2)\omega^2$$

$$\Rightarrow |KE_f - KE_i| = \frac{1}{2}\left(\frac{I_1 I_2}{I_1 + I_2}\right)(\omega_1 - \omega_2)^2$$

**Sol16.** $\frac{1}{f_1} = (\mu_1 - 1)\left(\frac{1}{\infty} - \frac{1}{(-R)}\right) = \frac{\mu_1 - 1}{R}$

$$\frac{1}{f_2} = (\mu_2 - 1)\left[\frac{1}{(-R)} - \frac{1}{\infty}\right] = -\frac{\mu_2 - 1}{R}$$

$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{\mu_1 - \mu_2}{R}$$

$$\Rightarrow \frac{R}{f_{eq}} = \mu_1 - \mu_2$$

**Sol17.** $I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = 4I_0$

**Sol18.** $[F] = MLT^{-2} \Rightarrow M = FL^{-1}T^2$

$$[\rho] = ML^{-3} = FL^{-1}T^2L^{-3} = FL^{-4}T^2$$

**Sol19.** $\frac{(v_{rms})_{O_2}}{(v_{rms})H_2} = \sqrt{\frac{M_{H_2}}{M_{O_2}}} = \sqrt{\frac{2}{32}}$

$$\Rightarrow (v_{rms})_{H_2} = 4 \times (v_{rms})_{O_2} = 4 \times 160 = 640 m/s$$

**Sol20.** Time taken by first drop to reach the ground,

$$t_1 = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 9.8}{9.8}} = \sqrt{2}s$$

Time for which second drop has fallen,

$$t_2 = t_1 - \frac{t_1}{2} = \sqrt{2} - \frac{1}{\sqrt{2}}s$$

Height of the second drop

$$= 9.8 - \frac{1}{2} \times 9.8 \times \left(\sqrt{2} - \frac{1}{\sqrt{2}}\right)^2 = 7.35m$$

## Section-B

**Sol1.** When switch is open,

$$R_{eq} = \frac{3R}{2}$$

When switch is closed,

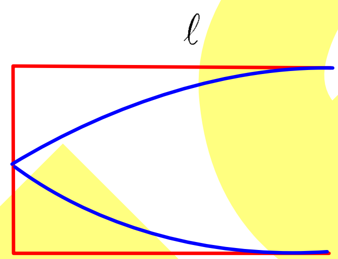
$$R'_{eq} = 2 \times \frac{R \times 2R}{R + 2R} = \frac{4R}{3}$$

$$\frac{R_{eq}}{R'_{eq}} = \frac{3R/2}{4R/3} = \frac{9}{8} = \frac{x}{8}$$
$$\Rightarrow x = 9$$

**Sol2.** $\frac{T_1}{400} = \frac{300}{240} \Rightarrow T_1 = 500K$

**Sol3.** $B = \frac{E}{c} = \frac{6}{3\times10^{8}} = 2\times10^{-8}T$
$$\Rightarrow x = 2$$

**Sol4.** $\frac{\lambda}{4} = \ell \Rightarrow \lambda = 4\ell$




$$f = \frac{v}{\lambda} = \frac{v}{4\ell} \Rightarrow \ell = \frac{v}{4f}$$
$$\ell = \frac{34}{4\times25} = 0.34m = 34cm$$

**Sol5.** No. of different wavelengths
$$= \frac{n\times(n-1)}{2} = \frac{6\times5}{2} = 15$$

**Sol6.** Maximum current through zener diode,
$$i = \frac{2}{10} = 0.2A$$
When unregulated voltage is 14V,
Potential difference across $R_S$ ,
$$V_S = 14 - 10 = 4V$$
$$R_S = \frac{V_S}{i} = \frac{4}{0.2} = 20\Omega$$

**Sol7.** Amplitude of $y_1 = A_1 = 10$
Amplitude of $y_2 = A_2 = \sqrt{(5)^2 + (5\sqrt{3})^2}$
$$\Rightarrow \frac{A_1}{A_2} = \frac{10}{10} = 1$$

**Sol8.** Velocity of bob after collision,
$$v_1 = \sqrt{5gR} = \sqrt{5\times10\times0.5} = 5m/s$$
From Conservation of Momentum,
$$\frac{10}{1000}\times v = \frac{-10}{1000}\times100 + 1\times5$$
$$\Rightarrow v = 400m/s$$

**Sol9.** $\cos\phi_1 = \frac{R}{\sqrt{R^2 + (3R)^2}} = \frac{1}{\sqrt{10}}$

$$\cos\phi_2 = \frac{R}{\sqrt{R^2+(3R-R)^2}} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{\cos\phi_2}{\cos\phi_1} = \frac{\sqrt{10}}{\sqrt{2}} = \sqrt{5} \Rightarrow x = 1$$

**Sol10.** For $W_1$,

$$\frac{(m+10+20)\times 10}{8\times 10^{-7}} \le 1.25\times 10^9$$

$\Rightarrow m \le 70kg$

For $W_2$,

$$\frac{(m+10)\times 10}{4\times 10^{-7}} \le 1.25\times 10^9$$

$\Rightarrow m \le 40kg$

Thus, $m \le 40kg$

## CHEMISTRY
### Section-A

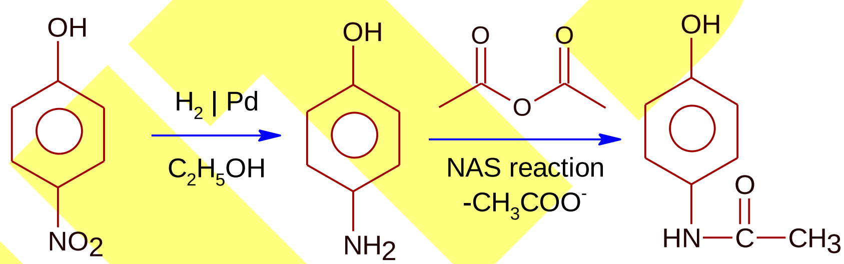
**Sol1.** (1) $Na_2O_2 + 2H_2O \longrightarrow 2NaOH + H_2O_2$

(2) $BaO_2 \cdot 8H_2O$ gives $H_2O_2$ after evaporation

(3) $PbO_2 + 2H_2O \longrightarrow Pb(OH)_4$

(4) $SnO_2 + 2H_2O \longrightarrow Sn(OH)_4$

**Sol2.**




**Sol3.** $2KMnO_4 \xrightarrow{\Delta} \underset{(green)}{K_2\overset{+6}{Mn}O_4} + \underset{(black)}{MnO_2} + O_2$

$Mn^{6+}$ has one unpaired electron so paramagnetic and has green colour.

