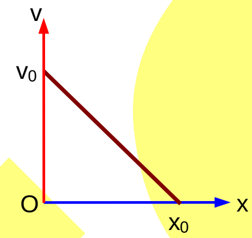
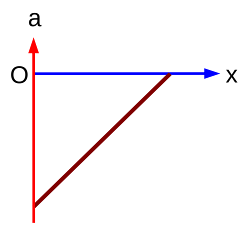
Paper-18-03-2021-Evening Shift

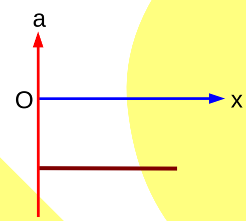
PHYSICS

SECTION – A

Q1. The velocity –displacement graph of a particle is shown in the figure. The acceleration – displacement graph of the same particle is represented by :



(A)


(B)


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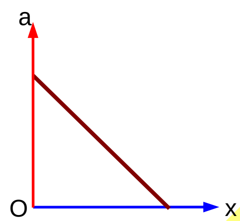
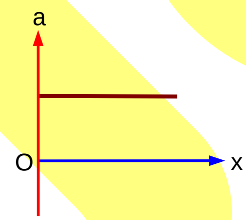


(D)


Q2. Which of the following statements are correct ?

(A) Electric monopoles do not exist whereas magnetic monopoles exist .

(B) Magnetic field lines due to a solenoid at its ends and outside can not be completely straight and confined

(C) Magnetic field lines are completely confined within a toroid.

(D) Magnetic field lines inside bar magnet are not parallel.

(E) χ = -1 is the condition for a perfect diamagnetic material , where χ is its magnetic susceptibility.

Choose the correct answer from the options given below :

(A) (C) and (E) only
(B) (A) and (B) only
(C) (B) and (D) only
(D) (B) and (C) only

Q3. The speed of electrons in a scanning electron microscope is $1\times10^7\ m/s$. If protons having the same speed are used instead of electrons , then the resolving power of scanning proton microscope will be changed by a factor of

(A) 1837
(B) $\sqrt{1837}$
(C) $\frac{1}{\sqrt{1837}}$
(D) $\frac{1}{1837}$

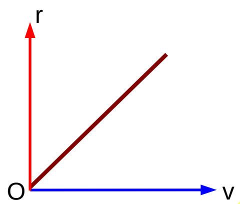
Q4. A proton and an α-particle, having kinetic energy K_p and K_α respectively, enter into a magnetic field at right angles. The ratio of the radii of trajectory of proton to that of α-particle is 2 : 1. The ratio of $K_p : K_\alpha$ is

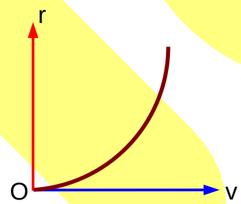
(A) $1:8$ (B) $8:1$
(C) $1:4$ (D) $4:1$

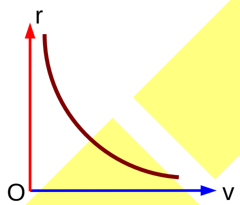
Q5. The time taken for the magnetic energy to reach 25% of its maximum value, when solenoid of resistance R, inductance L is connected to a battery, is:

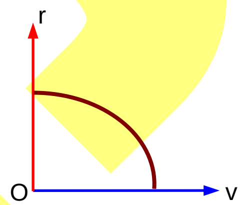
(A) $\frac{L}{R}\ell n(2)$ (B) $\frac{L}{R}\ell n(10)$
(C) Infinite (D) $\frac{L}{R}\ell n(5)$

Q6. A particle of mass m moves in a circular orbit under the central potential field, $V(r) = -\frac{C}{r}$, where C is a constant. The correct radius – velocity graph of the particle's motion is ;

(A)


(B)


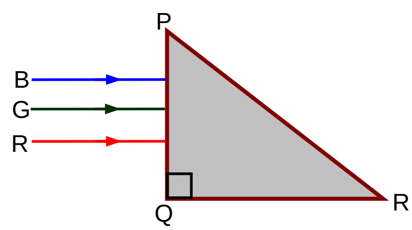
(C)


(D)


Q7. For an adiabatic expansion of an ideal gas, the fractional change in its pressure is equal to (where γ is the ratio of specific heats):

(A) $-\gamma\frac{dV}{V}$ (B) $\frac{dV}{V}$
(C) $-\gamma\frac{V}{dV}$ (D) $-\frac{1}{\gamma}\frac{dV}{V}$

Q8. Three rays of light, red (R), green(G) and blue (B) are incident on the face PQ of a right angled prism PQR as shown in the figure.
The refractive indices of material of the prism for red, green and blue wavelength are 1.27, 1.42 and 1.49 respectively. The colour of ray(s) emerging out of the face PR is



(A) red (B) blue and green
(C) green (D) blue

Q9. The angular momentum of a planet of mass M moving around the sun in an elliptical orbit is $\vec{L}$. The magnitude of the areal velocity of the planet is ;

(A) $\frac{2L}{M}$ (B) $\frac{L}{2M}$

(C) $\frac{4L}{M}$ (D) $\frac{L}{M}$

Q10. The decay of a proton to neutron is :

(A) possible only inside the nucleus
(B) not possible but neutron to proton conversion is possible
(C) not possible as proton mass is less than the neutron mass
(D) always possible as it is associated only with β^+ decay

Q11. The function of time representing a simple harmonic motion with time period of $\frac{\pi}{\omega}$ is :

(A) $\sin(\omega t)+\cos(\omega t)$ (B) $\sin^2(\omega t)$

(C) $3\cos\left(\frac{\pi}{4}-2\omega t\right)$ (D) $\cos(\omega t)+\cos(2\omega t)+\cos(3\omega t)$

Q12. If the angular velocity of earth's spin is increased such that the bodies at the equator start floating , the duration of the day would be approximately :

$\left[Take\ \ g=10\,ms^{-2}, the\ \ radius\ of\ \ earth,\ R=6400\times10^3\,m,\ Take\ \pi=3.14\right]$

(A) 1200 minutes (B) does not change
(C) 60 minutes (D) 84 minutes

Q13. The correct relation between α (ratio of collector current to emitter current) and β(ratio of collector current to base current) of a transistor is :

(A) $\beta=\frac{1}{1-\alpha}$ (B) $\alpha=\frac{\beta}{1+\beta}$

(C) $\alpha=\frac{\beta}{1-\alpha}$ (D) $\beta=\frac{\alpha}{1+\alpha}$

Q14. An ideal gas in a cylinder is separated by a piston in such a way that the entropy of one part is S_1 and that of the other part is S_2. Given that $S_1 > S_2$. If the piston is removed then the total entropy of the system will be :

(A) S_1-S_2 (B) $\frac{S_1}{S_2}$

(C) $S_1\times S_2$ (D) S_1+S_2

Q15. An object of mass m_1 collides with another object of mass m_2 , which is at rest. After collision the objects move with equal speeds in the opposite direction. The ratio of the masses m_2: m_1 is :

(A) $1:1$ (B) $3:1$
(C) $1:2$ (D) $2:1$

Q16. In a series LCR circuit , the inductive reactance (X_L) is 10 Ω , and the capacitive reactance (X_C) is 4 Ω . The resistance (R) in the circuit is 6 Ω. The power factor of the circuit is :

(A) $\frac{1}{2}$ (B) $\frac{\sqrt{3}}{2}$

(C) $\frac{1}{\sqrt{2}}$ (D) $\frac{1}{2\sqrt{2}}$

Q17. Consider a uniform wire of mass M and length L . It is bent into a semi-circle. Its moment of inertia about a line perpendicular to the plane of the wire passing through centre is :

(A) $\frac{ML^2}{\pi^2}$ (B) $\frac{2ML^2}{5\pi^2}$

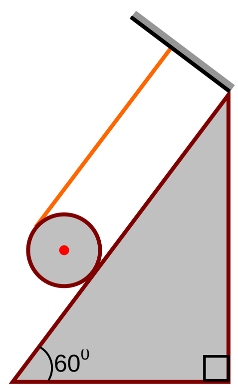
(C) $\frac{ML^2}{2\pi^2}$ (D) $\frac{ML^2}{4\pi^2}$

Q18. A plane electromagnetic wave propagating along y-direction, can have the following pair of electric field $\left(\vec{E}\right)$ and magnetic field $\left(\vec{B}\right)$ components :

(A) E_x, B_y or E_y, B_x (B) E_y, B_x or E_x, B_y

(C) E_y, B_y or E_z, B_z (D) E_x, B_z or E_z, B_x

Q19. A solid cylinder of mass m is wrapped with an inextensible light string and , is placed on a rough inclined plane as shown in the figure. The frictional force acting between the cylinder and the inclined plane is:
[the coefficient of static friction, μ_s is 0.4]



(A) 0 (B) $\frac{7mg}{2}$

(C) 5mg (D) $\frac{mg}{5}$

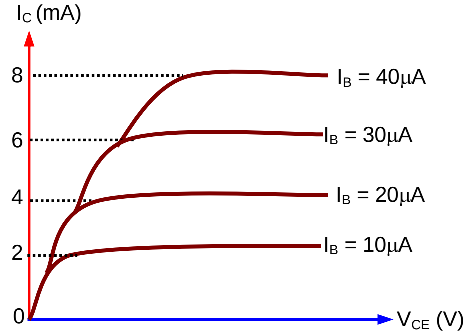
Q20. Consider a sample of oxygen behaving like an ideal gas. At 300K, the ratio of root mean square (rms) velocity to the average velocity of gas molecule would be :
(Molecular weight of oxygen is 32 g / mol ; R = 8.3 JK^{-1} mol^{-1})

(A) $\sqrt{\frac{8\pi}{3}}$ (B) $\sqrt{\frac{8}{3}}$

(C) $\sqrt{\frac{3}{3}}$ (D) $\sqrt{\frac{3\pi}{8}}$

SECTION – B

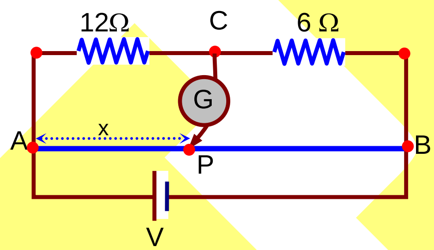
Q1. The typical output characteristics curve for a transistor working in the common –emitter configuration is shown in the figure.



The estimated current gain from the figure is……………….

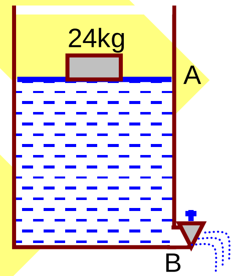
Q2. A TV transmission tower antenna is at a height of 20m. Suppose that the receiving antenna is at
(i) ground level
(ii) a height of 5m
The increase in the antenna range in case (ii) relative to case (i) is n %.
The value of n , to the nearest integer, is……………….

Q3. Consider a 72 cm long wire AB as shown in the figure. The galvanometer jockey is placed at P on AB at a distance x cm from A. The galvanometer shows zero deflection.



The value of x , to nearest integer, is……………….

Q4. Consider a water tank as shown in the figure. It's cross-sectional area is 0.4 m^2. The tank has an opening B near the bottom whose cross- sectional area is 1cm^2. A load of 24 kg is applied on the water at the top when the height of the water level is 40 cm above the bottom, the velocity of water coming out the opening B is v ms^{-1}.

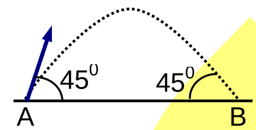


The value of v , to the nearest integer, is……………….
[Take value of g to be 10 ms^{-2}]

Q5. Two wires of same length and thickness having specific resistance 6 Ω cm and 3 Ω cm respectively are connected in parallel. The effective resistivity is ρ Ω cm. The value of ρ , to the nearest integer, is……………….

