Paper-17-03-2021-Morning Shift

PHYSICS

SECTION – A

Q1. A current of 10 A exists in a wire of cross-sectional area of $5 mm^2$ with a drift velocity of $2\times10^{-3}\ ms^{-1}$. The number of free electrons in each cubic meter of the wire …………..

(A) 2×10^{25} (B) 1×10^{23}
(C) 2×10^{6} (D) 625×10^{25}

Q2. An electron of mass m and a photon have same energy E. The ratio of wavelength of electron to that of photon is : (c being the velocity of light)

(A) $\frac{1}{c}\left(\frac{E}{2m}\right)^{1/2}$ (B) $\left(\frac{E}{2m}\right)^{1/2}$
(C) $c(2mE)^{1/2}$ (D) $\frac{1}{c}\left(\frac{2m}{E}\right)^{1/2}$

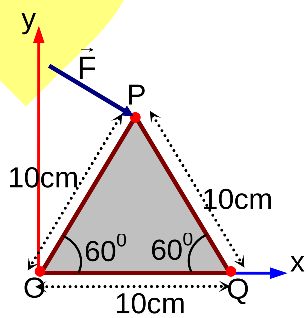
Q3. The thickness at the centre of a plano- convex lens is 3 mm and the diameter is 6cm. If the speed of light in the material of the lens is $2\times10^{8} ms^{-1}$.The focal length of the lens is ………..

(A) 15 cm (B) 0.30 cm
(C) 30 cm (D) 1.5 cm

Q4. The vernier-scale used for measurement has a positive zero error of 0.2 mm. If while taking a measurement it was noted that '0' on the vernier- scale lies between 8.5 cm and 8.6 cm, vernier coincidence is 6, then the correct value of measurement is …………….. cm. (least count = 0.01 cm)

(A) 8.36 cm (B) 8.58 cm
(C) 8.54 cm (D) 8.56 cm

Q5. A triangular plate is shown. A force $\vec{F}=4\hat{i}-3\hat{j}$ is applied at point P. The torque at point P with respect to point 'O' and 'Q' are :



(A) $15+20\sqrt{3}, 15-20\sqrt{3}$
(B) $-15-20\sqrt{3}, 15-20\sqrt{3}$
(C) $-15+20\sqrt{3}, 15+20\sqrt{3}$
(D) $15-20\sqrt{3}, 15+20\sqrt{3}$

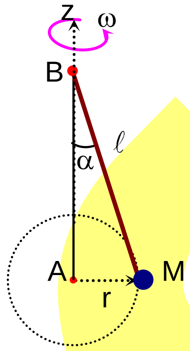
Q6. A car accelerates from rest at a constant rate α for some time after which it decelerates at a constant rate β to come to rest. If the total time elapsed is t seconds, the total distance traveled is :

(A) $\frac{4\alpha\beta}{(\alpha+\beta)}t^2$ (B) $\frac{\alpha\beta}{4(\alpha+\beta)}t^2$
(C) $\frac{2\alpha\beta}{(\alpha+\beta)}t^2$ (D) $\frac{\alpha\beta}{2(\alpha+\beta)}t^2$

Q7. A boy is rolling a 0.5 kg ball on the frictionless floor with the speed of $20 ms^{-1}$. The ball gets deflected by an obstacle on the way. After deflection it moves with 5% of its initial kinetic energy. What is the speed of the ball now?

(A) $14.41 ms^{-1}$ (B) $19.0 ms^{-1}$
(C) $1.00 ms^{-1}$ (D) $4.47 ms^{-1}$

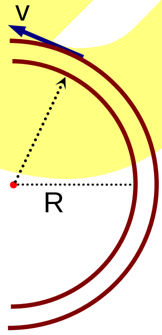
Q8. A mass M hangs on a massless rod of length ℓ which rotates at a constant angular frequency. The mass M moves with steady speed in a circular path of constant radius. Assume that the system is in steady circular motion with constant angular velocity ω. The angular momentum of M about point A is L_A which lies in the positive z direction and the angular momentum of M about point B is L_B. The correct statement for this system is:



(A) L_A is constant, both in magnitude and direction
(B) L_B is constant in direction with varying magnitude
(C) L_A and L_B are both constant in magnitude and direction
(D) L_B is constant, both in magnitude and direction

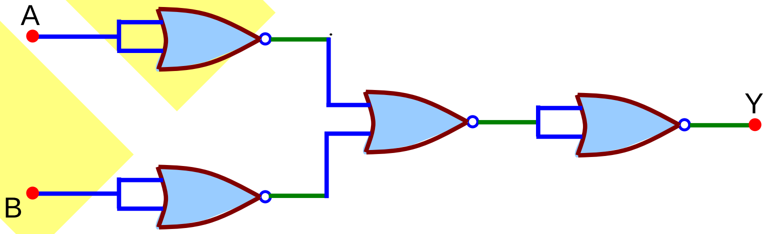
Q9. Which level of the single ionized carbon has the same energy as the ground state energy of hydrogen atom?
(A) 6 (B) 8
(C) 4 (D) 1

Q10. A modern grand - prix racing car of mass m is traveling on a flat track in a circular arc of radius R with a speed υ. If the coefficient of static between the tyres and the track is μ_s, then the magnitude of negative lift F_L acting downwards on the car is :
(Assume forces on the four tyres are identical and g = acceleration due to gravity)



(A) $m\left(\frac{\upsilon^2}{\mu_s R} - g\right)$ (B) $-m\left(g + \frac{\upsilon^2}{\mu_s R}\right)$
(C) $m\left(g - \frac{\upsilon^2}{\mu_s R}\right)$ (D) $m\left(\frac{\upsilon^2}{\mu_s R} + g\right)$

Q11. The output of the given combination gates represents:
(A) NOR Gate (B) NAND Gate
(C) XOR Gate (D) AND Gate



Q12. A polyatomic ideal gas has 24 vibrational modes. What is the value of γ ?
(A) 1.30 (B) 1.03
(C) 10.3 (D) 1.37

Q13. A solenoid of 1000 turns per meter has a core with relative permeability 500. Insulated windings of the solenoid carry an electric current of 5 A. The magnetic flux density produced by the solenoid is: (permeability of free space $= 4\pi \times 10^{-7} H/m$)

(A) $2 \times 10^{-3} \pi T$
(B) πT
(C) $10^{-4} \pi T$
(D) $\frac{\pi}{5} T$

Q14. A Carnot's engine working between 400 K and 800 K has a work output of 1200J per cycle. The amount of heat energy supplied to the engine from the source in each cycle is:

(A) 3200 J
(B) 2400 J
(C) 1800 J
(D) 1600 J

Q15. When two soap bubbles of radii a and b (b > a) coalesce, the radius of curvature of common surface is:

(A) $\frac{ab}{a+b}$
(B) $\frac{ab}{b-a}$
(C) $\frac{b-a}{ab}$
(D) $\frac{a+b}{ab}$

Q16. For what value of displacement the kinetic energy and potential energy of a simple harmonic oscillation become equal?

(A) x = 0
(B) $x = \pm A$
(C) $x = \pm \frac{A}{\sqrt{2}}$
(D) $x = \frac{A}{2}$

Q17. An AC current is given by $I = I_1 \sin \omega t + I_2 \cos \omega t$. A hot wire ammeter will give a reading:

(A) $\frac{I_1 + I_2}{2\sqrt{2}}$
(B) $\sqrt{\frac{I_1^2 + I_2^2}{2}}$.
(C) $\frac{I_1 + I_2}{\sqrt{2}}$
(D) $\sqrt{\frac{I_1^2 - I_2^2}{2}}$

Q18. Two ideal polyatomic gages at temperatures T_1 and T_2 are mixed so that there is no loss of energy. If F_1 and F_2, m_1 and m_2, n_1 and n_2 be the degrees of freedom, masses, number of molecules of the first and second gas respectively, the temperature of mixture of these two gases is:

(A) $\frac{n_1T_1 + n_2T_2}{n_1 + n_2}$
(B) $\frac{n_1F_1T_1 + n_2F_2T_2}{n_1F_1 + n_2F_2}$
(C) $\frac{n_1F_1T_1 + n_2F_2T_2}{F_1 + F_2}$
(D) $\frac{n_1F_1T_1 + n_2F_2T_2}{n_1 + n_2}$

Q19. Two identical metal of thermal conductivities K_1 and K_2 respectively are connected in series. The effective thermal conductivity of the combination is:

(A) $\frac{K_1 + K_2}{K_1K_2}$
(B) $\frac{K_1 + K_2}{2K_1K_2}$
(C) $\frac{2K_1K_2}{K_1 + K_2}$
(D) $\frac{K_1K_2}{K_1 + K_2}$

Q20. If an electron is moving in the n[th] orbit of the hydrogen atom, then its velocity (υ_n) for the n[th] orbit is given as :

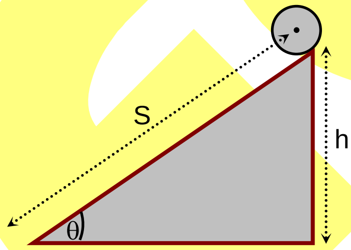
(A) $\upsilon_n \alpha \frac{1}{n}$ (B) $\upsilon_n \alpha n^2$

(C) $\upsilon_n \alpha n$ (D) $\upsilon_n \alpha \frac{1}{n^2}$

Section-B

Q1. A parallel plate capacitor whose capacitance C is 14 pF is charged by a battery to a potential difference $V = 12V$ between its plates. The charging battery is now disconnected and a porcelain plate with $k = 7$ is inserted between the plates, then the plate would oscillate back and forth between the plates with a constant mechanical energy of ………pJ. (Assume no friction)

Q2. The following bodies,
(1) a ring
(2) a disc
(3) a solid cylinder
(4) a solid sphere
of same mass 'm' and radius 'R' are allowed to roll down without slipping simultaneously from the top of the inclined plane. The body which will reach first at the bottom of the inclined plane is ----------.
[Mark the body as per their respective numbering given in the question]



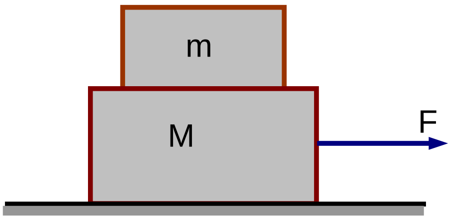
Q3. The angular speed of truck wheel is increased from 900 rpm to 2460 rpm in 26 seconds. The number of revolutions by the truck engine during this time is --------. (Assuming the acceleration to be uniform) .

Q4. For VHF signal broadcasting…….. km^2 of maximum service area will be covered by an antenna tower of height 30m, if the receiving antenna is placed at ground. Let radius of the earth be 6400km. (Round off to the Nearest Integer) (Take π as 3.14)

Q5. The radius in kilometre to which the radius of earth (R = 6400 km) to be compressed so that the escape velocity is increased 10 times is ------------.

