Paper-16-03-2021-Morning Shift

PHYSICS

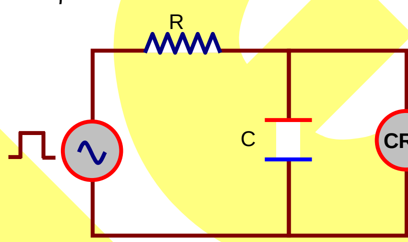
SECTION – A

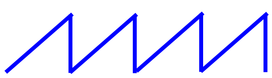
Q1. A conducting wire of length 'ℓ', area of cross-section A and electric resistivity ρ is connected between the terminals of a battery. A potential difference V is developed between its ends, causing an electric current.
If the length of the wire of the same material is doubled and the area of cross-section is halved, the resultant current would be :

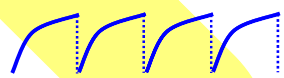
(A) $4\frac{VA}{\rho\ell}$ (B) $\frac{1}{4}\frac{\rho\ell}{VA}$

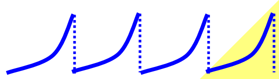
(C) $\frac{3}{4}\frac{VA}{\rho\ell}$ (D) $\frac{1}{4}\frac{VA}{\rho\ell}$

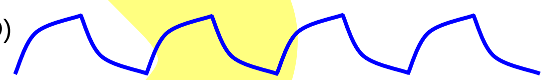
Q2. An RC circuit as shown in the figure is driven by a AC source generating a square wave. The output wave pattern monitored by CRO would look close to :



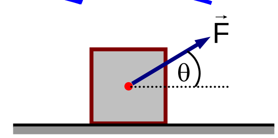
(A) 

(B) 

(C) 

(D) 

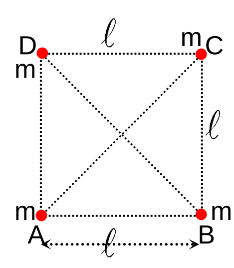
Q3. A block of mass m slides along a floor while a force of magnitude F is applied to it at an angle θ as shown in figure. The coefficient of kinetic is μ_K. Then, the block's acceleration 'a' is given by :
(g is acceleration due to gravity)



(A) $\frac{F}{m}\cos\theta + \mu_K\left(g - \frac{F}{m}\sin\theta\right)$ (B) $\frac{F}{m}\cos\theta - \mu_K\left(g + \frac{F}{m}\sin\theta\right)$

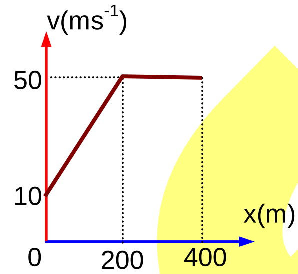
(C) $\frac{F}{m}\cos\theta - \mu_K\left(g - \frac{F}{m}\sin\theta\right)$ (D) $-\frac{F}{m}\cos\theta - \mu_K\left(g - \frac{F}{m}\sin\theta\right)$

Q4. Four equal masses, m each are placed at the corners of a square of length (ℓ) as shown in the figure. The moment of inertia of the system about an axis passing through A and parallel to DB would be :

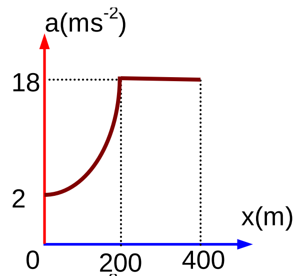


(A) $2m\ell^2$ (B) $m\ell^2$
(C) $\sqrt{3}m\ell^2$ (D) $3m\ell^2$

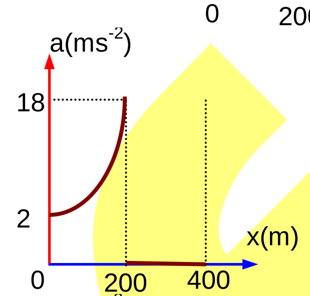
Q5. The velocity- displacement graph describing the motion of a bicycle is shown in the figure. The acceleration-displacement graph of the bicycle's motion is best described by:



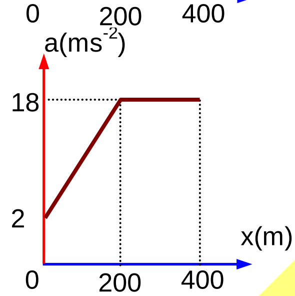
(A)



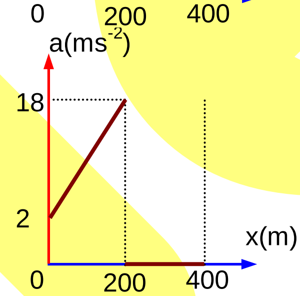
(B)



(C)



(D)



Q6. For changing the capacitance of a given parallel plate capacitor, a dielectric material of dielectric constant K is used, which has the same area as the plates of the capacitor. The thickness of the dielectric slab is $\frac{3}{4}d$, where 'd' is the separation between the plates of parallel plate capacitor. The new capacitance (C') in terms of original capacitance (C_0) is given by the following relation:

(A) $C' = \frac{4+K}{3}C_0$ (B) $C' = \frac{3+K}{4K}C_0$

(C) $C' = \frac{4}{3+K}C_0$ (D) $C' = \frac{4K}{K+3}C_0$

Q7. A plane electromagnetic wave of frequency 500 MHz is traveling in vacuum along y-direction. At a particular point in space and time, $\vec{B} = 8.0\times10^{-8}\hat{z}\,T$. The value of electric field at this point is:
(speed of light $= 3\times10^{8} ms^{-1}$)
$\hat{x}, \hat{y}, \hat{z}$ are unit vectors along x, y and z directions.