**Sol4.** $O_2(g) \xrightarrow{UV} O(g) + O(g)$

$O(g) + O_2(g) \xrightarrow{UV} O_3(g)$

Ozone in the stratosphere is a product of UV radiations acting on $O_2$ molecule.

**Sol5.** In the lyophilic colloids, the colloidal particles are extremely solvated.

**Sol6.** (1) Standard enthalpy of formation for alkali metal bromides becomes more negative on descending down the group.

(2) Standard enthalpy of formation for LiF is most negative among alkali metal fluorides.

(3) In case of CsI, lattice energy is less but $Cs^+$ having less hydration energy due to which it is less soluble in water.

(4) For alkali metal fluorides, the solubility in water increases from Li to Cs. LiF is least soluble in water.

**Sol7.** (a) Shows intra molecular H- bonding.
(b) Shows inter molecular H- bonding.
(c) It does not shows intermolecular H- bonding due to high steric hindrance at o- position of benzene ring.

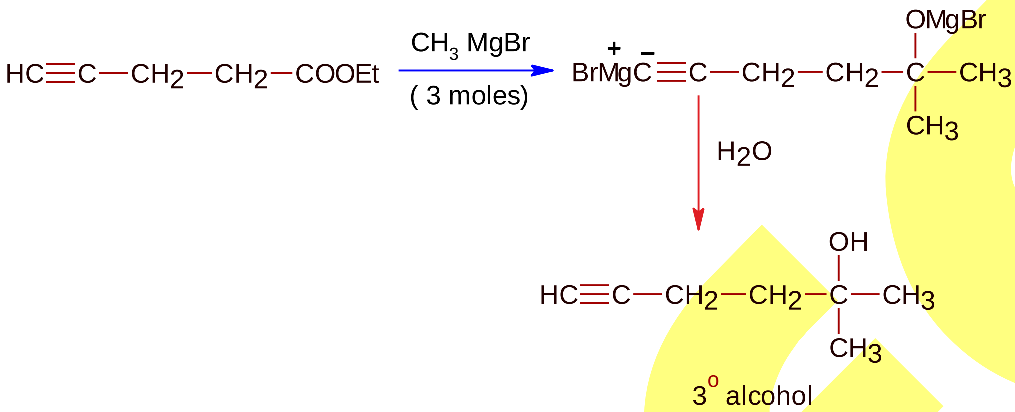
**Sol8.** (a) Calamine : $ZnCO_3$
(b) Malachite : $CuCO_3 \cdot Cu(OH)_2$
(c) Siderite : $FeCO_3$
(d) Sphalerite : ZnS

**Sol9.** When red phosphorus is heated in a sealed tube at 803 K, $\alpha$- black phosphorus is formed.

**Sol10.** Ionic radii of cations is smaller than anions, also more the positive charge less be the ionic radii and more the negative charge more be the ionic radii. Hence correct order of ionic radii is,

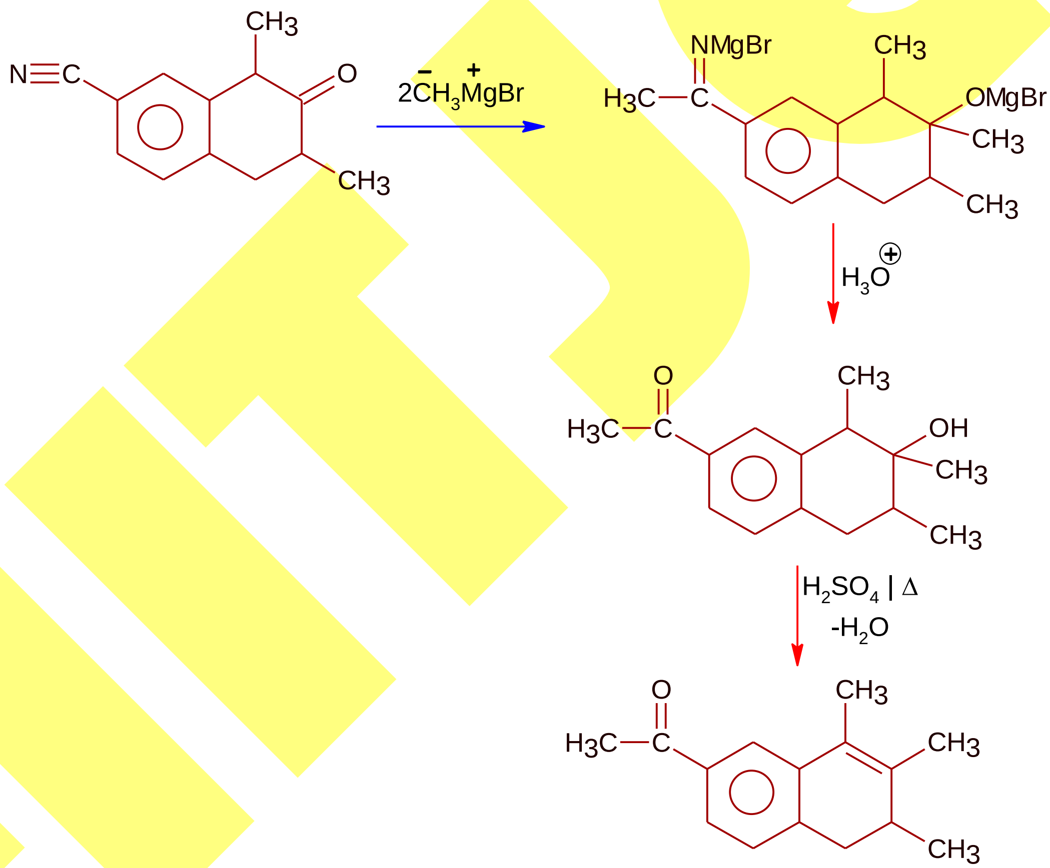
$P^{3-} > S^{2-} > Cl^{-} > K^{+} > Ca^{2+}$

**Sol11.**



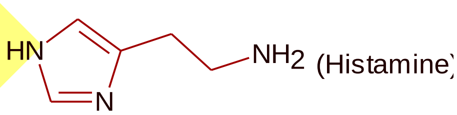

Here, ethyl pent-4-yn-oate utilizes 3 moles of $CH_3MgBr$.

