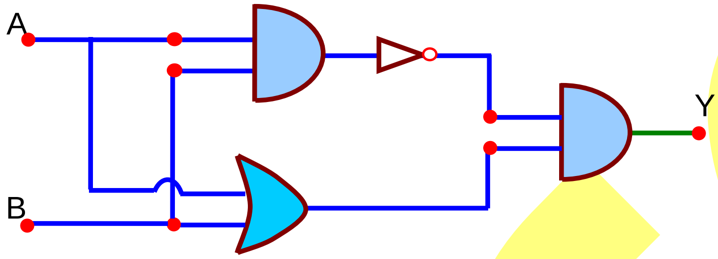
Paper-17-03-2021-Evening Shift

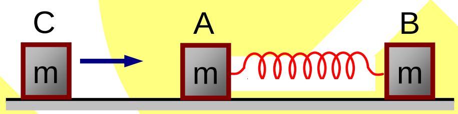
PHYSICS

Q1. Which one of the following will be the output of the given circuit?



(A) AND Gate
(B) NAND Gate
(C) NOR Gate
(D) XOR Gate

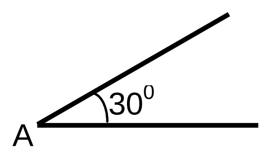
Q2. Two identical blocks A and B each of mass m resting on the smooth horizontal floor are connected by a light spring of natural L and spring constant K.



A third block C of mass m moving with a speed v along the line joining A and B collides with A. The maximum compression in the spring is

(A) $\sqrt{\frac{mv}{K}}$
(B) $\sqrt{\frac{mv}{2K}}$
(C) $\sqrt{\frac{m}{2K}}$
(D) $v\sqrt{\frac{m}{2K}}$

Q3. A sphere of mass 2 kg and radius 0.5 m is rolling with an initial speed of $1\,ms^{-1}$ goes up an inclined plane which makes an angle of 30^0 with the horizontal plane, without slipping. How long will the sphere take to return to the starting point A?



(A) 0.60 s
(B) 0.57 s
(C) 0.52 s
(D) 0.80 s

Q4. The velocity of a particle is $\upsilon = \upsilon_0 + gt + Ft^2$. Its position is x = 0 at t = 0; then its displacement after time (t = 1) is:

(A) $\upsilon_0 + \frac{g}{2} + \frac{F}{3}$
(B) $\upsilon_0 + \frac{g}{2} + F$
(C) $\upsilon_0 + 2g + 3F$
(D) $\upsilon_0 + g + F$

Q5. What happens to the inductive reactance and the current in a purely inductive circuit if the frequency is halved?
(A) Both, inducting reactance and current will be doubled.
(B) Inductive reactance will be halved and current will be doubled.
(C) Inductive reactance will be doubled and current will be halved.
(D) Both, inductive reactance and current will be halved.

Q6. Two identical photo-cathodes receive the light of frequencies f_1 and f_2 respectively. If the velocities of the photo-electrons coming out are υ_1 and υ_2 respectively, then

(A) $\upsilon_1^2 - \upsilon_2^2 = \frac{2h}{m}[f_1 - f_2]$ (B) $\upsilon_1^2 + \upsilon_2^2 = \frac{2h}{m}[f_1 + f_2]$

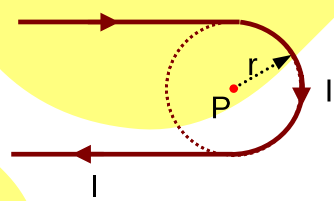
(C) $\upsilon_1 + \upsilon_2 = \left[\frac{2h}{m}(f_1 + f_2)\right]^{\frac{1}{2}}$ (D) $\upsilon_1 - \upsilon_2 = \left[\frac{2h}{m}(f_1 - f_2)\right]^{\frac{1}{2}}$

Q7. A rubber ball is released from a height of 5 m above the floor. It bounces back repeatedly, always rising to $\frac{81}{100}$ of the height through which it falls. Find the average speed of the ball. $\left(\text{Take } g = 10\,ms^{-2}\right)$

(A) 2.0 ms^{-1} (B) 2.50 ms^{-1}

(C) 3.0 ms^{-1} (D) 3.50 ms^{-1}

Q8. A hairpin like shape as shown in figure is made by bending a long current carrying wire. What is the magnitude of a magnetic field at point P which lies on the centre of the semicircle?



(A) $\frac{\mu_0 I}{4\pi r}(2+\pi)$ (B) $\frac{\mu_0 I}{2\pi r}(2+\pi)$

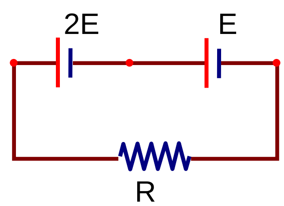
(C) $\frac{\mu_0 I}{4\pi r}(2-\pi)$ (D) $\frac{\mu_0 I}{2\pi r}(2-\pi)$

Q9. A carrier signal $C(t) = 25\sin\left(2.512\times10^{10}\,t\right)$ is amplitude modulated by a message signal $m(t) = 5\sin\left(1.57\times10^{8}\,t\right)$ and transmitted through an antenna. What will be the bandwidth of the modulated signal?

(A) 8 GHz (B) 2.01 GHz

(C) 50 MHz (D) 1987.5 MHz

Q10. Two cells of emf 2E and E with internal resistance r_1 and r_2 respectively are connected in series to an external resistor R (see figure). The value of R, at which the potential difference across the terminals of the first cell becomes zero is



(A) $\frac{r_1}{2} - r_2$ (B) $\frac{r_1}{2} + r_2$

(C) $r_1 + r_2$ (D) $r_1 - r_2$

Q11. **Match List -I with List –II**

	List – I	List - II
(A)	Phase difference between current and voltage in a purely resistive AC circuit	(i) $\frac{\pi}{2}$; current leads voltage
(B)	Phase difference between current	(ii) Zero

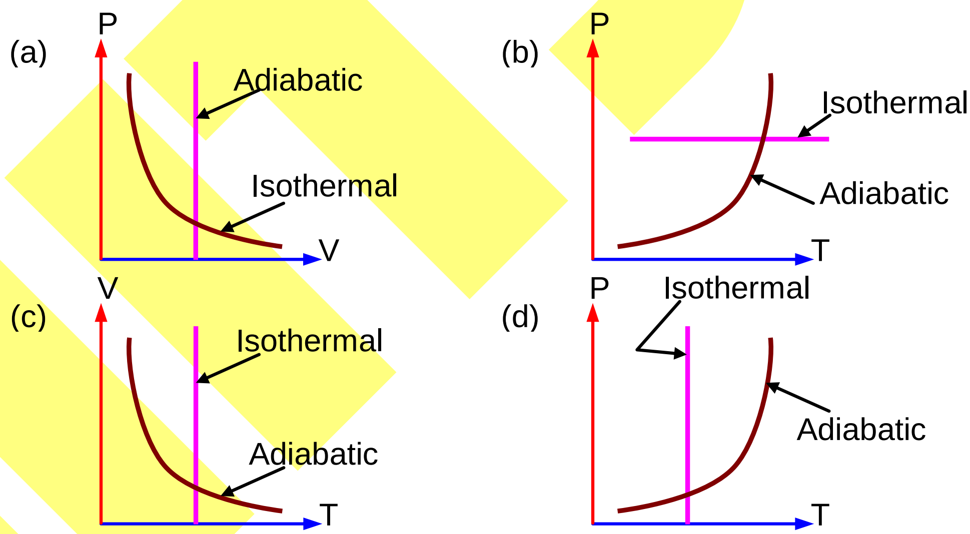
	and voltage in a pure inductive AC circuit	
(C)	Phase difference between current and voltage in a pure capacitive AC circuit	(iii) $\frac{\pi}{2}$; current lags voltage
(D)	Phase difference between current and voltage in an LCR series circuit	(iv) $\tan^{-1}\left(\frac{X_C - X_L}{R}\right)$

Choose the most appropriate answer from the options given below :
(A) **(a) – (ii), (b) – (iii), (c) – (iv), (d) – (i)**
(B) **(a) – (ii), (b) – (iii), (c) – (i), (d) – (iv)**
(C) **(a) – (i), (b) – (iii), (c) – (iv), (d) – (ii)**
(D) **(a) – (ii), (b) – (iv), (c) – (iii), (d) – (i)**

Q12. If one mole of the polyatomic gas is having two vibrational modes and β is the ratio of molar specific heats for polyatomic gas $\left(\beta = \frac{C_P}{C_V}\right)$ then the value of β is :
(A) 1.02
(B) 1.25
(C) 1.35
(D) 1.2

Q13. The atomic hydrogen emits a line spectrum consisting of various series. Which series of hydrogen atomic spectra is lying in the visible region?
(A) Paschen series
(B) Balmer series
(C) Lyman series
(D) Brackett series

Q14. Which one is the correct option for the two different thermodynamic processes?



(A) (c) and (a)
(B) (b) and (c)
(C) (c) and (d)
(D) (a) only

Q15. A sound wave of frequency 245 Hz travels with the speed of 300 ms^{-1} along the positive x –axis. Each point of the wave moves to and for through a total distance of 6 cm. What will be the mathematical expression of this traveling wave?
(A) $Y(x,t) = 0.03\left[\sin 5.1x - \left(1.5 \times 10^3\right)t\right]$
(B) $Y(x,t) = 0.03\left[\sin 5.1x - \left(0.2 \times 10^3\right)t\right]$

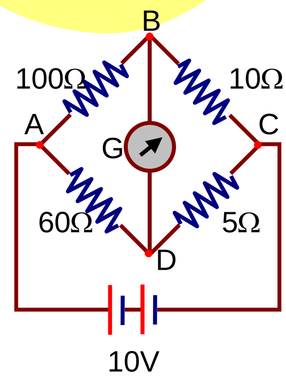
(C) $Y(x,t) = 0.06\left[\sin 5.1x - (1.5\times10^3)t\right]$

(D) $Y(x,t) = 0.06\left[\sin 0.8x - (0.5\times10^3)t\right]$

Q16. An object is located at 2 km beneath the surface of the water. If the fractional compression $\frac{\Delta V}{V}$ is 1.36%, the ratio of hydraulic stress to the corresponding hydraulic strain will be ----------.
[Given: density of water is 1000 kgm^{-3} and g = 9.8 ms^{-2}.]
(A) 1.44×10^7 Nm^{-2}
(B) 1.96×10^7 Nm^{-2}
(C) 2.26×10^9 Nm^{-2}
(D) 1.44×10^9 Nm^{-2}

Q17. A geostationary satellite is orbiting around an arbitrary planet 'P' at a height of 11R above the surface of 'P', R being the radius of 'P'. The time period of another satellite in hours at a height of 2R from the surface of 'P' is ---------. 'P' has the time period of 24 hours.
(A) $6\sqrt{2}$
(B) 3
(C) 5
(D) $\frac{6}{\sqrt{2}}$

Q18. The four arms of a Wheatstone bridge have resistances as shown in the figure. A galvanometer of 15 Ω resistance is connected across BD. Calculate the current through the galvanometer when a potential difference of 10V is maintained across AC.
(A) 4.87 μA
(B) 2.44 mA
(C) 4.87 mA
(D) 2.44 μA

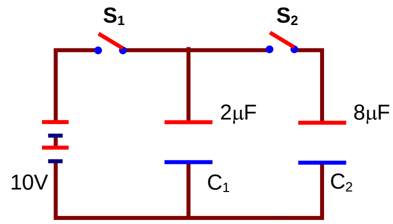


Q19. A block of mass 1 kg attached to a spring is made to oscillate with initial amplitude of 12cm. After 2 minutes the amplitude decreases to 6cm. Determine the value of the damping constant for this motion. (take ln 2 = 0.693)
(A) 3.3×10^2 kg s^{-1}
(B) 1.16×10^2 kg s^{-1}
(C) 0.69×10^2 kg s^{-1}
(D) 5.7×10^{-3} kg s^{-1}