Q6. The radius of a sphere is measured to be (7.50±0.85) cm. Suppose the percentage error in its volume is x. The value of x , to the nearest integer, is……………….

Q7. The projectile motion of a particle of mass 5g is shown in the figure. The initial velocity of the projectile is $5\sqrt{2}$ ms^{-1} and air resistance is assumed to be negligible. The magnitude of change in momentum between the points A to B is
$x \times 10^{-2}\, kg\, m\, s^{-1}$
The value of x , to the nearest integer, is………………..



Q8. A galaxy is moving away from the earth at a speed 286 kms-1. the shift in wavelength of a redline at 630 nm is $x \times 10^{-10}\ m$. The value of x , to the nearest integer, is………………..
[Take the value of speed of light c, as 3×10^{8} ms^{-1}]

Q9. A ball of mass 4 kg , moving with a velocity 10 ms^{-1}, collides with a spring of length 8m and force constant 100 Nm^{-1}. The length of compressed spring is x m . The value of x , to the nearest integer, is………………..

Q10. An infinite number of point charges, each carrying 1 μC charge , are placed along y-axis at y = 1 m , 2 m , 4 m , 8 m, …………………...The total force on a 1 C point charge , placed at the origin is $x \times 10^{3}\ N$ The value of x , to the nearest integer, is………………..

$$\left[Take\, \frac{1}{4\pi\varepsilon_0} = 9 \times 10^{9}\ Nm^2 / C^2 \right]$$

CHEMISTRY

SECTION A

Q1. Given below are two statements:

Statement I: Bohr's theory accounts for the stability and line spectrum of Li^+ ion.

Statement II: Bohr's theory was unable to explain the splitting of spectral lines in the presence of a magnetic field.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

(A) Both statement **I** and statement **II** are true.
(B) Statement **I** is true but statement **II** is false.
(C) Both statement **I** and statement **II** are false
(D) Statement **I** is false but statement **II** is true.

Q2. Match **List – I** with **List – II:**

List – I	**List – II**
(a) Be	(i) Treatment of cancer
(b) Mg	(ii) Extraction of metals
(c) Ca	(iii) Incendiary bombs and signals
(d) Ra	(iv) Windows of X-ray tubes
	(v) Bearings for motor engines.

Choose the **most appropriate** answer from the option given below:

(A) (a)- (iii), (b)-(iv), (c)-(v), (d)-(ii)
(B) (a)-(iv), (b)-(iii), (c)-(i), (d)-(ii)
(C) (a)-(iii), (b)-(iv), (c)-(ii), (d)-(v)
(D) (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)

Q3. Given below are two statements:

Statement I : C_2H_5OH and AgCN both can generate nucleophile.

Statement II : KCN and AgCN both will generate nitrile nucleophile with all reaction conditions.

Choose the **most appropriate** option:

(A) Both statement **I** and statement **II** are false.
(B) Statement **I** is false but statement **II** is true.
(C) Both statement **I** and statement **II** are true.
(D) Statement **I** is true but statement **II** is false.

Q4. The oxide that shows magnetic property is:

(A) SiO_2
(B) Na_2O
(C) Mn_3O_4
(D) MgO

Q5. In basic medium H_2O_2 exhibits which of the following reactions?

(A) $Mn^{2+} \rightarrow Mn^{4+}$
(B) $I_2 \rightarrow I^-$
(C) $PbS \rightarrow PbSO_4$

Choose the **most appropriate** answer from the options given below:

(A) (A), (C) only
(B) (B) only
(C) (A), (B) only
(D) (A) only

Q6. A hard substance melts at high temperature and is an insulator in both solid and in molten state. This solid is most likely to be a/ an:

(A) Ionic solid
(B) Metallic solid
(C) Molecular solid
(D) Covalent solid

Q7. Given below are two statements:

Statement I : Non- biodegradable wastes are generated by the thermal power plants.

Statement II: Bio-degradable detergents leads to eutrophication.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

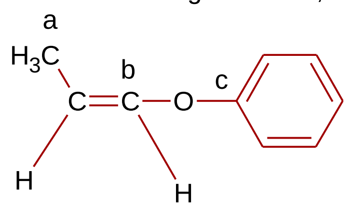
(A) Statement **I** is true but statement **II** is false

(B) Both Statement **I** and Statement **II** are false.

(C) Statement **I** is false but Statement **II** is true.

(D) Both Statement **I** and Statement **II** are true.

Q8. In the following molecule,



Hybridisation of Carbon a, b and c respectively are:

(A) sp^3, sp^2, sp^2
(B) sp^3, sp, sp
(C) sp^3, sp, sp^2
(D) sp^3, sp^2,sp

Q9. Match **List – I** with **List – II:**

List – I (Class of Chemicals)	**List – II** (Example)
(a) Antifertility drug	(i) Meprobamate
(b) Antibiotic	(ii) Alitame
(c) Tranquilizer	(iii) Norethindrone
(d) Artificial Sweetener	(iv) Salvarsan

Choose the **most appropriate** match:

(A) (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)
(B) (a)-(iii), (b)-(iv), (c)-(i),(d)-(ii)
(C) (a)-(ii), (b)-(iii),(c)-(iv), (d)-(i)
(D) (a)-(ii), (b)-(iv), (c)-(i), (d)- (iii)

Q10. The oxidation states of nitrogen in NO, NO_2, N_2O and NO_3^- are in the order of:

(A) $NO_2 > NO_3^- > NO > N_2O$
(B) $NO_3^- > NO_2 > NO > N_2O$
(C) $NO > NO_2 > N_2O > NO_3^-$
(D) $N_2O > NO_2 > NO > NO_3^-$

Q11. The secondary valency and the number of hydrogen bonded water molecule(s) in $CuSO_4.5H_2O$ respectively, are:

(A) 6 and 4
(B) 4 and 1
(C) 5 and 1
(D) 6 and 5

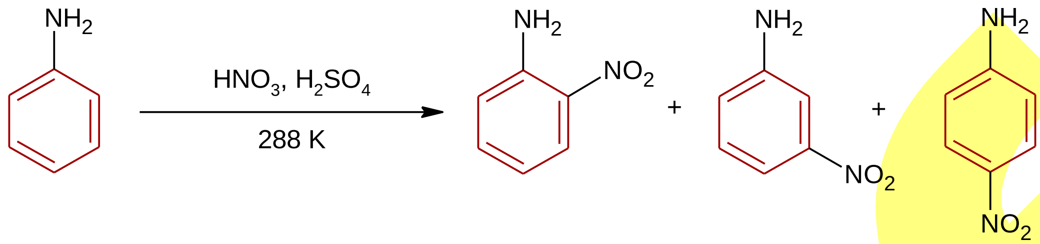
Q12. Deficiency of vitamin K causes:

(A) Cheilosis
(B) Increase in blood clotting time
(C) Decrease in blood clotting time
(D) Increase in fragility of RBC's

Q13. In the reaction of hypobromite with amide, the carbonyl carbon is lost as:

(A) CO_3^{2-}
(B) HCO_3^-
(C) CO
(D) CO_2

Q14.

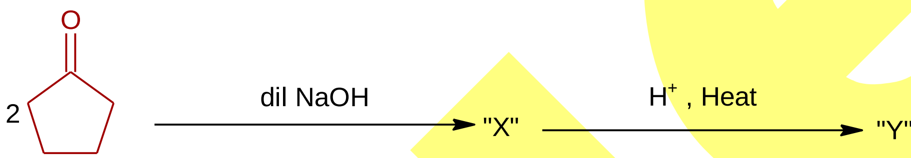


A B C

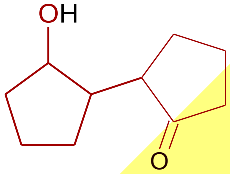
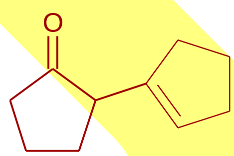
Consider the given reaction, percentage yield of:

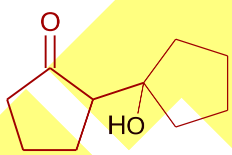
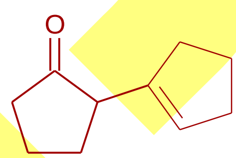
(A) B > C> A
(B) C> B> A
(C) A> C> B
(D) C> A> B

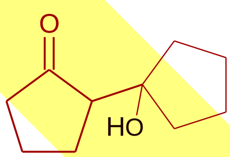
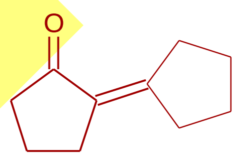
Q15.

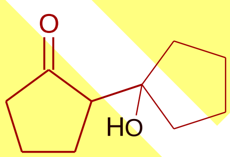
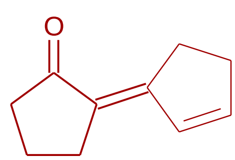


Consider the above reaction, the product 'X" and 'Y' respectively are:

(A)  , 

(B)  , 

(C)  , 

(D)  , 

Q16. Match **List – I** with **List – II**:

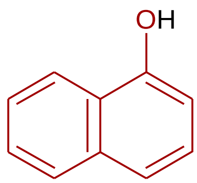
List – I	**List – II**
(a) Mercury	(i) Vapour phase refining
(b) Copper	(ii) Distillation Refining
(c) Silicon	(iii) Electrolytic Refining
(d) Nickel	(iv) Zone Refining

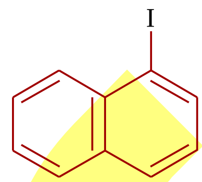
Choose the **most appropriate** answer from the option given below:

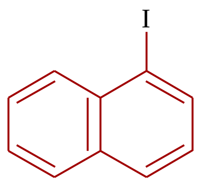
(A) (a)- (ii), (b)-(iii), (c)-(iv), (d)-(i)
(B) (a)-(ii), (b)-(iv), (c)-(iii), (d)-(i)
(C) (a)-(i), (b)-(iv), (c)-(ii), (d)- (iii)
(D) (a)-(ii), (b)-(iii), (c)-(i), (d)-(iv)

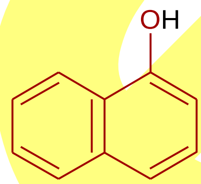
Q17. The first ionization energy of magnesium is smaller as compared to that of elements X and Y, but higher than that of Z. The elements X, Y and Z, respectively are:
(A) argon, chlorine and sodium
(B) chlorine, lithium and sodium
(C) neon, sodium and chlorine
(D) argon, lithium and sodium

Q18. Main products formed during a reaction of 1- methoxy naphthalene with hydroiodic acid are:

(A)  and CH_3OH

(B)  and CH_3I

(C)  and CH_3OH

(D)  and CH_3I

Q19. The charges on the colloidal CdS sol and TiO_2 sol are, respectively:
(A) Positive and negative
(B) Positive and positive
(C) Negative and positive
(D) Negative and negative

Q20. An organic compound "A" on treatment with benzene sulphonyl chloride gives compound B. B is soluble in dil. NaOH solution. Compound A is:
(A) $C_6H_5-NHCH_2CH_3$
(B) $C_6H_5-CH(CH_3)-NH_2$
(C) $C_6H_5-N-(CH_3)_2$
(D) $C_6H_5-CH_2\ NH\ CH_3$

SECTION B

Q1. A solute A dimerizes in water. The boiling point of a 2 molal solution of A is 100.52°C. The percentage association of A is_________. (Round off to the nearest integer).
[Use: K_b for water = 0.52 K kg mol^{-1}, Boiling point of water = 100°C]

Q2. The number of species below that have two lone pairs of electrons in their central atom is _____________. (Round off to the nearest integer).
SF_4, BF_4^-, ClF_3, AsF_3, PCl_5, BrF_5, XeF_4, SF_6

Q3. A xenon compound 'A' upon partial hydrolysis gives XeO_2F_2. The number of lone pair of electrons present in compound A is________. (Round off to the nearest integer).