(A) $24\hat{x}$ V / m (B) $2.6\hat{x}$ V / m

(C) $-24\hat{x}$ V / m (D) $-2.6\hat{y}$ V / m

Q8. Time period of a simple pendulum is T inside a lift when the lift is stationary. If the lift moves upwards with an acceleration g / 2, the time period of pendulum will be:

(A) $\sqrt{\frac{2}{3}}T$ (B) $\sqrt{3}T$

(C) $\frac{T}{\sqrt{3}}$ (D) $\sqrt{\frac{3}{2}}T$

Q9. The stopping potential in the context of photoelectric depends on the following property of incident electromagnetic radiation:

(A) Frequency (B) Amplitude

(C) Intensity (D) Phase

Q10. A bar magnet of length 14 cm is placed in the magnetic meridian with its north pole pointing towards the geographic north pole. A neutral point is obtained at a distance of 18 cm from the centre of the magnet. If $B_H = 0.4\,G$, the magnetic moment of the magnet is $\left(1G = 10^{-4}T\right)$

(A) $2.880\times10^{3}\,JT^{-1}$ (B) $2.880\,JT^{-1}$

(C) $2.880\times10^{2}\,JT^{-1}$ (D) $28.80\,JT^{-1}$

Q11. A block of 200 g mass moves with a uniform speed in a horizontal circular groove, with vertical side walls of radius 20cm. If the block takes 40 s to complete one round, the normal force by the side walls of the groove is:

(A) $6.28\times10^{-3}N$ (B) 0.0314 N

(C) $9.859\times10^{-2}N$ (D) $9.859\times10^{-4}N$

Q12. The volume V of an enclosure contains a mixture of three gases, 16 g of oxygen, 28 g of nitrogen and 44 g of carbon dioxide at absolute temperature T. Consider R as universal gas constant. The pressure of the mixture of gases is :

(A) $\frac{3RT}{V}$ (B) $\frac{5}{2}\frac{RT}{V}$

(C) $\frac{88RT}{V}$ (D) $\frac{4RT}{V}$

Q13. The pressure acting on a submarine is 3×10^{5} Pa at a certain depth. If the depth is doubled, the percentage in the pressure acting on the submarine would be:
(Assume that atmospheric pressure is 1×10^{5} Pa density of water is $10^{3}kgm^{-3}, g = 10ms^{-2}$)

(A) $\frac{3}{200}\%$ (B) $\frac{5}{200}\%$

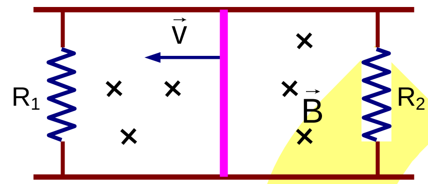
(C) $\frac{200}{3}\%$ (D) $\frac{200}{5}\%$

Q14. The maximum and minimum distances of a comet from the Sun are $1.6\times10^{12}m$ and $8.0\times10^{10}m$ respectively. If the speed of the comet at the nearest point is $6\times10^{4}ms^{-1}$, the speed at the farthest point is:

(A) $3.0\times10^{3}m/s$ (B) $1.5\times10^{3}m/s$

(C) $4.5\times10^{3}m/s$ (D) $6.0\times10^{3}m/s$

Q15. A conducting bar of length L is free to slide on two parallel conducting rails as shown in the figure. Two resistors R_1 and R_2 are connected across the ends of the rails. There is a uniform magnetic field $\vec{B}$ pointing into the page. An external agent pulls the bar to the left at a constant speed v.



The correct statement about the directions of induced currents I_1 and I_2 flowing through R_1 and R_2 respectively is:
(A) I_1 is in anticlockwise direction and I_2 is in clockwise direction
(B) Both I_1 and I_2 are in anticlockwise direction
(C) Both I_1 and I_2 are in clockwise direction
(D) I_1 is in clockwise direction and I_2 is in anticlockwise direction

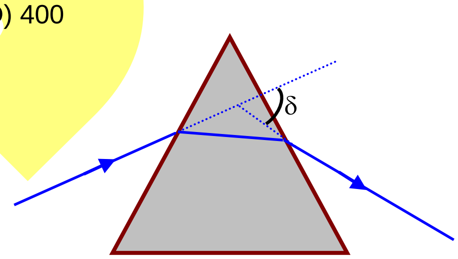
Q16. One main scale division of a vernier callipers is 'a' cm and n^{th} division of the vernier scale coincide with $(n-1)^{th}$ division of the main scale. The least count of the callipers in mm is :

(A) $\frac{10na}{(n-1)}$ (B) $\frac{10a}{(n-1)}$

(C) $\frac{10a}{n}$ (D) $\left(\frac{n-1}{10n}\right)a$

Q17. A 25 m long antenna is mounted on an antenna tower. The height of the antenna tower is 75m. The wavelength (in meter) of the signal transmitted by this antenna would be:
(A) 100 (B) 300
(C) 200 (D) 400

Q18. The angle of deviation through a prism is minimum when



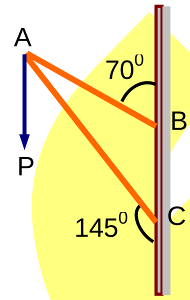
(A) Incident ray and emergent ray are symmetric to the prism
(B) The refracted ray inside the prism becomes parallel to its base
(C) Angle if incidence is equal to that of the angle of emergence
(D) When angle of emergence is double the angle of incidence
Choose the correct answer from the options given below :
(A) Statements (B) and (C) are true
(B) Statements (A), (B) and (C) are true
(C) Only statement (D) is true
(D) Only statements (A) and (B) are true