**Sol12.**

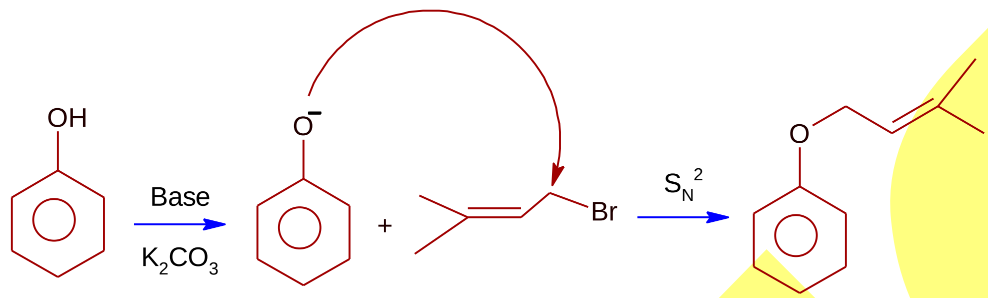



**Sol13.** $\underset{C_{12}H_{22}O_{11}}{\text{Sucrose}} \xrightarrow{H_2O/H^+} \alpha - D(+)\text{glucose} + \beta - D(-)\text{fructose}$

**Sol14.** Histamine stimulate the secretion of HCl



**Sol15.**



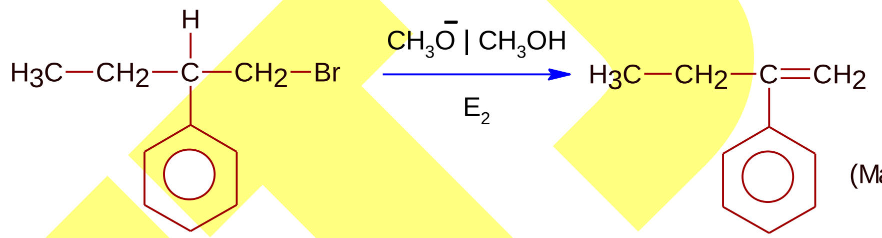

**Sol16.** $O_2F_2$ oxidises plutonium to $PuF_6$ and the reaction is used in removing plutonium as $PuF_6$ from spent nuclear fuel.

**Sol17.** $CH_3CH_2CH_2Br \xrightarrow{Mg} CH_3CH_2CH_2MgBr \xrightarrow{CO_2}$
$CH_3CH_2CH_2COOMgBr \xrightarrow{H_3O^+} CH_3CH_2CH_2COOH$ (butanoic acid)

**Sol18.** In Seliwanoff's reagent, Cu is not present. In Barfoed, Biuret and Benedict's reagent Cu is present.

**Sol19.** $Cr^{+3} + NaOH(dil) \rightarrow Cr_2O_3 \cdot (H_2O)_n$

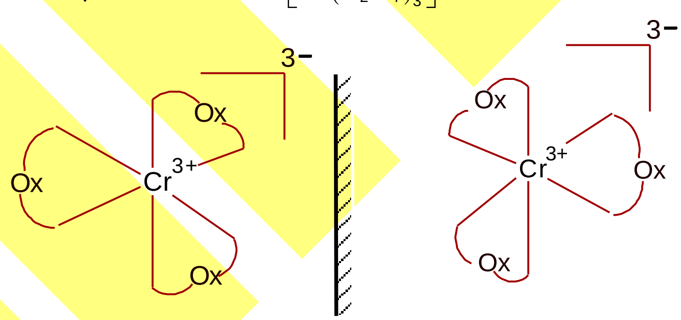
**Sol20.**




(Major Product)

## Section-B

**Sol1.** Two optical isomers of $\left[Cr(C_2O_4)_3\right]^{3-}$ are shown:




**Sol2.** $K_{700} = 6.36 \times 10^{-3} s^{-1}$

$K_{600} = x \times 10^{-6} s^{-1}$

$E_a = 209 KJ/mol$

$$\log\left(\frac{K_2}{K_1}\right) = \frac{-Ea}{2.303R}\left[\frac{1}{T_2} - \frac{1}{T_1}\right]$$

$$\log\left(\frac{K_{700}}{K_{600}}\right) = \frac{-Ea}{2.303R}\left[\frac{1}{700} - \frac{1}{600}\right]$$

$$\log\left[\frac{6.36\times10^{-3}}{K_{600}}\right]=\frac{209\times1000}{2.303\times8.31}\left[\frac{100}{700\times600}\right]$$

$\log K_{600}=-4.79$

$K_{600}=10^{-4.79}=1.62\times10^{-5}$

$=16.2\times10^{-6}$

x = 16 (Nearest integer)

**Sol3.** Moles of $NH_4HS$ initially taken = $\frac{5.1}{51}=0.1\text{mol}$

$$NH_4HS(S)+NH_3(g)+H_2S(g)$$

| | | | |
|---|---|---|---|
| $t=0$ | 0.1mol | 0 | 0 |
| $t=\infty$ | $0.1(1-0.2)$ | $0.1\times0.2$ | $0.1\times0.2$ |

$$P_{NH_3}=\frac{nRT}{V}=\frac{0.1\times0.2\times0.082\times300}{2}=0.246\text{atm}=P_{H_2S}$$

$K_p=P_{NH_3}\times P_{H_2S}=(0.246)^2=0.060516=6.05\times10^{-2}$

x = 6 (Nearest integer).