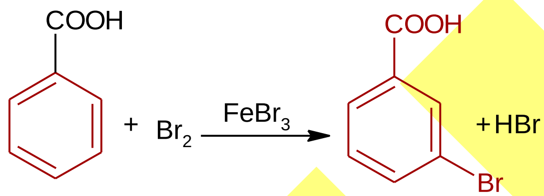
Q4. In Tollen's test for aldehyde, the overall number of electron(s) transferred to the Tollen's reagent formula $[Ag(NH_3)_2]^+$ per aldehyde group to form silver mirror is ________. (Round off to the nearest integer).

Q5. 10.0 mL of Na_2CO_3 solution is titrated against 0.2 M HCl solution.
The following titre values were obtained in 5 readings:
4.8 mL, 4.9 mL 5.0 mL 5.0 mL and 5.0 mL
Based on these readings, and convention of titrimetric estimation the concentration of Na_2CO_3 solution is__________ mM.
(Round off to the nearest integer).

Q6. The molar conductivities at infinite dilution of barium chloride, sulphuric acid and hydrochloric acid are 280, 860 and 426 $Scm^2\ mol^{-1}$ respectively. The molar conductivity at infinite dilution of barium sulphate is _________ $S\ cm^2\ mol^{-1}$.
(Round off to the nearest integer).

Q7. A reaction has a half life of 1 min. The time required for 99.9% completion of the reaction is __________ min. (Round off to the nearest integer).
[Use ln 2 = 0.69; ln 10 = 2.3]

Q8.



Consider the above reaction where 6.1 g of Benzoic acid is used to get 7.8g of m- bromo benzoic acid. The percentage yield of the product is________.
(Round off to the nearest integer).
[Given : Atomic masses : C : 12.0u, H:1.0u, O:16.0u, Br:80.0 u]

Q9. The gas phase reaction
$2A(g) \rightleftharpoons A_2(g)$
At 400 K has $\Delta G^o = +25.2\ kJ\,mol^{-1}$.
The equilibrium constant K_C for this reaction is________ $\times 10^{-2}$.
(Round off to the nearest integer).
[Use : R = 8.3 J $mol^{-1}\ K^{-1}$, ln10 = 2.3, $\log_{10} 2 = 0.30$, 1atm = 1 bar]
[antilog (–0.3) = 0.501]

Q10. The solubility of $CdSO_4$ in water is 8.0×10^{-4} mol L^{-1}. Its solubility in 0.01 M H_2SO_4 solution is ________ $\times 10^{-6}$ mol L^{-1}.
(Round off to the nearest integer).
(Assume that solubility is much less than 0.01 M)

MATHEMATICS
SECTION A

Q1. Let the centroid of an equilateral triangle ABC be at the origin. Let one of the sides of the equilateral triangle be along the straight line $x+y=3$. If R and r be the radius of circumcircle and incircle respectively of ΔABC, then $(R+r)$ is equal to :

(A) $7\sqrt{2}$ (B) $2\sqrt{2}$
(C) $\frac{9}{\sqrt{2}}$ (D) $3\sqrt{2}$

Q2. A pole stands vertically inside a triangular park ABC. Let the angel of elevation of the top of the pole from each corner of the park be $\frac{\pi}{3}$. If the radius of the circumcircle of ΔABC is 2, then the height of the pole is equal to :

(A) $\frac{1}{\sqrt{3}}$ (B) $\sqrt{3}$
(C) $2\sqrt{3}$ (D) $\frac{2\sqrt{3}}{3}$

Q3. Define a relation R over a class of n x n real matrices A and B as "ARB iff there exists a non-singular matrix P such that $PAP^{-1}=B$". Then which of the following is true?
(A) R is reflexive, symmetric but not transitive
(B) R is symmetric, transitive but no reflexive
(C) R is reflexive, transitive but not symmetric
(D) R is an equivalence relation

Q4. Consider a hyperbola $H: x^2-2y^2=4$. Let the tangent at a point $P(4,\sqrt{6})$ meet the x-axis at Q and latus rectum at $R(x_1,y_1), x_1>0$. If F is a focus of H which is nearer to the point P, then the area of ΔQFR is equal to :

(A) $4\sqrt{6}-1$ (B) $4\sqrt{6}$
(C) $\sqrt{6}-1$ (D) $\frac{7}{\sqrt{6}}-2$

Q5. Let $f: R-\{3\}\to R-\{1\}$ be defined by $f(x)=\frac{x-2}{x-3}$.
Let $g: R\to R$ be given as $g(x)=2x-3$. Then, the sum of all the values of x for which $f^{-1}(x)+g^{-1}(x)=\frac{13}{2}$ is equal to :

(A) 2 (B) 7
(C) 5 (D) 3

Q6. Let in a series of 2n observations, half of them are equal to a and remaining half are equal to –a. Also by adding a constant b in each of these observations, the mean and standard deviation of new set become 5 and 20, respectively. Then the value of a^2+b^2 is equal to :
(A) 650 (B) 925
(C) 425 (D) 250

Q7. Let $\vec{a}$ and $\vec{b}$ are two non-zero vectors perpendicular to each other and $|\vec{a}| = |\vec{b}|$. If $|\vec{a} \times \vec{b}| = |\vec{a}|$, then the angle between the vectors $\left(\vec{a} + \vec{b} + \left(\vec{a} \times \vec{b}\right)\right)$ and $\vec{a}$ is equal to :

(A) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$
(B) $\sin^{-1}\left(\frac{1}{\sqrt{6}}\right)$
(C) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$
(D) $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$

Q8. If $15\sin^4\alpha + 10\cos^4\alpha = 6$, for some $\alpha \in R$, then the value of $27\sec^6\alpha + 8\mathrm{cosec}^6\alpha$ is equal to :

(A) 500
(B) 350
(C) 400
(D) 250

Q9. If P and Q are two statements, then which of the following compound statement is a tautology?

(A) $\left((P \Rightarrow Q) \wedge \sim Q\right) \Rightarrow Q$
(B) $\left((P \Rightarrow Q) \wedge \sim Q\right) \Rightarrow (P \wedge Q)$
(C) $\left((P \Rightarrow Q) \wedge \sim Q\right) \Rightarrow P$
(D) $\left((P \Rightarrow Q) \wedge \sim Q\right) \Rightarrow \sim P$

Q10. Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} = (y+1)\left((y+1)e^{x^2/2} - x\right), 0 < x < 2.1$, with $y(2) = 0$. Then the value of $\frac{dy}{dx}$ at x = 1 is equal to:

(A) $\frac{-e^{3/2}}{\left(e^2+1\right)^2}$
(B) $-\frac{2e^2}{\left(1+e^2\right)^2}$
(C) $\frac{e^{5/2}}{\left(1+e^2\right)^2}$
(D) $\frac{5e^{1/2}}{\left(e^2+1\right)^2}$

Q11. Let S_1 be the sum of first 2n terms of an arithmetic progression. Let S_2 be the sum of first 4n terms of the same arithmetic progression. If $(S_2 - S_1)$ is 1000, then the sum of the first 6n terms of the arithmetic progression is equal to :

(A) 5000
(B) 1000
(C) 7000
(D) 3000

Q12. Let $f : R \to R$ be a function defined as

$$f(x) = \begin{cases} \frac{\sin(a+1)x + \sin 2x}{2x} & , \text{ if } x < 0 \\ b & , \text{ if } x = 0 \\ \frac{\sqrt{x+bx^3} - \sqrt{x}}{bx^{5/2}} & , \text{ if } x > 0 \end{cases}$$

If f is continuous at x = 0, then the value of a + b is equal to :

(A) –2
(B) –3
(C) $-\frac{5}{2}$
(D) $-\frac{3}{2}$

Q13. Let a tangent be drawn to the ellipse $\frac{x^2}{27}+y^2=1$ at $\left(3\sqrt{3}\cos\theta, \sin\theta\right)$ where $\theta \in \left(0, \frac{\pi}{2}\right)$. Then the value of θ such that the sum of intercepts on axes made by this tangent is minimum is equal to :

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{8}$ (D) $\frac{\pi}{3}$

Q14. Let the system of linear equations

$4x+\lambda y+2z=0$

$2x-y+z=0$

$\mu x+2y+3z=0,\ \lambda, \mu \in R$

has a non-trivial solution. Then which of the following is true?

(A) $\mu=6, \lambda \in R$ (B) $\mu=-6, \lambda \in R$

(C) $\lambda=2, \mu \in R$ (D) $\lambda=3, \mu \in R$

Q15. Let $g(x)=\int_0^x f(t)dt$, where f is continuous function in $[0,3]$ such that $\frac{1}{3}\le f(t)\le 1$ for all $t\in[0,1]$ and $0\le f(t)\le\frac{1}{2}$ for all $t\in(1,3]$. The largest possible interval in which $g(3)$ lies is :

(A) $[1,3]$ (B) $\left[\frac{1}{3},2\right]$

(C) $\left[-1,-\frac{1}{2}\right]$ (D) $\left[-\frac{3}{2},-1\right]$

Q16. In a triangle ABC, if $|\overrightarrow{BC}|=8, |\overrightarrow{CA}|=7, |\overrightarrow{AB}|=10$, then the projection of the vector $\overrightarrow{AB}$ on $\overrightarrow{AC}$ is equal to :

(A) $\frac{85}{14}$ (B) $\frac{25}{4}$

(C) $\frac{127}{20}$ (D) $\frac{115}{16}$

Q17. Let a complex number be $w=1-\sqrt{3}i$. Let another complex number z be such that $|zw|=1$ and $\arg(z)-\arg(w)=\frac{\pi}{2}$. Then the area of the triangle with vertices origin, z and w is equal to :

(A) 4 (B) $\frac{1}{4}$

(C) $\frac{1}{2}$ (D) 2

Q18. The area bounded by the curve $4y^2=x^2(4-x)(x-2)$ is equal to :

(A) $\frac{\pi}{16}$ (B) $\frac{\pi}{8}$

(C) $\frac{3\pi}{2}$ (D) $\frac{3\pi}{8}$

Q19. Let in a Binomial distribution, consisting of 5 independent trials, probabilities of exactly 1 and 2 successes be 0.4096 and 0.2048 respectively. Then the probability of getting exactly 3 successes is equal to :

(A) $\frac{40}{243}$ (B) $\frac{80}{243}$

(C) $\frac{32}{625}$ (D) $\frac{128}{625}$

Q20. Let $S_1 : x^2 + y^2 = 9$ and $S_2 : (x-2)^2 + y^2 = 1$. Then the locus of center of a variable circle S which touches S_1 internally and S_2 externally always passes through the points :

(A) $\left(\frac{1}{2}, \pm\frac{\sqrt{5}}{2}\right)$ (B) $\left(0, \pm\sqrt{3}\right)$

(C) $(1, \pm 2)$ (D) $\left(2, \pm\frac{3}{2}\right)$

SECTION B

Q1. Let ${}^nC_r$ denote the binomial coefficient of x^r in the expansion of $(1+x)^n$. If $\sum_{k=0}^{10}\left(2^2 + 3k\right){}^nC_k = \alpha.3^{10} + \beta.2^{10}, \alpha, \beta \in R$, then $\alpha + \beta$ is equal to……

Q2. Let I be an identity matrix of order 2×2 and $P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$. Then the value of $n \in N$ for which $P^n = 5I - 8P$ is equal to……

Q3. Let $f : R \to R$ satisfy the equation $f(x+y) = f(x).f(y)$ for all $x, y \in R$ and $f(x) \neq 0$ for any $x \in R$. If the function f is differentiable at $x = 0$ and $f'(0) = 3$, then $\lim_{h\to 0}\frac{1}{h}\left(f(h) - 1\right)$ is equal to……

Q4. Let the mirror image of the point $(1, 3, a)$ with respect to the plane $\vec{r}.\left(2\hat{i} - \hat{j} + \hat{k}\right) - b = 0$ be $(-3, 5, 2)$. Then, the value of $|a+b|$ is equal to……..