Q20. Two particles A and B of equal masses are suspended from two massless springs of spring constants K_1 and K_2 respectively. If the maximum velocities during oscillations are equal, the ratio of the amplitude of A and B is
(A) $\sqrt{\frac{K_1}{K_2}}$
(B) $\sqrt{\frac{K_2}{K_1}}$
(C) $\frac{K_1}{K_2}$
(D) $\frac{K_2}{K_1}$

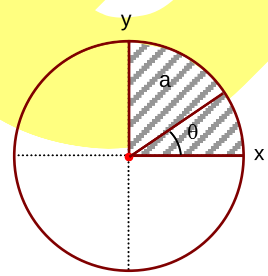
SECTION-B

Q1. A $2\mu F$ capacitor C_1 is first charged to a potential difference of 10V using a battery. Then the battery is removed and the capacitor is connected to an uncharged capacitor C_2 of $8\mu F$. The charge in C_2 on equilibrium condition is --------- μC. (Round off to the Nearest integer)



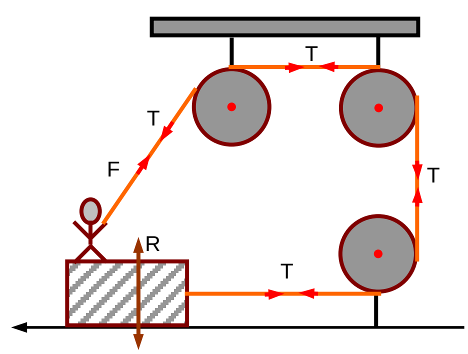
Q2. A body of mass 1 kg rests on a horizontal floor with which it has a coefficient of static friction $\frac{1}{\sqrt{3}}$. It is desired to make the body move by applying the minimum possible force F N. The value of F will be -------------. (Round off to Nearest Integer)
[Take g = 10 ms^{-2}]

Q3. The disc of mass M with uniform surface mass density σ is shown in the figure. The centre of mass of the quarter disc (the shaded area) is at the position $\frac{x}{3}\frac{a}{\pi}, \frac{x}{3}\frac{a}{\pi}$ where x is---------. (Round off to the Nearest integer)
[a is an area as shown in the figure]



Q4. The image of an object placed in air formed by a convex refracting surface is at a distance of 10 m behind the surface. The image is real and is at $\frac{2^{rd}}{3}$ of the distance of the object from the surface. The wavelength of light inside the surface is $\frac{2}{3}$ times the wavelength in air. The radius of the curved surface is $\frac{x}{13}$m. The value of 'x' is -----------.

Q5. A boy of mass 4 kg is standing on a piece of wood having mass 5 kg. If the coefficient of friction between the wood and the floor is 0.5, the maximum force that boy can exert on the rope so that the piece of wood does not move from its place is ------------ N. (Round off to the Nearest Integer)
[Take g = 10 ms^{-2}]



Q6. The electric field intensity produced by the radiation coming from a 100W bulb at a distance of 3 m is E. The electric field intensity produced by the radiation coming from 60 W at the same distance is

$\sqrt{\frac{x}{5}}$ E . Where the value of x = ----------.

Q7. Sea-water at a frequency $f = 9 \times 10^2 Hz$, has permittivity $\varepsilon = 80\varepsilon_0$ and resistivity $\rho = 0.25\Omega m$. Imagine a parallel plate capacitor is immersed in seawater and is driven by an alternating voltage source $V(t) = V_0 \sin(2\pi ft)$. Then the conduction current density become 10^x times the displacement current density after time $t = \frac{1}{800}s$. The value of x is --------. $\left(\text{Given}: \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 Nm^2C^{-2}\right)$

Q8. Suppose you have taken a dilute solution of oleic acid in such a way that its concentration becomes $0.01 cm^3$ of oleic acid per cm^3 of the solution. Then you make a thin film of this solution (monomolecular thickness) of area $4 cm^2$ by considering 100 spherical drops of radius $\left(\frac{3}{40\pi}\right)^{\frac{1}{3}} \times 10^{-3} cm$. Then the thickness of oleic acid will be $x \times 10^{-14} m$. Where x is -----------.

Q9. A particle of mass m moves in a circular orbit in a central potential field $U(r) = U_0 r^4$. If Bohr's quantization conditions are applied, radii of possible orbitals r_n vary with $n^{\frac{1}{\alpha}}$, where α is---------.

Q10. The electric field in a region is given by $\vec{E} = \frac{2}{5}E_0\hat{i} + \frac{3}{5}E_0\hat{j}$ with $E_0 = 4.0 \times 10^3 \frac{N}{C}$. The flux of this field through a rectangular surface area $0.4 m^2$ parallel to the Y- Z plane is -------- Nm^2C^{-1}.

CHEMISTRY

SECTION A

Q1. The functional groups that are responsible for the ion- exchange property of cation and anion exchange resins, respectively, are:
(A) $-SO_3H$ and $-NH_2$
(B) $-SO_3H$ and $-COOH$
(C) $-NH_2$ and $-COOH$
(D) $-NH_2$ and $-SO_3H$

Q2. The set that represents the pair of neutral oxides of nitrogen is:
(A) N_2O and N_2O_3
(B) N_2O and NO_2
(C) NO and NO_2
(D) NO and N_2O

Q3. Which of the following statement(s) is (are) **incorrect** reason for eutrophication?
(A) Excess usage of fertilisers
(B) Excess usage of detergents
(C) Dense plant population in water bodies
(D) Lack of nutrients in water bodies that prevent plant growth
Choose the **most appropriate** answer from the options given below:
(A) (C) only
(B) (A) only
(C) (D) only
(D) (B) and (D) only

Q4. The common positive oxidation states for an element with atomic number 24, are:
(A) +1 and +3 to + 6
(B) +1 and +3
(C) +2 to +6
(D) +1 to + 6

Q5. Match **List – I** with **List – II**

List – I	**List – II**
(a) Haematite	(i) $Al_2O_3.xH_2O$
(b) Bauxite	(ii) Fe_2O_3
(c) Magnetite	(iii) $CuCO_3.Cu(OH)_2$
(d) Malachite	(iv) Fe_3O_4

Choose the **correct** answer from the option given below:
(A) (a)-(iv), (b)-(i), (c)-(ii), (d)-(iii)
(B) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)
(C) (a)-(i), (b)-(iii), (c)-(ii), (d)-(iv)
(D) (a)-(ii), (b)-(iii), (c)-(i), (d)-(iv)

Q6. Fructose is an example of:
(A) Heptose
(B) Aldohexose
(C) Ketohexose
(D) Pyranose

Q7. Match **List – I** with **List –II**

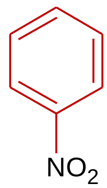
List – I **Chemical Compound**	**List – II** **Used as**
(a) Sucralose	(ii) Synthetic detergent
(b) Glyceryl ester of stearic acid	(ii) Artifical sweetener
(c) Sodium benzoate	(iii) Antiseptic
(d) Bithionol	(iv) Food preservative

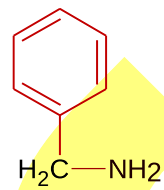
Choose the **correct** match:
(A) (a)-(iii), (b)- (ii), (c)-(iv), (d)- (i)
(B) (a)-(i), (b)-(ii), (c)-(iv), (d)-(iii)
(C) (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)
(D) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)

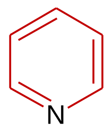
Q8. For the coagulation of a negative sol, the species below, that has the highest flocculating power is :

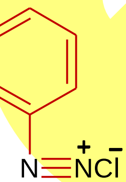
(A) PO_4^{3-} (B) Na^+
(C) Ba^{2+} (D) SO_4^{2-}

Q9. Nitrogen can be estimated by Kjeldahl's method for which of the following compound?

(A)


(B)


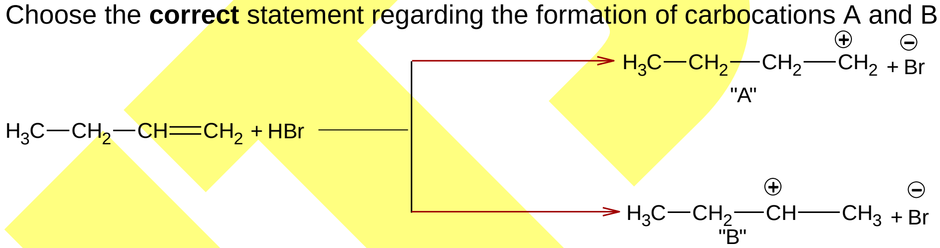
(A)


(B)


Q10. The set of elements that differ in mutual relationship from those of the other sets is:

(A) Be – Al (B) B – Si
(C) Li – Na (D) Li – Mg

Q11. Choose the **correct** statement regarding the formation of carbocations A and B given.

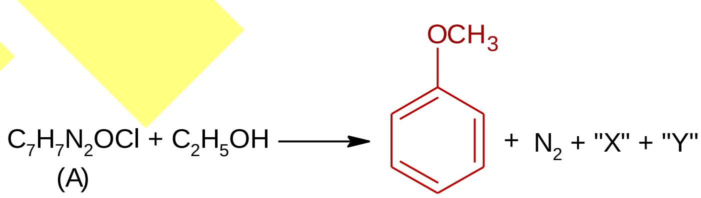


(A) Carbocation A is more stable and formed relatively at slow rate
(C) Carbocation B is more stable and formed relatively at faster rate
(C) Carbocation B is more stable and formed relatively at slow rate
(D) Carbocation A is more stable and formed relatively at faster rate

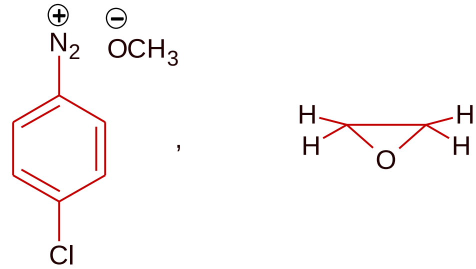
Q12. One of the by-products formed during the recovery of NH_3 from solvay process is:

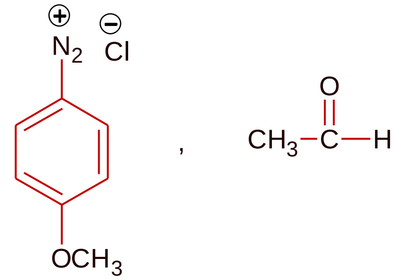
(A) $NaHCO_3$ (B) $CaCl_2$
(C) $Ca(OH)_2$ (D) NH_4Cl

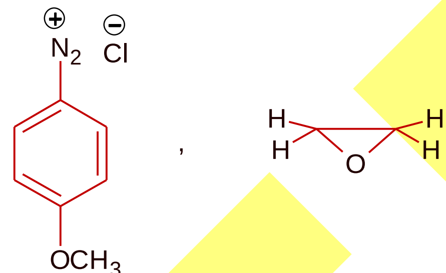
Q13.