**Sol4.**

| | $C_3H_8(g)$ + | $5O_2(g) \longrightarrow$ | $3CO_2(g)$ + | $4H_2O(\ell)$ |
|---|---|---|---|---|
| $t=0$ ; | 2.27 mol | 31.25 mol | 0 | 0 |
| $t=\infty$ ; | 0 | 19.9mol | 6.81mol | 9.08 mol |

$$X_{CO_2}=\frac{6.81}{19.9+6.81+9.08}=0.1902=19.02\times10^{-2}$$

x =19(Nearest integer)

**Sol5.** Minimum temperature at which reaction becomes spontaneous is,

$$T_{min}=\frac{\Delta H^0}{\Delta S^0}$$

$\Delta H^o{}_{min}=\left[\Delta H_f^o(Fe)+\Delta H_f^o(CO)\right]-\left[\Delta H_f^o(FeO)+\Delta H_f^o(C)\right]$

$=[0-110.5]-[-266.3+0]$

$=155.8\,KJ/mol$

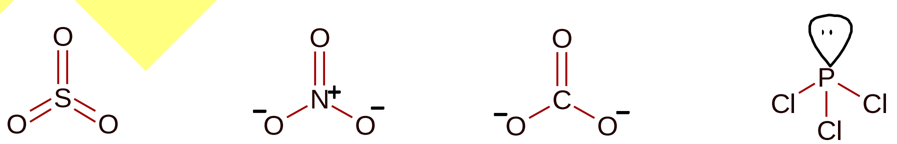
$\Delta S^o=\left[\Delta S^o(Fe)+\Delta S^o(CO)\right]-\left[\Delta S^o(FeO)+\Delta S^o(C)\right]$

$=(27.28+197.6)-(57.49+5.74)\,J/molK$

$=161.65\,J/molK$

$$T_{min}=\frac{155.8\times10^3}{161.65}K=963.8K\approx964K$$

**Sol6.**




Trigonal Planar — Trigonal Planar — Trigonal Planar — Pyramidal

**Sol7.** $\kappa = \frac{1}{R} \times \frac{\ell}{A} = \left[\left(\frac{1}{1500}\right) \times 1.14\right] S\,cm^{-1} = \frac{1.14}{1500} S\,cm^{-1}$

$\lambda_m = \frac{\kappa}{M} \times 1000\,Scm^2\,mol^{-1}$

$\lambda_m = 1000 \times \frac{\left(\frac{1.14}{1500}\right)}{0.001} Scm^2\,mol^{-1} = 760 Scm^2\,mol^{-1}$

**Sol8.** Energy of emitted photon in 0.1 sec = $10^{-4}$ J

$n \times \frac{hc}{\lambda} = 10^{-4}$

$\frac{n \times 6.63 \times 10^{-34} \times 3 \times 10^8}{1000 \times 10^{-9}} = 10^{-4}$

$n = 5.02 \times 10^{14} = 50.2 \times 10^{13} \approx 50 \times 10^{13}$

**Sol9.** Applying : $(n_1 + n_2)_{initial} = (n_1 + n_2)_{final}$

Assuming the system attains a final temperature of T ( Such that 300 < T < 60 )
(Heat lost by $N_2$ of container I) = (Heat gained by $N_2$ of container II)

$n_1 Cm(300 - T) = n_2 Cm(T - 60)$

$\left(\frac{2.8}{28}\right)(300 - T) = \frac{0.2}{28}(T - 60)$

$14(300 - T) = T - 60$

$T = \frac{14 \times 300 + 60}{15} K$

$T = 284K$

$(n_1 + n_2)_{final} = \frac{3}{28} mol$

$\frac{P}{RT}(V_1 + V_2) = \frac{3}{28}$

$P = \left(\frac{3}{28}\right) \times 8.31 \times \frac{284}{3 \times 10^{-3}} \times 10^{-5} bar = 0.84287 bar$

$P = 84.28 \times 10^{-2} bar$

$\approx 84 \times 10^{-2} bar$

**Sol10.** Molality (m) $= \frac{(40/180)}{0.2} = \left(\frac{10}{9}\right) molal$

$\Delta T_f = T_f - T_f' = 1.86 \times \frac{10}{9} K$

$T_f' = 273.15 - 1.86 \times \frac{10}{9} K$

$= 271.08K \approx 271K$

## MATHEMATICS
### Section-A

**Sol1.** $(p \wedge q) \Rightarrow ((r \wedge q) \wedge p)$

$\sim (p \wedge q) \vee ((r \wedge q) \wedge p)$

$\sim (p \wedge q) \vee ((r \wedge q) \wedge (p \wedge q))$

$[\sim (p \wedge q) \vee (p \wedge q)] \wedge [\sim (p \wedge q) \vee (r \wedge p)]$

$\sim (p \wedge q) \vee (r \wedge p)$

$\Rightarrow (p \wedge q) \Rightarrow (r \wedge p)$

**Sol2.** $f(x) = \tan^{-1}(\sin x + \cos x)$

as $x \in \left[0, \frac{\pi}{2}\right]$ so $1 \le \sin x + \cos x \le \sqrt{2}$

so $f(x) \in \left[\tan^{-1} 1, \tan^{-1}\sqrt{2}\right]$

$\Rightarrow M = \tan^{-1}\sqrt{2}$ & $m = \tan^{-1} 1 = \frac{\pi}{4}$

Now, $\tan(M - m) = \tan\left(\tan^{-1}\sqrt{2} - \frac{\pi}{4}\right)$

$= \frac{\sqrt{2} - 1}{1 + \sqrt{2}.1} = \frac{\sqrt{2} - 1}{\sqrt{2} + 1} \times \frac{\sqrt{2} - 1}{\sqrt{2} - 1} = 3 - 2\sqrt{2}$

**Sol3.** Given $2\ell + 2m - n = 0$ ........(i)

$mn + n\ell + \ell m = 0$ .........(ii)

& we have $\ell^2 + m^2 + n^2 = 1$..........(iii)