Q19. For an electromagnetic wave traveling in free space, the relation between average energy densities due to electric (U_e) and magnetic (U_m) fields is :
(A) $U_e > U_m$ (B) $U_e = U_m$
(C) $U_e \neq U_m$ (D) $U_e < U_m$

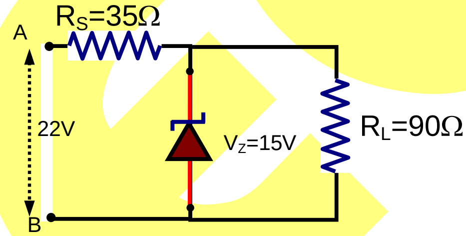
Q20. In thermodynamics, heat and work are :
(A) Point functions (B) Extensive thermodynamic state variables
(C) Path functions (D) Intensive thermodynamic state variables

Section-B

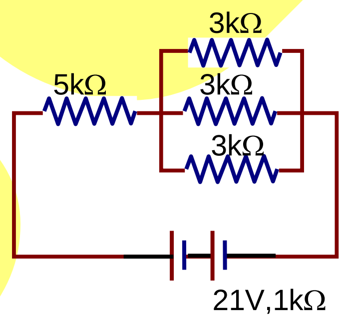
Q1. Consider a frame that is made up of two thin massless rods AB and AC as shown in the figure. A vertical force $\vec{P}$ of magnitude 100N is applied at point A of the frame. Suppose the force is $\vec{P}$ resolved parallel to the arms AB and AC of the frame. The magnitude of the resolved component along the arm AC is xN.
The value of x, to the nearest integer, is------------.
[Given : $\sin(35^0) = 0.573$, $\cos(35^0) = 0.819$
$\sin(110^0) = 0.939$, $\cos(110^0) = -0.342$]



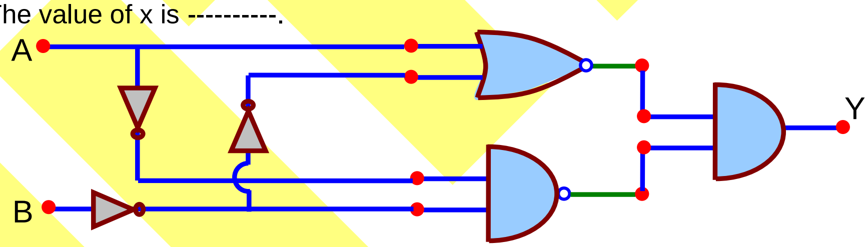
Q2. The value of power dissipated across the zener diode $(V_z = 15V)$ connected in the circuit as shown in the figure is $x \times 10^{-1}$ watt.
The value of x, to the nearest integer, is--------.



Q3. In the figure given, the electric current flowing through the 5 kΩ resistor is 'x' mA. The value of x to the nearest integer is --------.



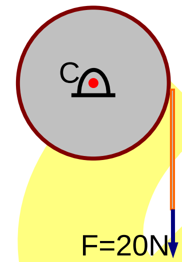
Q4. In the logic circuit shown in the figure, if input A and B are 0 to 1 respectively, then output at Y would be 'x'.
The value of x is ----------.



Q5. The resistance $R = \frac{V}{I}$, where $V = (50 \pm 2)V$ and $I = (20 \pm 0.2)A$. The percentage error in R is 'x' %.
The value of 'x' to the nearest integer is ----------.

Q6. A sinusoidal voltage of peak value 250V is applied to a series LCR circuit, in which $R = 8\Omega, L = 24mH$ and $C = 60\mu F$. The value of power dissipated at resonant condition is 'x' kW. The value of x to the nearest integer is-----------.

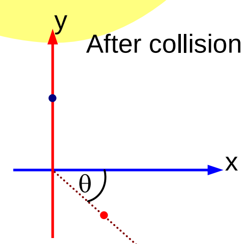
Q7. Consider a 20 kg uniform circular disk of radius 0.2 m. It is pin supported at its centre and is at rest initially. The disk is acted upon by a constant force $F = 20\,N$ through a massless string wrapped around its periphery as shown in the figure. Suppose the disk makes n number of revolutions to attain an angular speed of 50 rad s^{-1}. The value of n, to the nearest integer, is ---------.
[Given : In one complete revolution, the disk rotates by 6.28 rad]



Q8. The first three spectral of H-atom in the Balmer series are given $\lambda_1, \lambda_2, \lambda_3$ considering the Bohr atomic model, the wave lengths of first and third spectral lines $\left(\frac{\lambda_1}{\lambda_3}\right)$ are related by a factor of approximately 'x' $\times 10^{-1}$.
The value of x, to the nearest integer, is-----------.

Q9. A fringe width of 6 mm was produced for two slits separated by 1 mm apart. The screen is placed 10 m away. The wavelength of light used is 'x' nm.
The value of 'x' to the nearest integer is----------.

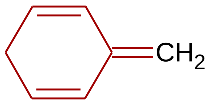
Q10. A ball of mass 10 kg moving with a velocity $10\sqrt{3}\,ms^{-1}$ along X – axis, hits another ball of mass 20 kg which is at rest. After collision, the first ball comes to rest and the second one disintegrates into two equal pieces. One of the pieces starts moving along Y– axis at a speed of 10m/s. The second piece starts moving at a speed of 20m/s at an angle θ (degree) with respect to the X –axis.
The configuration of pieces after collision is shown in the figure.
The value of θ to the nearest integer is----------.