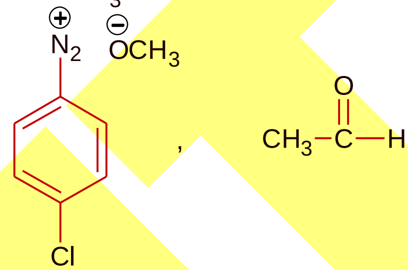


In the above reaction, the structural formula of (A), "X" and "Y" respectively are:

(A)  , HCl

(B)  , HCl

(C)  , H_2O

(D)  , H_2O

Q14. During which of the following processes, does entropy decrease?
(A) Freezing of water to ice at 0°C
(B) Freezing of water to ice at –10°C
(C) $N_2(g) + 3H_2(g) \rightarrow 2NH_3(g)$
(D)Adsorption of CO(g) on lead surface
(E) Dissolution of NaCl in water
Choose the **correct** answer from the options given below:
(A) (A) and (E) only
(B) (A), (B), (C) and (D) only
(C) (A), (C) and (E) only
(D) (B) and (C) only

Q15. The correct pair(s) of the ambident nuclophiles is (are):
(A) AgCN / KCN
(B) RCOOAg / RCOOK
(C) $AgNO_2$ / KNO_2
(D) AgI / KI

(A) (B) only
(B) (B) and (C) only
(C) (A) only
(D) (A) and (C) only

Q16. Match **List – I** with **List – II:**

List – I	**List – II**
(a) $\left[Co(NH_3)_6\right]\left[Cr(CN)_6\right]$	(i) Linkage isomerism
(b) $\left[Co(NH_3)_3(NO_2)_3\right]$	(ii) Solvate isomerism
(c) $\left[Cr(H_2O)_6\right]Cl_3$	(iii) Co-ordination isomerism
(d) $cis-\left[CrCl_2(ox)_2\right]^{3-}$	(iv) Optical isomerism

Choose the **correct** answer from the options given below:
(A)(a)-(iv), (b)-(ii), (c)-(iii),(d)-(i)
(B) (a)-(i), (b)-(ii), (c)-(iii), (d)-(iv)
(C) (a)-(ii), (b)-(i), (c)-(iii), (d)-(iv)
(D) (a)- (iii), (b)-(i), (c)-(ii), (d)-(iv)

Q17. Given below are two statements:
Statement I : 2-methylbutane on oxidation with $KMnO_4$ gives 2-methylbutan-2-ol.
Statement II: n-alkanes can be easily oxidised to corresponding alcohols with $KMnO_4$.
Choose the **correct** option:
(A) Both statement **I** and statement **II** are correct
(B) Both statement **I** and statement **II** are incorrect
(C) Statement **I** is correct but statement **II** is incorrect
(D) Statement **I** is incorrect but statement **II** is correct

Q18.

$$\underset{\text{Sucrose}}{C_{12}H_{22}O_{11}} + H_2O \xrightarrow{\text{Enzyme A}} \underset{\text{Glucose}}{C_6H_{12}O_6} + \underset{\text{Fructose}}{C_6H_{12}O_6}$$

$$C_6H_{12}O_6 \xrightarrow{\text{Enzyme B}} 2C_2H_5OH + 2CO_2$$

In the above reactions, the enzyme A and enzyme B respectively are:
(A) Invertase and Zymase
(B) Invertase and Amylase
(C) Zymase and Invertase
(D) Amylase and Invertase

Q19. Primary, secondary and tertiary amines can be separated using:
(A) Para –Toluene sulphonyl chloride
(B) Acetyl amide
(C) Chloroform and KOH
(D) Benzene sulphonic acid

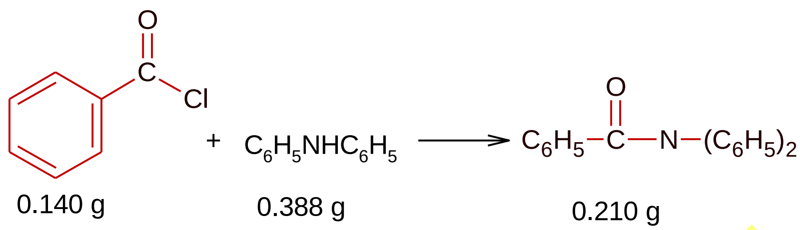
Q20. Amongst the following, the linear species is:
(A) O_3
(B) Cl_2O
(C) NO_2
(D) N_3^-

SECTION B

Q1. The reaction $2A + B_2 \rightarrow 2AB$ is an elementary reaction.
For a certain quantity of reactants, if the volume of the reaction vessel is reduced by a factor of 3, the rate of reaction increases by a factor of ______.
(Round off to the nearest integer).

Q2. In the ground state of atomic Fe(Z = 26), the spin- only magnetic moment is __$\times 10^{-1}$ BM.
(Round off to the nearest integer).
[Given $\sqrt{3} = 1.73, \sqrt{2} = 1.41$]

Q3.



Consider the above reaction. The percentage yield of amide product is _______.
(Round off to the nearest integer).
(Given : Atomic mass : C:12.0u, H:1.0 u, N: 14.0 u, O : 16.0 u, Cl: 35.5 u)

Q4. A KCl solution of conductivity 0.14 Sm^{-1} shows a resistance of 4.19 Ω in a conductivity cell. If the same cell is filled with an HCl solution, the resistance drops to 1.03Ω. The conductivity of the HCl solution is __________$\times 10^{-2}$S m^{-1}.
(Round off to the nearest integer).

Q5. On complete reaction of $FeCl_3$ with oxalic acid in aqueous solution containing KOH, resulted in the formation of product A. The secondary valency of Fe in the product A is________.
(Round off to the nearest integer).

Q6. A 1 molal $K_4Fe(CN)_6$ solution has a degree of dissociation of 0.4. Its boiling point is equal to that of another solution which contains 18.1 weight percent of a non electrolytic solute A. The molar mass of A is ___________ u. (Round off to the nearest integer).
[Density of water = 1.0 g cm^{-3}]

Q7. The number of chlorine atoms in 20 mL of chlorine gas at STP is_____10^{21}(Round off to the nearest integer).
[Assume chlorine is an ideal gas at STP
R = 0.083 L bar mol^{-1} K^{-1}, $N_A = 6.023 \times 10^{23}$]

Q8. Consider the reaction $N_2O_4(g) \rightleftharpoons 2NO_2(g)$. The temperature at which $K_C = 20.4$ and $K_p = 600.1$, is_______ K. (Round off to the nearest integer).
[Assume all gases are ideal and R = 0.0831 L bar mol^{-1} K^{-1}]

Q9. The total number of C-C sigma bond / s in mesityl oxide ($C_6H_{10}O$) is_______. (Round off to the nearest integer).

Q10. KBr is doped with 10^{-5} mole percent of $SrBr_2$. The number of cationic vacancies in 1 g of KBr crystal is __________ 10^{14}. (Round off to the nearest integer).
[Atomic Mass : K: 39.1 u, Br : 79.9 u, $N_A = 6.023 \times 10^{23}$]

MATHEMATICS
SECTION A

Q1. Let S_1,S_2 and S_3 be three sets defined as

$S_1 = \{ z \in C : |z-1| \le \sqrt{2} \}$

$S_2 = \{z \in C : Re((1-i)z) \ge 1\}$

$S_3 = \{z \in C: Im(z) \le 1\}$

Then the set $S_1 \cap S_2 \cap S_3$

(A) Is a singleton
(B) Has exactly two elements
(C) Has infinitely many elements
(D) Has exactly three elements

Q2. If the Boolean expression $(p \wedge q) \odot (p \otimes q)$ is a tautology, then $\odot$ and $\otimes$ are respectively by:

(A) $\rightarrow, \rightarrow$
(B) $\wedge, \rightarrow$
(C) $\vee, \rightarrow$
(D) $\wedge, \vee$

Q3. Let O be the origin. Let $\overrightarrow{OP} = x\hat{i} + y\hat{i} - \hat{k}$ and $OQ = -\hat{i} + 2\hat{j} + 3x\hat{k}, x, y \in R, x > 0$, be such that $\left|\overrightarrow{PQ}\right| = \sqrt{20}$ and the vector $\overrightarrow{OP}$ is perpendicular to $\overrightarrow{OQ}$. If $\overrightarrow{OR} = 3\hat{i} + z\hat{j} - 7\hat{k}$, $z \in R$, is coplanar with $\overrightarrow{OP}$ and $\overrightarrow{OQ}$, then the value of $x^2 + y^2 + z^2$ is equal to:

(A) 9
(B) 2
(C) 7
(D) 1

Q4. If the curve y=y(x) is the solution of the differential equation

$2\left(x^2 + x^{5/4}\right)dy - y\left(x + x^{1/4}\right)dx = 2x^{9/4}dx, x > 0$ which passes through the point

$\left(1, 1 - \frac{4}{3}\log_e 2\right)$, then the value of y(16) is equal to:

(A) $4\left(\frac{31}{3} - \frac{8}{3}\log_e 3\right)$
(B) $\left(\frac{31}{3} + \frac{8}{3}\log_e 3\right)$
(C) $\left(\frac{31}{3} - \frac{8}{3}\log_e 3\right)$
(B) $4\left(\frac{31}{3} + \frac{8}{3}\log_e 3\right)$

Q5. Let a computer program generate only the digits 0 and 1 to form a string of binary numbers with probability of occurrence of 0 at even places be $\frac{1}{2}$ and probability of occurrence of 0 at the odd place be $\frac{1}{3}$. Then the probability that '10' is followed by 01 is equal to :

(A) $\frac{1}{18}$
(B) $\frac{1}{3}$
(C) $\frac{1}{6}$
(D) $\frac{1}{9}$

Q6. Consider the function f:R→R defined by f(x) = $\begin{cases}\left(2-\sin\left(\frac{1}{x}\right)\right)|x|, x \neq 0 \\ 0 \qquad\qquad\qquad\quad, x = 0\end{cases}$. Then f is:

(A) Monotonic on $(-\infty,0)$ only
(B) Monotonic on $(-\infty,0)\cup(0,\infty)$
(C) Monotonic on $(0,\infty)$ only
(D) Not monotonic on $(-\infty,0)$ and $(0,\infty)$

Q7. Let y=y(x) be in the solution of the differential equation cosx(3sinx+cosx+3) dy=(1+y sinx(3sinx+cosx+3))dx, $0 \le x \le \frac{\pi}{2}, y(0)=0$. Then $y\left(\frac{\pi}{3}\right)$ is equal to:

(A) $2\log_e\left(\frac{2\sqrt{3}+10}{11}\right)$
(B) $2\log_e\left(\frac{2\sqrt{3}+9}{6}\right)$
(C) $2\log_e\left(\frac{\sqrt{3}+7}{2}\right)$
(D) $2\log_e\left(\frac{3\sqrt{3}-8}{4}\right)$

Q8. The value of the limit $\lim\limits_{\theta\to 0}\frac{\tan\left(\pi\cos^2\theta\right)}{\sin\left(2\pi\sin^2\theta\right)}$ is equal to:

(A) 0
(B) $-\frac{1}{2}$
(C) $\frac{1}{4}$
(D) $-\frac{1}{4}$

Q9. The value of
$\lim\limits_{n\to\infty}\frac{[r]+[2r]+.....+[nr]}{n^2}$,
Where r is a non-zero real number and [r] denotes the greatest integer less than or equal to r, is equal to:

(A) $\frac{r}{2}$
(B) 2r
(C) r
(D) 0

Q10. The number of solutions of the equation $\sin^{-1}\left[x^2+\frac{1}{3}\right]+\cos^{-1}\left[x^2-\frac{2}{3}\right]=x^2$, for $x\in[-1,1]$ and [x] denotes the greatest less than or equal to x, is:
(A) 2
(B) 0
(C) Infinite
(D) 4

Q11. Let f:R→R be defined as f(x) = $e^{-x}\sin x$. If F : [0,1] →R is a differentiable function such that $F(x)=\int_0^x f(t)dt$, then the value of $F(x)=\int_0^1 (F'(x)+f(x))e^x dx$ lies in the interval

(A) $\left[\frac{330}{360},\frac{331}{360}\right]$
(B) $\left[\frac{335}{360},\frac{336}{360}\right]$

(C) $\left[\frac{327}{360},\frac{329}{360}\right]$ (D) $\left[\frac{331}{360},\frac{334}{360}\right]$

Q12. If the sides AB, BC and CA of triangle ABC have 3,5 and 6 interior points respectively, then the total number of triangles that can be constructed using these points as vertices, is equal to:
(A) 360 (B) 333
(C) 364 (D) 240

Q13. If the equation of plane passing through the mirror image of a point (2,3,1) with respect to line $\frac{x+1}{2}=\frac{y-3}{1}=\frac{z+2}{-1}$ and containing the line $\frac{x-2}{3}=\frac{1-y}{2}=\frac{z+1}{1}$ is, $\alpha x+\beta y+\gamma z=24$ then $\alpha+\beta+\gamma$ is equal to:
(A) 20 (B) 19
(C) 18 (D) 21