Q5. Let P be a plane containing the line $\frac{x-1}{3} = \frac{y+6}{4} = \frac{z+5}{2}$ and parallel to the line $\frac{x-3}{4} = \frac{y-2}{-3} = \frac{z+5}{7}$. If the point $(1, -1, \alpha)$ lies on the plane P, then the value of $|5\alpha|$ is equal to….

Q6. Let P(x) be a real polynomial of degree 3 which vanishes at $x = -3$. Let P(x) have local minima at x = 1, local maxima at $x = -1$ and $\int_{-1}^{1} P(x)dx = 18$, then the sum of all the coefficients of the polynomial $P(x)$ is equal to……

Q7. The term independent of x in the expansion of $\left[\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right]^{10}$, $x \neq 1$, is equal to……

Q8. If $\sum_{r=1}^{10} r!\left(r^3+6r^2+2r+5\right)=\alpha(11!)$, then the value of α is equal to…….

Q9. If $f(x)$ and $g(x)$ are two polynomials such that the polynomial $P(x)=f\left(x^3\right)+x\,g\left(x^3\right)$ is divisible by x^2+x+1, then $P(1)$ is equal to…….

Q10. Let $y=y(x)$ be the solution of the differential equation $xdy-ydx=\sqrt{\left(x^2-y^2\right)}dx$, $x \geq 1$, with $y(1)=0$. If the area bounded by the line $x=1$, $x=e^{\pi}$, $y=0$ and $y=y(x)$ is $\alpha e^{2\pi}+\beta$, then the value of $10(\alpha+\beta)$ is equal to……

ANSWER: Paper-Jee-Main-18-03-2021-Evening Shift

PHYSICS	CHEMISTRY	MATHEMATICS
Section-A	SECTION-A	**SECTION – A**
Ans1. A	**Ans1.** D	**Ans1. C**
Ans2. A	**Ans2.** D	**Ans2. C**
Ans3. A	**Ans3.** D	**Ans3. D**
Ans4. D	**Ans4.** C	**Ans4. D**
Ans5. A	**Ans5.** C	**Ans5. C**
Ans6. C	**Ans6.** D	**Ans6. C**
Ans7. A	**Ans7.** D	**Ans7. C**
Ans8. A	**Ans8.** A	**Ans8. D**
Ans9. B	**Ans9.** B	**Ans9. D**
Ans10. A	**Ans10.** B	**Ans10. A**
Ans11. B, C	**Ans11.** B	**Ans11. D**
Ans12. D	**Ans12.** B	**Ans12. D**
Ans13. B	**Ans13.** A	**Ans13. A**
Ans14. D	**Ans14.** B	**Ans14. A**
Ans15. B	**Ans15.** C	**Ans15. B**
Ans16. C	**Ans16.** A	**Ans16. A**
Ans17. A	**Ans17.** A	**Ans17. C**
Ans18. D	**Ans18.** D	**Ans18. C**
Ans19. D	**Ans19.** C	**Ans19. C**
Ans20. D	**Ans20.** B	**Ans20. D**
Section-B	SECTION-B	**SECTION – B**
Ans1. 200	**Ans1.** 100	**Ans1. 19**
Ans2. 50	**Ans2.** 2	**Ans2. 6**
Ans3. 48	**Ans3.** 1	**Ans3. 3**
Ans4. 3	**Ans4.** 2	**Ans4. 1**
Ans5. 4	**Ans5.** 50	**Ans5. 38**
Ans6. 34	**Ans6.** 288	**Ans6. 8**
Ans7. 5	**Ans7.** 10	**Ans7. 210**
Ans8. 6	**Ans8.** 78	**Ans8. 160**
Ans9. 2	**Ans9.** 166	**Ans9. 0**
Ans10. 12	**Ans10.** 64	**Ans10. 4**

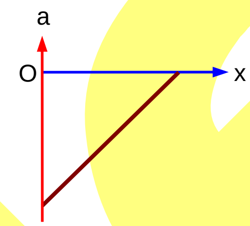
SOLUTION: Paper-Jee-Main-18-03-0-2021-Evening Shift
PHYSICS
SECTION – A

Sol1.

Equation of curve is

$$\frac{v}{v_0}+\frac{x}{x_0}=1 \Rightarrow v=-\left(\frac{v_0}{x_0}\right)x+v_0$$

$$\Rightarrow a=\frac{dv}{dt}=-\left(\frac{v_0}{x_0}\right)(v)=-\left(\frac{v_0}{x_0}\right)\left(-\left(\frac{v_0}{x_0}\right)x+v_0\right) \Rightarrow a=\left(\frac{v_0}{x_0}\right)^2 x-\frac{v_0^2}{x_0}$$



Concept involved: **Graph of kinematics**
Topic: **Kinematics**
Difficulty level: **Moderate**
Note: **IIT-Jee-2005**
Point of Error: **Writing Equation of straight line and differentiation**

Sol2. Basic Fact
Concept involved: **Electric and magnetic field lines**
Topic: **Magnetics and Magnetic Material**
Difficulty level: **Moderate**
Point of Error: **Fact**

Sol3. As we know that resolving power (R) of a microscope is given as

$$R=\frac{2\mu\sin\theta}{1.22\lambda}\text{.........(1)}$$

According to de-Broglie's hypothesis, we can write

$$\lambda=\frac{h}{mv}\text{............(2)}$$

With the help of equations (1) and (2) , we can write

$$R=\frac{2\mu mv\sin\theta}{1.22h} \Rightarrow \frac{R_p}{R_e}=\frac{m_p}{m_e}=1837 \Rightarrow R_p=1837R_e$$

Concept involved: **Resolving Power of Microscope**
Topic: **Optics**
Difficulty level: **Moderate**
Point of Error: **Formula**

Sol4. As we know that radius of trajectory is given as

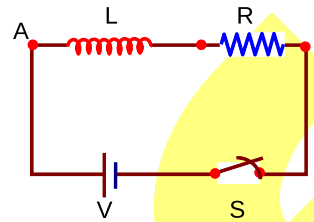
$$R=\frac{mv}{qB}=\frac{\sqrt{2mK}}{qB} \Rightarrow K=\frac{R^2q^2B^2}{2m}$$

$$\Rightarrow \frac{K_p}{K_\alpha}=\frac{R_p^2q_p^2B^2}{2m_p}\times\frac{2m_\alpha}{R_\alpha^2q_\alpha^2B^2}=\left(\frac{m_\alpha}{m_p}\right)\times\left(\frac{R_p}{R_\alpha}\right)^2\times\left(\frac{q_p}{q_\alpha}\right)^2$$

$$\Rightarrow \frac{K_p}{K_\alpha}=(4)\times(2)^2\times\left(\frac{1}{2}\right)^2=4:1$$

Concept involved: **Motion of charged particle in magnetic Field**
Topic: **Magnetics**
Difficulty level: **Easy**

Sol5. As we know that the current in L-R - circuit is given as



$I=\frac{V}{R}\left(1-e^{-\frac{t}{\tau}}\right)$, where $\tau=\frac{L}{R}$

$E=\frac{LI^2}{2}=\frac{LV^2}{2R^2}\left(1-e^{-\frac{t}{\tau}}\right)^2 \Rightarrow$ Energy stored in inductor

According to Question, we can write

$\frac{1}{4}\left(\frac{LV^2}{2R^2}\right)=\frac{LV^2}{2R^2}\left(1-e^{-\frac{t}{\tau}}\right)^2 \Rightarrow \frac{1}{4}=\left(1-e^{-\frac{t}{\tau}}\right)^2 \Rightarrow \frac{1}{2}=1-e^{-\frac{t}{\tau}} \Rightarrow e^{-\frac{t}{\tau}}=\frac{1}{2}$

$\Rightarrow \frac{t}{\tau}=\ell n(2) \Rightarrow t=\tau \ell n(2)=\frac{L}{R}\ell n(2)$

Concept involved: Energy stored in inductor in L-R - circuit
Topic: EMI
Difficulty level: Moderate
Point of Error: Formula

Sol6. $U=mV(r)=-\frac{Cm}{r}$

$F=-\frac{dU}{dr}=\frac{Cm}{r^2} \Rightarrow$ The force which provides required centripetal force

$\Rightarrow \frac{mv^2}{r}=\frac{Cm}{r^2} \Rightarrow r=\frac{C}{v^2}$

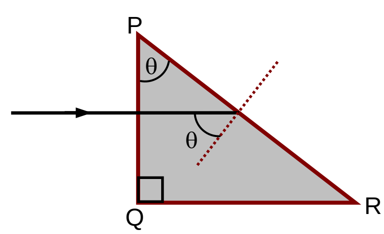
Concept involved: Dynamics of circular motion
Topic: Work ,power and Energy
Difficulty level: Moderate

Sol7. As we know that for adiabatic expansion

$PV^{\gamma}=C \Rightarrow V^{\gamma}\frac{dP}{dV}+P\left(\gamma V^{\gamma-1}\right)=0 \Rightarrow \frac{dP}{dV}=-\gamma\frac{P}{V} \Rightarrow \frac{dP}{P}=-\gamma\frac{dV}{V}$

Concept involved: Adiabatic Process
Topic: Heat and Thermodynamics
Difficulty level: Easy

Sol8. For light ray to come out from face PR of prism



$\theta_C > i=\theta \Rightarrow \sin\theta_C > \sin\theta \Rightarrow \frac{1}{\mu} > \sin\theta$

$\Rightarrow \mu < \frac{1}{\sin\theta}$

Note: If we assume $\theta=45^0 \Rightarrow \mu < 1.414$, then red colour light ray will come out from face PR of prism

Concept involved: Total internal refraction
Topic: Optics
Difficulty level: Moderate
Note: IIT-Jee-1989

Sol9. A= Area swept $\Rightarrow \frac{dA}{dt}=\frac{1}{2}r^2\frac{d\theta}{dt}=\frac{1}{2}\left(\frac{Mr^2\omega}{M}\right)=\frac{L}{2M}$

Concept involved: Kepler's Second Law

Topic: Gravitation
Difficulty level: Easy

Sol10. Basic Fact
Concept involved: β^+ decay
Topic: Modern Physics
Difficulty level: Easy

Sol11. (A) $\sin(\omega t)+\cos(\omega t)=\sqrt{2}\sin\left(\omega t+\frac{\pi}{4}\right)\Rightarrow T=\frac{2\pi}{\omega}$

(B) $\sin^2(\omega t)=\frac{1}{2}-\frac{1}{2}\cos(2\omega t)\Rightarrow T=\frac{2\pi}{2\omega}=\frac{\pi}{\omega}$

(C) $3\cos\left(\frac{\pi}{4}-2\omega t\right)\Rightarrow T=\frac{2\pi}{2\omega}=\frac{\pi}{\omega}$

(D) $\cos(\omega t)+\cos(2\omega t)+\cos(3\omega t)$

Time period of $\cos(\omega t)=\frac{2\pi}{\omega}$

Time period of $\cos(2\omega t)=\frac{2\pi}{2\omega}$

Time period of $\cos(3\omega t)=\frac{2\pi}{3\omega}$

Time period of combined function $=\frac{2\pi}{\omega}$

Concept involved: Basic equation of SHM
Topic: SHM
Difficulty level: Easy

Sol12. $g_e=g-\omega^2R\Rightarrow 0=g-\omega^2R\Rightarrow \omega=\sqrt{\frac{g}{R}}$

$$\Rightarrow T=\frac{2\pi}{\omega}=2\pi\sqrt{\frac{R}{g}}=2\times3.14\times\sqrt{\frac{6400\times10^3}{10}}$$

$$\Rightarrow T=2\times3.14\times80\times10\,s=83.733\min\simeq 84\min$$

Concept involved: Variation of g[Circular motion of earth]
Topic: Laws of motion
Difficulty level: Moderate

Sol13. As we know that for a transistor

$$I_E=I_C+I_B\Rightarrow\frac{I_E}{I_C}=1+\frac{I_B}{I_C}\Rightarrow\frac{1}{\alpha}=1+\frac{1}{\beta}=\frac{1+\beta}{\beta}\Rightarrow\alpha=\frac{\beta}{1+\beta}$$

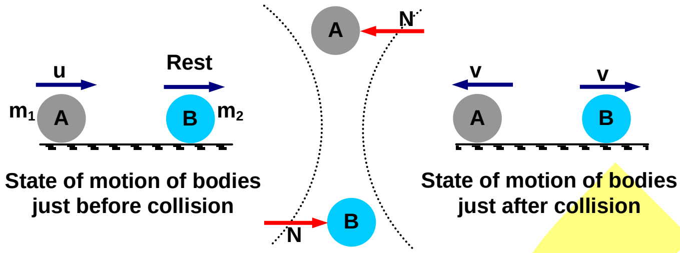
$$\Rightarrow\frac{1}{\beta}=\frac{1}{\alpha}-1=\frac{1-\alpha}{\alpha}\Rightarrow\beta=\frac{\alpha}{1-\alpha}$$

Concept involved: Transistor
Topic: Semiconductor and devices
Difficulty level: Easy

Sol14. $S=S_1+S_2$
Concept involved: Entropy

Topic: Heat and thermodynamics
Difficulty level: Moderate

Sol15.