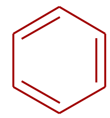


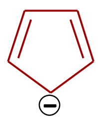
CHEMISTRY

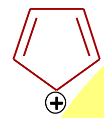
SECTION A

Q1. Among the following, the aromatic compounds are:

(A) 

(B) 

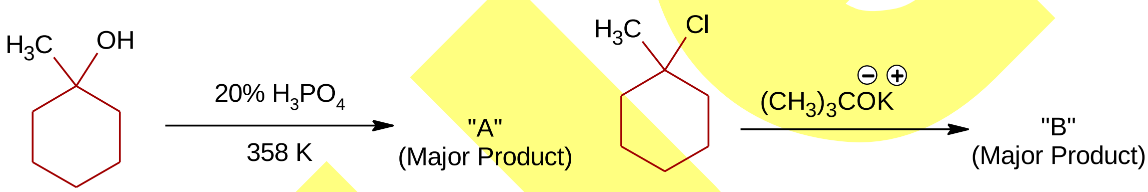
(C) 

(D) 

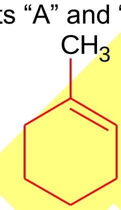
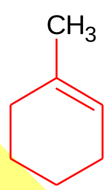
Choose the **correct** answer from the following options:

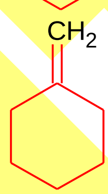
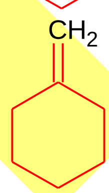
(A) (B), (C) and (D) only
(B) (A) and (B) only
(C) (A), (B) and (C) only
(D) (B) and (C) only

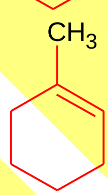
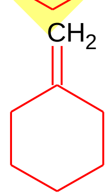
Q2.

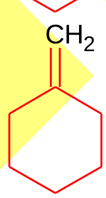
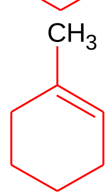


The products "A" and "B" formed in above reactions are:

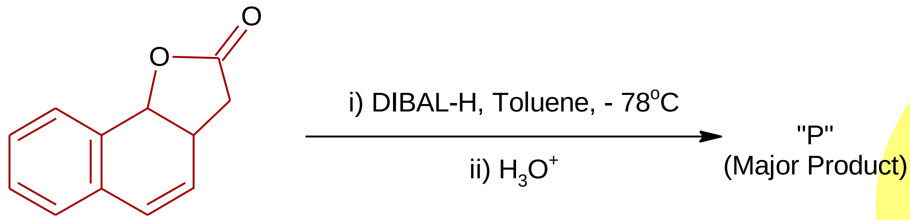
(A) A –  B – 

(B) A –  B – 

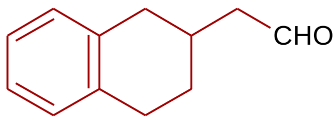
(C) A –  B – 

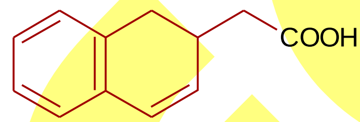
(D) A –  B – 

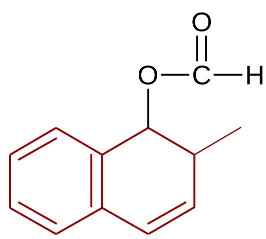
Q3.

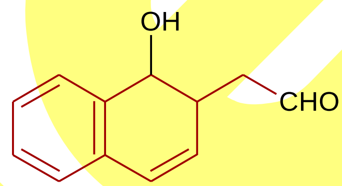


The product “P” in the above reaction is:

(A) 

(B) 

(C) 

(D) 

Q4. **Assertion A :** Enol form of acetone [CH_3COCH_3] exists in <0.1% quantity. However, the enol form of acetyl acetone [$CH_3COCH_2OCCH_3$] exists in approximately 15% quantity.

Reason R : Enol form of acetylacetone is stabilized by intramolecular hydrogen bonding, which is not possible in enol form of acetone.

Choose the **correct** statement:

(A) Both **A** and **R** are true and **R** is the correct explanation of **A**
(B) **A** is true but **R** is false
(C) **A** is false but **R** is true
(D) Both **A** and **R** are true but **R** is not the correct explanation of **A**

Q5. Match **List – I with List – II**:

	List – I		**List – II**
	Industrial process		**Application**
(a)	Haber’s process	(i)	HNO_3 synthesis
(b)	Ostwald’s process	(ii)	Aluminium extraction
(c)	Contact process	(iii)	NH_3 sythesis
(d)	Hall-Heroult process	(iv)	H_2SO_4 synthesis

Choose the **correct** answer from the options given below:

(A) (a)- (ii), (b)-(iii), (c)-(iv), (d)-(i)
(B) (a)-(iv), (b)-(i), (c)-(ii), (d)-(iii)
(C) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)
(D) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

Q6. Given below are two statement: one is labelled as Assertion **A** and the other is labelled as Reason **R**:

Assertion **A**: Size of Bk^{3+} ion is less than Np^{3+} ion.

Reason **R** : The above is a consequence of the lanthanoid contraction.