Q14. Two tangents are drawn from a point P to the circle $x^2 + y^2 - 2x - 4y + 4 = 0$, such that the angle between these tangents is $\tan^{-1}\left(\frac{12}{5}\right)$, where $\tan^{-1}\left(\frac{12}{5}\right)\in(0,\pi)$. If the centre of the circle is denoted by C and these tangents touch the circle at points A and B, then the ratio of the areas of ΔPAB and ΔCAB is:
(A) 9:4 (B) 3:1
(C) 2:1 (D) 11:4

Q15. If x, y, z are in arithmetic progression with common difference d, $x\neq 3d$, and the determinant of the matrix $\begin{bmatrix}3 & 4\sqrt{2} & x\\ 4 & 5\sqrt{2} & y\\ 5 & k & z\end{bmatrix}$ is zero, then the value of k^2 is:
(A) 6 (B) 72
(C) 36 (D) 12

Q16. Let L be a tangent line to parabola $y^2=4x-20$ at $(6,2)$. If L is also a tangent to the ellipse $\frac{x^2}{2}+\frac{y^2}{b}=1$, then the value of b is equal to:
(A) 14 (B) 11
(C) 20 (D) 16

Q17. The number of solution of the equation $x+2\tan x=\frac{\pi}{2}$ in the interval $[0,2\pi]$ is:
(A) 5 (B) 4
(C) 3 (D) 2

Q18. If the integral $\int_0^{10}\frac{|\sin 2\pi x|}{e^{x-[x]}}dx=\alpha e^{-1}+\beta e^{-\frac{1}{2}}+\gamma$ where α,β,γ are integers and [x] denotes the greatest integer less than or equal to x, then the value of $\alpha+\beta+\gamma$ is equal to:
(A) 10 (B) 25
(C) 20 (D) 0

Q19. Let the tangent to the circle $x^2+y^2=25$ at the point R(3,4) meet x-axis and y-axis at points P and Q, respectively. If r is the radius of the circle passing through the origin O and having centre at the incentre of the triangle OPQ, then r^2 is equal to:

(A) $\frac{529}{29}$ (B) $\frac{585}{66}$
(C) $\frac{125}{72}$ (D) $\frac{625}{72}$

Q20. The value of $\sum_{r=0}^{6}\left({}^6C_r \cdot {}^6C_{6-r}\right)$ is equal to:

(A) 1024 (B) 924
(C) 1324 (D) 1124

SECTION B

Q1. Let $I_n=\int_1^e x^{19}(\log|x|)^n dx$, where $n\in N$. If $(20)I_{10}=\alpha I_9+\beta I_8$, for natural numbers α and β then $\alpha-\beta$ equals to__________.

Q2. Let $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B=\begin{bmatrix} \alpha \\ \beta \end{bmatrix}\neq\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ such that AB = B and a + d = 2021, then the value of ad–bc is equal to________________.

Q3. If 1, $\log_{10}\left(4^x-2\right)$ and $\log_{10}\left(4^x+\frac{18}{5}\right)$ are in arithmetic progression for a real number x, then the value of the determinant $\begin{vmatrix} 2\left(x-\frac{1}{2}\right) & x-1 & x^2 \\ 1 & 0 & x \\ x & 1 & 0 \end{vmatrix}$ is equal to:

Q4. Let $\tan\alpha$, $\tan\beta$ and $\tan\gamma$; $\alpha,\beta,\gamma\neq\frac{(2n-1)\pi}{2}, n\in N$ be the slopes of three line segments OA, OB and OC, respectively, where O is origin. If circumcentre of ΔABC coincides with origin and its orthocenter lies on y-axis, then the value of $\left(\frac{\cos3\alpha+\cos3\beta+\cos3\gamma}{\cos\alpha\cos\beta\cos\gamma}\right)$ is equal to__________.

Q5. Let P be an arbitrary point having sum of the squares of the distance from the planes $x+y+z=0$, $\ell x-nz=0$ and $x-2y+z=0$ equal to 9. If the locus of the point P is $x^2+y^2+z^2=9$, then the value of $\ell-n$ is equal to______.

Q6. Let $\vec{x}$ be a vector in the plane containing vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$. If the vector $\vec{x}$ is perpendicular to $\left(3\hat{i} + 2\hat{j} - \hat{k}\right)$ and its projection on $\vec{a}$ is $\frac{17\sqrt{6}}{2}$, then the value of $|\vec{x}|^2$ is equal to _________.

Q7. Let f:[–1,1] →R be defined as f(x) = $ax^2 + bx + c$ for all x∈[–1,1], where a, b, c∈R such that f(–1) = 2, f′(–1) =1 and for x∈(–1,1) the maximum value of f″(x) is $\frac{1}{2}$. If $f(x) \le \alpha$, x∈[–1,1], then the least value of α is equal to_______.

Q8. Consider a set of 3n numbers having variance 4. In this set, the mean of first 2n numbers is 6 and the mean of the remaining n number is 3. A new set is constructed by adding 1 into each of first 2n numbers, and subtracting 1 from each of the remaining n numbers. If the variance of the new set is k, then 9k is equal to__________.

Q9. Let f: [–3,1] → R be given as

$$f(x) = \begin{cases} \min\left\{(x+6), x^2\right\}, & -3 \le x \le 0 \\ \max\left\{\sqrt{x}, x^2\right\}, & 0 \le x \le 1 \end{cases}$$

If the area bounded by y=f(x) and x-axis is A, then the value of 6A is equal to________.

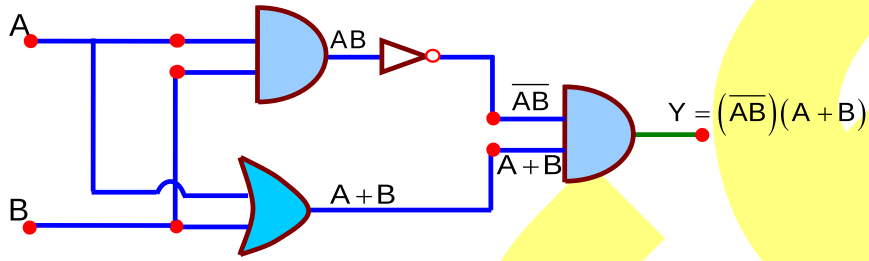
Q10. Let the coefficients of third, fourth and fifth terms in the expansion of $\left(x + \frac{a}{x^2}\right)^n, x \ne 0$, be in the ratio 12:8:3. Then the term independent of x in the expansion is equal to _______.

ANSWER: Paper-Jee-Main-17-03-2021-Evening Shift

PHYSICS	CHEMISTRY	MATHEMATICS
Section-A	**SECTION – A**	**SECTION – A**
Ans1. D	**Ans1.** A	**Ans1. C**
Ans2. D	**Ans2.** D	**Ans2. A**
Ans3. B	**Ans3.** C	**Ans3. A**
Ans4. A	**Ans4.** C	**Ans4. A**
Ans5. B	**Ans5.** B	**Ans5. D**
Ans6. A	**Ans6.** C	**Ans6. D**
Ans7. B	**Ans7.** D	**Ans7. A**
Ans8. A	**Ans8.** C	**Ans8. B**
Ans9. C	**Ans9.** B	**Ans9. A**
Ans10. A	**Ans10.** C	**Ans10. B**
Ans11. B	**Ans11.** B	**Ans11. A**
Ans12. D	**Ans12.** B	**Ans12. B**
Ans13. B	**Ans13.** B	**Ans13. B**
Ans14. C	**Ans14.** B	**Ans14. A**
Ans15. A	**Ans15.** D	**Ans15. B**
Ans16. D	**Ans16.** D	**Ans16. A**
Ans17. B	**Ans17.** C	**Ans17. C**
Ans18. C	**Ans18.** A	**Ans18. D**
Ans19. D	**Ans19.** A	**Ans19. D**
Ans20. B	**Ans20.** D	**Ans20. B**
Section-B	**SECTION – B**	**SECTION – B**
Ans1. 16	**Ans1.** 27	**Ans1. 1**
Ans2. 5	**Ans2.** 49	**Ans2. 2020**
Ans3. 4	**Ans3.** 77	**Ans3. 2**
Ans4. 30	**Ans4.** 57	**Ans4. 144**
Ans5. 30	**Ans5.** 6	**Ans5. 0**
Ans6. 3	**Ans6.** 85	**Ans6. 486**
Ans7. 6	**Ans7.** 1	**Ans7. 5**
Ans8. 25	**Ans8.** 354	**Ans8. 68**
Ans9. 3	**Ans9.** 5	**Ans9. 41**
Ans10. 640	**Ans10.** 5	**Ans10. 4**

SOLUTION: Paper-Jee-Main-17-03-0-2021-Evening Shift
PHYSICS
SECTION – A

Sol1.



$$Y = \left(\overline{AB}\right)(A+B) = \left(\overline{A}+\overline{B}\right)(A+B) = \overline{A}B + A\overline{B}\,(\text{XOR Gate})$$

Sol2. If collision is elastic, C comes to rest after collision. When compression in spring is maximum, velocities of A and B are same, (say v).
Using conservation of Mechanical Energy, we can write

$$\frac{1}{2}mv^2 = 2x\frac{1}{2}mv^2 + \frac{1}{2}kx^2 \Rightarrow x = v\sqrt{\frac{m}{2k}}$$

Sol3. $a = \dfrac{g\sin\theta}{1+\dfrac{I}{mr^2}} = \dfrac{10\sin 30^0}{1+\dfrac{2}{5}} = \dfrac{25}{7}\,m/s^2$

$$t = \frac{2v}{a} = \frac{2\times 1}{\frac{25}{7}} = 0.57\,s.$$

Sol4. $S = x - x_0 = \int_0^1 v\,dt = \int_0^1 \left(v_0 + gt + Ft^2\right)dt = v_0 + \dfrac{g}{2} + \dfrac{F}{3}$

Sol5. $X_L = \omega L$ and $i_0 = \dfrac{V_0}{\omega L}$.

If ω is halved, X_L is halved while i_0 is doubled

Sol6. $\dfrac{1}{2}mv_1^2 = hf_1 - \phi$ ....(1)

$\dfrac{1}{2}mv_2^2 = hf_2 - \phi$ ....(2)

With the help of equation (1) and (2), we can write

$$v_1^2 - v_2^2 = \frac{2h}{m}(f_1 - f_2)$$

Sol7. Let $h = 5\,m$ and $e = 0.9 \Rightarrow e^2 = 0.81$

Distance traveled, $d = h + 2e^2h + 2e^4h + \ldots = h + \dfrac{2he^2}{1-e^2} = h\left(\dfrac{1+e^2}{1-e^2}\right)$.

Time taken, $t = \sqrt{\dfrac{2h}{g}} + 2x\sqrt{\dfrac{2e^2h}{g}} + 2x\sqrt{\dfrac{2e^4h}{g}} + \ldots = \sqrt{\dfrac{2h}{g}}\left(1 + 2e + 2e^2 + \ldots\right)$

$$= \sqrt{\frac{2h}{g}}\left(1 + \frac{2e}{1-e}\right) = \sqrt{\frac{2h}{g}}\left(\frac{1+e}{1-e}\right)$$

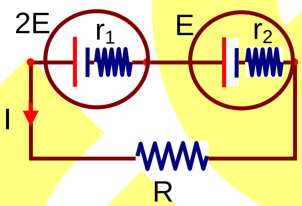
Average speed $= \dfrac{d}{t} = \sqrt{\dfrac{gh}{2}}.\dfrac{1+e^2}{(1+e)^2} = 5\times\dfrac{1.81}{(1.9)^2} = 2.50\,m/s.$

Sol8. $B = 2\times\frac{\mu_0 i}{4\pi r} + \frac{\mu_0 i}{4r} = \frac{\mu_0 i}{4\pi r}(2+\pi)$

Sol9. Frequency of message signal, $f_m = \frac{1.57\times10^8}{2\times3.14} = 2.5\times10^7\,\text{Hz}$

Bandwidth $= 2f_m = 5\times10^7\,\text{Hz} = 50\text{MHz}$

Sol10.