With the help of Conservation of linear momentum, we can write

$$m_1 u = (m_2 - m_1)v \text{ ....(1)}$$

With the help of definition of e, we can write

$$e = \frac{v_s}{v_a} = \frac{2v}{u} \Rightarrow u = \frac{2v}{e} \text{ ....(2)}$$

Putting the value of e in equation (1) , we have

$$m_1 \frac{2v}{e} = (m_2 - m_1)v \Rightarrow 2m_1 = em_2 - em_1 \Rightarrow \frac{m_2}{m_1} = \frac{2+e}{e} = 1 + \frac{2}{e} > 2$$

From options, only possible answer is B

Concept involved: Collision
Topic: Centre of mass and collision
Difficulty level: Difficult

Sol16. $z = \sqrt{R^2 + (X_C - X_L)^2} = \sqrt{(6)^2 + (4-10)^2} = 6\sqrt{2}\,\Omega$

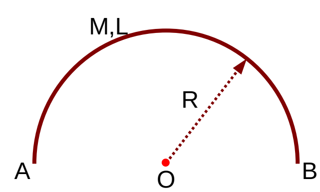
Power factor $= \cos\phi = \frac{R}{z} = \frac{6}{6\sqrt{2}} = \frac{1}{\sqrt{2}}$

Concept involved: Power Factor of LCR-circuit
Topic: Alternating Current
Difficulty level: Easy

Sol17.

$$\pi R = L \Rightarrow R = \frac{L}{\pi}$$

$$I_O = MR^2 = M\left(\frac{L}{\pi}\right)^2 = \frac{ML^2}{\pi^2}$$



Concept involved: Moment of Inertia
Topic: Rotation
Difficulty level: Moderate

Sol18. As we know that direction of propagation of electromagnetic wave is perpendicular to plane containing mutually perpendicular electric field and magnetic field , so option D will be correct answer.

Concept involved: Propagation of EMW
Topic: EMW
Difficulty level: Easy

Sol19. As we know that for rolling without slipping on inclined plane, the minimum value of coefficient of static friction will be

$$\mu_{\min} = \frac{I\tan\theta}{I + mR^2}$$

$$\Rightarrow \mu_{\min} = \frac{\frac{mR^2}{2}\tan\theta}{\frac{mR^2}{2} + mR^2} = \frac{\tan\theta}{3} = \frac{\tan 60^0}{3} = \frac{\sqrt{3}}{3} = \frac{1.732}{3} = 0.5773$$

Since given coefficient of static friction is less than $\mu_{\min}$, so body will perform rolling with slipping and kinetic friction will act

$$F_K = \mu N = \mu mg\cos\theta = (0.4)\times mg\cos 60^0 = \frac{mg}{5}$$

Concept involved: Rolling motion on inclined plane
Topic: Rotation
Difficulty level: Difficult

Sol20. As we know that

$$\text{v}_{\text{rms}} = \sqrt{\frac{3RT}{M}} \text{ and } \text{v}_{\text{av}} = \sqrt{\frac{8RT}{\pi M}}$$

$$\Rightarrow \frac{\text{v}_{\text{rms}}}{\text{v}_{\text{av}}} = \frac{\sqrt{\frac{3RT}{M}}}{\sqrt{\frac{8RT}{\pi M}}} = \sqrt{\frac{3\pi}{8}}$$

Concept involved: Kinetic theory of gases
Topic: Heat and thermodynamics
Difficulty level: Easy
SECTION – B

Sol1. $\beta = \frac{I_C}{I_B} = \frac{2\times10^{-3}}{10\times10^{-6}} = 200$

Sol2. d_{1m} = Distance covered by transmitting antenna + Distance covered by receiving antenna

$$\Rightarrow d_{1m} = \sqrt{2R_e h_{transmitting}} + \sqrt{2R_e h_{receiving}}$$

$$\Rightarrow d_{1m} = \sqrt{2\times64\times10^5\times20} + \sqrt{2\times64\times10^5\times5} = 16000 + 8000 = 24000m$$

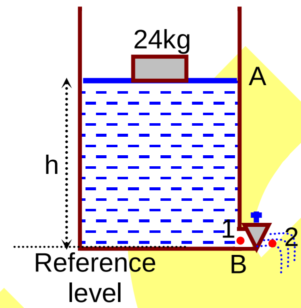
When $h_{receiving}$ =0 then

$$d_{2m} = \sqrt{2\times64\times10^5\times20} = 16000\,m$$

% increment $= \frac{8000}{16000}\times100 = 50$

Sol3. $\frac{12}{x} = \frac{6}{72-x} \Rightarrow x = 144 - 2x \Rightarrow x = \frac{144}{3} = 48cm$

Sol4. **Using Bernoulli's equation at point-1 and point-2, we can write**



$$\frac{v_1^2}{2}+\frac{P_1}{\rho}+gh_1=\frac{v_2^2}{2}+\frac{P_2}{\rho}+gh_2$$

$$\Rightarrow \frac{v_1^2}{2}+\frac{P_0+\rho gh+\frac{mg}{A}}{\rho}+g\times 0=\frac{v_2^2}{2}+\frac{P_0}{\rho}+g\times 0$$

$$\Rightarrow \frac{v_1^2}{2}+\frac{\rho gh+\frac{mg}{A}}{\rho}=\frac{v_2^2}{2}\ \ldots\ldots\ldots(1)$$

Using Continuity equation , we can write

$$Av_1=av_2 \Rightarrow v_1=\frac{a}{A}v_2$$

Putting the value of v_1 in equation(1),we have

$$\Rightarrow \frac{a^2}{A^2}\frac{v_2^2}{2}+\frac{\rho gh+\frac{mg}{A}}{\rho}=\frac{v_2^2}{2} \Rightarrow \left(\frac{A^2-a^2}{A^2}\right)v_2^2=\frac{2\left(\rho gh+\frac{mg}{A}\right)}{\rho}$$

$$\Rightarrow v_2=\sqrt{\left(\frac{A^2}{A^2-a^2}\right)\left\{\frac{2\left(\rho gh+\frac{mg}{A}\right)}{\rho}\right\}}=3.09\,m/s\approx 3\,m/s$$

Sol5. $$R_{eq}=\frac{R_1R_2}{R_1+R_2}\Rightarrow \frac{\rho(\ell)}{2A}=\frac{\left\{\frac{\rho_1\ell}{A}\right\}\left\{\frac{\rho_2\ell}{A}\right\}}{\left\{\frac{\rho_1\ell}{A}\right\}+\left\{\frac{\rho_2\ell}{A}\right\}}\Rightarrow \rho=\frac{2\rho_1\rho_2}{(\rho_1+\rho_2)}=\frac{2\times 6\times 3}{9}=4\Omega cm$$

Sol6. $$V=\frac{4\pi}{3}r^3$$

$$\Rightarrow \frac{\Delta V}{V}=3\frac{\Delta r}{r}\Rightarrow \text{Relative error}$$

% error in volume $=\frac{\Delta V}{V}\times 100=3\frac{\Delta r}{r}\times 100=3\times\frac{0.85}{7.50}\times 100=33.999\approx 34$

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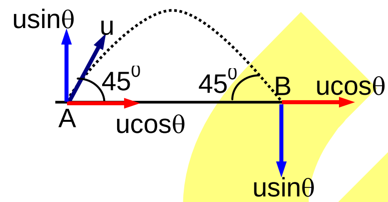
Sol7.

$$\left|\Delta\vec{p}\right| = 2mu\sin\theta = 2\times5\times10^{-3}\times5\sqrt{2}\times\frac{1}{\sqrt{2}} = 5\times10^{-2}\,kg\,m\,s^{-1}$$

$$\Rightarrow x = 5$$



Sol8. $$\Delta\lambda = \frac{v}{c}\times\lambda = \frac{286}{3000000}\times630\times10^{-9} = 6.006\times10^{-10} \approx 6\times10^{-10}\,m$$

Sol9. $$\frac{1}{2}kx^2 = \frac{1}{2}mv^2 \Rightarrow x = v\sqrt{\frac{m}{k}} = 10\times\sqrt{\frac{4}{100}} = 10\times\frac{2}{10} = 2\,m$$

Sol10. $$F = \frac{qQ}{4\pi\varepsilon_0 r_1^2} + \frac{qQ}{4\pi\varepsilon_0 r_2^2} + \frac{qQ}{4\pi\varepsilon_0 r_3^2} + \frac{qQ}{4\pi\varepsilon_0 r_4^2} + \ldots\ldots\ldots\ldots$$

$$\Rightarrow F = \frac{qQ}{4\pi\varepsilon_0}\left[\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{4^2}+\frac{1}{8^2}+\ldots\ldots\ldots\right] \Rightarrow F = \frac{qQ}{4\pi\varepsilon_0}\left[\frac{1}{1-\frac{1}{4}}\right]$$

$$\Rightarrow F = \frac{qQ}{4\pi\varepsilon_0}\left[\frac{4}{3}\right] = 10^{-6}\times1\times9\times10^{9}\times\frac{4}{3} = 12\times10^{3}\,N$$

CHEMISTRY

SECTION – A

Sol1. Bohr's theory accounts for the line spectrum of single electron species but Li^+ has two electrons.
Bohr's theory fails to explain splitting of spectral lines in presence of magnetic field i.e. Zeeman effect.

Sol2. Be is used in X ray tube windows.
Mg is used in incendiary bombs & signals.
Compounds of Ca i.e $CaCO_3$ is used in extraction of metals like Fe.
Radium (Ra) is used in treatment of cancer in radiotherapy.

Sol3. Both C_2H_5OH and AgCN can generate nucleophile.
KCN generate nitrile as nucleophile while AgCN generate isonitrile as nucleophile in nucleophilic substitution reaction.

Sol4. Mn_3O_4 is mixed oxide which contains MnO and Mn_2O_3, so Mn is in +2 and +3 oxidation state respectively. Both Mn^{+2} & Mn^{3+} has unpaired electrons so Mn_3O_4 will show magnetic property. While all other oxides have no unpaired electrons either on cation or on anion.

Sol5. In basic medium H_2O_2 can behaves as both oxidizing and reducing agent, so it can oxidize Mn^{2+} to Mn^{4+} and reduce I_2 to I^-.

Sol6. Covalent solids have high melting point due to strong covalent bonds, also they are insulator in both solid as well as in molten state.

Sol7. Non- biodegradable wastes are generated by thermal power plants which produce fly ash.
Bio-degradable detergents leads to eutrophication by decreasing oxygen level in water.