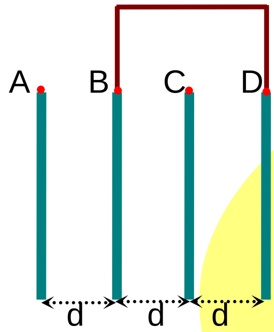
Q6. If 2.5×10^{-6} N average force is exerted by a light wave on a non- reflecting surface of 30 cm^2 area during 40 minutes of time span, the energy flux of light just before it falls on the surface is ---------- W / cm^2. (Round off to the Nearest Integer)
(Assume complete absorption and normal incidence conditions are there)

Q7. Two blocks $(m = 0.5 kg$ and $M = 4.5 kg)$ are arranged on a horizontal frictionless table as shown in figure. The coefficient of static friction between the two blocks is $\frac{3}{7}$. Then the maximum horizontal force that can be applied on the block so that blocks move together is ---------N. (Round off to the Nearest Integer) [Take g as 9.8 ms^{-2}]



Q8. The equivalent resistance of series combination of two resistors is 's'. When they are connected in parallel, the equivalent resistance is 'p'. If s = np, then the minimum value for n is ----------. (Round off to the Nearest Integer)

Q9. Four identical rectangular plates with length, $\ell = 2\,\text{cm}$ and breadth, $b = \frac{3}{2}\,\text{cm}$ are arranged as shown in figure. The equivalent capacitance between A and C is $\frac{x\varepsilon_0}{d}$. The value of x is ----------. (Round off to the nearest integer)



Q10. Consider two identical springs each of spring constant k and negligible mass compared to the mass M as shown. Fig. 1 shows one of them and Fig. 2 shows their series combination. The ratios of time period of oscillation of the two SHM is $\frac{T_b}{T_a} = \sqrt{x}$, where value of x is -----------. (Round off to the Nearest Integer)

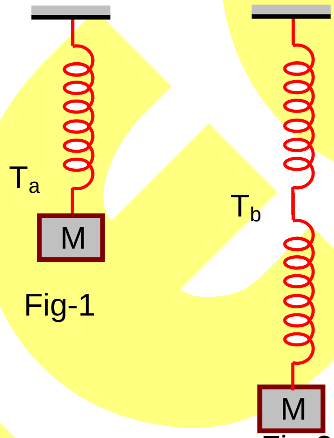


Fig-1

Fig-2

CHEMISTRY

SECTION A

Q1. Reducing smog is a mixture of :

(A) Smoke, fog and $CH_2 = CH - CHO$

(B) Smoke, fog and O_3

(C) Smoke, fog and N_2O_3

(D) Smoke, fog and SO_2

Q2. Which of the following compound CANNOT act as a Lewis base?

(A) ClF_3

(B) PCl_5

(C) NF_3

(D) SF_4

Q3. Given below are two statements:

Statements I : Potassium permanganate on heating at 573 K forms potassium manganate.

Statement II : Both potassium permanganate and potassium manganate are tetrahedral and paramagnetic in nature.

In the light of the above statements, choose the most appropriate answer from the options given below:

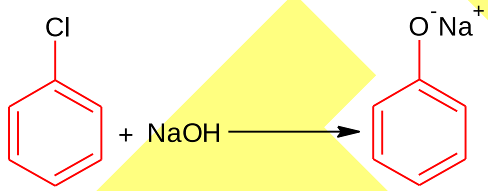
(A) Statement **I** is true but statement **II** is false

(B) Both statement **I** and statement **II** are true

(C) Both statement **I** and statement **II** are false

(D) Statement **I** is false but statement **II** is true

Q4.



The above reaction requires which of the following reaction conditions?

(A) 573 K, 300 atm

(B) 623 K, Cu, 300 atm

(C) 573 K, Cu, 300 atm

(D) 623 K, 300 atm

Q5. A central atom in a molecule has two lone pairs of electrons and forms three single bonds. The shape of this molecule is:

(A) Trigonal pyramidal

(B) Planar triangular

(C) T- shaped

(D) See-saw

Q6. Given below are two statements:

Statement I : Retardation factor (R_f) can be measured in meter / centimeter.

Statement II: R_f value of a compound remains constant in all solvents.

Chose the most appropriate answer from the options given below:

(A) Statement **I** is true but statement **II** is false

(B) Both statement **I** and statement **II** are false

(C) Both statement **I** and statement **II** are true

(D) Statement **I** is false but statement **II** is true

Q7. Mesityl oxide is a common name of:

(A) 2,4- Dimethyl pentan-3-one

(B) 3-Methyl cyclohexane carbaldehyde

(C) 2-Methyl cyclohexanone

(D) 4-Methyl pent-3-en-2-one

|

Q8. The absolute value of the electron gain enthalpy of halogens satisfies :

(A) F > Cl > Br > I

(B) Cl > F > Br > I

(C) I > Br > Cl > F

(D) Cl > Br > F > I

Q9. A colloidal system consisting of a gas dispersed in a solid is called a/an:

(A) gel
(B) aerosol
(C) solid sol
(D) foam

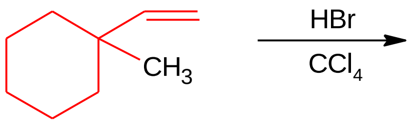
Q10. The **INCORRECT** statement(s) about heavy water is (are)

(A) used as a moderator in nuclear reactor
(B) obtained as by-product in fertilizer industry
(C) used for the study of reaction mechanism
(D) has a higher dielectric constant than water

Choose the correct answer from the options given below:

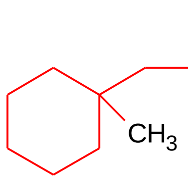
(A) (A) (D) only
(B) (B) (C) only
(C) (B) only
(D) (B) and (D) only

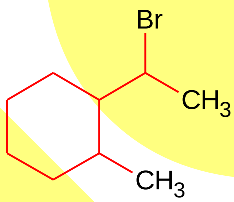
Q11.

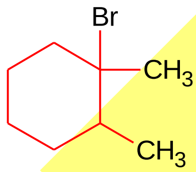


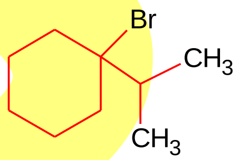
A
(Major Product)

Product " A" in the above chemical reaction is:

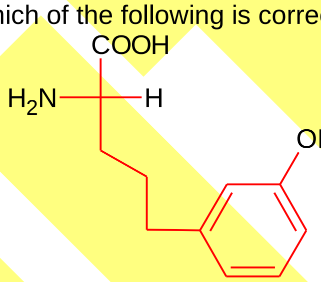
(A)


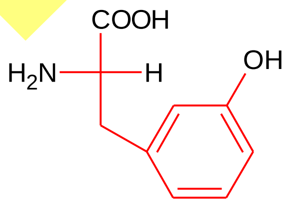
(B)


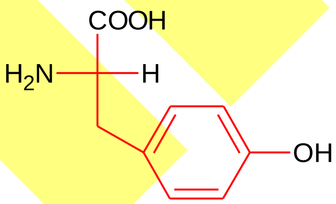
(C)


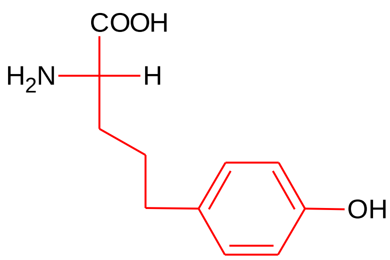
(D)


Q12. Which of the following is correct structure of tyrosine?

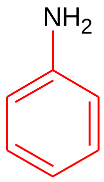
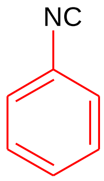
(A)


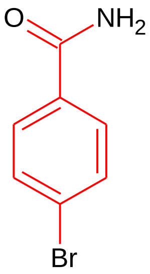
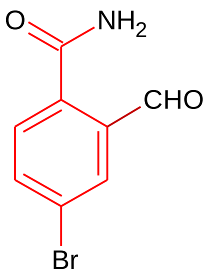
(B)


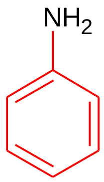
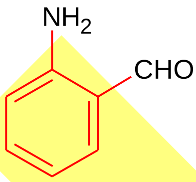
(C)


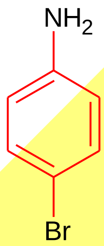
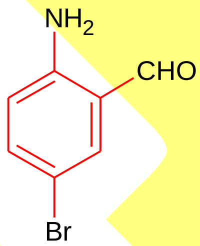
(D)


Q13. Hoffmann bromomide degradation of benzamide gives product A, which upon heating with $CHCl_3$ and NaOH gives product B.
The structures of A and B are:

(A) A –  B – 

(B) A –  B – 

(C) A –  B – 

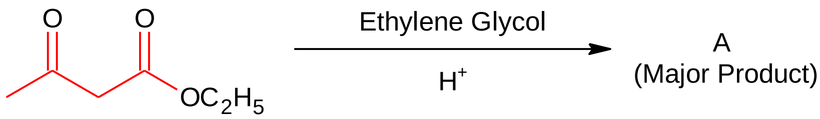
(D) A –  B – 

Q14. The correct order of conductivity of ions in water is:
(A) $Rb^+ > Na^+ > K^+ > Li^+$
(B) $Cs^+ > Rb^+ > K^+ > Na^+$
(C) $K^+ > Na^+ > Cs^+ > Rb^+$
(D) $Na^+ > K^+ > Rb^+ > Cs^+$

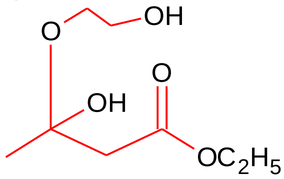
Q15. What is the spin- only magnetic moment value (BM) of divalent metal ion with atomic number 25, in it's aqueous solution?
(A) 5.26
(B) zero
(C) 5.92
(D) 5.0

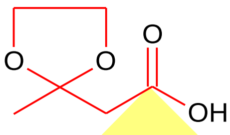
Q16. With respect to drug-enzyme interaction, identify the wrong statement.
(A) Competitive inhibitor binds to the enzyme's active site
(B) Allosteric inhibitor changes the enzyme's active site
(C) Non- Competitive inhibitor binds to the allosteric site
(D) Allosteric inhibitor competes with the enzyme's active site

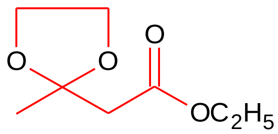
Q17.

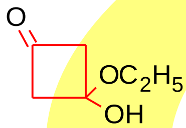


The product "A" in the above reaction is:

(A)


(B)


(C)


(D)


Q18. Which of the following reaction is an example of ammonolysis?