$I = \frac{3E}{R + r_1 + r_2}$

$2E - Ir_1 = 0 \Rightarrow 2E - \frac{3Er_1}{R + r_1 + r_2} = 0 \Rightarrow R = \frac{r_1}{2} - r_2$

Sol11. In purely resistive AC circuit, if $V = V_0\sin(\omega t)$ then

$I = \frac{V_0}{X_C}\sin\left(\omega t - \frac{\pi}{2}\right)$

In LCR series AC circuit, phase difference between current and voltage, $\phi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$.

Sol12. Each vibrational mode contributes two degrees of freedom.

$f = 3 + 3 + 4 = 10$

$\beta = 1 + \frac{2}{f} = 1.2$

Sol13. Balmer series lies in the visible region.

Sol14. Pressure decreases with increase in volume, in both isothermal and adiabatic process.
In adiabatic process, volume decreases and pressure increases with increase in temperature

Sol15. $\omega = 2\pi f = 2\times3.14\times245 = 1.5386\times10^3\,\text{rad/s} \approx 1.5\times10^3\,\text{rad/s}$

$k = \frac{\omega}{v} = \frac{1.53\times10^3}{300} = 5.1\ \text{m}^{-1}$

$A = \frac{0.06}{2} = 0.03\,\text{m}$

Sol16. $B = \frac{-\Delta P}{\frac{\Delta V}{V}} = \frac{\rho g h}{\frac{\Delta V}{V}} = \frac{10^3\times9.8\times2\times10^3}{1.36\times10^{-2}} = 1.44\times10^9\,\text{N/m}^2$

Sol17. $\frac{T}{24} = \left(\frac{3}{12}\right)^{3/2} \Rightarrow T = 3\,\text{hours}$

Sol18. Let, $V_A = 10\,V$, $V_B = x\ V$, $V_C = 0\ V$, and $V_D = y\ V$

Sum of currents away from B is zero, so,

$\frac{x-10}{100} + \frac{x-y}{15} + \frac{x-0}{10} = 0 \Rightarrow 53x - 20y = 30$ ………. (1)

Sum of the currents away from D is zero, so,

$\frac{y-10}{60} + \frac{y-x}{15} + \frac{y-0}{5} = 0 \Rightarrow 17y - 4x = 10$ ………….. (2)

Solving equations (1) and (2), we can write

$x = 0.865$ **and** $y = 0.792$,

$$i_G = \frac{x-y}{15} = 4.87\,mA$$

Sol19. $A = A_0 e^{\frac{-bt}{m}}$

$$\Rightarrow \frac{b}{m}t = \ell n\left(\frac{A_0}{A}\right) \Rightarrow b = \frac{m}{t_0}\ell n\left(\frac{A_0}{A}\right) = \frac{1}{2\times 60}\ell n\left(\frac{12}{6}\right) = 5.775\times 10^{-3}\,kg/s$$

Sol20. $A_1\sqrt{\frac{K_1}{m}} = A_2\sqrt{\frac{K_2}{m}} \Rightarrow \frac{A_1}{A_2} = \sqrt{\frac{K_2}{K_1}}$

Section – B

Sol1. Let charge on C_2 be q μ C.

$$\frac{q}{C_2} = \frac{C_1\times 10 - q}{C_1} \Rightarrow \frac{q}{8} = \frac{2\times 10 - q}{2} \Rightarrow q = 16$$

Sol2. $F_{min} = \frac{\mu mg}{\sqrt{1+\mu^2}} = \frac{\frac{1}{\sqrt{3}}\times 1\times 10}{\sqrt{1+\left(\frac{1}{\sqrt{3}}\right)^2}} = 5N$

Sol3. $x_{CM} = y_{CM} = \frac{4a}{3\pi}$

Sol4. $\mu = \frac{\lambda_{air}}{\lambda} = \frac{3}{2}$

$v = 10m$

$u = -\frac{3}{2}v = -15m$

$$\frac{\mu}{v} - \frac{1}{\mu} = \frac{\mu - 1}{R} \Rightarrow \frac{3/2}{10} - \frac{1}{-15} = \frac{\frac{3}{2}-1}{R} \Rightarrow R = \frac{30}{13}m.$$

Sol5. Assume the part of rope in hand of the boy is vertical. Using FBD of boy and piece of wood together, we can write

$f_S = T$

$R + T = 90 \Rightarrow R = 90 - T.$

For the piece of wood not to move,

$f_S \le \mu_s R \Rightarrow T \le 0.5(90 - T) \Rightarrow T \le 30N$

Sol6. $E^2 \;\alpha\; P$ [P is power of the bulb]

$$\frac{E'}{E} = \left(\frac{60}{100}\right)^{1/2} \Rightarrow E' = \sqrt{\frac{3}{5}}\,E$$

Sol7. Conduction current density,

$$J_C = \frac{E}{\rho} = \frac{V}{\rho d} = \frac{V_0\sin(2\pi ft)}{\rho d}$$

Displacement current density, $J_d = \in\frac{dE}{dt} = \frac{\in}{d}\frac{dV}{dt} = \frac{2\pi f\in}{d}V_0\cos(2\pi ft)$

$$\frac{J_c}{J_d} = \frac{\tan(2\pi ft)}{2\pi f \in \rho} = \frac{\tan\left(2\pi \times \frac{900}{800}\right)}{2\pi \times 9 \times 10^2 \times 80 \in_0 \times 0.25} = 10^6$$

Sol8. $4t = 100 \times \frac{4}{3}\pi \times \frac{3}{40\pi} \times 10^{-9} \times 0.01 \Rightarrow t = 2.5 \times 10^{-11} cm = 25 \times 10^{-14} m.$

Sol9. $\frac{mv^2}{r} = \left|\frac{dU}{dr}\right| = 4|U_0|r^3$ ....(1)

$mvr = \frac{nh}{2\pi}$ .... (2)

With the help of equation (1) and (2), we can write

$$r = \left(\frac{nh}{4\pi\sqrt{|U_0|}}\right)^{\frac{1}{3}} \Rightarrow r \ \alpha \ n^{\frac{1}{3}}$$

Sol10. $\phi = \frac{2}{5} \times 4 \times 10^3 \times 0.4 = 640 Nm^2C^{-1}$

CHEMISTRY

SECTION – A

Sol1. $-SO_3H$ is cation exchanger and $-NH_2$ is anion exchanger.

Sol2. NO, N_2O oxides are neutral oxide.
NO_2, N_2O_3 are acidic oxides

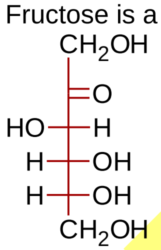
Sol3. Eutrophication is the process in which a water body become overly enriched with nutrients, leading to plentiful growth of simple plant life.

Sol4. $Cr(24) \longrightarrow [Ar]4s^1 3d^5$
Chromium shows common oxidation state from +2 to +6.

Sol5.

Haematite :	Fe_2O_3
Bauxite :	$Al_2O_3.xH_2O$
Magnetite :	Fe_3O_4
Malachite :	$CuCO_3$. $Cu(OH)_2$

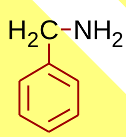
Sol6. Fructose is a ketohexose.



Sol7. Sucralose : Artificial sweetner
Glyceryl ester of stearic acid: Sodium stearate which is synthetic detergent
Sodium benzoate : Food preservative
Bithionol : Antiseptic

Sol8. For coagulation of negative sol, the species Ba^{2+} that has the highest flocculating power.

Sol9.

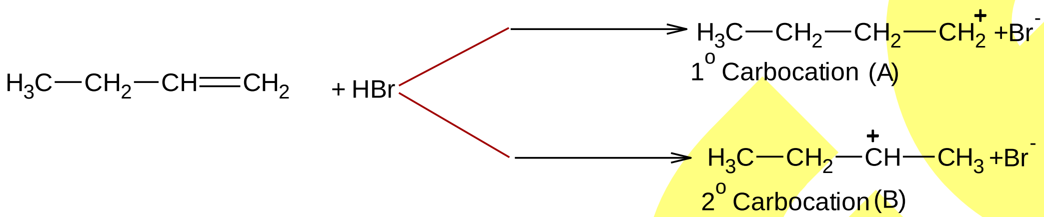


Kjeldahl method is not applicable to
→compound containing nitrogen in nitro group
→Azo group
→Pyridine

Sol10. Diagonal relationship shows in period (2) and period (3)

Li – Na not shows diagonal relationship

Sol11.



B is more stable than A, so Ea for second step is lower hence B form faster than A.

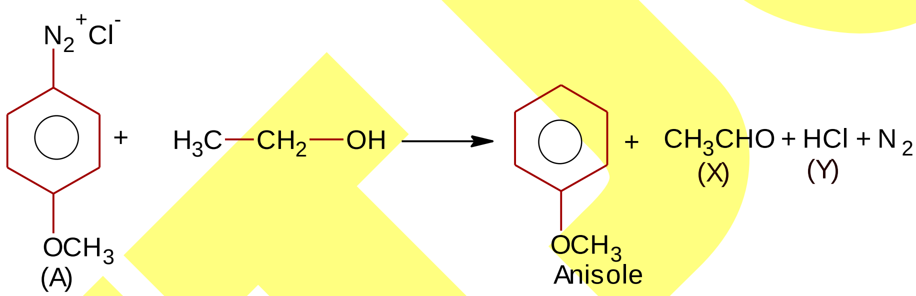
Sol12. $NaCl + H_2O + NH_3 + CO_2 \rightarrow NH_4Cl + NaHCO_3$

$2NaHCO_3 \rightarrow Na_2CO_3 + CO_2 + H_2O$

$2NH_4Cl + Ca(OH)_2 \rightarrow CaCl_2 + 2NH_3 + H_2O$

$CaCl_2$ is byproduct.

Sol13.



Sol14. Water $\xrightarrow{0^oC}$ ice ΔS = -ve

$H_2O(\ell) \xrightarrow{-10^oC}$ ice ΔS = -ve

$N_2(g) + 3H_2(g) \rightarrow 2NH_3(g)$ ΔS = -ve

Adsorption ΔS = -ve

Sol15. Ambident nucleophiles are $CN^- \,\&\, NO_2^-$.

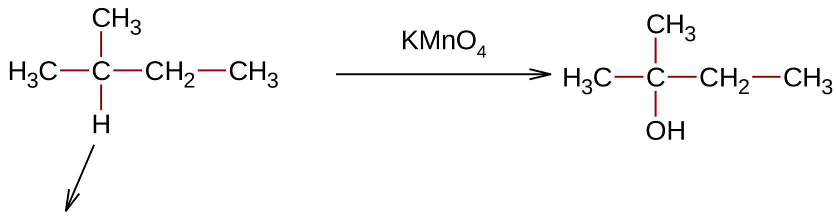
Sol16. $[Co(NH_3)_6][Cr(CN)_6]$ $\Rightarrow$ Co-ordination isomerism

$[Co(NH_3)_3(NO_2)_3]$ $\Rightarrow$ Linkage isomerism

$[Cr(H_2O)_6]Cl_3$ $\Rightarrow$ Solvate isomerism

Cis – $[CrCl_2(Ox)_2]^{3-}$ $\Rightarrow$ Optical isomerism

Sol17.



Tertiary Hydrogen
$KMnO_4$ oxidises alkanes with 3°H to alcohols.
n-alkanes do not contain 3°H so do not oxidized by $KMnO_4$.

Sol18. $\underset{\text{Sucrose}}{C_{12}H_{22}O_{11}} + H_2O \xrightarrow{\text{Invertase}} \underset{\text{Glucose}}{C_6H_{12}O_6} + \underset{\text{Fructose}}{C_6H_{12}O_6}$

$\underset{\text{Glucose}}{C_6H_{12}O_6} + H_2O \xrightarrow{\text{Zymase}} 2C_2H_5OH + 2CO_2$

Sol19. Para – Toluene sulphonyl chloride
1° amine → Precipitate that is soluble with NaOH
2° amine → Preciptate that is insoluble with NaOH
3° amine → No reaction.

Sol20.