Sol8. Hybridisation of carbon a, b, and c respectively are sp^3, sp^2 and sp^2.

Sol9.
Antifertility drug $\rightarrow$ Norethindrone
Antibiotic $\rightarrow$ Salvarsan
Tranquilizer $\rightarrow$ Meprobamate
Artificial Sweetener $\rightarrow$ Alitame

Sol10.

	Oxidation state of N
NO	+2
NO_2	+4
N_2O	+1
NO_3^-	+5

So , order of oxidation state is
$NO_3^- > NO_2 > NO > N_2O$

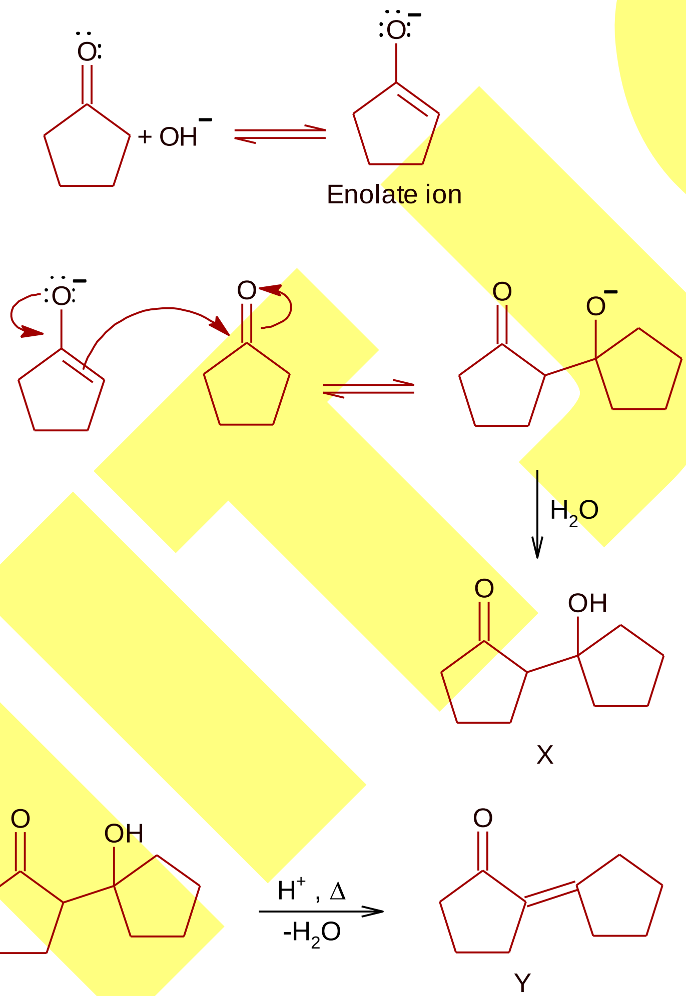
Sol11. $CuSO_4 . 5H_2O$ is blue vitriol which is represented as $[Cu(H_2O)_4]. H_2O.SO_4$
Here, secondary valencies are 4 H_2O and $1H_2O$ molecule outside coordination sphere is involved in hydrogen bonding.

Sol12. Deficiency of vitamin K causes increase in blood clotting time.

Sol13. In Hoffmann bromamide reaction, hypobromite ion react with amide and in this reaction carbonyl group is lost as CO_3^{2-} in form of Na_2CO_3 .

Sol14. During nitration of aniline, meta- nitroaniline is also formed as product due to formation of $-NH_3^+$ group. The percentage of p,m and o product is 51%, 47% and 2% respectively.

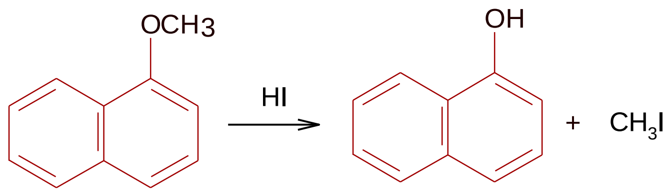
Sol15.



Sol16. Mercury has low boiling point so is refined by distillation method.
Copper refining is done through electrolytic refining.
Silicon is refined by zone refining method.
Nickel is refined by vapour phase refining.

Sol17. First ionization enthaly of Mg is smaller than Ar and Cl but higher than Na.

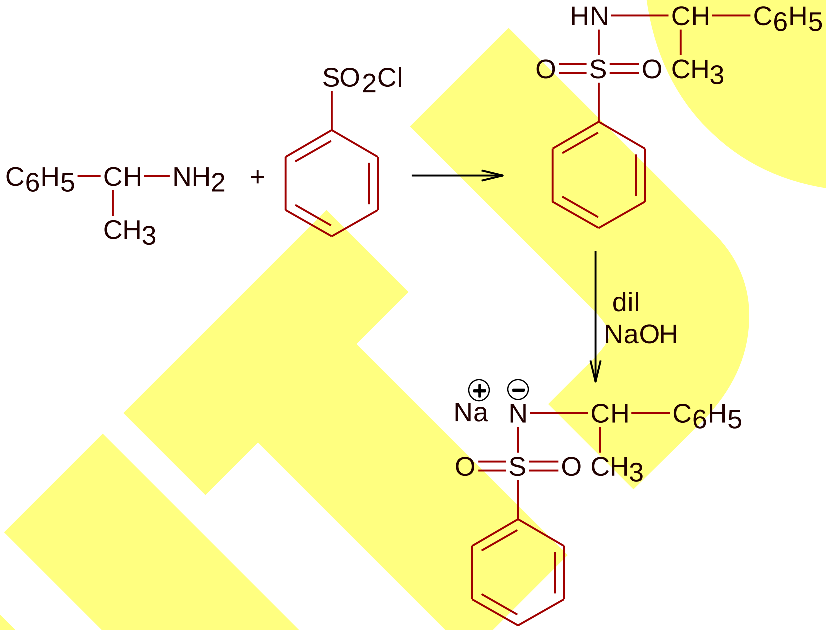
Sol18.



1- methoxy naphthalene

Sol19. Metal sulphide sols are negatively charged while metal oxide sols are positively charged sols. So, CdS is negative while TiO_2 is positively charged sol.

Sol20. Only 1° amine react with benzene sulphonyl chloride to give a compound which is soluble in alkali.



Soluble in NaOH

SECTION - B

Sol1. m=2 molal

$\Delta T_b = 100.52 - 100$

$= 0.52^oC$

Using, $\Delta T_b = iK_bm$

$0.52 = i \times 0.52 \times 2$

i =0.5

Now using,

$$\alpha = \frac{1-i}{1-1/n}$$

Where , n=2 (dimerisation)

$$\alpha = \frac{1-0.5}{1-0.5} = 1$$

So, percentage association = 100%

Sol2. ClF_3 and XeF_4 have two lone pairs of electrons on central atom.

Sol3. Partial hydrolysis of XeF_6 gives XeO_2F_2.
$XeF_6 + 2H_2O \rightarrow XeO_2F_2 + 4HF$
Compound A is XeF_6, so number of lone pair on Xe is 1

Sol4. In Tollen's test for aldehyde, aldehyde is oxidized to carboxylic acid salt as:
$$R-CHO + H_2O \longrightarrow R-COO^- + 3H^+ + 2e^-$$
So; $2e^-$ are transferred per aldhyde group.

Sol5. $V_{Na_2CO_3} = 10$ mL
Volume of HCl used will be 5 mL (average of titre values)
Meq of HCl = meq of Na_2CO_3
$(M \times \text{n-factor} \times V)_{HCl} = (M \times \text{n-factor} \times V)_{Na_2CO_3}$
$0.2 \times 1 \times 5 = M \times 2 \times 10$
M= 0.05 M
M= 50 mM

Sol6. $\lambda^o_{BaCl_2} = 280\ Scm^2mol^{-1}$
$\lambda^o_{H_2SO_4} = 860\ Scm^2mol^{-1}$
$\lambda^o_{HCl} = 426\ Scm^2mol^{-1}$
$\lambda^o_{BaSO_4} = \lambda^o_{BaCl_2} + \lambda^o_{H_2SO_4} - 2\lambda^o_{HCl}$
$= 280 + 860 - 2 \times 426$
$= 280\ Scm^2\ mol^{-1}$

Sol7. Half life, $t_{1/2} = 1$min
Let, time of 99.9% completion of reaction be 't' min
Let the reaction is of first order
$$K = \frac{2.303}{t}\log_{10}\frac{[R]_0}{[R]}$$
$[R] = 0.001\ [R]_0$
$$\frac{0.693}{1} = \frac{2.303}{t}\log_{10}10^3$$
$$t = \frac{2.303}{0.693} \times 3 \text{ min}$$
t =9.99 min
Nearest integer is 10.

Sol8. Moles of benzoic acid $= \frac{6.1}{121}$
$= 0.05$
Theoretical moles of m- bromobenzoic acid = 0.05
Observed moles of m- bromobenzoic acid = $\frac{7.8}{200}$
$= 0.039$

% yield = $\frac{0.039}{0.05} \times 100$

= 78%

Sol9. $\Delta G^o = + 25.2$ kJ / mol

Using $\Delta G^o = -2.3\ RT \log K_p$

$25.2 \times 10^3 = -2.3 \times 8.3 \times 400 \log K_p$

$-3.3 = \log K_p$

$$\log \frac{1}{2\times10^3} = \log K_p$$

$$K_p = \frac{1}{2\times10^3}$$

Using; $K_P = K_C (RT)$

$$\frac{1}{2\times10^3} = K_C(0.083\times400)^{-1}$$

$$K_C = 16.6\times10^{-3}$$

$$K_C = 166\times10^{-2}$$

Sol10. Solubility of $CdSO_4$ is water ; S = $8\times10^{-4}M$

Using $K_{sp} = S^2$

$K_{sp} = 64\times10^{-8}M$

Now,

$CdSO_4 \rightleftharpoons$	Cd^{2+} +	SO_4^{2-}
$'S_1'M$	-	-
-	'S_1'M	$'S_1 + 0.01'M$

$H_2SO_4 \rightleftharpoons 2H^+ + SO_4^{2-}$

0.01M

Using, $K_{sp} = [Cd^{2+}][SO_4^{2-}]$

$64\times10^{-8} = S_1(S_1 + 0.01)$

$S_1 << 0.01$

So, $S_1 = 64\times10^{-6}$ M.

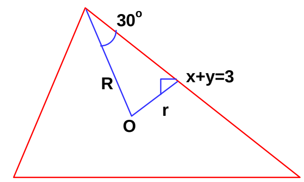
MATHEMATICS
SECTION – A

Sol1. $r = \left|\frac{0+0-3}{\sqrt{2}}\right| = \frac{3}{\sqrt{2}}$

again $\frac{r}{R} = \sin 30^\circ = \frac{1}{2}$

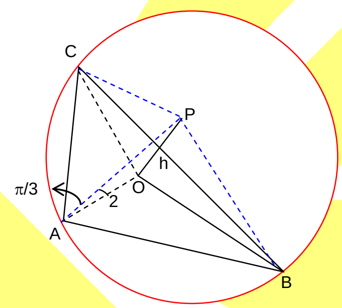
$R = 2r$.

$\therefore\ R + r = 3r = \frac{9}{\sqrt{2}}$



Sol2. $\frac{h}{2} = \tan\frac{\pi}{3} = \sqrt{3}$.

$\Rightarrow\ h = 2\sqrt{3}$



Sol3. $PAP^{-1} = B$ P is non singular

now $PAP^{-1} = A$ If $P = I$

$\therefore$ we can say ARA. $\Rightarrow$ Reflexive

$PAP^{-1} = B$

$\therefore\ P^{-1}PAP^{-1}P = P^{-1}BP$

$\Rightarrow\ A = P^{-1}BP$

Now PAP^{-1}

$\Rightarrow PP^{-1}BPP^{-1} = B$

$\therefore$ ARB $\Rightarrow$ BRA

$\therefore$ R is symmetric

$ARB \Rightarrow PAP^{-1} = B$ - (I)

$BRC \Rightarrow PBP^{-1} = C$ - (II)

$\Rightarrow P\left(PAP^{-1}\right)P^{-1} = C$

$\Rightarrow \left(P^2\right)A\left(P^2\right)^{-1} = C$

$\Rightarrow$ ARC.