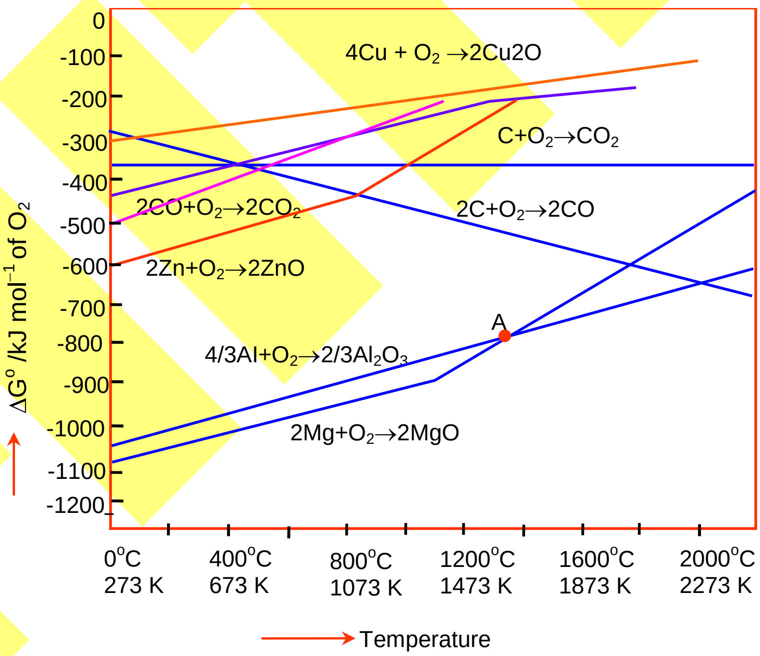
(A) $C_6H_5CH_2CN \xrightarrow{[H]} C_6H_5CH_2CH_2NH_2$

(B) $C_6H_5COCl + C_6H_5NH_2 \longrightarrow C_6H_5CONHC_6H_5$

(C) $C_6H_5NH_2 \xrightarrow{HCl} C_6H_5\overset{+}{N}H_3\overset{-}{Cl}$

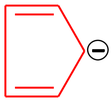
(D) $C_6H_5CH_2Cl + NH_3 \longrightarrow C_6H_5CH_2NH_2$

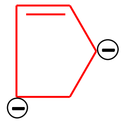
Q19. The point of intersection and sudden increase in the slope, in the diagram given below, respectively, indicates:

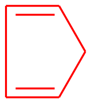


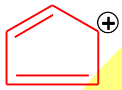
(A) $\Delta G = 0$ and reduction of the metal oxide
(B) $\Delta G = 0$ and melting or boiling point of the metal oxide
(C) $\Delta G > 0$ and decomposition of the metal oxide
(D) $\Delta G < 0$ and decomposition of the metal oxide

Q20. Which of the following is an aromatic compound ?

(A) 

(B) 

(C) 

(D) 

SECTION B

Q1. The oxygen dissolved in water exerts a partial pressure of 20 kPa in the vapour above water. The molar solubility of oxygen in water is ____________ $\times 10^{-5}$ mol dm^{-3}. (Round off to the nearest integer).
[Given : Henry's law constant = $K_H = 8.0 \times 10^4$ kPa for O_2.
Density of water with dissolved oxygen = 1.0kg dm^{-3}]

Q2. For a certain first order reaction 32% of the reactant is left after 570s. The rate constant of this reaction is __________ $\times 10^{-3}$ s^{-1}. (Round off to the nearest integer).
[Given : $\log_{10}2 = 0.301$, ln10 = 2.303]

Q3. The pressure exerted by a non- reactive gaseous mixture of 6.4 g of methane and 8.8 g of carbon dioxide in a 10 L vessel at 27°C is_____________ kPa.
(Round off to the nearest integer).
[Assume gases are ideal, R = 8.314 J mol^{-1} K^{-1}
Atomic masses : C : 12.0 u, H: 1.0 u, O: 16.0 u]

Q4. 0.01 moles of a weak acid HA (Ka = 2.0×10^{-6}) is dissolved in 1.0 L of 0.1 M HCl solution .The degree of dissociation of HA is ________________ $\times 10^{-5}$.
(Round off to the nearest integer).
[Neglect volume change on adding HA.
Assume degree of dissociation <<1]

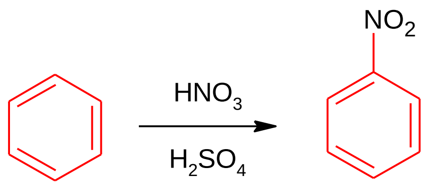
Q5. The reaction of white phosphorous on boiling with alkali in inert atmosphere resulted in the formation of product 'A'. The reaction of 1 mol of 'A' with excess of $AgNO_3$ in aqueous medium gives ______________ mol(s) of Ag.
(Round off to the nearest integer).

Q6. 15 mL of aqueous solution of Fe^{2+} in acidic medium completely reacted with 20 ml. of 0.03 M aqueous $Cr_2O_7^{2-}$. The molarity of the Fe^{2+} solution is _________ $\times 10^{-2}$ M
(Round off to the nearest integer).

Q7. A certain orbital has n= 4 and m_ℓ = –3. The number of radial nodes in this orbital is __________. (Round off to the nearest integer).

Q8. The mole fraction of a solute in a 100 molal aqueous solution is __________ $\times 10^{-2}$.
(Round off to the nearest integer).
[Given : Atomic masses : H : 1.0u, O : 16.0u]

Q9.



In the above reaction, 3.9 g of benzene on nitration gives 4.92 g of nitrobenzene. The percentage yield of nitrobenzene in the above reaction is_____________ %
(Round off to the nearest integer).
(Given atomic mass : C: 12.0u, H:1.0u, O:16.0u, N:14.0u)

Q10. The standard enthalpies of formation of Al_2O_3 and CaO are –1675 kJ mol^{-1} and –635 kJ mol^{-1} respectively.
For the reaction
$3CaO + 2Al \rightarrow 3Ca + Al_2O_3$ the standard reaction enthalpy $\Delta_r H^0 =$ __________kJ.
(Round off to the nearest integer).

MATHEMATICS
SECTION A

Q1. Two dices are rolled. If both dices have six faces numbered 1,2,3,5,7 and 11, then the probability that the sum of the numbers on the top faces is less than or equal to 8 is :

(A) $\frac{1}{2}$ (B) $\frac{17}{36}$
(C) $\frac{4}{9}$ (D) $\frac{5}{12}$

Q2. Which of the following is true for $y(x)$ that satisfies the differential equation $\frac{dy}{dx} = xy - 1 + x - y;\ y(0) = 0$:

(A) $y(1) = e^{\frac{1}{2}} - 1$ (B) $y(1) = 1$
(C) $y(1) = e^{\frac{1}{2}} - e^{-\frac{1}{2}}$ (D) $y(1) = e^{-\frac{1}{2}} - 1$

Q3. The system of equation $kx + y + z = 1$, $x + ky + z = k$ and $x + y + zk = k^2$ has no solution if k is equal to :

(A) 0 (B) 1
(C) –2 (D) –1

Q4. Team 'A' consists of 7 boys and n girls and Team 'B' has 4 boys and 6 girls. If a total of 52 single matches can be arranged between these two teams when a boy plays against a boy and a girl plays against a girl, then n is equal to :

(A) 2 (B) 5
(C) 6 (D) 4

Q5. The inverse of $y = 5^{\log x}$ is :

(A) $x = y^{\log 5}$ (B) $x = y^{\frac{1}{\log 5}}$
(C) $x = 5^{\log y}$ (D) $x = 5^{\frac{1}{\log y}}$

Q6. If $\cot^{-1}(\alpha) = \cot^{-1}2 + \cot^{-1}8 + \cot^{-1}18 + \cot^{-1}32 + ...$ upto 100 terms, then α is :

(A) 1.03 (B) 1.02
(C) 1.00 (D) 1.01

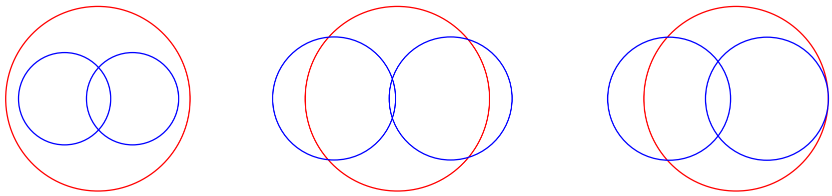
Q7. The equation of the plane which contains the y-axis and passes through the point $(1, 2, 3)$ is :

(A) $x + 3z = 10$ (B) $3x + z = 6$
(C) $3x - z = 0$ (D) $x + 3z = 0$

Q8. If the Boolean expression $(p \Rightarrow q) \Leftrightarrow (q * (\sim p))$ is a tautology, then the Boolean expression $p * (\sim q)$ is equivalent to :

(A) $\sim q \Rightarrow p$ (B) $p \Rightarrow \sim q$
(C) $p \Rightarrow q$ (D) $q \Rightarrow p$

Q9. In a school, there are three types of games to be played. Some of the students play two types of games, but none play all the three games. Which Venn diagrams can justify the above statement ?



(A) Q and R　　(B) None of these
(C) P and R　　(D) P and Q

Q10. In a triangle PQR, the co-ordinates of the points P and Q are $(-2,4)$ and $(4,-2)$ respectively. If the equation of the perpendicular bisector of PR is $2x-y+2=0$, then the centre of the circumcircle of the ΔPQR is :

(A) $(0,2)$　　(B) $(-2,-2)$
(C) $(-1,0)$　　(D) $(1,4)$

Q11. Choose the incorrect statement about the two circles whose equations are given below :

$x^2+y^2-10x-10y+41=0$ and $x^2+y^2-16x-10y+80=0$

(A) Distance between two centres is the average of radii of both the circles.
(B) Both circles' centres lie inside region of one another.
(C) Circles have two intersection points.
(D) Both circles pass through the centre of each other.