In the light of the above statements, choose the **correct** answer from the options given below:

(A) Both **A** and **R** are true but **R** is not the correct explanation of **A**
(B) **A** is true but **R** is false
(C) Both **A** and **R** are true and **R** is the correct explanation of **A**
(D) **A** is false but **R** is true

Q7. Given below are two statements:
Statement I : The E^o value for Ce^{4+} / Ce^{3+} is + 1.74V
Statement II : Ce is more stable in Ce^{4+} state than Ce^{3+} state
In the light of the above statements, choose the most **appropriate** answer from the options given below:
(A) Statement **I** is incorrect but statement **II** is correct
(B) Statement **I** is correct but statement **II** is incorrect
(C) Both statement **I** and statement **II** are correct
(D) Both statement **I** and statement **II** are incorrect

Q8. Which among the following pairs of Vitamins is stored in our body relatively for longer duration?
(A) Thiamine and Vitamin A
(B) Ascorbic acid and Vitamin D
(C) Thiamine and Ascorbic acid
(D) Vitamin A and Vitamin D

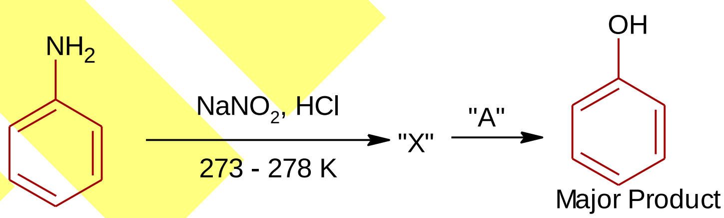
Q9. Which of the following is **Lindlar catalyst**?
(A) Partially deactivated palladised charcoal
(B) Cold dilute solution of $KMnO_4$
(C) Zinc chloride and HCl
(D) Sodium and Liquid NH_3

Q10. Given below are two statements : one is labblled as **Assertion A** and the other is labelled as **Reasons R :**
Assertion A : The H-O-H bond angle in water molecule is 104.5º.
Reason R : The lone pair- lone pair repulsion of electrons is higher than the bond pair- bond pair repulsion.
In the light of the above statements, choose the **correct** answer from the options given below:
(A) **A** is false but **R** is true
(B) Both A and **R** are true, and **R** is the correct explanation of **A**
(C) A is true but **R** is false
(D) Both **A** and **R** are true, but **R** is not the correct explanation of **A**

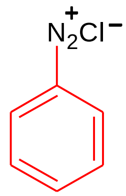
Q11. In chromatography technique, the purification of compound is independent of:
(A) Physical state of the pure compound
(B) Solubility of the compound
(C) Mobility or flow of solvent system
(D) Length of the column or TLC plate

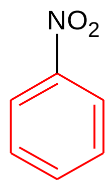
Q12. The type of pollution that gets increased during the day time and in the presence of O_3 is:
(A) Acid rain
(B) Oxidising smog
(C) Reducing smog
(D) Global warming

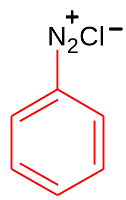
Q13.

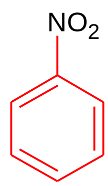


In the above chemical reaction, intermediate “X” and reagent / condition “A” are:

(A) X –  ; A – H_2O / NaOH

(B) X –  ; A – H_2O / NaOH

(C) X –  ; A – H_2O / Δ

(D) X –  ; A – H_2O / Δ

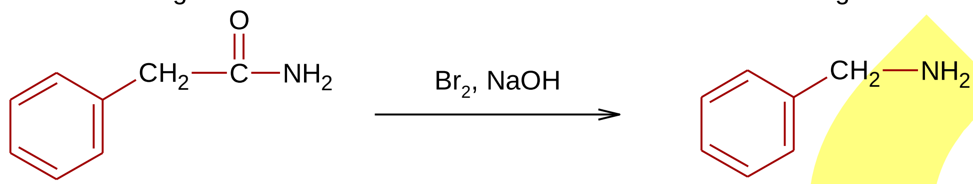
Q14. Match **List – I** with **List – II**

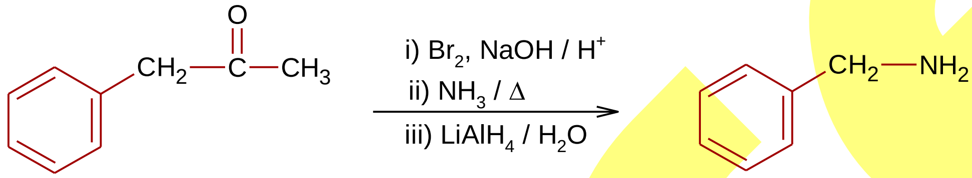
List – I **Name of oxo acid**	**List – II** **Oxidation state of "P"**
(a) Hypophosphorous acid	(i) + 5
(b) Orthophosphoric acid	(ii) + 4
(c) Hypophosphoric acid	(iii) + 3
(d) Orthophosphorous acid	(iv) + 2
	(v) + 1

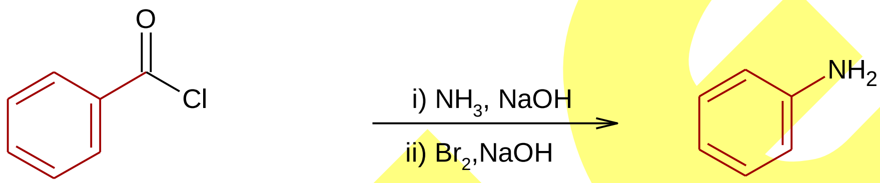
Choose the **correct** answer from the options given below:

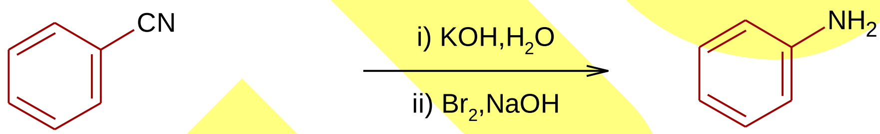
(A) (a) - (v), (b)-(iv), (c)-(ii), (d)-(iii)
(B) (a)-(iv), (b)-(i), (c)-(ii), (d)-(iii)
(C) (a) -(iv), (b)-(v), (c)-(ii), (d)-(iii)
(D) (a) - (v), (b)-(i), (c)-(ii), (d)-(iii)

Q15. Which of the following reaction **DOES NOT** involve Hoffmann bromamide degradation?