(i) $\Rightarrow 2(\ell + m) = n$

(ii) $\Rightarrow \ell m + n(\ell + m) = 0$

$2\ell^2 + 2m^2 + 5\ell m = 0$

$2\left(\frac{\ell}{m}\right)^2 + 2 + 5\left(\frac{\ell}{m}\right) = 0$

$\frac{\ell}{m} = -2, \frac{-1}{2}$

(a) $\frac{\ell}{m} = -2$

(i) $\Rightarrow \frac{2\ell}{m} + 2 - \frac{n}{m} = 0$

$\frac{n}{m} = -2$

So, $(\ell, m, n) = (-2m, m, -2m)$

$= (-2, 1, -2)$

(b) $\frac{\ell}{m} = \frac{-1}{2}$ gives $n = -2\ell$

$(\ell, m, n) = (\ell, -2\ell, -2\ell) = (1, -2, -2)$

Now, $\cos\theta = \frac{-2 - 2 + 4}{3 \times 3} = 0$

$\Rightarrow \theta = \frac{\pi}{2}$

**Sol4.** $I = \int_0^1 \frac{\sqrt{x}dx}{(1+x)(1+3x)(3+x)}$

Put $x = t^2$ then $dx = 2tdt$

$$I = \int_0^1 \frac{dt}{(1+t^2)(3+t^2)} - \int_0^1 \frac{dt}{(1+3t^2)(3+t^2)}$$

$$= \frac{1}{2}\int_0^1 \frac{(3+t^2)-(t^2+1)}{(1+t^2)(3+t^2)}dt + \frac{1}{8}\int_0^1 \frac{(1+3t^2)-3(3+t^2)}{(1+3t^2)(3+t^2)}dt$$

$$= \frac{1}{2}\int_0^1 \frac{dt}{1+t^2} - \frac{1}{2}\int \frac{dt}{3+t^2} + \frac{1}{8}\int_0^1 \frac{dt}{3+t^2} - \frac{3}{8}\int_0^1 \frac{dt}{1+3t^2}$$

$$= \frac{1}{2}\left(\tan^{-1} t\right)_0^1 - \frac{3}{8\sqrt{3}}\left(\tan^{-1}\frac{t}{\sqrt{3}}\right)_0^1 - \frac{8}{8\sqrt{3}}\left(\tan^{-1}\sqrt{3}t\right)_0^1$$

$$= \frac{\pi}{8}\left(1-\frac{\sqrt{3}}{2}\right)$$

**Sol5.** $\lim_{x\to\infty}\left(\sqrt{x^2-x+1}-ax\right) = b$

$$\lim_{x\to\infty}\left|\frac{x^2-x+1-a^2x^2}{\sqrt{x^2-x+1}+ax}\right| = b$$

$$\lim_{x\to\infty}\frac{x^2(1-a^2)-x+1}{\sqrt{x^2-x+1}+ax} = b$$

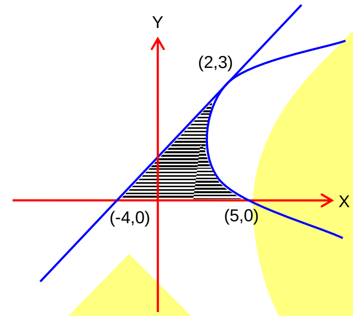
For existence of limit $1-a^2 = 0$ i.e. a = 1 only

$$\lim_{x\to\infty}\frac{1-x}{\sqrt{x^2-x+1}+x} = b$$

$$\lim_{x\to\infty}\frac{\frac{1}{x}-1}{\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}+1} = b$$

$\Rightarrow b = \frac{-1}{2}$

So, $(a,b) = \left(1,-\frac{1}{2}\right)$




**Sol6.** $(y-2)^2=(x-1)$

$2(y-2)\frac{dy}{dx}=1$

$\Rightarrow \frac{dy}{dx}(2,3)=\frac{1}{2(3-2)}=\frac{1}{2}$

Equation of tangent at P(2,3) :

$y-3=\frac{1}{2}(x-2)$

$2y-6=x-2$

$x-2y+4=0$

$Q(-4,0)$

Required area =

$\int_0^3\left((y-2)^2+1-(2y-4)\right)dy=9$

**Sol7.** $L.R.=\frac{|3\times2+4x-3-5|}{\sqrt{3^2+4^2}}=\frac{11}{5}$

Equation of family of parabolas

$(x-h)^2=\frac{11}{5}(y-k)$

Differentiate $2(x-h)=\frac{11}{5}\frac{dy}{dx}$

Again differentiate $2=\frac{11}{5}\frac{d^2y}{dx^2}$

$11\frac{d^2y}{dx^2}=10$

**Sol8.** Equation of the plane will be $\left\{\vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right)-1\right\}+\lambda\left\{\vec{r}.\left(2\hat{i}+3\hat{j}-\hat{k}\right)+4\right\}=0$

$\vec{r}.\left\{(1+2\lambda)i+(1+3\lambda)\hat{j}+(1-\lambda)\hat{k}\right\}+(4\lambda-1)=0$

$\Rightarrow(1+2\lambda)x+(1+3\lambda)y+(1-\lambda)z+(4\lambda-1)=0$ …………(i)

(i) is parallel to x-axis so its d.r.s will be (1,0,0)

$\Rightarrow 1+2\lambda=0$ so $\lambda=\frac{-1}{2}$

Hence required equation will be

$\vec{r}\left\{-\frac{1}{2}\hat{j}.+\frac{3}{2}\hat{k}\right\}+(-3)=0$

$\vec{r}.\left(\hat{j}-3\hat{k}\right)+6=0$

**Sol9.** $\left(3x^2+4x+3\right)^2-(k+1)\left(3x^2+4x+3\right)\left(3x^2+4x+2\right)+k\left(3x^2+4x+2\right)^2=0$

Let $3x^2+4x+3=a$ & $3x^2+4x+2=b$ $\Rightarrow b=a-1$

So, (i) becomes $a^2-(k+1)ab+kb^2=0$

$(a-kb)(a-b)=0 \Rightarrow a=kb$ or $a=b \rightarrow$ not possible

$\Rightarrow 3x^2+4x+3=k(3x^2+4x+2)$

$\Rightarrow 3(k-1)x^2+4(k-1)x+2k-3=0$

For real roots $D \geq 0$

$16(k-1)^2-12(k-1)(2k-3) \geq 0$

$\Rightarrow k \in \left[1, \frac{5}{2}\right]$

Since $k \neq 1$

So, $k \in \left(1, \frac{5}{2}\right]$

**Sol10.** $|A| = \begin{vmatrix} [x+1] & [x+2] & [x+3] \\ [x] & [x+3] & [x+3] \\ [x] & [x+2] & [x+4] \end{vmatrix} = \begin{vmatrix} [x]+1 & [x]+2 & [x]+3 \\ [x] & [x]+3 & [x]+3 \\ [x] & [x]+2 & [x]+4 \end{vmatrix}$