$\therefore$ R is transitive.

$\therefore$ R is an equivalence Relation .

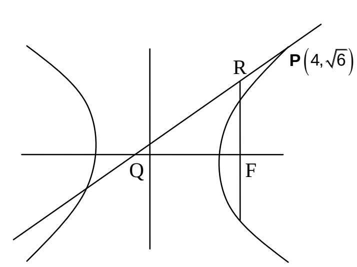
Sol4. $\frac{x^2}{4} - \frac{y^2}{2} = 1$

$e = \sqrt{1 + \frac{2}{4}} = \frac{\sqrt{6}}{2}$

$F\left(\sqrt{6}, 0\right)$

Equation of tangent at P is

$x - \frac{\sqrt{6}\,y}{2} = 1$



$\therefore$ It cuts x axis at $Q(1,0)$

latus rectum at F , $x=\sqrt{6}$ cuts the tangent at.

$$\Rightarrow \sqrt{6}-\frac{\sqrt{6}\,y}{2}=1$$

$$y=\frac{2\left(\sqrt{6}-1\right)}{\sqrt{6}}.$$

Area of ΔQFR $=\frac{1}{2}\left(\sqrt{6}-1\right)\left(\frac{\sqrt{6}-1}{\sqrt{6}}\right)$

$$=\frac{7-2\sqrt{6}}{\sqrt{6}}=\frac{7}{\sqrt{6}}-2$$

Sol5. $f(x)=\frac{x-2}{x-3}, x\neq 3.$ $\quad$ $g(x)=2x-3$

$y=\frac{x-2}{x-3}$ $\quad$ $y=2x-3$

$\Rightarrow x=\frac{3y-2}{y-1}=f^{-1}(y)$ $\quad$ $\Rightarrow x=\frac{y+3}{2}=g^{-1}(y)$

$\therefore$ $f^{-1}(x)+g^{-1}(x)=\frac{13}{2}$

$\Rightarrow \frac{3x-2}{x-1}+\frac{x+3}{2}=\frac{13}{2}.$

$\Rightarrow x^2-5x+6=0 \Rightarrow x=2,3$

$\therefore$ sum of possible values of x is 5

Sol6. Mean = $\bar{x}=\frac{na-na}{2n}=0$

Variance $=\frac{1}{2n}\sum_{i=1}^{2n}x_i^{\,2}-\bar{x}^{\,2}=\frac{2na^2}{2n}=a^2$

If b added to all of salvation then

mean = $0+b=b=5$

$(S.D.)^2=a^2=(2\,0).^2$

$\therefore$ $a^2+b^2=$ 400+25 = 425

Sol7. $|\vec{a}|=|\vec{b}|$ $\quad$ $\vec{a}.\vec{b}=0$

$\left|\vec{a}\times\vec{b}\right|=|\vec{a}|\left|\vec{b}\right|=|\vec{a}|$

$\Rightarrow \left|\vec{b}\right|=1=|\vec{a}|$

Let $\vec{c}=\vec{a}+\vec{b}+\vec{a}\times\vec{b}$

$|\vec{c}|^2=\left|\vec{a}+\vec{b}\right|^2+\left|\vec{a}\times\vec{b}\right|^2+2\left(\vec{a}+\vec{b}\right).\left(\vec{a}\times\vec{b}\right)$

$=|\vec{a}|^2+\left|\vec{b}\right|^2+|\vec{a}|^2$

$= 2|\vec{a}|^2 + |\vec{b}|^2 = 3$

$\therefore\ \vec{c} = \sqrt{3}.$

Again $\vec{c}.\vec{a} = |\vec{a}|^2 + (\vec{a}\times\vec{b}).\vec{a}$

$= 1 + 0 = 1$

$\therefore$ angle between $\vec{c}\ \&\ \vec{a}$

$\cos\theta = \dfrac{\vec{c}.\vec{a}}{|\vec{c}||\vec{a}|} = \dfrac{1}{\sqrt{3}}$

$\Rightarrow\ \theta = \cos^{-1}\left(1/\sqrt{3}\right)$

Sol8. $15\sin^4\alpha + 10\cos^4\alpha = 6,\ \alpha \in R$

$\Rightarrow 15\tan^4\alpha + 10 = 6\sec^4\alpha = 6\left(1+\tan^2\alpha\right)^2$

$= 6\left(1 + 2\tan^2\alpha + \tan^4\alpha\right)$

$\Rightarrow 9\tan^4\alpha - 12\tan^2\alpha + 4 = 0$

$\Rightarrow \left(3\tan^2\alpha - 2\right)^2 = 0$

$\Rightarrow \tan^2\alpha = \dfrac{2}{3}\qquad \Rightarrow \sin^2\alpha = \dfrac{2}{5},\qquad \cos^2\alpha = 3/5$

$\therefore 27\sec^6\alpha + 8\cos^6\alpha = 27\left(5/3\right)^3 + 8\left(5/2\right)^3$

$= 125 + 125 = 250$

Sol9.

P	Q	$P\Rightarrow Q$	$\sim Q$	$(P\Rightarrow Q)\wedge\sim Q$	$\sim P$	$((P\Rightarrow Q)\wedge\sim Q)\Rightarrow\sim P$
T	T	T	F	F	F	T
T	F	F	T	F	F	T
F	T	T	F	F	T	T
F	F	T	T	T	T	T

Sol10. $\dfrac{dy}{dx} = (y+1)\left[(y+1)e^{x^2/2} - x\right]$ - (I)

$\Rightarrow \dfrac{1}{(y+1)^2}\dfrac{dy}{dx} + \dfrac{x}{(y+1)} = e^{x^2/2}$

Let $\dfrac{1}{y+1} = t \Rightarrow \qquad -\dfrac{1}{(y+1)^2}\dfrac{dy}{dx} = \dfrac{dt}{dx}$

(I) $\Rightarrow\ \dfrac{dt}{dx} - xt = -e^{x^2/2}$

solution

$I.F. = e^{-\int x\,dx} = e^{-\frac{x^2}{2}}$

$\dfrac{e^{-x^2/2}}{y+1} = -x + c$

$y(2) = 0 \Rightarrow e^{-2} = c - 2 \Rightarrow c = 2 + e^{-2}$

$$\frac{e^{-x^2/2}}{y+1} = 2 + e^{-2} - x.$$

$$x = 1, \Rightarrow y+1 = \frac{e^{3/2}}{1+e^2}$$

(I) $\Rightarrow$

$$\therefore \frac{dy}{dx} = \frac{e^3}{\left(1+e^2\right)^2} \times e^{1/2} - \frac{e^{3/2}}{1+e^2} = -\frac{e^{3/2}}{\left(1+e^2\right)^2}$$

Sol11. $S_1 = n\left[a + (2n-1)d\right]$ **- (I)**

$S_2 = 2n\left[a + (4n-1)d\right]$ .....(ii)

$S_2 - S_1 = 2n\left[a+(4n-1)d\right] - n\left[a+(2n-1)d\right]$

$= n\left[a+(6n-1)d\right] = 1000$

$\therefore S_{6n} = 3n\left[a+(6n-1)d\right] = 3000$

Sol12. LHL at $x = 0$

$$\lim_{x\to 0} \frac{2\sin\left\{\left(\frac{a+3}{2}\right)x\right\}\cos\left(\frac{a-1}{2}\right)x}{\left\{\left(\frac{a+3}{2}\right)x\right\}} \times \frac{\left(\frac{a+3}{2}\right)x}{2x} = b$$

$\Rightarrow \quad \frac{a+3}{2} = b \Rightarrow \quad a - 2b + 3 = 0$ - (I)

RHL at $x = 0$

$$\lim_{x\to 0} \frac{\sqrt{x+bx^3} - \sqrt{x}}{b\,x^{5/2}} = b$$

$$\lim_{x\to 0} \frac{b\,x^3}{bx^{5/2}\left[\sqrt{x+bx^3} + \sqrt{x}\right]} = b$$

$$\Rightarrow \lim_{x\to 0} \frac{1}{\sqrt{1+bx^2} + 1} = b$$

$\Rightarrow \quad b = \frac{1}{2}$

$\Rightarrow \quad a = 2b - 3 = -2$

$a + b = -2 + \frac{1}{2} = -3/2$

Sol13. Equation of tangent to $\frac{x^2}{27} + y^2 = 1$ at $\left(3\sqrt{3}\cos\theta, \sin\theta\right)$

$\Rightarrow \quad \frac{x}{3\sqrt{3}}\cos\theta + \frac{y}{1}\sin\theta = 1 \qquad \theta \in \left(0, \frac{\pi}{2}\right)$

$\therefore$ sum of intercepts on coordinate axes

$z = 3\sqrt{3}\sec\theta + \operatorname{cosec}\theta$

$\therefore \frac{dz}{d\theta} = 3\sqrt{3}\sec\theta\tan\theta - \text{cosec}\,\theta\cot\theta$

for z min ; $\frac{dz}{d\theta} = 0 \Rightarrow 3\sqrt{3}\sec\theta\tan\theta = \text{cosec}\,\theta\cot\theta$

$\Rightarrow \tan^3\theta = \frac{1}{3\sqrt{3}} \Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$

$\therefore \theta = \frac{\pi}{6}.$

$\theta > \frac{\pi}{6},\ \frac{dz}{d\theta} > 0\ \&\ \theta < \frac{\pi}{6},\ \frac{dz}{d\theta} < 0$

$\therefore$ z in minimum for $\theta = \frac{\pi}{6}.$

Sol14. System of equation has not trivial solutions

$$\therefore \Delta = \begin{vmatrix} 4 & \lambda & 2 \\ 2 & -1 & 1 \\ \mu & 2 & 3 \end{vmatrix} = 0$$

$R_1 - 2R_2$

$$\Rightarrow \begin{vmatrix} 0 & \lambda+2 & 0 \\ 2 & -1 & 1 \\ \mu & 2 & 3 \end{vmatrix} = 0$$

$\Rightarrow (\lambda+2)[\mu-6] = 0$

$\therefore \mu = 6 \quad \& \quad \lambda \in R$

Sol15. $\frac{1}{3} \le f(t) \le 1 \qquad \forall t \in [0, 1]$

$0 \le f(t) \le 1/2 \qquad \forall t \in (1, 3]$

$g(3) = \int_0^1 f(t)dt + \int_1^3 f(t)dt.$ - (1)

Now $\frac{1}{3} \le \int_0^1 f(t)dt \le 1$ - (2)

& $0(3-1) \le \int_1^3 f(t) \le \frac{1}{2}(3-1)$

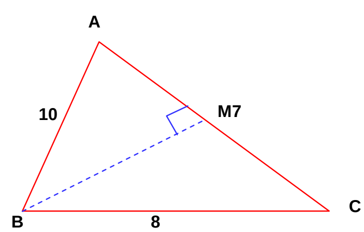
$\Rightarrow 0 \le \int_0^1 f(t)dt \le 1$ - (3)

(2) + (3)

$\Rightarrow \frac{1}{3} \le \int_0^3 f(t)dt \le 2$

$\Rightarrow g(3) \in \left[1/3, 2\right]$

Sol16. $\left|\overrightarrow{BC}\right| = 8\ \left|\overrightarrow{CA}\right| = 7 \qquad \left|\overrightarrow{AB}\right| = 10$