(A)


(B)


(C)


(D)


Q16. Given below are two statements:
Statement I : Both $CaCl_2.6H_2O$ and $MgCl_2.8H_2O$ undergo dehydration on heating.
Statement II : BeO is amphoteric whereas the oxides of other elements in the same group are acidic.
In the light of the above statements, choose the **correct** answer from the options given below:
(A) Statement **I** is false but statement **II** is true
(B) Statement **I** is true but statement **II** is false
(C) Both statement **I** and statement **II** are false
(D) Both statement **I** and statement **II** are true

Q17. A group 15 element, which is a metal and forms a hydride with strongest reducing power among group 15 hydrides. The element is:
(A) Bi
(B) Sb
(C) As
(D) P

Q18. The process that involves the removal of sulphur from the ores is:
(A) Smelting
(B) Leaching
(C) Refining
(D) Roasting

Q19. Given below are two statements:
Statement I : H_2O_2 can act as both oxidizing and reducing agent in basic medium.
Statement II: In the hydrogen economy, the energy is transmitted in the form of dihydrogen.
In the light of the above statements, choose the **correct** answer from the options given below:
(A) Both statement **I** and statement **II** are false
(B) Statement **I** is false but statement **II** is true
(C) Both statement **I** and statement **II** are true
(D) Statement **I** is true but statement **II** is false

Q20. The functions of antihistamine are:
(A) Analgesic and antacid
(B) Antiallergic and Analgesic
(C) Antacid and antiallergic
(D) Antiallergic and antidepressant

SECTION B

Q1. A certain element crystallises in a bcc lattice of unit cell edge length 27 $\overset{o}{A}$. If the same element under the same conditions crystallises in the fcc lattice, the edge length of the unit cell in $\overset{o}{A}$ will be ___________. (Round off to the nearest Integer).
[Assume each lattice point has a single atom] [Assume $\sqrt{3} = 1.73, \sqrt{2} = 1.41$]

Q2. $2MnO_4^- + bC_2O_4^{2-} + cH^+ \rightarrow xMn^{2+} + yCO_2 + zH_2O$
If the above equation is balanced with integer coefficients, the value of c is_________.
(Round off to the nearest Integer).

Q3. Two salts A_2X and MX have the same value of solubility product of 4.0×10^{-12}. The ratio of their molar solubilities i.e $\frac{S(A_2X)}{S(MX)}$ = ____________.
(Round off to the Nearest Integer).

Q4. Complete combustion of 750g of an organic compound provides 420 g of CO_2 and 210 g of H_2O. The percentage composition of carbon and hydrogen in organic compound is 15.3 and _________ respectively. (Round off to the nearest Integer).

Q5. A 6.50 molal solution of KOH (aq) has a density of 1.89 g cm^{-3}. The molarity of the solution is________ mol dm^{-3}. (Round off to the nearest Integer).
[Atomic masses : K = 39.0u, O = 16.0u, H= 1.0u]

Q6. AB_2 is 10% dissociated in water to A^{2+} and B^-. The boiling point of a 10.0 molal aqueous solution of AB_2 is_________ ºC. (Round off to the nearest Integer).
[Given : Molal elevation constant of water K_b = 0.5 K kg mol^{-1}, boiling point of pure water = 100ºC]

Q7. For the reaction A(g) $\rightleftharpoons$ B(g) at 495 K, $\Delta_rG^o = -9.478$kJ mol^{-1}.
If we start the reaction in a closed container at 495 K with 22 millimoles of A, the amount of B in the equilibrium mixture is __________ millimoles
(Round off to the nearest Integer).
[R=8.314 J mol^{-1} K^{-1}; ln 10 = 2.303]

Q8. The equivalents of ethylene diamine required to replace the neutral ligands from the coordination sphere of the trans-complex of $CoCl_3.4NH_3$ is _________.
(Round off to the Nearest Integer).

Q9. The decomposition of formic acid on gold surface follows first order kinetics. If the rate constant at 300 K is 1.0×10^{-3} S^{-1} and the activation energy Ea = 11.488 kJ mol^{-1}, the rate constant at 200 K is ___________ $\times10^{-5}$ s^{-1}. (Round off to the nearest Integer).
[Given R = 8.314 J $mol^{-1}K^{-1}$]

Q10. When light of wavelength 248 nm falls on a metal of threshold energy 3.0eV, the de-Broglie wavelength of emitted electrons is__________ $\overset{o}{A}$.
(Round off to the nearest Integer).
[Use : $\sqrt{3} = 1.73, h = 6.63\times10^{-34}$ Js
$m_e = 9.1\times10^{-31}kg : c = 3.0\times10^{8}ms^{-1}; 1eV = 1.6\times10^{-19}$ J]

MATHEMATICS
SECTION A

Q1. Let a complex number $z, |z| \neq 1$, satisfy $\log_{\frac{1}{\sqrt{2}}}\left(\frac{|z|+11}{(|z|-1)^2}\right) \leq 2$. Then, the largest value of $|z|$ is equal to.........

(A) 6
(B) 7
(C) 5
(D) 8

Q2. Consider three observations a, b and c such that $b = a + c$. If the standard deviation of $a+2, b+2, c+2$ is d, then which of the following is true?