Q12. If the fourth term in the expansion of $\left(x+x^{\log_2 x}\right)^7$ is 4480, then the value of x where $x\in N$ is equal to :

(A) 1　　(B) 2
(C) 3　　(D) 4

Q13. The value of $4+\cfrac{1}{5+\cfrac{1}{4+\cfrac{1}{5+\cfrac{1}{4+....\infty}}}}$ is :

(A) $2+\frac{2}{5}\sqrt{30}$　　(B) $4+\frac{4}{\sqrt{5}}\sqrt{30}$
(C) $5+\frac{2}{5}\sqrt{30}$　　(D) $2+\frac{4}{\sqrt{5}}\sqrt{30}$

Q14. The value of $\lim\limits_{x\to 0^+}\frac{\cos^{-1}\left(x-[x]^2\right).\sin^{-1}\left(x-[x]^2\right)}{x-x^3}$, where [x] denotes the greatest integer $\le x$ is :

(A) $\frac{\pi}{4}$　　(B) π
(C) $\frac{\pi}{2}$　　(D) 0

Q15. The line $2x-y+1=0$ is a tangent to the circle at the point $(2,5)$ and the centre of the circle lies on $x-2y=4$. Then, the radius of the circle is :

(A) $5\sqrt{3}$　　(B) $4\sqrt{5}$

(C) $5\sqrt{4}$ (D) $3\sqrt{5}$

Q16. Let $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{b} = 7\hat{i} + \hat{j} - 6\hat{k}$.
If $\vec{r} \times \vec{a} = \vec{r} \times \vec{b}$, $\vec{r}.(\hat{i} + 2\hat{j} + \hat{k}) = -3$, then $\vec{r}.(2\hat{i} - 3\hat{j} + \hat{k})$ is equal to :

(A) 10 (B) 12
(C) 8 (D) 13

Q17. Which of the following statements is incorrect for the function $g(\alpha)$ for $\alpha \in R$ such that

$$g(\alpha) = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin^\alpha x}{\cos^\alpha x + \sin^\alpha x} dx$$

(A) $g(\alpha)$ is an even function

(B) $g(\alpha)$ has an inflection point at $\alpha = -\frac{1}{2}$

(C) $g(\alpha)$ is a strictly increasing function

(D) $g(\alpha)$ is a strictly decreasing function

Q18. The sum of possible values of x for $\tan^{-1}(x+1) + \cot^{-1}\left(\frac{1}{x-1}\right) = \tan^{-1}\left(\frac{8}{31}\right)$ is :

(A) $-\frac{32}{4}$ (B) $-\frac{30}{4}$

(C) $-\frac{33}{4}$ (D) $-\frac{31}{4}$

Q19. The area of the triangle vertices $A(z), B(iz)$ and $C(z+iz)$ is :

(A) $\frac{1}{2}|z|^2$ (B) $\frac{1}{2}|z+iz|^2$

(C) 1 (D) $\frac{1}{2}$

Q20. If $A = \begin{pmatrix} 0 & \sin\alpha \\ \sin\alpha & 0 \end{pmatrix}$ and $\det\left(A^2 - \frac{1}{2}I\right) = 0$, then a possible value of α is :

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$

(C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$

SECTION B

Q1. If $(2021)^{3762}$ is divided by 17, then the remainder is ……..

Q2. If $f(x) = \sin\left(\cos^{-1}\left(\frac{1-2^{2x}}{1+2^{2x}}\right)\right)$ and its first derivative with respect to x is $-\frac{b}{a}\log_e 2$ when $x = 1$, where a and b are integers, then the minimum value of $|a^2 - b^2|$ is……..

Q3. Let there be three independent events E_1, E_2 and E_3. The probability that only E_1 occurs is α, only E_2 occurs is β and only E_3 occurs is γ. Let 'p' denote the probability of none of events occurs that satisfies the equations $(\alpha-2\beta)p=\alpha\beta$ and $(\beta-3\gamma)p=2\beta\gamma$. All the given probabilities are assumed to lie in the interval $(0,1)$.

Then, $\frac{\text{Probability of occurrence of } E_1}{\text{Probability of occurrence of } E_3}$ is equal to……

Q4. If $A=\begin{bmatrix}2 & 3\\ 0 & -1\end{bmatrix}$, then the value of $\det(A^4)+\det\left(A^{10}-(\text{Adj}(2A))^{10}\right)$ is equal to…..

Q5. The minimum distance between any two points P_1 and P_2 while considering point P_1 on one circle and point P_2 on the other circle for the given circles' equations

$x^2+y^2-10x-10y+41=0$

$x^2+y^2-24x-10y+160=0$ is………..

Q6. If $\vec{a}=\alpha\hat{i}+\beta\hat{j}+3\hat{k}$,

$\vec{b}=-\beta\hat{i}-\alpha\hat{j}-\hat{k}$ and

$\vec{c}=\hat{i}-2\hat{j}-\hat{k}$

such that $\vec{a}.\vec{b}=1$ and $\vec{b}.\vec{c}=-3$, then $\frac{1}{3}\left((\vec{a}\times\vec{b}).\vec{c}\right)$ is equal to……

Q7. The maximum value of z in the following equation $z=6xy+y^2$, where $3x+4y\le 100$ and $4x+3y\le 75$ for $x\ge 0$ and $y\ge 0$ is………

Q8. If the equation of the plane passing through the line of intersection of the planes $2x-7y+4z-3=0$, $3x-5y+4z+11=0$ and the point $(-2,1,3)$ is $ax+by+cz-7=0$, then the value of $2a+b+c-7$ is…….

Q9. If $[.]$ represents the greatest integer function, then the value of

$\left|\int_0^{\sqrt{\frac{\pi}{2}}}\left[[x^2]-\cos x\right]dx\right|$ is……..

Q10. If the function $f(x)=\frac{\cos(\sin x)-\cos x}{x^4}$ is continuous at each point in its domain and $f(0)=\frac{1}{k}$, then k is ……..

ANSWER: Paper-Jee-Main-16-03-2021-Morning Shift

PHYSICS	CHEMISTRY	MATHEMATICS
Section-A	**SECTION – A**	**SECTION – A**
Ans1. D	**Ans1.** D	**Ans1.** **B**
Ans2. A	**Ans2.** B	**Ans2.** **D**
Ans3. C	**Ans3.** A	**Ans3.** **C**
Ans4. C	**Ans4.** D	**Ans4.** **D**
Ans5. B	**Ans5.** C	**Ans5.** **A,C**
Ans6. D	**Ans6.** B	**Ans6.** **D**
Ans7. D	**Ans7.** D	**Ans7.** **C**
Ans8. A	**Ans8.** B	**Ans8.** **D**
Ans9. A	**Ans9.** C	**Ans9.** **B**
Ans10. A	**Ans10.** A	**Ans10.** **B**
Ans11. B	**Ans11.** D	**Ans11.** **B**
Ans12. B	**Ans12.** C	**Ans12.** **B**
Ans13. B	**Ans13.** A	**Ans13.** **A**
Ans14. B	**Ans14.** B	**Ans14.** **C**
Ans15. B	**Ans15.** C	**Ans15.** **D**
Ans16. C	**Ans16** D	**Ans16.** **B**
Ans17. B	**Ans17.** C	**Ans17.** **B,C,D(Bonus)**
Ans18. B	**Ans18.** D	**Ans18.** **A**
Ans19. C	**Ans19.** B	**Ans19.** **A**
Ans20. A	**Ans20.** A	**Ans20.** **C**
Section-B	**SECTION – B**	**SECTION – B**
Ans1. 864	**Ans1.** 25	**Ans1.** **4**
Ans2. 4	**Ans2.** 2	**Ans2.** **481**
Ans3. 728	**Ans3.** 150	**Ans3.** **6**
Ans4. 1206	**Ans4.** 2	**Ans4.** **16**
Ans5. 64	**Ans5.** 4	**Ans5.** **1**
Ans6. 25	**Ans6.** 24	**Ans6.** **2**
Ans7. 21	**Ans7.** 0	**Ans7.** **904.017**
Ans8. 4	**Ans8.** 64	**Ans8.** **4**
Ans9. 2	**Ans9.** 80	**Ans9.** **1**
Ans10. 2	**Ans10.** 230	**Ans10.** **6**

SOLUTION: Paper-Jee-Main-17-03-0-2021-Morning Shift
PHYSICS
SECTION – A

Sol1. $i = neAV_d$

$$\Rightarrow n = \frac{i}{eAV_d} = \frac{10}{1.6\times10^{-19}\times5\times10^{-6}\times2\times10^{-3}} = 625\times10^{25}$$

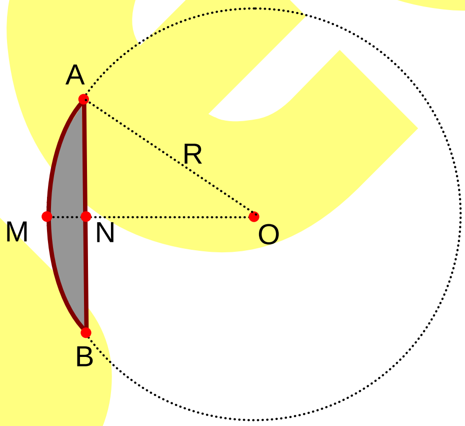
Sol2. $\lambda_e = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$ and $\lambda_P = \frac{hc}{E}$

$$\Rightarrow \frac{\lambda_e^2}{\lambda_P^2} = \frac{\frac{h^2}{2Em}}{\frac{h^2c^2}{E^2}} = \frac{E}{2mc^2} \Rightarrow \frac{\lambda_e}{\lambda_P} = \frac{1}{c}\left(\frac{E}{2m}\right)^{\frac{1}{2}}$$

Sol3. $\mu = \frac{c}{v_m} = \frac{3\times10^8}{2\times10^8} = 1.5$

Here MN = 3mm = 0.3 cm ,
AN = BN =6 cm

With the help of right angled triangle ANO ,we can write



$$(AO)^2 = (AN)^2 + (ON)^2$$
$$\Rightarrow R^2 = (3)^2 + (R-0.3)^2$$
$$\Rightarrow R^2 = 9 + R^2 + 0.09 - 0.6R$$
$$\Rightarrow 0.6R = 9.09 \Rightarrow R = \frac{9.09}{0.6} = 15.15 \text{ cm} \approx 15 \text{ cm}$$

With the help of Lens Maker's formula, we can write

$$\frac{1}{f} = \left(\frac{\mu-\mu_m}{\mu_m}\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) = \left(\frac{1.5-1}{1}\right)\left(\frac{1}{15}-\frac{1}{\infty}\right) = \frac{1}{30} \Rightarrow f = 30 \text{ cm}$$

Sol4. Positive zero error $= 0.02$ cm
Reading $= 8.5 + 6\times0.01 = 8.56$ cm
Actual reading $= 8.56 - 0.02 = 8.54$ cm

Sol5. **For Torque about point-O**

$\vec{r}_{PO} = 5\hat{i} + 5\sqrt{3}\hat{j}$

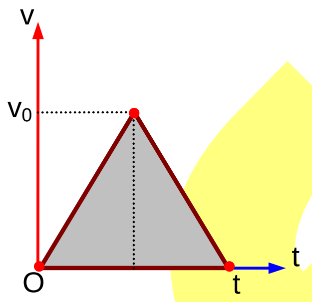
$$\vec{\tau}_O = \vec{r}_{PO}\times\vec{F} = \left(5\hat{i}+5\sqrt{3}\hat{j}\right)\times\left(4\hat{i}-3\hat{j}\right) = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 5 & 5\sqrt{3} & 0\\ 4 & -3 & 0\end{vmatrix} = \left(-15-20\sqrt{3}\right)\hat{k}$$

For Torque about point-Q

$\vec{r}_{PQ} = -5\hat{i} + 5\sqrt{3}\hat{j}$

$$\vec{\tau}_Q = \vec{r}_{PQ}\times\vec{F} = \left(-5\hat{i}+5\sqrt{3}\hat{j}\right)\times\left(4\hat{i}-3\hat{j}\right) = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -5 & 5\sqrt{3} & 0\\ 4 & -3 & 0\end{vmatrix} = \left(+15-20\sqrt{3}\right)\hat{k}$$

Sol6. $t_1 + t_2 = t$



$\frac{v_0}{t_1} = \alpha$, and $\frac{v_0}{t_2} = \beta$

$\Rightarrow \frac{v_0}{\alpha} + \frac{v_0}{\beta} = t_1 + t_2 = t \Rightarrow v_0 = \frac{t}{\frac{1}{\alpha}+\frac{1}{\beta}} = \frac{\alpha\beta t}{\alpha+\beta}$

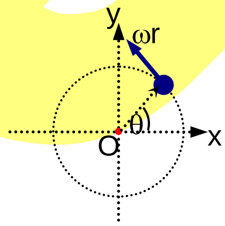
Distance traveled = Area under speed –time graph $= \frac{1}{2} \times t \times \frac{\alpha\beta t}{\alpha+\beta} = \frac{\alpha\beta}{2(\alpha+\beta)} t^2$

Sol7. $K_i = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v}{r}\right)^2 = \frac{7}{10}mv^2 = 140\,J$

$K_f = 0.05K_i = \frac{7}{10}mv_f^2 \Rightarrow 7 = \frac{7}{10} \times \frac{1}{2} \times v_f^2 \Rightarrow v_f^2 = 20$

$\Rightarrow v_f = \sqrt{20} \approx 4.47\ ms^{-1}$

Sol8.