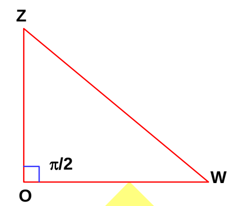
$a = 8, \quad b = 7, \quad c = 10$

$\cos A = \frac{49+100-64}{2(7)(10)} = \frac{17}{28}$

Projection of AB along AC $= AB\cos A$

$$=10\times\frac{17}{28}=\frac{85}{14}$$

Sol17. $\omega=1-\sqrt{3}i$, **z is such that** $|z\omega|=1$



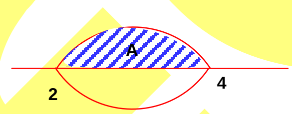
$|\omega|=2 \qquad \Rightarrow|z||\omega|=1$

$\Rightarrow|z|=\frac{1}{2}$

$\therefore$ Area of Regard triangle

$=\frac{1}{2}|\omega||z|=\frac{1}{2}$

Sol18. $4y^2=x^2(4-x)(x-2)\geq 0$



$\Rightarrow x\in[2,4]$

$y=\pm\frac{1}{2}x\sqrt{(4-x)(x-2)}$

$A=\frac{1}{2}\int_2^4 x\sqrt{(4-x)(x-2)}\,dx$

$=\frac{1}{2}\int_2^4 x\sqrt{6x-x^2-8}\,dx$

$=\frac{1}{2}\int_2^4 (x-3+3)\sqrt{1-(x-3)^2}\,dx.$

$=\frac{1}{2}\int_2^4 (x-3)\sqrt{1-(x-3)^2}\,dx+\frac{3}{2}\int_2^4\sqrt{1-(x-3)^2}\,dx.$

put $x-3=\sin\theta$

$\Rightarrow dx=\cos\theta d\theta$

$A=\frac{1}{2}\int_{-\pi/2}^{\pi/2}\sin\theta\cos^2\theta\,d\theta+\frac{3}{2}\int_{-\pi/2}^{\pi/2}\cos^2\theta\,d\theta$

$=0+\frac{3}{2}\int_0^{\pi/2}(1+\cos2\theta)d\theta$

$A=\frac{3}{2}\left(\theta+\frac{\sin2\theta}{2}\right)_0^{\frac{\pi}{2}}$

$=\frac{3\pi}{4}.$

$\therefore$ Required total area $=2A=\frac{3\pi}{2}$

Sol19. p = probability of getting success in a single trial
q = probability of getting failure in a single trial

$p+q=1$

given ${}^5C_1p^1q^4=0.4096$ - (I)

& ${}^5C_2p^2q^3=0.2048$ - (II)

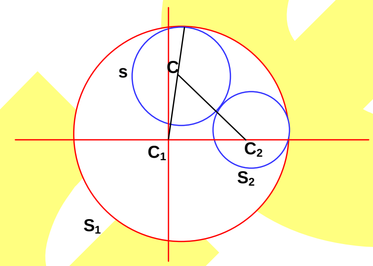
$\frac{{}^5C_1pq^4}{{}^5C_2p^2q^3}=\frac{0.4096}{0.2048}=2$

$\Rightarrow\frac{q}{p}=4\Rightarrow p=\frac{1}{5}\quad q=\frac{4}{5}$

$\therefore\ p(x=3) = 5c_3 p^3 q^2 = 10 \times \frac{1}{125} \times \frac{16}{25}$

$= \frac{32}{625}$

Sol20. Let centre of Variable circle is at $C(h,k)$ & radius = r



$Cc_1 = 3 - r$

$Cc_2 = 1 + r$

$Cc_1 + Cc_2 = 4$ constant.

$\therefore$ Locus of C is an ellipse.

$\sqrt{(h-2)^2 + k^2} = 4 - \sqrt{h^2 + k^2}$

Or, point $\left(2, \pm\frac{3}{2}\right)$ satisfying the expression

SECTION – B

Sol1. $\sum_{k=0}^{10}\left(2^2 + 3k\right)\ {}^{n}C_k = 4\sum_{k=0}^{10} {}^{10}C_k + 3\sum_{k=0}^{10} k\ {}^{10}C_k$ $\qquad n = 10(\text{assumption})$.

$= 4.2^{10} + 3\sum_{k=0}^{10} k \times \frac{10}{k}\ {}^{9}C_{k-1}$

$= 4.2^{10} + 30 \times 2^9$

$= 2^{10}(4 + 15) = 19 \times 2^{10} = \alpha \times 3^{10} + \beta \times 2^{10}$

Compaining $\alpha = 0 \quad \beta = 19$

$\therefore \alpha + \beta = 19$

Sol2. $P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$

Its characteristic eqution

$\begin{vmatrix} 2-\lambda & -1 \\ 5 & -3-\lambda \end{vmatrix} = 0 \Rightarrow \lambda^2 + \lambda - 1 = 0$

$\therefore\ P^2 = I - P$ - (I)

$P^3 = P - P^2 = P - (I - P) = 2P - I$

$P^4 = 2P^2 - P = 2(I - P) - P = 2I - 3P.$

$P^5 = 2P - 3P^2 = 2P - 3(I - P) = 5P - 3I$

$P^6 = 5P^2 - 3P = 5(I - P) - 3P = 5I - 8P$

$\therefore\ N = 6$

Sol3. $f(x+y) = f(x).f(y)$

$\Rightarrow f(x) = \lambda^x.$

$f'(x) = \lambda^x \ell n\lambda$

& $f'(0) = \ell n\lambda = 3$ & $\lambda = e^3.$

$\therefore f(x) = e^{3x}.$

$$\therefore \lim_{x\to 0}\left(\frac{e^{3x}-1}{3x}\right)\times 3 = 3.$$

Sol4. $P(1,3,a)$ **Plane 2x-y+z-b=0 ------- - (1)**

$Q(-3,5,2)$

$\overrightarrow{PQ}$ is collinear with $2\hat{i}-\hat{j}+\hat{k}$

& $\frac{-4}{2}=\frac{2}{-1}=\frac{2-a}{1} \Rightarrow$ a = 4

Mid point of P and Q is $M(-1,4,3)$ lies on

(I) $\Rightarrow$ $-2-4+3-b=0$

$\Rightarrow$ b = - 3

Sol5. Equation of plane is

$a(x-1)+b(y+6)+c(z+5)=0$

$\therefore 4a-3b+7c=0$ (1)

& $3a+4b+2c=0$ (2)

$\Rightarrow \frac{a}{-34}=\frac{b}{13}=\frac{c}{25}$

$\therefore$ plane $-34(x-1)+13(y+6)+25(z+5)=0$

Now $(1,-1,\alpha)$ lies on it

$\therefore 65+25(\alpha+5)=0$

$25\alpha=-5(25+13)$

$5\alpha=-38$

$|5\alpha|=38$

Sol6. $P'(x)=a(x-1)(x+1)=a(x^2-1)$

$\therefore$ $P(x)=a\left(\frac{x^3}{3}-x\right)+b$

$P(-3)=0$

$\Rightarrow 0=a(-9+3)+b$

$\Rightarrow b=6a$

$\int_{-1}^{1}P(x)dx=18$

$\Rightarrow$ $\int_{-1}^{1}\left[a\left(\frac{x^3}{3}-x\right)+b\right]dx=18$

$\Rightarrow 2b=18$ $\Rightarrow b=9$

$\therefore$ $a=3/2$

$\therefore$ $P(x)=\frac{1}{2}x^3-\frac{3}{2}x+9$

$\therefore$ Sum of all coefficients

$=\frac{1}{2}-\frac{3}{2}+9=8$

Sol7. $\left[\dfrac{x+1}{x^{2/3}-x^{1/3}+1}-\dfrac{x-1}{\sqrt{x}\left(\sqrt{x}-1\right)}\right]^{10}$

$\Rightarrow\left[\left(x^{1/3}+1\right)-\dfrac{\left(\sqrt{x}+1\right)}{\sqrt{x}}\right]^{10}$

$=\left[x^{1/3}+1-x^{-1/2}\right]^{10}$

general term $={}^{10}C_r x^{\frac{10-r}{3}}\times x^{-\frac{r}{2}}={}^{10}C_r x^{\frac{20-5r}{6}}$

For independent of $\dfrac{20-5r}{6}=0\Rightarrow r=4$

$\therefore$ coefficent $={}^{10}C_4=210$

Sol8. $\sum_{r=1}^{10} r!\left(r^3+6r^2+2r+5\right)$

$\left(r+1\right)\left(r+2\right)\left(r+3\right)=r^3+6r^2+11r+6$

$=\sum_{r=1}^{10} r!\left[\left(r+1\right)\left(r+2\right)\left(r+3\right)-9r-1\right]$

$=\sum_{r=1}^{10}\left(r+3\right)!-9r!\left(r+1-1\right)-r!$

$=\sum_{r=1}^{10}\left(r+3\right)!-9\left(r+1\right)!+9r!-r!$

$=\sum_{r=1}^{10}\left[\left(r+3\right)!-r!\right]-9\sum_{r=1}^{10}\left[\left(r+1\right)!-r!\right]$

$=\left(11!+12!+13!-1!-2!-3!\right)-9\left(11!-1!\right)$

$=13!+12!+13!-9!-9!11!+9!$

$=13!+12!+11!-9\left(11!\right)$

$=13!+4\left(11!\right)=11!\left(12\times13+4\right)=160\left(11!\right)$

Sol9. $P\left(x\right)=f\left(x^3\right)+x\,g\left(x^3\right)$ - (I)

$x^2+x+1=\left(x-w\right)\left(x-w^2\right)$ is factor of (1)

$\therefore\ P\left(w\right)=f\left(1\right)+wg\left(1\right)=0$ - (2)

$P\left(w^2\right)=f\left(1\right)+w^2g\left(1\right)=0$ - (3)

$\left(2\right)\times w-\left(3\right)$

& $f\left(1\right)w-f\left(1\right)=0\ \&\ f\left(1\right)\left(w-1\right)=0$

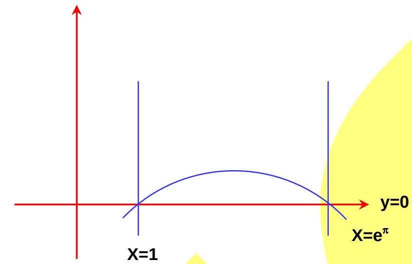
$\Rightarrow f\left(1\right)=0$

Again (2) + (3)

& $2f\left(1\right)-g\left(1\right)=0\Rightarrow g\left(1\right)=2f\left(1\right)$

$\therefore\ P\left(1\right)=f\left(1\right)+g\left(1\right)=3f\left(1\right)=3\times0=0$

Sol10.



$$x\,dy - y\,dx = \sqrt{x^2 - y^2} \quad x \ge 1.$$

$$d\left(\frac{y}{x}\right) = \frac{1}{x}\sqrt{1-\left(\frac{y}{x}\right)^2}$$

$$\Rightarrow \int \frac{d(y/x)}{\sqrt{1-\left(\frac{y}{x}\right)^2}} = \int \frac{dx}{x} \Rightarrow \text{Sin}^{-1}\left(\frac{y}{x}\right) = \ln x + c$$

$$\text{Sin}^{-1}(0) = 0 + c \Rightarrow c = 0$$

$$y = x\sin(\ell n x)$$

$$x = 1, \quad y = 0$$

$$x = e^{\pi} \quad y = 0$$

Required area $A = \int_1^{e^{\pi}} x\sin(\ell n x)dx.$

Let $\ln x = t \Rightarrow \frac{1}{x}dx = dt.$

x	1	e^{π}
t	0	π

$$A = \int_0^{\pi} e^{2t}\sin t\,dt = \left[\frac{e^{2t}(2\sin t - \cos t)}{5}\right]_0^{\pi} = \frac{e^{2\pi}(1+1)}{5} = \frac{2e^{2\pi}}{5} = \alpha e^{2\pi} + \beta$$

$$\therefore \alpha = \frac{2}{5}, \beta = 0$$

$$\therefore 10(\alpha + \beta) = 10\left(\frac{2}{5} + 0\right) = 4$$