$\overset{-}{N}=\overset{+}{N}=\overset{-}{N}$
Linear

$\left.\begin{matrix} O_3 \\ NO_2 \\ Cl_2O \end{matrix}\right\}$ Bent-shape

SECTION – B

Sol1. $2A + B_2 \longrightarrow 2AB$

Rate, $R = K[A]^2[B_2]$

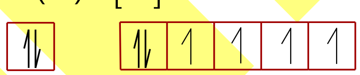
On reducing V by factor of 3, concentration becomes thrice.

$R' = K(3[A])^2(3[B_2])$

$= 27K[A]^2[B_2]$

$R' = 27R$

Sol2. $Fe(26) \to [Ar]4s^2 3d^6$



n = 4 (unpaired electrons)

$\mu = \sqrt{n(n+2)}$ B.M

$= \sqrt{4(4+2)}$

$= \sqrt{4 \times 6}$

$= \sqrt{24}$

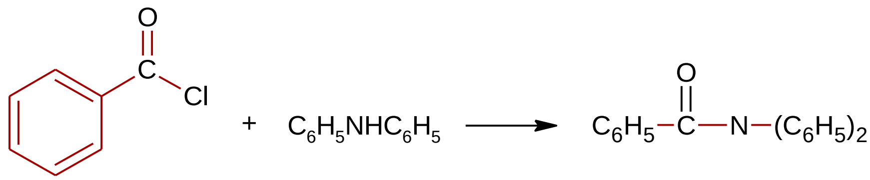
= 4.89B.M

$= 48.9 \times 10^{-1}$ BM

Ans $= 49$ (Rounded off)

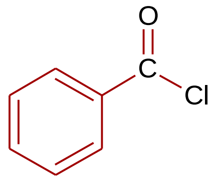
Sol3.

Limiting reagent



0.140 g 0.388 g 0.210 g

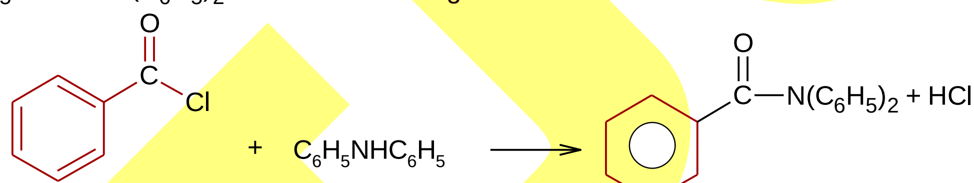
M.W⇒



⟹ 140.5 gm/ mol

$C_6H_5NHC_6H_5$ ⟹ 169 gm / mol

$C_6H_5-\overset{O}{\overset{||}{C}}-N-(C_6H_5)_2$ ⟹ 273 gm / mol



Mole $\frac{0.140}{140.5}$ $\frac{0.388}{169}$

Wt of amide product produced $\Rightarrow \frac{0.140}{140.5}\times 273 = 0.272g$

$\%Yield = \frac{0.210}{0.272}\times 100$

= 77.20%

Ans =77 (Rounded off)

Sol4. $\kappa = \frac{G^*}{R}$

$G^* \to$ cell constant

$G^* = \kappa \times R$

Since G^* is constant

So; $(\kappa \times R)_{KCl} = (\kappa \times R)_{HCl}$

$0.14 \times 4.19 = \kappa \times 1.03$

κ for HCl solution $= 0.569 Sm^{-1} = 56.9 \times 10^{-2} Sm^{-1}$

Ans = 57 (Rounded off)

Sol5. $FeCl_3 + 3H_2C_2O_4 + 3KOH \rightarrow K_3\left[Fe\left(C_2O_4\right)_3\right] + 3HCl + 3H_2O$
(A)
Secondary valency $\Rightarrow$ 6

Sol6. $K_4\left[Fe\left(CN\right)_6\right] \rightleftharpoons 4K^+ + \left[Fe\left(CN\right)_6\right]^{4-}$
$\alpha = 0.4$
$\%W/W = 18.1\%$
$\rho_{H_2O} = 1g/cm^3$
$i \text{ for } K_4\left[Fe\left(CN\right)_6\right]; \alpha = \frac{i-1}{n-1}$
$0.4 = \frac{i-1}{5-1}$
$i = 2.6$
Molality of solution;
$m = \frac{18.1}{M \times \left(100 - 18.1\right)} \times 1000$
M – Molar mass of A
ΔT_b for $K_4\left[Fe\left(CN\right)_6\right]$ solution = ΔT_b for another solution
Using, $\Delta T_b = iK_b m$
$2.6 \times K_b \times 1 = 1 \times K_b \times \frac{18.1}{M \times \left(100 - 18.1\right)} \times 1000$
$2.6 = \frac{18.1 \times 1000}{M \times 81.9}$
M = 85.

Sol7. PV = nRT
$1 \times 0.020 = n \times 0.0831 \times 273$
$n = 8.8 \times 10^{-4}$

P = 1 bar
V = 20ml
= 0.020L
R = 0.083 L bar mol^{-1} K^{-1}
N_A = 6.023×10^{23}

$n = \frac{N}{N_A}$
$N = 8.8 \times 10^{-4} \times 6.022 \times 10^{23}$
$= 5.3 \times 10^{20}$ molecules of Cl_2
Cl atom $= 5.3 \times 10^{20} \times 2$
$= 1.06 \times 10^{21}$
Ans = 1 (Rounded off)

Sol8. $N_2O_4(g) \rightleftharpoons 2NO_2(g)$ $\quad K_c = 20.4$

$K_p = 600.1$

Using, $K_p = K_c(RT)^{\Delta ng}$ $\quad \Delta n_g = 2-1 = 1$

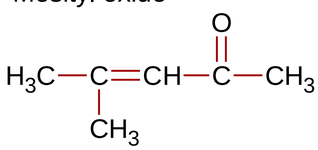
$600.1 = 20.4(0.0831 \times T)^1$

$T = \dfrac{600.1}{20.4 \times 0.0831}$

$= 253.99\,K$

$= 254K$ (Rounded off)

Sol9. Mesityl oxide



C-C sigma bonds are 5.

Sol10. Here; 100 mol of KBr is doped with 10^{-5} mol of $SrBr_2$

1mol KBr contains 10^{-7} mol of $SrBr_2$

119g KBr contains 10^{-7} mol of $SrBr_2$

1g KBr contains $= \dfrac{10^{-7}}{119}$ mol $SrBr_2$

1 Sr^{2+} produces 1 cation vacancy

So; number of cation vacancies $= \dfrac{10^{-7}}{119} \times 6.023 \times 10^{23}$

$= 5.06 \times 10^{14}$

Ans = 5 (Rounded off)

MATHEMATICS
SECTION – A

Sol1.

$S_1 : |z-1| \le \sqrt{2}$

locus of z is circle : $(x-1)^2 + y^2 = 2$

centre $(1, 0)$, radius $= \sqrt{2}$

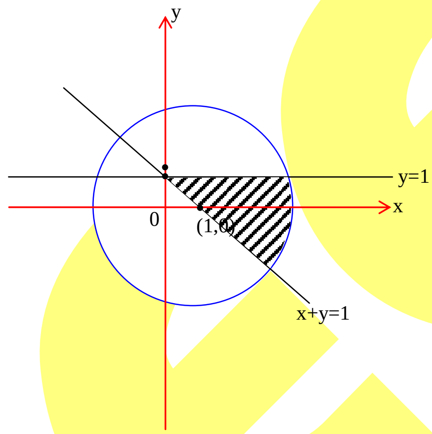
$S_2 : \mathrm{Re}((1-i)z) \ge 1$

locus of z is a line $x + y \ge 1$

$S_3 : \mathrm{Im}(z) \le 1$

locus of z is a line $y \le 1$

Hence set $S_1 \cap S_2 \cap S_3$ has infinitely many elements.



Sol2. Truth table of $(p \wedge q) \to (p \to q)$

p	q	$p \wedge q$	$p \to q$	$(p \wedge q) \to (p \to q)$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

Sol3. $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = (-1-x)\hat{i} + (2-y)\hat{j} + (3x+1)\hat{k}$

$|\overrightarrow{PQ}| = \sqrt{20} \Rightarrow (1+x)^2 + (2-y)^2 + (3x+1)^2 = 20$ …………(i)

$\because \overrightarrow{OP}.\overrightarrow{OQ} = 0 \Rightarrow -x + 2y - 3x = 0 \Rightarrow y = 2x$ ………(ii)

From (i) & (ii), $x = \pm 1, y = \pm 2$

$\because \overrightarrow{OR}$ is coplanar with $\overrightarrow{OP}$ and $\overrightarrow{OQ}$

$$\therefore \left[\overrightarrow{OR}\ \overrightarrow{OP}\ \overrightarrow{OQ}\right] = 0 \Rightarrow \begin{vmatrix} 3 & z & -7 \\ x & y & -1 \\ -1 & 2 & 3x \end{vmatrix} = 0$$

$\Rightarrow 3(3xy + 2) + z(1 - 3x^2) - 7(2x + y) = 0$ ……..(iii)

Put $x = 1, y = 2$ in (iii), $z = -2$

$\therefore x^2 + y^2 + z^2 = 1 + 4 + 4 = 9$

Sol4. $2\left(x^2 + x^{5/4}\right)dy - y\left(x + x^{1/4}\right)dx = 2x^{9/4}dx, x > 0$

$2x^{\frac{5}{4}}\left(x^{3/4} + 1\right)dy - yx^{1/4}\left(x^{3/4} + 1\right)dx = 2x^{9/4}dx$

$2x\left(x^{3/4} + 1\right)dy - y\left(x^{3/4} + 1\right)dx = 2x^2dx$

$$\frac{dy}{dx} - \frac{y}{2x} = \frac{x}{x^{3/4} + 1}$$

$$\therefore \text{I.F} = e^{\int \frac{-1}{2x}dx} = e^{\frac{-1}{2}\ln x} = \frac{1}{\sqrt{x}}$$

$$\frac{y}{\sqrt{x}} = \int \frac{\sqrt{x}dx}{x^{3/4}+1} = \int \frac{x^{1/2}}{x^{3/4}+1}dx$$

Put $x^{3/4}+1 = t \Rightarrow \frac{3}{4}x^{-\frac{1}{4}}dx = dt$

$$\frac{y}{\sqrt{x}} = \frac{4}{3}\left[x^{\frac{3}{4}}+1-\ln\left(1+x^{\frac{3}{4}}\right)\right]+C$$

$$= \frac{4}{3}\left[t-\ln t\right]+C$$

$$\frac{y}{\sqrt{x}} = \frac{4}{3}\left[x^{\frac{3}{4}}+1-\ln\left(1+x^{\frac{3}{4}}\right)\right]+C$$

Put $x = 1, y = 1-\frac{4}{3}\ln 2$

$$1-\frac{4}{3}\ln 2 = \frac{8}{3}-\frac{4}{3}\ln 2 + C$$

$$C = -\frac{5}{3}$$

$$\therefore \frac{y}{4} = \frac{4}{3}\left[\left(2^4\right)^{\frac{3}{4}}+1-\ln\left(1+\left(2^4\right)^{\frac{3}{4}}\right)\right]-\frac{5}{3}$$

$$= \frac{4}{3}\left[8+1-\ln\left(1+8\right)\right]-\frac{5}{3}$$

$$\frac{y}{4} = -12-\frac{5}{3}-\frac{4}{3}\ln 9 = \left(\frac{31}{3}-\frac{8}{3}\ln 3\right)$$

$$y = 4\left(\frac{31}{3}-\frac{8}{3}\ln 3\right)$$

Sol5. P(O at even place) = $\frac{1}{2}$, P('O' at odd place) = $\frac{1}{3}$

P(1 at even place) = $1-\frac{1}{2}=\frac{1}{2}$, P(1 at odd place) = $1-\frac{2}{3}=\frac{2}{3}$

$\therefore$ P(10 is followed by 01) = $\frac{2}{3}\times\frac{1}{2}\times\frac{1}{3}\times\frac{1}{2}+\frac{1}{2}\times\frac{1}{3}\times\frac{1}{2}\times\frac{2}{3}=\frac{1}{18}+\frac{1}{18}=\frac{1}{9}$

Sol6. $f(x) = \begin{cases} \left(2-\sin\frac{1}{x}\right)|x|, & x \neq 0 \\ 0, & x = 0 \end{cases}$

$$= \begin{cases} x\left(\sin\frac{1}{x}-2\right) & , \quad x<0 \\ 0 & , \quad x=0 \\ x\left(2-\sin\frac{1}{x}\right) & , \quad x>0 \end{cases}$$

$$f'(x)=\begin{cases}-2+\sin\frac{1}{x}-\frac{1}{x}\cos\frac{1}{x}, & x<0\\ 2-\sin\frac{1}{x}+\frac{1}{x}\cos\frac{1}{x}, & x>0\end{cases}$$

$f'(0)$ is undefined.