(A) $b^2 = 3(a^2 + c^2) - 9d^2$
(B) $b^2 = a^2 + c^2 + 3d^2$
(C) $b^2 = 3(a^2 + c^2 + d^2)$
(D) $b^2 = 3(a^2 + c^2) + 9d^2$

Q3. The number of elements in the set $\{x \in R : (|x|-3)|x+4| = 6\}$ is equal to :

(A) 3
(B) 1
(C) 4
(D) 2

Q4. The range of $a \in R$ for which the function
$f(x) = (4a-3)(x + \log_e 5) + 2(a-7)\cot\left(\frac{x}{2}\right)\sin^2\left(\frac{x}{2}\right), x \neq 2n\pi, n \in N$ has critical points is:

(A) $(-\infty, -1]$
(B) $(-3, 1)$
(C) $[1, \infty)$
(D) $\left[-\frac{4}{3}, 2\right]$

Q5. If for $x \in \left(0, \frac{\pi}{2}\right)$, $\log_{10}\sin x + \log_{10}\cos x = -1$ and $\log_{10}(\sin x + \cos x) = \frac{1}{2}(\log_{10}{}^{n} - 1), n > 0$, then the value of n is equal to :

(A) 16
(B) 12
(C) 9
(D) 20

Q6. Let the functions $f : R \to R$ and $g : R \to R$ be defined as :

$$f(x) = \begin{cases} x+2, & x < 0 \\ x^2, & x \geq 0 \end{cases} \text{ and } g(x) = \begin{cases} x^3, & x < 1 \\ 3x-2, & x \geq 1 \end{cases}$$

Then, the number of points in R where $(f\circ g)(x)$ is NOT differentiable is equal to :

(A) 0
(B) 1
(C) 2
(D) 3

Q7. Let the position vectors of two points P and Q be $3\hat{i} - \hat{j} + 2\hat{k}$ and $\hat{i} + 2\hat{j} - 4\hat{k}$ respectively. Let R and S be two points such that the direction rations of lines PR and QS are $(4, -1, 2)$ and $(-2, 1, -2)$, respectively. Let lines PR and QS intersect at T. If the vector $\overrightarrow{TA}$ is perpendicular to both $\overrightarrow{PR}$ and $\overrightarrow{QS}$ and the length of vector $\overrightarrow{TA}$ is $\sqrt{5}$ units, then the modulus of a position vector of A is :

(A) $\sqrt{227}$
(B) $\sqrt{482}$
(C) $\sqrt{5}$
(D) $\sqrt{171}$

Q8. If $y = y(x)$ is the solution of the differential equation, $\frac{dy}{dx} + 2y\tan x = \sin x, y\left(\frac{\pi}{3}\right) = 0$, then the maximum value of the function $y(x)$ over R is equal to :

(A) $\frac{1}{2}$
(B) $\frac{1}{8}$
(C) $-\frac{15}{4}$
(D) 8

Q9. Let a vector $\alpha\hat{i} + \beta\hat{j}$ be obtained by rotating the vector $\sqrt{3}\hat{i} + \hat{j}$ by an angle 45° about the origin in counterclockwise direction in the first quadrant. Then the area of triangle having vertices $(\alpha,\beta), (0,\beta)$ and $(0,0)$ is equal to :

(A) $2\sqrt{2}$
(B) $\frac{1}{2}$
(C) 1
(D) $\frac{1}{\sqrt{2}}$

Q10. If n is the number or irrational terms in the expansion of $\left(3^{1/4} + 5^{1/8}\right)^{60}$, then (n – 1) is divisible by:

(A) 7
(B) 30
(C) 8
(D) 26

Q11. If the three normals drawn to the parabola, $y^2 = 2x$ pass through the point $(a, 0)\, a \neq 0$, then 'a' must be greater than :

(A) $\frac{1}{2}$
(B) 1
(C) $-\frac{1}{2}$
(D) -1

Q12. If for a > 0, the feet of perpendiculars from the points $A(a,-2a,3)$ and $B(0,4,5)$ on the plane $lx + my + nz = 0$ are points $C(0,-a,-1)$ and D respectively, then the length of line segment CD is equal to :

(A) $\sqrt{41}$
(B) $\sqrt{31}$
(C) $\sqrt{66}$
(D) $\sqrt{55}$

Q13. The number of roots of the equation,
$(81)^{\sin^2 x} + (81)^{\cos^2 x} = 30$ in the interval $[0, \pi]$ is equal to :

(A) 2
(B) 8
(C) 3
(D) 4

Q14. Which of the following Boolean expression is a tautology?

(A) $(p \wedge q) \to (p \to q)$
(B) $(p \wedge q) \vee (p \to q)$
(C) $(p \wedge q) \wedge (p \to q)$
(D) $(p \wedge q) \vee (p \vee q)$

Q15. The locus of the midpoints of the chord of the circle, $x^2 + y^2 = 25$ which is tangent to the hyperbola, $\frac{x^2}{9} - \frac{y^2}{16} = 1$ is :

(A) $(x^2+y^2)^2 - 9x^2 + 144y^2 = 0$
(B) $(x^2+y^2)^2 - 9x^2 + 16y^2 = 0$
(C) $(x^2+y^2)^2 - 9x^2 - 16y^2 = 0$
(D) $(x^2+y^2)^2 - 16x^2 + 9y^2 = 0$

Q16. Let $A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}, i = \sqrt{-1}$. Then, the system of linear equation $A^8 \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$ has :