$\vec{L}_A = Mvr\,\hat{k} = M\omega r^2\,\hat{k}$

$\vec{L}_B = \left(r\cos\theta\,\hat{i} + \sin\theta\,\hat{j} - \ell\cos\alpha\,\hat{k}\right) \times (M\omega r)\left(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j}\right)$

Where $\theta = \omega t$

Magnitude $\vec{L}_B$ is constant and direction changes with time

Sol9. **As we know that**

$E_{Ground}(H-atom) = -13.6eV$

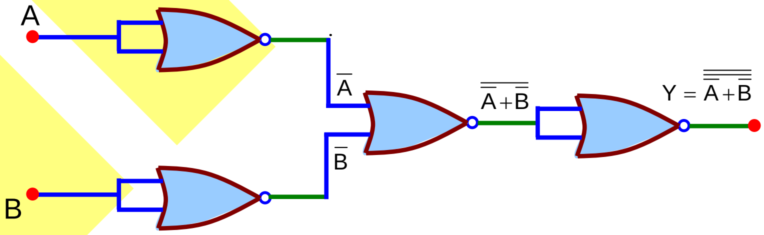
According to Question, we can write

$E_{Carbon} = E_{Ground}(H-atom) \Rightarrow -13.6eV = -13.6 \times \frac{6^2}{n^2} \Rightarrow n = 6$

Sol10. $N = mg - F_L$

$f_S = \frac{mv^2}{R} \le \mu_S N = \mu_S(mg - F_L) \Rightarrow |-F_L| = m\left(\frac{v^2}{\mu_S R} - g\right)$

Sol11.



$Y = \overline{\overline{\overline{A}+\overline{B}}} = \overline{A}+\overline{B} = \overline{AB} \Rightarrow$ NAND Gate

Sol12. Since each vibrational mode has two degree of freedom , so

$f = f_{Translational} + f_{Rotational} + f_{Vibrational} = 3 + 3 + 48 = 54$

$\gamma = 1 + \frac{2}{f} = 1 + \frac{2}{54} = \frac{28}{27} \approx 1.03$

Sol13. $B = \mu ni = 500 \times 4\pi \times 10^{-7} \times 1000 \times 5 = \pi$ Tesla

Sol14. $\eta = 1 - \frac{T_C}{T_H} = 1 - \frac{400}{800} = \frac{1}{2} \Rightarrow \eta = \frac{W}{Q} = \frac{1}{2} \Rightarrow Q = 2W = 2400\,J$.

Sol15. $P_1 - P_0 = \frac{4T}{a}$

$$P_2 - P_0 = \frac{4T}{b}$$

$$\Rightarrow P_1 - P_2 = 4T\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$P_1 - P_2 = \frac{4T}{r} = 4T\left(\frac{1}{a} - \frac{1}{b}\right) \Rightarrow r = \frac{ab}{b-a}$$

Sol16. **As we know that**

$$K = \frac{1}{2}mA^2\omega^2\cos^2\omega t = \frac{1}{2}m\omega^2\left(A^2 - x^2\right) \text{ and } U = \frac{1}{2}mA^2\omega^2\sin^2\omega t = \frac{1}{2}m\omega^2x^2$$

According to Question, we can write

$$K = U \Rightarrow \frac{1}{2}m\omega^2\left(A^2 - x^2\right) = \frac{1}{2}m\omega^2x^2 \Rightarrow A^2 - x^2 = x^2 \Rightarrow x = \pm\frac{A}{\sqrt{2}}$$

Sol17. A hot wire ammeter reads rms value of current, so

$$I = I_1\sin\omega t + I_2\cos\omega t = \sqrt{I_1^2 + I_2^2}\sin(\omega t + \phi) = I_0\sin(\omega t + \phi) \quad \text{, where } I_0 = \sqrt{I_1^2 + I_2^2}$$

$$\Rightarrow I_{rms} = \frac{I_0}{\sqrt{2}} = \sqrt{\frac{I_1^2 + I_2^2}{2}}$$

Sol18. $U_1 = \left(\frac{n_1}{N_A}\right)\left(\frac{F_1R}{2}\right)T_1$ and $U_2 = \left(\frac{n_2}{N_A}\right)\left(\frac{F_2R}{2}\right)T_2$

$$U = U_1 + U_2 \Rightarrow \frac{(n_1 + n_2)}{N_A}\frac{(FR)}{2}T = \left(\frac{n_1}{N_A}\right)\left(\frac{F_1R}{2}\right)T_1 + \left(\frac{n_2}{N_A}\right)\left(\frac{F_2R}{2}\right)T_2 \quad \ldots(1)$$

$$F = \frac{n_1F_1 + n_2F_2}{n_1 + n_2} \quad \ldots\ldots\ldots\ldots(2)$$

With the help of equations (1) and (2), we can write

$$\Rightarrow T = \frac{n_1F_1T_1 + n_2F_2T_2}{n_1F_1 + n_2F_2}$$

Sol19. $R_{eq} = R_1 + R_2 = \frac{\ell}{K_1A} + \frac{\ell}{K_2A} = \frac{\ell}{A}\left(\frac{1}{K_1} + \frac{1}{K_2}\right) = \frac{2\ell}{KA}$

$$\Rightarrow K = \frac{2K_1K_2}{K_1 + K_2}$$

Sol20. $v_n \,\alpha\, \frac{1}{n}$

Section-B

Sol1. $Q = CV \Rightarrow$ Charge on the Capacitor

$$U_{in} = \frac{Q^2}{2C} = \frac{CV^2}{2} = \frac{14 \times 10^{-12} \times 12 \times 12}{2} = 1008\text{ pJ}$$

When Battery is disconnected and a porcelain plate with $k = 7$ is inserted between the plates, the capacitance of system becomes $C' = kC$

$$U_f = \frac{Q^2}{2C'} = \frac{(CV)^2}{2kC} = \frac{CV^2}{2k} = \frac{14\times10^{-12}\times12\times12}{2\times7} = 144 \text{ pJ}$$

Mechanical energy available for oscillation $= U_{in} - U_f = 1008 - 144 = 864$ pJ

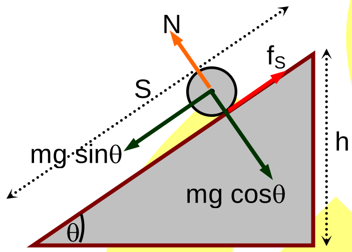
Sol2.

$mg - f_S = ma$

$f_S R = I\alpha = \frac{Ia}{R} \Rightarrow f_S = \frac{Ia}{R^2}$

$\Rightarrow a = \frac{mg}{m + \frac{I}{R^2}}$

$S = \frac{1}{2}at^2 \quad \Rightarrow t \ \alpha \ \frac{1}{\sqrt{a}}$



FBD of Body

Body	Moment of inertia(I)	a
Ring	$I = mR^2$	$\frac{g\sin\theta}{2} = 0.50 g\sin\theta$
Disc	$I = \frac{mR^2}{2}$	$\frac{2g\sin\theta}{3} = 0.67 g\sin\theta$
Solid Cylinder	$I = \frac{mR^2}{2}$	$\frac{2g\sin\theta}{3} = 0.67 g\sin\theta$
Solid Sphere	$I = \frac{2mR^2}{5}$	$\frac{5g\sin\theta}{7} = 0.71 g\sin\theta$

$\Rightarrow$ So solid sphere will came first.

Sol3. $\omega = \omega_0 + \alpha t \Rightarrow \alpha = \frac{\omega - \omega_0}{t}$

$$\omega^2 = \omega_0^2 + 2\alpha\theta \Rightarrow \theta = \frac{\omega^2 - \omega_0^2}{2\alpha} = \frac{\omega^2 - \omega_0^2}{2\left(\frac{\omega - \omega_0}{t}\right)} \Rightarrow \theta = \frac{(\omega + \omega_0)t}{2}$$

$$\Rightarrow \text{Number of revolution} = \frac{\theta}{2\pi} = \frac{(\omega + \omega_0)t}{2\pi\times2} = \frac{2\pi\times3360\times26}{2\pi\times2\times60} = 728$$

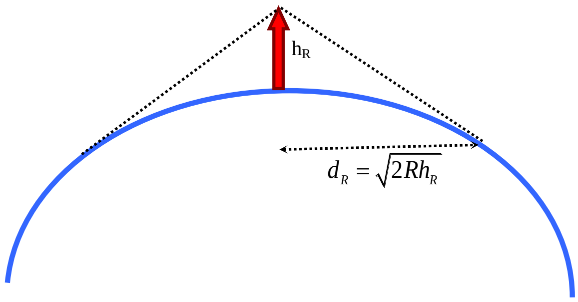
Sol4.

$$d_R = \sqrt{2Rh_R}$$

$$\text{Area} = \pi(d_R)^2 = \pi(2Rh_R)$$

$$= 3.14\times2\times6400\times\frac{30}{1000}$$

$$= 1205.76\,km \approx 1206\,km$$



Sol5. $v_e = \sqrt{\frac{2Gm}{R}}$

According to Question, we can write

$$10v_e = \sqrt{\frac{2Gm}{R'}} \Rightarrow R' = \frac{R}{100} = 64\text{Km}$$

Sol6. $F = \frac{\Delta P}{\Delta t} = \left(\frac{n}{\Delta t}\right)\frac{E}{c} \Rightarrow \frac{\left(\frac{n}{\Delta t}\right)E}{A} = \frac{Fc}{A} = \frac{2.5\times10^{-6}\times3\times10^{8}}{30} = 25\,W/cm^2$