Sol7. $\cos x(3\sin x+\cos x+3)dy=(1+y\sin z(3\sin x+\cos x+3)dx$

$$\frac{dy}{dx}-\frac{\sin x}{\cos x}y=\frac{1}{\cos x(3\sin x+\cos x+3)}$$

$$\therefore \text{I.F}=e^{-\int\frac{\sin x}{\cos x}dx}=\cos x$$

$$\therefore y\cos x=\int\frac{dx}{3\sin x+\cos x+3}=\int\frac{\sec^2\frac{x}{2}dx}{6\tan\frac{x}{2}+1-\tan^2\frac{x}{2}+3+3\tan^2\frac{x}{2}}$$

$$=\int\frac{\sec^2\frac{x}{2}dx}{2\tan^2\frac{x}{2}+6\tan\frac{x}{2}+4}\qquad \text{put }\tan\frac{x}{2}=t\ \ \sec^2\frac{x}{2}dx=2dt$$

$$=\int\left(\frac{1}{t+1}-\frac{1}{t+2}\right)dt=\ln|t+1|-\ln|t+2|+\ln c$$

$$\therefore y\cos x=\ln\left|\frac{\tan\frac{x}{2}+1}{\tan\frac{x}{2}+2}\right|+\ln c\ \ldots\ldots\ldots\ldots(i)$$

Put $y(0)=0$ in (i)

$c=2$

$$\therefore y\cos x=\ln\left(\left|\frac{\tan\frac{x}{2}+1}{\tan\frac{x}{2}+2}\right|\times 2\right)$$

$$\therefore y\left(\frac{\pi}{3}\right)\times\frac{1}{2}=\ln\left(\left(\frac{\frac{1}{\sqrt3}+1}{\frac{1}{\sqrt3}+2}\right)\times 2\right)=\ln\left(\left(\frac{\sqrt3+2}{2\sqrt3+1}\right)\right)$$

$$\therefore y\left(\frac{\pi}{3}\right)=2\ln\left(\frac{2\sqrt3+2}{2\sqrt3+1}\right)=2\ln\left(\frac{2\sqrt3+10}{11}\right)$$

Sol8. $\underset{\theta\to0}{\text{Lt}}\dfrac{\tan(\pi\cos^2\theta)}{\sin(2\pi\sin^2\theta)}=\underset{\theta\to0}{\text{Lim}}$

$$=\underset{\theta\to0}{\text{Lim}}\frac{\tan(\pi-\pi\sin^2\theta)}{2\sin(\pi\sin^2\theta)\cos(\pi\sin^2\theta)}=\frac{-1}{2}\underset{\theta\to0}{\text{Lim}}\frac{\tan(\pi\sin^2\theta)}{\sin(\pi\sin^2\theta)\times1}=-\frac{1}{2}$$

Sol9. $\underset{n\to\infty}{\text{Lt}} \dfrac{[r]+[2r]+.....+[nr]}{n^2}$

$= r-1<[r]\le r$

$2r-1<[2r]\le 2r$

$3r-1<[3r]\le 3r$

adding $\dfrac{nr-1<[nr]\le nr}{\dfrac{n(n+1)}{2}-n<[r]+[2r]+.....+[nr]\le \dfrac{n(n+1)r}{2}}$

$\therefore \dfrac{n(n+1)r}{2n^2}-\dfrac{1}{n}<\dfrac{[r]+[2r]+....+[nr]}{n^2}\le\dfrac{n(n+1)r}{2n^2}$

$\therefore \underset{n\to\infty}{\text{Lt}}\left(\dfrac{\left(1+\dfrac{1}{n}\right)r}{2}-\dfrac{1}{h}\right)<\underset{n\to\infty}{\text{Lt}}\dfrac{[r]+[2r]+....+[nr]}{n^2}\le\underset{n\to\infty}{\text{Lt}}\dfrac{\left(1+\dfrac{1}{2}\right)r}{2}$

By sandwich theorem, $\therefore \underset{n\to\infty}{\text{Lt}}\dfrac{[r]+[2r]+.....+[nr]}{n^2}=\dfrac{r}{2}$

Sol10. $\sin^{-1}\left[x^2+\dfrac{1}{3}\right]+\cos^{-1}\left[x^2-\dfrac{2}{3}\right]=x^2$

$\left[x^2+\dfrac{1}{3}\right]=\left[x^2-\dfrac{2}{3}+1\right]=\left[x^2-\dfrac{2}{3}\right]+1=0,1$

$\Rightarrow\left[x^2-\dfrac{2}{3}\right]=0,-1\Rightarrow\left[x^2+\dfrac{1}{3}\right]=1,0$

$\therefore$ LHS = $\{\pi\}$ and RHS $=x^2\in[0,1]$

$\therefore \text{LHS}\neq\text{RHS}$

Hence equation has no solution.

Sol11. $f(x)=e^{-x}\sin x$

$F(x)=\int_0^x f(t)dt\Rightarrow F'(x)=f(x)$

$\Rightarrow F'(x)=e^{-x}\sin x$

$\therefore I=\int_0^1(F'(x)+f(x))e^x dx=2\int_0^1\sin x dx=2(1-\cos 1)$

$=2\left(1-\left(1-\dfrac{1}{2!}+\dfrac{1}{4!}-\dfrac{1}{6!}+.....\right)\right)=1-\dfrac{1}{12}+\dfrac{1}{360}-....$

$\therefore \dfrac{330}{360}<I<\dfrac{331}{360}$

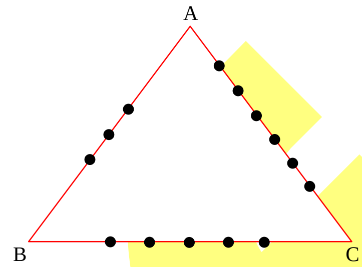
$=\dfrac{360-30+1}{360}=\dfrac{331}{360}$

Sol12. Total number of triangle

$= {}^3C_1 \times {}^5C_1 \times {}^6C_1 + {}^3C_2\left({}^5C_1 + {}^6C_1\right) + {}^5C_2\left({}^6C_1 + {}^3C_1\right)$

$+ {}^6C_2\left({}^3C_1 + {}^5C_1\right)$

$= 90 + 33 + 90 + 120 = 333$

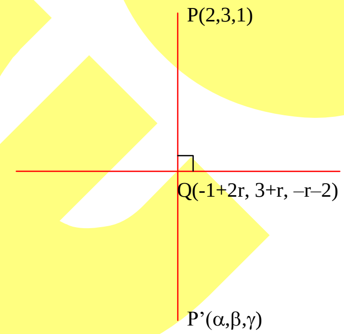


<u>IInd Method</u>

Required number of triangles

$= {}^{14}C_3 - {}^3C_3 - {}^5C_3 - {}^6C_3 = 364 - 1 - 10 - 20 = 333$

Sol13. $\frac{x+1}{2} = \frac{y-3}{1} = \frac{z+2}{-1} = r$



$\therefore$ Direction ratios of $PQ = 2r-3, r, -3-r$

$\therefore 2(2r-3) + r + 3 + r = 0$

$6r = 3 \Rightarrow r = \frac{1}{2}$

$\therefore Q\left(0, \frac{7}{2}, \frac{-5}{2}\right)$

$\therefore P'(-2, 4, -6)$

Equation of plane containing the line $\frac{x-2}{3} = \frac{1-y}{2} = \frac{z+1}{1}$ is

$a(x-2) + b(y-1) + c(z+1) = 0$ ……..(i)

Plane passes through $(-2, 4, -6)$

$-4a + 3b - 5c = 0 \Rightarrow 4a - 3b + 5c = 0$ ……(ii)

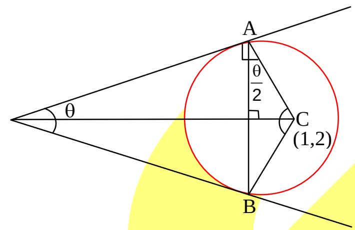
and also $3a - 2b + c = 0$ ……(iii)

solving (ii) & (iii), $\frac{a}{7} = \frac{b}{11} = \frac{c}{1}$

$\therefore$ From (i), $7x + 11y + z = 24$

$\therefore \alpha + \beta + \gamma = 7 + 11 + 1 = 19$

Sol14. $x^2 + y^2 - 2x - 4y + 4 = 0$



$\theta = \tan^{-1}\frac{12}{5}$

$\tan\theta = \frac{12}{5}$

$\frac{2\tan\frac{\theta}{2}}{1-\tan\frac{2\theta}{2}} = \frac{12}{5}$

$\Rightarrow \tan\frac{\theta}{2} = \frac{2}{3}, -\frac{3}{2}$ (rejected)

$\Rightarrow \tan\frac{\theta}{2} = \frac{AC}{AP} = \frac{2}{3}$

$\frac{\text{area of triangle } \Delta PAB}{\text{area of triangle } \Delta ABC} = \frac{\frac{1}{2}AP^2\sin\theta}{\frac{1}{2}AC^2\sin(\pi-\theta)} = \left(\frac{AP}{AC}\right)^2 = \frac{9}{4}$

Sol15. $\because$ x,y,z are in A.P
$\therefore 2y = x + z$

Given $\begin{vmatrix} 3 & 4\sqrt{2} & x \\ 4 & 5\sqrt{2} & y \\ 5 & k & z \end{vmatrix} = 0$

$R_1 \to R_1 + R_3 - 2R_2$

$\begin{vmatrix} 0 & 4\sqrt{2}+k-10\sqrt{2} & 0 \\ 4 & 5\sqrt{2} & y \\ 5 & k & z \end{vmatrix} = 0$

$\Rightarrow \left(k - 6\sqrt{2}\right)(5y - 4z) = 0$

$\Rightarrow k = 6\sqrt{2}$ or $y = \frac{4z}{5}$ (not possible as $x \neq 3d$)

$\Rightarrow k = 6\sqrt{2} \Rightarrow k^2 = 72$

Sol16. $y^2 = 4x - 20$ ……..(i) and $\frac{x^2}{2} + \frac{y^2}{b} = 1$……….(ii)
Equation of tangent to the parabola is

$2y = 4\left(\frac{x+6}{2}\right) - 20$

$\Rightarrow y = x - 4$ ……..(iii)

From (ii) & (iii)

$\frac{x^2}{2} + \frac{(x-4)^2}{b} = 1 \Rightarrow \left(\frac{1}{2} + \frac{1}{b}\right)x^2 - \frac{8}{b}x + \frac{16}{b} - 1 = 0$

This is the quadratic equation in x.
For condition of tangency, D = 0

$$\frac{64}{b^2}-4\left(\frac{1}{2}+\frac{1}{b}\right)\left(\frac{16}{b}-1\right)=0\Rightarrow b=14$$

Sol17. $2\tan x=\frac{\pi}{2}-x$

Number of points of intersection of $y=\tan x$ and $y=\frac{\pi}{4}-\frac{x}{2}$ is 3 in $\left[0,2\pi\right]$.

Total number of solution is 3.