$R_1 \rightarrow R_1 - R_3$ & $R_2 \rightarrow R_2 - R_3$

$\begin{vmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ [x] & [x]+2 & [x]+4 \end{vmatrix} = 192$

$\Rightarrow [x] = 62$

$\Rightarrow x \in [62, 63)$

**Sol11.** $y(x) = \cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right), x \in \left(\frac{\pi}{2}, \pi\right)$

$= \cot^{-1}\left(\frac{\sin\frac{x}{2}+\cos\frac{x}{2}+\sin\frac{x}{2}-\cos\frac{x}{2}}{\sin\frac{x}{2}+\cos\frac{x}{2}-\sin\frac{x}{2}+\cos\frac{x}{2}}\right) = \cot^{-1}\tan\frac{x}{2} = \cot^{-1}\cot\left(\frac{\pi}{2}-\frac{x}{2}\right) = \frac{\pi}{2}-\frac{x}{2}$

$\frac{dy}{dx} = \frac{-1}{2}$

**Sol12.** Locus of point of intersection of perpendicular tangent will be its director circle & director circle of parabola be its directrix.
Given parabola $y^2 = 16(x-3)$ so equation of directrix be x – 3 = - 4 i.e., x + 1 = 0

**Sol13.** $ar(\Delta ABC) = \frac{1}{2}\begin{vmatrix} a & 0 & 1 \\ b & 2b+1 & 1 \\ 0 & b & 1 \end{vmatrix} = \frac{1}{2}\left[a(2b+1-b)+(b^2)\right] = \frac{1}{2}\left[ab+a+b^2\right]$

Given $ar(\Delta ABC)=1$ so $|ab+a+b^2| = 2$

$\Rightarrow a(b+1)+b^2 = \pm 2$

$a = \frac{-b^2+2}{b+1}, \frac{-b^2-2}{b+1}$

So sum = $\frac{-2b^2}{b+1}$

**Sol14.** $x+y+z=4$

$3x+2y+5z=3$

$9x+4y+(28+[\lambda])z=[\lambda]$

$$\Delta=\begin{vmatrix}1 & 1 & 1\\ 3 & 2 & 5\\ 9 & 4 & 28+[\lambda]\end{vmatrix}=56+2[\lambda]-20-(84+3[\lambda]-45)+(-6)$$

$=-[\lambda]-9$

If $[\lambda]+9\neq 0$ then unique solution

& if $[\lambda]+9=0$ then $\Delta_1=\Delta_2=\Delta_3=0$ so infinite solution will exist

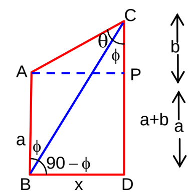
Hence $\lambda\in R$

**Sol15.** Required probability = probability of both getting 0 head or 1 head or 2 head or 3 head

$$=\left({}^3C_0\left(\frac{1}{2}\right)^{3-0}\left(\frac{1}{2}\right)^0\right)^2+\left({}^3C_1\left(\frac{1}{2}\right)^{3-1}\left(\frac{1}{2}\right)^1\right)^2+\left({}^3C_2\left(\frac{1}{2}\right)^{3-2}\left(\frac{1}{2}\right)^2\right)^2+\left({}^3C_3\left(\frac{1}{2}\right)^{3-3}\left(\frac{1}{2}\right)^3\right)^2$$

$=\frac{5}{16}$

**Sol16.** In $\Delta BCD$, $\tan\phi=\frac{x}{a+b}$ ………(i)




In $\Delta APC$, $\tan(\theta+\phi)=\frac{x}{b}$ ………..(ii)

Now $\tan\theta=\tan(\theta+\phi-\phi)$

$$=\frac{\tan(\theta+\phi)-\tan\phi}{1+\tan(\theta+\phi)\tan\phi}$$

$$\frac{\frac{x}{b}-\frac{x}{a+b}}{1+\frac{x}{b}\cdot\frac{x}{a+b}}=\frac{ax}{b(a+b)+x^2}$$

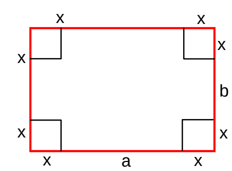
given , $\tan\theta=\frac{1}{2}$ so $\frac{ax}{b(a+b)+x^2}=\frac{1}{2}$

$\Rightarrow x^2-2ax+b(a+b)=0$

**Sol17.** Length of box = a – 2x

Breadth = b – 2x

& height = x




Let volume $V=(a-2x)(b-2x)x$

For minimum volume $\frac{dv}{dx}=0$

$(a-2x)(b-2x)-2(a-2x)x-2(b-2x)x=0$

$ab-2(a+b)x+4x^2-2ax+4x^2-2bx+4x^2=0$

$12x^2 - 4(a+b)x + ab = 0$

$$x = \frac{4(a+b) \pm \sqrt{16(a+b)^2 - 48ab}}{24} = \frac{4(a+b) \pm 4\sqrt{a^2+b^2-ab}}{24}$$

$x = \left(a+b-\sqrt{a^2+b^2-ab}\,/\,6\right)$ for minimum

Since $x = \left\{(a+b)+\sqrt{a^2+b^2-ab}\right\}/6$ not possible because maxima occurs

**Sol18.** number of elements in $A \cap B = 5$ which is

$(0,0)(1,0)(1,1)(1,-1)(2,0)$

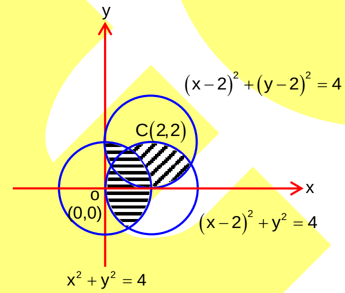
Similarly number of elements in $A \cap C = 5$ which is