Sol7. Combined acceleration $a = \frac{F}{M+m}$

$\Rightarrow f_S = ma = \frac{mF}{M+m} \le \mu mg$

$\Rightarrow F \le \mu(M+m)g \Rightarrow F \le 21\,N$

Sol8. $s = R_1 + R_2$ and $p = \frac{R_1R_2}{R_1+R_2}$

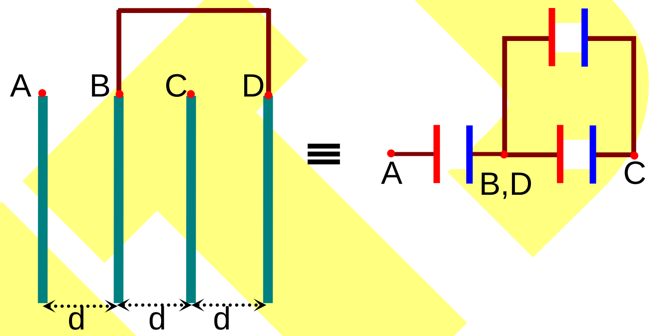
$s = np \Rightarrow R_1 + R_2 = n\left(\frac{R_1R_2}{R_1+R_2}\right)$

$\Rightarrow (R_1 - R_2)^2 + (4-n)R_1R_2 = 0 \;\Rightarrow n = 4 + \frac{(R_1-R_2)^2}{R_1R_2}$

Minimum value of $n = 4$

When $(R_1 - R_2)^2 = 0 \Rightarrow R_1 = R_2$

Sol9.



$C_{AC} = \frac{2\varepsilon_0 A}{3d} = \frac{2\varepsilon_0\times2\times10^{-2}\times\frac{3}{2}\times10^{-2}}{3d} = \frac{2\varepsilon_0}{d}\times\times10^{-4}\ F$

Sol10. For Figure-1:

$T_a = 2\pi\sqrt{\frac{m}{k}}$

For Figure-2:

$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} = \frac{2}{k} \Rightarrow T_b = 2\pi\sqrt{\frac{2m}{k}}$

$\Rightarrow \frac{T_b}{T_a} = \sqrt{2}$

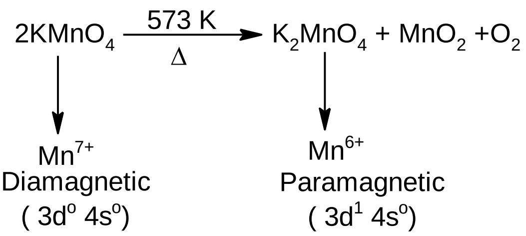
CHEMISTRY

SECTION – A

Sol1. SO_2 acts as reducing agent which is present in reducing smog i.e smoke, fog and SO_2 gas.

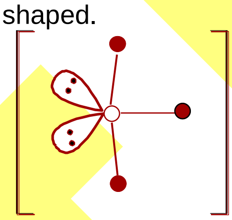
Sol2. Due to non-availability of lone pair on P in PCl_5, it cannot act as Lewis base.

Sol3.



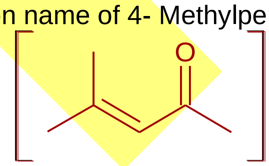
Sol4. This is nucleophilic aromatic substitution reaction; aryl halides are very less reactive toward this reaction so reaction takes place at high temperature i.e 623K and high pressure of 300 atm.

Sol5. Central atom having two lone pairs and three bond pairs reflects sp^3d hybridization and corresponding T- shaped.



Sol6. R_f, Retardation factor varies according to the solvent and its characteristics and it is unitless.

Sol7. Mesityl oxide is the common name of 4- Methylpent-3-en-2-one.

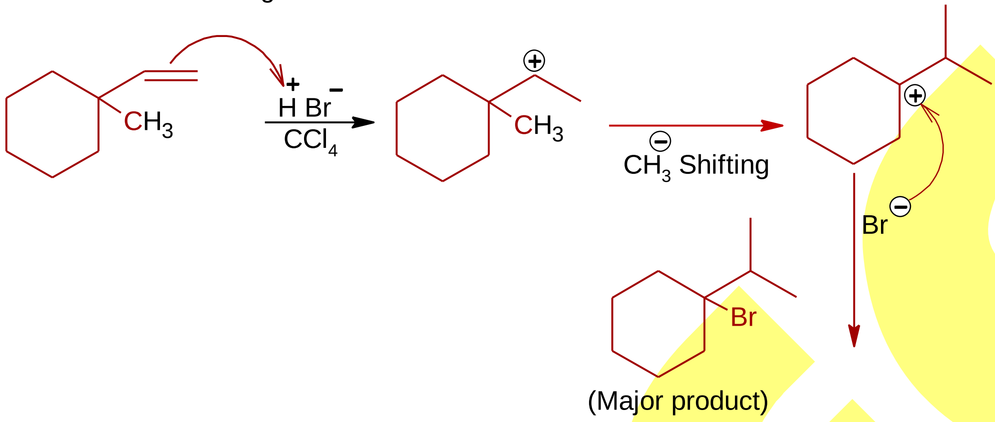


Sol8. Due to larger size of Cl-atom, addition of e^- in outer most orbit produces less electronic repulsion with respect to F-atom. Following are the electron gain enthalpies in (KJ) of halogens.
F→ –328, Cl→ –349, Br→ –325, I→ –295

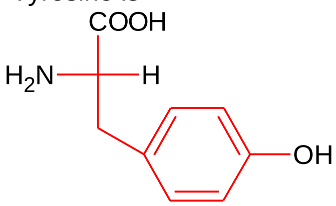
Sol9. Solid sol is the colloidal system consisting of gas dispersed in solid, eg- pumice stone, foam rubber.

Sol10. Except option (D), all are the important characteristics of D_2O. D_2O is useful for the isotopic levelling effect in reaction mechanism. Dielectric constant of H_2O and D_2O are respectively 80 and 60.

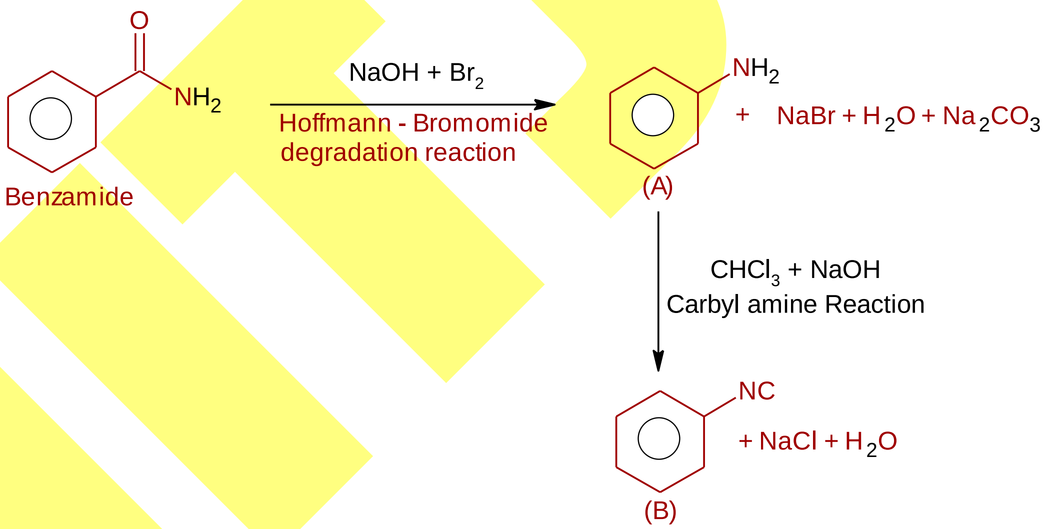
Sol11. Carbocationic rearrangement based reaction.



Sol12. Tyrosine is



Sol13.



Sol14. In aqueous medium all alkali metal cations converted into hydrated form and more intensify +ve charge cation have more hydration and hence size of cation increases after hydration. Therefore Li^{+} cation has larger size in hydrated form and least conductivity.
Order of conductivity of alkali metal ions are, $Li^+ < Na^+ < K^+ < Rb^+ < Cs^+$

Sol15. Divalent metal ion of atomic Number (Z) = 25 is $Mn^{2+}_{(25)}$

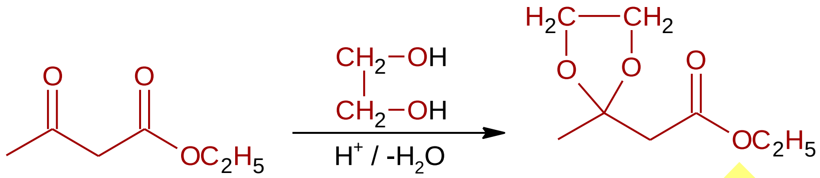
$Mn^{2+}_{(25)} = ...3d^5\,4s^o$

(n=5, five unpaired electron)

$\mu = \sqrt{n(n+2)} = \sqrt{5 \times (5+2)} = \sqrt{35} = 5.92\ \text{BM}$

Sol16. Allosteric inhibitors are the such type of drug- enzyme inhibitors which bind the active site of enzyme due to steric factor.

Sol17. More reactive carbonyl i.e ketone masked with ethylene glycol at first in acidic medium.



Sol18. In ammonolysis process bond cleavage carried out in the presence of NH_3.
When halide compound treated with NH_3, X^- substituted by NH_2^- called nucleophilic substitution reaction.

Sol19. At point of intersection ΔG for two processes becomes equal, so ΔG for reduction becomes zero.
Sudden increase in slope indicates change in state of metal oxide i.e solid to liquid or liquid to vapour.

Sol20. In aromatic compound, overall delocalization processed by the electron according to Huckel's rule. Here total delocalized electrons are = 6

SECTION – B

Sol1. Partial Pressure of O_2 = $K_H \times$ solubility (K_H = Henry's constant)

$\therefore$ solubility = $\frac{P_{O_2}}{K_H} = \frac{20}{8.0\times10^4} = 2.5\times10^{-4} = 25\times10^{-5}$ M

Ans = 25

Sol2. $Kt = 2.303 \log_{10}\frac{A_0}{A_t}$

$K\times570 = 2.303\log_{10}\frac{100}{32}$

$\therefore K = \frac{2.303}{570}\left[\log_{10}10^2 - \log_{10}2^5\right]$

$= \frac{2.303}{570}[2-5\times0.301] = \frac{2.303}{570}\times0.495 = 0.002 = 2.0\times10^{-3}\ s^{-1}$

Ans = 2

Sol3. Mole of CH_4 = $\frac{6.4}{16} = 0.4$ and mole of CO_2 = $\frac{8.8}{44} = 0.2$

Total mole = (0.4 +0.2) = 0.6 mole of non-reacting mixture of gas

Using ; $P = \frac{nRT}{V}$

$\therefore P = \frac{0.6\times8.314\times300}{10} = 149.65kPa$

Ans = 150 (Rounded off)

Sol4.