Sol18. $\int_0^{10}\frac{\left[\sin 2\pi x\right]}{e^{x-[x]}}dx=10\int_0^{1}\frac{\left[\sin 2\pi x\right]}{e^{x-[x]}}dx$

[$\because$ period of $\sin 2\pi x$ and $\{x\}$ is 1]

$$=10\int_0^{1/2}o\,dx+10\int_{1/2}^{1}\frac{-1}{e^x}dx=-10\left[\frac{e^{-x}}{-1}\right]_{1/2}^{1}=10\left(\frac{1}{e}-\frac{1}{e^{1/2}}\right)$$

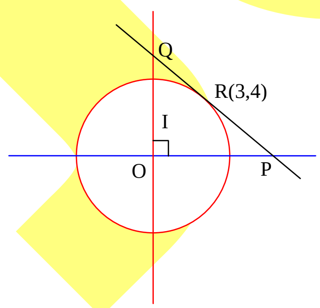
$=10e^{-1}-10e^{-1/2}$

$\therefore \alpha=10,\beta=-10\ \&\ \gamma=0$

$\therefore \alpha+\beta+\gamma=10-10+0=0$

Sol19. Equation of tangent at P(3, 4) is

$T=0\Rightarrow 3x+4y=25$



$\therefore P\left(\frac{25}{3},0\right)$ and $Q\left(0,\frac{25}{4}\right)$

$$I\left(\frac{0+0+\frac{25}{3}\times\frac{25}{4}}{\frac{25}{3}+\frac{25}{4}+\frac{125}{12}},\frac{\frac{25}{4}\times\frac{25}{3}+0+0}{\frac{25}{3}+\frac{25}{4}+\frac{125}{12}}\right)$$

$\therefore I\left(\frac{25}{12},\frac{25}{12}\right)$

$\therefore OI=r=\sqrt{\frac{625}{144}+\frac{625}{144}}=\frac{25}{12}\sqrt{2}$

$\Rightarrow r^2=\frac{625}{72}$

Sol20. $\sum_{r=0}^{6}{}^6C_r\cdot{}^6C_{6-r}=\sum_{r=0}^{6}\left({}^6C_r\right)^2$

$(1+x)^6={}^6C_0+{}^6C_1+x+....+{}^6C_0x^6$

$(x+1)^6={}^6C_0x^6+{}^6C_1x^5+....+{}^6C_6$

$(1+x)^{12}=\left({}^6C_0+{}^6C_1x+....{}^6C_6x^6\right)\left({}^6C_0x^6+{}^6C_1x^5+....+{}^6C_6\right)$

= weff of x^6 in $(1+x)^{12}={}^{12}C_6=924$

SECTION – B

Sol1. $In=\int_1^e x^{19}\left(\log|x|\right)^n dx, n\in N$

Put $\ln|x| = t \Rightarrow x = e^t, dx = e^t dt$

$\therefore In = \int_0^1 e^{20t} t^n dt$

$= \left(\frac{e^{20}}{20} t^n\right)_0^1 - \frac{n}{20}\int_0^1 e^{20t}.\, t^{n-1} dt$

$\therefore In = \frac{e^{20}}{20} - \frac{n}{20} I_{n-1} \Rightarrow 20I_n = e^{20} - nI_{n-1}$

$n = 10,\ 20I_{10} = e^{20} - 10I_9$ ....(i)

$n = 9,\ 20I_9 = e^{20} - 9I_8$ ....(ii)

From (i) & (ii), $20I_{10} = 10I_9 + 9I_8$

$\therefore \alpha = 10, \beta = 9 \ \therefore \alpha - \beta = 1$

Sol2. $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$AB = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} a\alpha + b\beta \\ c\alpha + d\beta \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$

$\Rightarrow a\alpha + b\beta = \alpha$ ....(i)

$c\alpha + d\beta = \beta$ ......(ii)

Solving (i) & (ii), we get $\frac{a-1}{c} = \frac{b}{d-1} \Rightarrow ad - bc = 2020$

Sol3. $\because 1, \log_{10}\left(4^x - 2\right), \log_{10}\left(4^x + \frac{18}{5}\right)$ are in A.P.

$\therefore 10,\ 4^x - 2,\ 4^x + \frac{18}{5}$ are in G.P.

$\therefore \left(4^x - 2\right)^2 = 10\left(4^x + \frac{18}{5}\right) \Rightarrow x = 2$

$\therefore \begin{vmatrix} 3 & 1 & 4 \\ 1 & 0 & 2 \\ 2 & 1 & 0 \end{vmatrix} = 3(-2) + 1(4-0) + 4(1-0) = 2$

Sol4. Since orthocenter and circumcentre both lies on y-axis then centroid will also lies on y-axis.

$\therefore \cos\alpha + \cos\beta + \cos\gamma = 0$

$\therefore \cos^3\alpha + \cos^3\beta + \cos^3\gamma = 3\cos\alpha\cos\beta\cos\gamma$

$\therefore \frac{\cos 3\alpha + \cos 3\beta + \cos 3\gamma}{\cos\alpha\cos\beta\cos\gamma}$

$= \frac{4\left(\cos^3\alpha + \cos^3\beta + \cos^3\gamma\right) - 3\left(\cos\alpha + \cos\beta + \cos\gamma\right)}{\cos\alpha\cos\beta\cos\gamma} = \frac{4 \times 3\cos\alpha\cos\beta\cos\gamma}{\cos\alpha\cos\beta\cos\gamma} = 12$

Hence $\left(\frac{\cos 3\alpha + \cos 3\beta + \cos 3\gamma}{\cos\alpha\cos\beta\cos\gamma}\right)^2 = 12^2 = 144$

Sol5. Let $P(\alpha, \beta, \gamma)$

A/q, $\left(\frac{\alpha+\beta+\gamma}{\sqrt{3}}\right)^2+\left(\frac{\ell\alpha-n\gamma}{\sqrt{\ell^2+n^2}}\right)^2+\left(\frac{\alpha-2\beta+\gamma}{\sqrt{6}}\right)^2=9$

$\therefore$ locus of (α,β,γ) is

$$\left(\frac{x+y+z}{\sqrt{3}}\right)^2+\left(\frac{lx-nz}{\sqrt{\ell^2+n^2}}\right)^2+\left(\frac{x-2y+z}{\sqrt{6}}\right)^2=9$$

$$\left(\frac{1}{2}+\frac{\ell^2}{\ell^2+n^2}\right)x^2+y^2+\left(\frac{1}{2}+\frac{n^2}{\ell^2+n^2}\right)z^2+2\left(\frac{1}{2}-\frac{\ell n}{\ell^2+n^2}\right)xz=9$$

A/q, $\frac{1}{2}+\frac{\ell^2}{\ell^2+n^2}=1$ and $\frac{1}{2}+\frac{n^2}{\ell^2+n^2}=1$ and $\frac{1}{2}-\frac{\ell n}{\ell^2+n^2}=0$

$\therefore \ell^2+n^2-2\ell n=0 \Rightarrow (\ell-n)^2=0 \Rightarrow \ell-n=0$

Sol6. Let $\vec{x}=m\left(\vec{a}+\lambda\vec{b}\right)$

A/q, $\therefore m\left(\vec{a}+\lambda\vec{b}\right).\left(3\hat{i}+2\hat{j}-\hat{k}\right)=0 \Rightarrow \lambda=-\frac{3}{8}$

Projection of $\vec{x}$ on $\vec{a}$ = $\vec{x}.\hat{a}=\frac{\vec{x}.\vec{a}}{|\vec{a}|}=\frac{17\sqrt{6}}{2}$

$$\Rightarrow m\left\{\frac{\left(\vec{a}+\lambda\vec{b}\right).\vec{a}}{\sqrt{6}}\right\}=\frac{17\sqrt{6}}{2} \Rightarrow m\left(6-\frac{3}{8}(-1)\right)=\frac{17\times 6}{2} \Rightarrow m=8$$

$$\therefore \vec{x}=8\left(\frac{13}{8}\hat{i}-\frac{14}{8}\hat{j}+\frac{11}{8}\hat{k}\right)=13\hat{i}-14\hat{j}+11\hat{k}$$

$\therefore |\vec{x}|^2=169+196+121=486$

Sol7. $f(x)=ax^2+bx+c \Rightarrow f(-1)=2$

$\Rightarrow a-b+c=2$

$f'(x)=2ax+b \Rightarrow f(-1)=1$

$\Rightarrow -2a+b=1$

$f''(x)=2a \Rightarrow f''(-1)=\frac{1}{2}$

$\Rightarrow a=\frac{1}{4}, b=\frac{3}{2}, c=\frac{13}{4}$

$f(x)=\frac{1}{4}\left(x^2+6x+13\right)$

$f(1)=\frac{1}{4}(1+6+13)=5$

Sol8. Let $x_1,x_2,......x_{2n}$ be the first 2n observations and $y_1,y_2,.....y_n$ be the last n observations

$\therefore \frac{\Sigma x_i}{2n}=6$ and $\frac{\Sigma y_i}{n}=3$

$\therefore \Sigma x_i=12n$ ....(i) and $\Sigma y_i=3n$ ....(ii)

$$\therefore \frac{\Sigma x_i + \Sigma y_i}{3n} = 5 \Rightarrow \Sigma x_i + \Sigma y_i = 15n$$

$$\frac{\Sigma x_i^2 + \Sigma y_i^2}{3n} - 5^2 = 4 \Rightarrow \Sigma x_i^2 + \Sigma y_i^2 = 87n$$

$$\text{A/q } \frac{\Sigma(x_i+1)+\Sigma(y_i-1)}{3n} = \frac{15n+2n-n}{3n} = \frac{16}{3} \text{ and } \frac{\Sigma(x_i+1)^2+\Sigma(y_i-1)^2}{3n} - \left(\frac{16}{3}\right)^2$$

$$= \frac{87n + 2(9n) + 3n}{3n} - \left(\frac{16}{3}\right)^2 = \frac{68}{9}$$

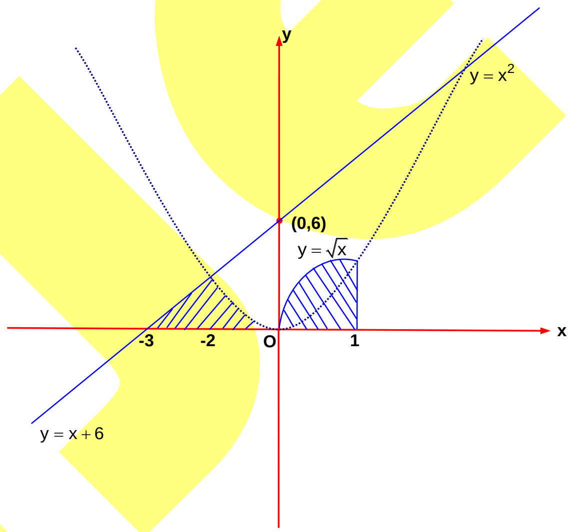
$$\therefore \frac{68}{9} = k \Rightarrow 9k = 68$$

Sol9. Area of shaded region,

$$A = \int_{-3}^{-2}(x+6)dx + \int_{-2}^{0} x^2 dx + \int_{0}^{1}\sqrt{x}dx$$

$$= \frac{7}{2} + \frac{8}{3} + \frac{2}{3} = \frac{41}{6}$$

$$\therefore 6A = 41$$



Sol10. $T_{r+1} = {}^nC_r\, x^{n-r}\left(\frac{a}{x^2}\right)^r$

$= {}^nC_r\, x^{n-r}.\, a^r.\, x^{-2r} = {}^nC_r x^{n-3r} a^r$

$\therefore T_3 = {}^nC_2 a^2 x^{n-6},\ T_4 = {}^nC_3 a^3 x^{n-9},\ T_5 = {}^nC_4 a^4 x^{n-12}$

$\therefore$ A/q $\frac{{}^nC_2 a^2}{{}^nC_3 a^3} = \frac{12}{8} = \frac{3}{2} \Rightarrow a(n-2) = 2$ ....(i)

and also $\frac{{}^nC_3 a^3}{{}^nC_4 a^4} = \frac{8}{3} \Rightarrow a(n-3) = \frac{3}{2}$ ....(ii)

Solving (i) & (ii), we get $n = 6, a = \frac{1}{2}$

$\because$ Term independent of x. $\therefore n - 3r = 0 \Rightarrow r = 2$

$\therefore {}^6C_2 \times \left(\frac{1}{2}\right)^2 = \frac{15}{4} = 3.75 \approx 4$