$(2,0)(2,2)(1,1)(2,1)(3,1)$

Hence number of relation from

$(A \cap B)$ to $(A \cap C) = 2^{5\times 5} = 2^{25}$

$\Rightarrow P = 25$




**Sol19.** $(2x - 10y^3)dy + ydx = 0$

$\frac{dx}{dy} + \frac{2x}{y} - 10y^2 = 0$

$\frac{dx}{dy} + \frac{2}{y}x = 10y^2 \rightarrow$ Linear differential equation

$P = \frac{2}{y},\ Q = 10y^2$

I.F. = $e^{\int P dy} = e^{\int \frac{2}{y} dy} = e^{2\ln y} = y^2$

Solution be $x.y^2 = \int 10y^2.y^2 dy + c = 2y^5 + c$

Put $x = 0,\ y = 1$ then c = -2

So, differential equation will be $xy^2 = 2y^5 - 2$

Now put $x = 2,\ y = \beta$ then $2\beta^2 = 2\beta^5 - 2$

or $\beta^5 - \beta^2 - 1 = 0$

So B will be roots of $y^5 - y^2 - 1 = 0$

**Sol20.** $y = \frac{x^2}{2} + \frac{2}{3}x^3 + \frac{3}{4}x^4 + \ldots\ldots$

$= \left(1-\frac{1}{2}\right)x^2 + \left(1-\frac{1}{3}\right)x^3 + \left(1-\frac{1}{4}\right)x^4 + \ldots\ldots$

$= \left(x^2 + x^3 + x^4 + \ldots..\right) + \left(-\frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4}\ldots\ldots\ldots\right)$

$y = \frac{x^2}{1-x} + \log(1-x) + x = \frac{x}{1-x} + \log(1-x)$

at $x = \frac{1}{2},\ y = 1 + \ln\frac{1}{2} = 1 - \ln 2$

$$e^{y+1} = e^{1-\ln 2+1} = e^{2-\ln 2} = \frac{e^2}{2}$$

## Section-B

**Sol1.** $\sin^4\theta + \cos^4\theta - \sin\theta\cos\theta = 0$

$\left(\sin^2\theta + \cos^2\theta\right)^2 - 2\sin^2\theta\cos^2\theta - \sin\theta\cos\theta = 0$

$-\dfrac{\left(2\sin\theta\cos\theta\right)^2}{2} - \dfrac{2\sin\theta\cos\theta}{2} + 1 = 0$

$\sin^2 2\theta + \sin 2\theta - 2 = 0$

$\sin 2\theta = 1 \ldots\ldots\ldots\ldots(\text{i})$

as $\theta \in [0, 4\pi]$ so $2\theta \in [0, 8\pi]$

(i) $2\theta = \dfrac{\pi}{2}, \dfrac{5\pi}{2}, \dfrac{9\pi}{2}, \dfrac{18\pi}{2}$

$\theta = \dfrac{\pi}{4}, \dfrac{5\pi}{4}, \dfrac{9\pi}{4}, \dfrac{18\pi}{4}$

So sum = $7\pi = s$

Hence $\dfrac{8s}{\pi} = 56$

**Sol2.** $S = \{1, 2, 3, 4, 5, 6, 9\}$

Elements of type $3n \to 3, 6, 9$

Type $3n+1 \to 1, 4$

$3n+2 \to 2, 5$

Number of subset of S containing one element which are not divisible by $3 = {}^2C_1 + {}^2C_1 = 4$ number of subset of S containing two numbers whose sum is not divisible by $3 = {}^3C_1 \times {}^2C_1 + {}^3C_1 \times {}^2C_1 + {}^2C_2 + {}^2C_2 = 14$

Number of subset of S containing 3 elements whose sum is not divisible by $3 = {}^3C_2 \times {}^2C_1 + {}^3C_2 \times {}^2C_1 + {}^2C_2 \times {}^3C_1 + {}^2C_2 \times {}^3C_1 + \left({}^2C_2 \times {}^2C_1\right) \times 2 = 22$

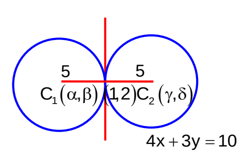
Number of subset containing 4 elements whose sum is not divisible by 3

$= {}^3C_2 \times {}^4C_1 + {}^3C_2\left({}^2C_2 + {}^2C_2\right) + \left({}^3C_1 \times {}^2C_1 \times {}^2C_2\right) \times 2 = 14$

Number of subset of S containing 6 elements = 4

Hence total subset = 80

**Sol3.** Slope of $C_1C_2 = \dfrac{3}{4} = \tan\theta$




By parametric form $C_1(1 + 5\cos\theta, 2 + 5\sin\theta)$

& $C_2(1 - 5\cos\theta, 2 - 5\sin\theta)$

$C_1\left(1 + 5 \times \dfrac{4}{5}.2 + 5 \times \dfrac{3}{5}\right)$ & $C_2\left(1 - 5 \times \dfrac{4}{5}.2 - 5 \times \dfrac{3}{5}\right)$

$C_1(5, 5)$ & $C_2(-3, -1)$

$\Rightarrow \alpha = 5, \beta = 5, r = -3, \delta = -1$

So, $\left|(\alpha+\beta)(r+\delta)\right| = 40$

**Sol4.** $3\times 7^{22} + 2\times 10^{22} - 44 = 3\times(1+6)^{22} + 2(1+9)^{22} - 44$

$= 3\left[1 + {}^{22}C_1\times 6 + {}^{22}C_2\times 6^2 + {}^{22}C_3\times 6^3 + \ldots\ldots {}^{22}C_{22}6^{22}\right]$

$+2\left[1 + {}^{22}C_1\times 9 + {}^{22}C_2\times 9^2 + {}^{22}C_3\times 9^3 + \ldots\ldots + {}^{22}C_{22}\times 9^{22}\right] - 44$

$= 18\left[{}^{22}C_1 + {}^{22}C_2\times 9 + {}^{22}C_3\times 9^2 + \ldots\ldots + {}^{22}C_{22}\times 9^{21}\right] - 39 +$