$HCl \longrightarrow H^+ \quad Cl^-$

$$HCl \longrightarrow H^+ + Cl^-$$

At equilibrium $\quad 0.1 \quad 0.1$

$$HA \underset{}{\overset{ka}{\rightleftharpoons}} H^+ + A^-$$

At equilibrium $\quad C\text{-}C\alpha \quad C\alpha+0.1 \quad C\alpha$

$$Ka = \frac{(C\alpha+0.1)\times C\alpha}{C(1-\alpha)} = \frac{0.1\times10^{-2}\alpha}{10^{-2}} = 0.1\alpha$$

$2\times10^{-6} = 10^{-1}\times\alpha \qquad \therefore \alpha = 2\times10^{-5}$

Ans = 2

Sol5. $P_4 + 3NaOH + 3H_2O \longrightarrow PH_3 + 3NaH_2PO_2$
A

Here ; NaH_2PO_2 is reducing agent.

$NaH_2PO_2 + AgNO_3 + H_2O \longrightarrow Ag + HNO3 + NaH_2PO_4$
1mole excess
Here; equivalents of NaH_2PO_2 = equivalents of Ag (n-factor of NaH_2PO_2 = 4)
$1 \times 4 = n \times 1$, so moles of Ag = 4

Sol6. $Cr_2O_7^{2-} + Fe^{2+} \xrightarrow{H^+} Cr^{3+} + Fe^{3+}$
n=6 n=1

$\text{Meq } Cr_2O_7^{--} = \text{MeqFe}^{2+}$

$20\times0.03\times6 = 15\times M\times1$

$$\therefore M = \frac{20\times0.03\times6}{15} = 0.24 = 24\times10^{-2}$$

Ans = 24

Sol7. n = 4 and the possible value of ℓ = 0, 1,2,3
For ℓ =3, and m = –3
Radial nodes = (n-ℓ-1) = (4-3-1)=0
Ans = 0

Sol8. $$\text{Molality} = \frac{\text{mole of solute}\times100}{\text{wt of sovent(gm)}}$$

$$100 = \frac{n_{solute}\times1000}{n_{solvent}\times\text{molwt(solvent)}}$$

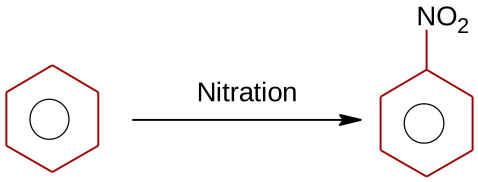
$$100 = \frac{n_{solute}\times1000}{(1-n_{solute})\times18}$$

$$\therefore \frac{n_{solute}}{1-n_{solute}} = \frac{18}{10} \qquad \therefore n_{solute}\times10 = 18-18n_{solute}$$

$$\therefore n_{solute} = \frac{18}{28} = 0.6428 = 64.28\times10^{-2}$$

Ans = 64 (Rounded off)

Sol9.



$\left(\frac{3.9}{78}\right)$ mole of benzene will produce $\left(\frac{3.9}{78}\right)$ mole of nitrobenzene in 100% conversion.

But produced mole of nitrobenzene is $\left(\frac{4.92}{123}\right)$ mole.

$$\% \text{ yield} = \frac{\left(\frac{4.92}{123}\right)\times 100}{\frac{3.9}{78}} = \frac{4.92\times 100\times 78}{123\times 3.9} = 80.0\%$$

Ans = 80

Sol10. $3CaO + 2Al \rightarrow 3Ca + Al_2O_3$

$$\Delta H = \sum \Delta H_{f(P)} - \sum \Delta H_{f(R)}$$

$$= \left[(-1675+0)-(-635\times 3+0)\right] = -1675+1905 = 230 \text{ KJ}$$

Ans = 230

MATHEMATICS
SECTION – A

Sol1. $n(s) = 6^2 = 36$

$E = (1,1),(1,2),(1,3),(1,5),(1,7),(2,1),(2,2)$

$(2,3),(2,5),(3,1),(3,2),(3,3),(3,5),(5,1),(5,2),(5,3),(7,1)$

$\therefore n(E) = 17$

Required prob. = $\frac{n(E)}{n(S)} = \frac{17}{36}$

Sol2. $\frac{dy}{dx} = xy - 1 + x - y = x(y+1) - (y+1) = (x-1)(y+1)$

$\Rightarrow \int \frac{dy}{1+y} = (x-1)dx \Rightarrow \ln(1+y) = \frac{x^2}{2} - x + C$

For $y(0) = 0, c = 0 \quad \therefore \ln(1+y) = \frac{x^2}{2} - x$

$1 + y = e^{\left(\frac{x^2}{2} - x\right)}$

$y(1) = e^{-\frac{1}{2}} - 1$

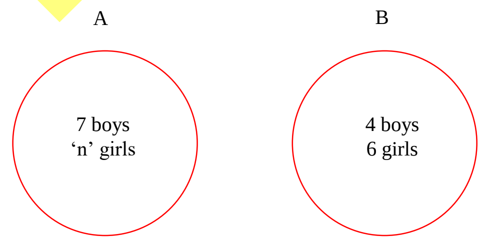
Sol3. $\Delta = \begin{vmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{vmatrix} = k(k^2 - 1) + 1(1-k) + 1(1-k) = 0$

$\Rightarrow k^3 - 3k + 2 = 0 \Rightarrow (k-1)^2(k+2) = 0$

$\therefore k = -2, 1$

If k = 1 then all the equations are identical. Hence k = -2 for no solution.

Sol4. A/q, $7 \times 4 + n \times 6 = 52 \Rightarrow 6n = 24 \Rightarrow n = 4$



Sol5. $y = 5^{\log x} \Rightarrow \log_5^y = \log x$

$x = 10^{\log_5^y} = y^{\log_5^{10}} = y^{\frac{1}{\log 5}} \Rightarrow$ for $y = f^{-1}(x), y = x^{\frac{1}{\log 5}} \Rightarrow x = y^{\log 5}$

Sol6. $\cot^{-1}\alpha = \sum_{r=1}^{100} \cot^{-1}(2r^2) = \sum_{r=1}^{100} \tan^{-1}\frac{2}{4r^2} = \sum_{r=1}^{100} \tan^{-1}\frac{(2r+1)-(2r-1)}{1+(2r+1)(2r-1)}$

$= \sum_{r=1}^{100}\left(\tan^{-1}(2r+1) - \tan^{-1}(2r-1)\right) = \tan^{-1}201 - \tan^{-1}1 = \tan^{-1}\left(\frac{201-1}{1+201}\right)$

$= \tan^{-1}\frac{200}{202} = \tan^{-1}\left(\frac{100}{101}\right) = \cot^{-1}\left(\frac{101}{100}\right)$

$\therefore \alpha = \frac{101}{100} = 1.01$

Sol7. Let equation of plane through (1,2,3) is

$a(x-1)+b(y-2)+c(z-3)=0$

Direction ratios = 0,1,0

$\therefore b=0$

Hence $a(x-1)+c(z-3)=0$

$ax+cz=a+3c=0$

$a=-3c$

$\therefore -3x+z=0 \Rightarrow 3x-z=0$

Sol8.

p	q	$p\to q$	$\sim p$	$q*\sim p$	$\sim q$	$\sim q\wedge p$	$\sim(\sim q\wedge p)$	$q\to p$	$p*\sim q$
T	T	T	F	T	F	F	T	T	T
T	F	F	F	F	T	T	F	T	T
F	T	T	T	T	F	F	T	F	F
F	F	T	T	T	T	F	T	T	T

$q*\sim p = qv\sim p$

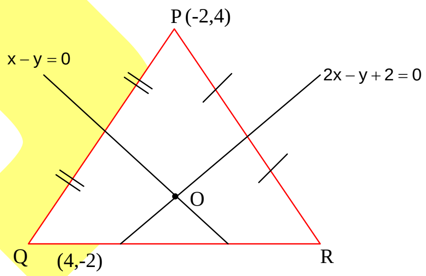
$p*(\sim q) = pv\sim q \Rightarrow q\to p$

Sol9. P,Q,R represents some students which play all three games. Hence no any option is correct.

Sol10. By property of triangle image of vertex of P is Q about the perpendicular side bisector of triangle. Hence according to question $x-y=0$ is a perpendicular side bisector of PQ.

Hence solving $x-y=0$ and $2x-y+2=0$

$\therefore O(-2,-2)$



Sol11. $S_1 = x^2+y^2-10x-10y+41=0$ ....(i) $C_1(5,5), r_1=3$

$S_2 = x^2+y^2-16x-10y+80=0$ ....(ii) $C_2(8,5), r_2=3$

Put $(8,5), S_1 = 64+25-80-50+41=0$

and $C_1(5,5), S_2 = 25+25-80-50+80=0$ both circles passes through centre of each other.

Sol12. $T_{r+1} = {}^7C_r\, x^{7-r}\left(x^{\log_2 x}\right)^r$

$\therefore {}^7C_3 x^4\left(x^{\log_2 x}\right)^3 = 4480$, taking $\log_2$ on both sides and put $\log_2^x = t$, we get

$\therefore x=2$

Sol13. Let $y = 4+\cfrac{1}{5+\cfrac{1}{4+\cfrac{1}{5+\cfrac{1}{4+....\infty}}}}$

$\Rightarrow y = 4+\cfrac{1}{5+\cfrac{1}{y}} = 4+\frac{y}{5.y+1}$

$$\Rightarrow 5y^2 - 20y - 4 = 0 \Rightarrow y = \frac{20 \pm \sqrt{480}}{10}, y > 0$$

$$\therefore y = 2 + \frac{2}{5}\sqrt{30}$$

Sol14. $\underset{x \to 0^+}{\text{Lt}} \dfrac{\cos^{-1}(x)\sin^{-1}(x)}{x - x^3} = \underset{x \to 0^+}{\text{Lt}} \dfrac{\cos^{-1}x.\sin^{-1}x}{(1-x^2)x} = \dfrac{\pi}{2}$

Sol15. Given equation of tangent is $2x - y + 1 = 0$

$\therefore$ equation of normal is $x + 2y = 12$

Solving with $x - 2y = 4$ we get centre at $(8,2)$ radius = $\sqrt{36+9} = 3\sqrt{5}$

Sol16. Let $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\vec{r} \times \vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ 2 & -3 & 4 \end{vmatrix} = \hat{i}(4y + 3z) + \hat{j}(2z - 4x) + \hat{k}(-3x - 2y)$$

$$\vec{r} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ 7 & 1 & -6 \end{vmatrix} = \hat{i}(-6y - z) + \hat{j}(7z + 6x) + \hat{k}(x - 7y)$$

$\because \vec{r} \times \vec{a} = \vec{r} \times \vec{b}$

$$\begin{array}{c|c|c} 4y + 3z = -6y - z & 2z - 4x = 7z + 6x & -3x - 2y = x - 7y \\ \therefore \quad 10y = -4z & 10x = -5z & 4x = 5y \\ & 2x = -z & \end{array}$$

and $\vec{r}.(\hat{i} + 2\hat{j} + \hat{k}) = -3$

$x + 2y + z = -3$

$\therefore \vec{r}.(2\hat{i} - 3\hat{j} + \hat{k}) = 2x - 3y + z = -10 + 12 + 10 = 12$

$x + \dfrac{x}{5} - 2x = -3 \Rightarrow x = -5,\ y = -4,\ z = 10$

Sol17. $g(\alpha) = \displaystyle\int_{\pi/6}^{\pi/3} \frac{\sin^\alpha x}{\cos^\alpha x + \sin^\alpha x} dx$

$$g(\alpha) = \int_{\pi/6}^{\pi/3} \frac{\sin^\alpha\left(\frac{\pi}{2} - x\right)dx}{\cos^\alpha\left(\frac{\pi}{2} - x\right) + \sin^\alpha\left(\frac{\pi}{2} - x\right)} dx \qquad \left[\because \int_a^b f(x)dx = \int_a^b f(a + b - x)dx\right.$$

$$2g(\alpha) = \int_{\pi/6}^{\pi/3} dx = \frac{\pi}{6}$$

$$\therefore g(\alpha) = \frac{\pi}{12}$$

Hence $g(\alpha)$ is constant function.

Sol18. $\tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}\frac{8}{31}$

$$\tan^{-1}\left(\frac{x+1+x-1}{1+x^2+1}\right)=\tan^{-1}\frac{8}{31}$$

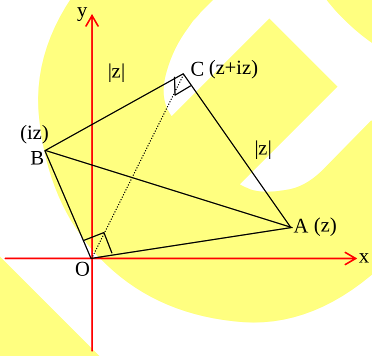
$$\frac{2x}{2-x^2}=\frac{8}{31}\Rightarrow x=\frac{1}{4},-8$$

at $x=\frac{1}{4}$ which does not satisfy.

$$\therefore x=-8=-\frac{32}{4}$$

Sol19. Area of ΔABC

$$=\frac{1}{2}|z|.|z|=\frac{1}{2}|z|^2$$



Sol20. $A^2=A.A=\begin{bmatrix}0 & \sin\alpha\\ \sin\alpha & 0\end{bmatrix}\begin{bmatrix}0 & \sin\alpha\\ \sin\alpha & 0\end{bmatrix}$

$$=\begin{bmatrix}\sin^2\alpha & 0\\ 0 & \sin^2\alpha\end{bmatrix}$$

$$A^2-\frac{1}{2}I=\begin{bmatrix}\sin^2\alpha-\frac{1}{2} & 0\\ 0 & \sin^2\alpha-\frac{1}{2}\end{bmatrix}$$

$$\therefore\left(\sin^2\alpha-\frac{1}{2}\right)^2=0\Rightarrow\sin^2\alpha=\frac{1}{2}\Rightarrow\alpha=\pm\frac{\pi}{4}$$

SECTION – B

Sol1. $2021\times2021\ldots\ldots\ldots\ldots(3762)$ times $\div17$

$15\times15\times\ldots\ldots\ldots\ldots(3762)$ times $\div17$

$(+2)(+2)(-2)(-2)\ldots\ldots\ldots\ldots(3762)$ times $\div17$

$16\times16\times16\ldots\ldots\ldots\ldots(940)$ times $\times2\times2\div17$

$(-1)(-1)(-1)\ldots\ldots\ldots\ldots(940)$ times $\times4\div17$

$\therefore$ Remainder = 4

Sol2. $f(x)=\sin\left(\cos^{-1}\left(\frac{1-2^{2r}}{1+2^{2x}}\right)\right)$

Put $2^x=\tan\theta$

$$\therefore f(x)=\frac{2.2^x}{1+4^x}$$

$$\therefore f'(1) = -\frac{12}{25}\ln 2$$

$$\therefore |a^2 - b^2| = |144 - 625| = 481$$

Sol3. Let probability of E_1, E_2, E_3 are a, b, c respectively.

$$\therefore a(1-b)(1-c) = \alpha, b(1-a)(1-c) = \beta, c(1-a)(1-b) = \gamma$$

$$\Rightarrow (1-a)(1-b)(1-c) = p$$

Also $(\alpha - 2\beta)p = \alpha\beta \Rightarrow a = 2b$ and $(\beta - 3\gamma)p = 2\beta\gamma$

$b = 3c$

Solving these equation we get $a = 6c \Rightarrow \frac{a}{c} = 6$

Sol4. $A^2 = A.A = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 2^2 & 2^2-1 \\ 0 & 1 \end{bmatrix}$

$$A^4 = \begin{bmatrix} 2^2 & 2^2-1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2^2 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2^4 & 2^4-1 \\ 0 & 1 \end{bmatrix}$$

$$A^{10} = \begin{bmatrix} 2^{10} & 2^{10}-1 \\ 0 & 1 \end{bmatrix}$$

$$2A = \begin{bmatrix} -4 & 6 \\ 0 & -2 \end{bmatrix}$$

$$\therefore adj(2A) = \begin{bmatrix} -2 & -6 \\ 0 & -4 \end{bmatrix} = -2\begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}$$

$$\therefore adj(2A)^{10} = 2^{10}\begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}^{10} = 2^{10}\begin{bmatrix} 1 & -(2^{10}-1) \\ 0 & 2^{10} \end{bmatrix} = 2^{10}\begin{bmatrix} 1 & -1023 \\ 0 & 1024 \end{bmatrix}$$

$$A^{10} - (adj\ 2A)^{10}\begin{bmatrix} 0 & 2''\times 1023 \\ 0 & 1-(1024)^2 \end{bmatrix}$$

$$\therefore \left|A^{10} - (adj\ 2A)^{10}\right| = 0$$

$$\det(A^4) + \det\left(A^{10} - (adj\ 2A)^{10}\right) = 16 - 0 = 16$$

Sol5. $x^2 + y^2 - 10x - 10y + 41 = 0$

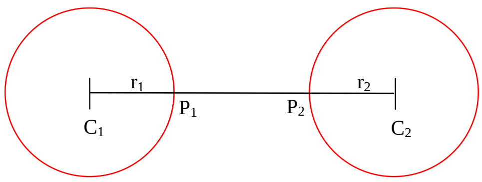
$C_1(5,5), r_1 = 3$

and $x^2 + y^2 - 24x - 10y + 160 = 0$

$C_2(12,5), r_2 = 3$

$C_1C_2 = \sqrt{49+0} = 7$

$\therefore P_1P_2 = 7 - (3+3) = 1$



Sol6. $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & \beta & 3 \\ -\beta & -\alpha & -1 \end{vmatrix} = \hat{i}(-\beta + 3\alpha) + \hat{j}(-3\beta + \alpha) + \hat{k}(-\alpha^2 + \beta^2)$ ....(i)

$\vec{a}.\vec{b} = 1 \Rightarrow -\alpha\beta - \alpha\beta - 3 = 1 \Rightarrow 2\alpha\beta = -4 \Rightarrow \alpha\beta = -2$ ....(ii)

$\vec{b}.\vec{c} = -3 \Rightarrow -\beta + 2\alpha + 1 = -3 \Rightarrow 2\alpha - \beta = -4$ ....(iii)

Solving (ii) & (iii), $\alpha = -1, \beta = 2$

$\therefore \vec{a}\times\vec{b} = -5\hat{i} - 7\hat{j} + 3\hat{k}$

$\frac{1}{3}\left(\left(\vec{a}\times\vec{b}\right).\vec{c}\right) = \frac{(-5+14-3)}{3} = 2$

Sol7. $z = 6xy + y^2 \Rightarrow x = \frac{z-y^2}{6y}$

$3\left(\frac{z-y^2}{6y}\right) + 4y \le 100 \Rightarrow z + 7y^2 = 200y$

$\Rightarrow z \le 200y - 7y^2 \;\&\; z \le \frac{225y - 7y^2}{2}$

$\therefore$ Range of $y \in \left(-\infty, \frac{10^4}{7}\right]$ and $y \in \left(-\infty, \frac{225^2}{56}\right]$

$\therefore Z_{max} = \frac{225^2}{56} = 904.017$

Sol8. Equation of plane passing line of intersection of two planes is

$2x - 7y + 4z - 3 + \lambda(3x - 5y + 4z + 11) = 0$

It passes through $(-2,1,3) \therefore \lambda = \frac{1}{6}$

$\therefore 15x - 47y + 28z - 7 = 0$

$\therefore 2a + b + c - 7 = 4$

Sol9. $\int_0^{\sqrt{\frac{\pi}{2}}} \left[\left[x^2\right] - \cos x\right]dx = \int_0^1 \left[-\cos x\right]dx + \int_1^{\sqrt{\frac{\pi}{2}}} \left[1 - \cos x\right]dx$

$= \int_0^1 \left(-1 - \left[\cos x\right]\right)dx + \int_1^{\sqrt{\frac{\pi}{2}}} dx + \int_1^{\sqrt{\frac{\pi}{2}}} \left(-1 - \left[\cos x\right]\right)dx$

$= 1 - \int_0^1 dx + \int_1^{\sqrt{\frac{\pi}{2}}} dx - \int_1^{\sqrt{\frac{\pi}{2}}} dx = -1$

Sol10. $\frac{1}{k} = \lim_{x\to 0} \frac{\cos(\sin x) - \cos x}{x^4} = \lim_{x\to 0} 2\frac{\sin\left(\frac{x+\sin x}{2}\right)\sin\left(\frac{x-\sin x}{2}\right)}{x^4}$

$= 2\lim_{x\to 0} \frac{(x+\sin x)(x-\sin x)}{4x^4}\left(\frac{0}{0}\right)$(L'Hospital rule)

$= \frac{1}{2}\lim_{x\to 0} \frac{2x - 2\sin x\cos x}{4x^3} = \frac{1}{8}\lim_{x\to 0} \frac{2x - \sin 2x}{x^3}\left(\frac{0}{0}\right) = \frac{1}{24}\lim_{x\to 0} \frac{2 - 2\cos 2x}{x^2}$

$= \frac{1}{12}\lim_{x\to 0} \frac{2\sin^2 x}{x^2} = \frac{1}{12}\times 2 = \frac{1}{6}$

$\therefore k = 6$