$18\left[{}^{22}C_1 + {}^{22}C_2\times 6 + {}^{22}C_3\times 6^2 + \ldots\ldots + {}^{22}C_{22}\times 6^{21}\right]$

$= -39$ on division by 18

$= (-54+15)$ on division by 18 = 15

**Sol5.** Normal vector to the given plane be

$2\hat{i} - \hat{j} + 3\hat{k}$ so

Equation of line QS :

$\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-4}{1} = \lambda$

So let $P(2\lambda+1, -\lambda+3, \lambda+4)$

Now P lies on given plane so

$4\lambda + 2 + \lambda - 3 + 8\lambda + 4 + 3 = 0$

$6\lambda + 6 = 0$ or $\lambda = -1$

So, $P(-1, 4, 3)$

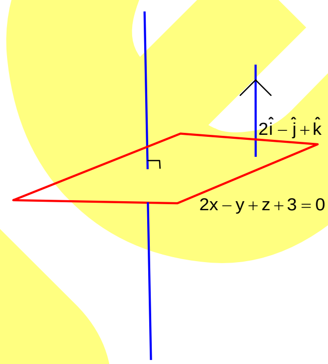
So, $S(-3, 5, 2)$

also given R lies on given plane so

$6 - 5 + \gamma + 3 = 0$ so $\gamma = -4$

So, $R(3, 5, -4)$

$SR^2 = 72$




**Sol6.** $\Sigma P(x) = 1 \Rightarrow k + 2k + 2k + 3k + k = 1$ so $k = \frac{1}{9}$

Now, $P\left(\frac{1<x<4}{x\angle 3}\right) = P\frac{(x=2)}{P(x\angle 3)} = \frac{\frac{2k}{9k}}{\frac{k}{9k} + \frac{2k}{9k}} = \frac{2}{3}$

$\Rightarrow P = \frac{2}{3}$

So, $5P = \lambda k$ gives $\frac{10}{3} = \lambda \times \frac{1}{9} \Rightarrow \lambda = 30$

**Sol7.** Let $z_1 = x_1 + iy_1$, $z_2 = x_2 + iy_2$ & $z = x + iy$ then

and $(z_1 - z_2) = \frac{\pi}{4}$ gives $\frac{y_1 - y_2}{x_1 - x_2} = 1$ or $y_1 - y_2 = x_1 - x_2$ ……….(i)

also $|z-3| = \text{Re}(z)$ gives $\sqrt{(x-3)^2 + y^2} = x$

or $y^2 - 6x + 9 = 0$ ……….(ii)

as $z_1$ & $z_2$ lies on (ii) so $y_1^2 - 6x_1 + 9 = 0$ …………(iii)

& $y_2 2 - 6x_2 + 9 = 0$ ..........(iv)

(iii) & (iv) $(y_1+y_2)(y_1-y_2)-6(x_1-x_2)=0$

$(x_1-x_2)(y_1+y_2-6)=0$ from (i)

$\Rightarrow y_1+y_2-6=0$ i.e., $y_1+y_2=6$

**Sol8.** Given variance of boys $\sigma_b^2 = 2$ & $\bar{x}_b = 12$ (average marks of boys)

& variance of girls $\sigma_g^2 = 2$ & $\mu \rightarrow$ average marks of girls

Now, $\bar{x}_g = \mu = \dfrac{50\times 15 - 12\times 20}{30} = 17$

Combined variance $\sigma^2 = \dfrac{n_1\sigma_b 2 + n_2\sigma_g 2}{n_1+n_2} + \dfrac{n_1.n_2}{(n_1+n_2)^2}(\bar{x}_b - \bar{x}_g)^2$

where $n_1$ & $n_2$ are number of boys & girls

$$\sigma^2 = \frac{20\times 2 + 30\times 2}{20+30} + \frac{20\times 30}{(20+30)^2}(12-17)^2 = 8$$

So, $\mu + \sigma^2 = 17 + 8 = 25$

**Sol9.** Since $A(\sec\theta, 2\tan\theta)$ & $B(\sec\phi, 2\tan\phi)$ lies on $2x^2 - y^2 = 2$ then

$2\sec^2\theta - 4\tan^2\theta = 2$ or $\sec^2\theta - 2\tan^2\theta = 1$

$\Rightarrow \tan^2\theta = 0$ so $\theta = 0$

Similarly $\phi = 0$ but $\theta + \phi = \dfrac{\pi}{2}$ (given) so not possible

Hence question is not correct

**Sol10.** Let $I = \displaystyle\int \frac{2e^x + 3e^{-x}}{4e^x + 7e^{-x}}dx = \int \frac{2e^{2x}+3}{4e^{2x}+7}dx$

$= \displaystyle\int \frac{2e^{2x}}{4e^{2x}+7}dx + \int \frac{3e^{-2x}}{4+7e^{-2x}}dx$

Put $4e^{2x}+7 = t$ $\quad 4+7e^{-2x} = \lambda$

$8e^{2x}dx = dt$ $\quad -14e^{-2x}dx = d\lambda$

$= \dfrac{1}{4}\displaystyle\int \frac{dt}{t} - \frac{3}{14}\int \frac{d\lambda}{\lambda} = \frac{1}{4}\ln t - \frac{3}{14}\ln\lambda + c$

$= \dfrac{1}{4}\ln(4e^{2x}+7) - \dfrac{3}{14}\ln(4+7e^{-2x}) + c$

$= \dfrac{1}{14}\left[\dfrac{7}{2}\ln(4e^{2x}+7) - 3\ln(4+7e^{-2x})\right] + c$

$= \dfrac{1}{14}\left[\dfrac{7}{2}\ln\left(e^x(4e^x+7e^{-x})\right) - 3\ln\left\{e^{-x}(4e^x+7e^{-x})\right\}\right] + c$

$= \dfrac{1}{14}\left[\dfrac{1}{2}\ln(4e^x+7e^{-x}) + \dfrac{13}{2}x\right] + c$

$\Rightarrow u = \dfrac{13}{2}, v = \dfrac{1}{2}$

so, $u + v = 7$