(A) Infinitely many solutions
(B) No solution
(C) Exactly two solutions
(D) A unique solution

Q17. Let [x] denote greatest integer less than or equal to x. If for $n \in N, (1 - x + x^3)^n = \sum_{j=0}^{3n} a_j x^j$, then $\sum_{j=0}^{\left[\frac{3n}{2}\right]} a_{2j} + 4 \sum_{j=0}^{\left[\frac{3n-1}{2}\right]} a_{2j+1}$ is equal to :

(A) 1
(B) n
(C) 2^{n-1}
(D) 2

Q18. Let $S_k = \sum_{r=1}^{k} \tan^{-1}\left(\frac{6^r}{2^{2r+1} + 3^{2r+1}}\right)$. Then $\lim_{k\to\infty} S_k$ is equal to :

(A) $\cot^{-1}\left(\frac{3}{2}\right)$
(B) $\tan^{-1}\left(\frac{3}{2}\right)$
(C) $\tan^{-1}(3)$
(D) $\frac{\pi}{2}$

Q19. Let P be a plane $lx + my + nz = 0$ containing the line, $\frac{1-x}{1} = \frac{y+4}{2} = \frac{z+2}{3}$. If plane P divides the line segment AB joining points $A(-3,-6,1)$ and $B(2,4,-3)$ in ratio k : 1 then the value of k is equal to :

(A) 4
(B) 2
(C) 1.5
(D) 3

Q20. A pack of cards has one card missing. Two cards are drawn randomly and are found to be spades. The probability that the missing card is not a spade, is :

(A) $\frac{22}{425}$
(B) $\frac{52}{867}$
(C) $\frac{3}{4}$
(D) $\frac{39}{50}$

SECTION B

Q1. Consider an arithmetic series and a geometric series having four initial terms from the set $\{11, 8, 21, 16, 26, 32, 4\}$. If the last terms of these series are the maximum possible four digit numbers, then the number of common terms in these two series is equal to……….

Q2. If the normal to the curve $y(x)=\int_0^x (2t^2-15t+10)dt$ at a point (a,b) is parallel to the line $x+3y=-5, a>1$, then the value of $|a+6b|$ is equal to ……….

Q3. Let $f:R\to R$ be a continuous function such that $f(x)+f(x+1)=2$, for all $x\in R$. If $I_1=\int_0^8 f(x)dx$ and $I_2=\int_{-1}^3 f(x)dx$, then the value of I_1+2I_2 is equal to………

Q4. Let the curve $y=y(x)$ be the solution of the differential equation, $\frac{dy}{dx}=2(x+1)$. If the numerical value of area bounded by the curve $y=y(x)$ and x-axis is $\frac{4\sqrt{8}}{3}$, then the value of $y(1)$ is equal to……..

Q5. Let z and ω be two complex numbers such that $\omega=z\bar{z}-2z+2$, $\left|\frac{z+i}{z-3i}\right|=1$ and $Re(\omega)$ has minimum value. Then, the minimum value of $n\in N$ for which ω^n is real, is equal to

Q6. If $\lim_{x\to 0}\frac{ae^x-b\cos x+ce^{-x}}{x\sin x}=2$, then a+b+c is equal to……..

Q7. Let $f:(0,2)\to R$ be defined as $f(x)=\log_2\left(1+\tan\left(\frac{\pi x}{4}\right)\right)$. Then, $\lim_{n\to\infty}\frac{2}{n}\left(f\left(\frac{1}{n}\right)+f\left(\frac{2}{n}\right)+....+f(1)\right)$ is equal to…….

Q8. The total number of 3 x 3 matrices A having entries from the set $\{0,1,2,3\}$ such that the sum of all the diagonal entries of AA^T is 9, is equal to……..

Q9. Let $P=\begin{bmatrix}-30 & 20 & 56\\ 90 & 140 & 112\\ 120 & 60 & 14\end{bmatrix}$ and $A=\begin{bmatrix}2 & 7 & \omega^2\\ -1 & -\omega & 1\\ 0 & -\omega & -\omega+1\end{bmatrix}$ where $\omega=\frac{-1+i\sqrt{3}}{2}$, and I_3 be the identity matrix of order 3. If the determinant of the matrix $(P^{-1}AP-I_3)^2$ is $\alpha\omega^2$, then the value of α is equal to…………

Q10. Let ABCD be a square of side of unit length. Let a circle C_1 centered at A with unit radius is drawn. Another circle C_2 which touches C_1 and the lines AD and AB are tangent to it, is also drawn. Let a tangent line from the point C to the circle C_2 meet the side AB at E. if the length of EB is $\alpha+\sqrt{3}\beta$, where α,β are integers, then $\alpha+\beta$ is equal to……….

ANSWER: Paper-Jee-Main-16-03-2021-Morning Shift

PHYSICS	CHEMISTRY	MATHEMATICS
Section-A	**SECTION- A**	**SECTION – A**
Ans1. D	**Ans1.** D	**Ans1. B**
Ans2. D	**Ans2.** C	**Ans2. A**
Ans3. C	**Ans3.** D	**Ans3. D**
Ans4. D	**Ans4.** A	**Ans4. D**
Ans5. D	**Ans5.** D	**Ans5. B**
Ans6. D	**Ans6.** B	**Ans6. B**
Ans7. C	**Ans7.** B	**Ans7. D**
Ans8. A	**Ans8.** D	**Ans8. B**
Ans9. A	**Ans9.** A	**Ans9. B**
Ans10. B	**Ans10.** B	**Ans10. D**
Ans11. D	**Ans11.** A	**Ans11. B**
Ans12. B	**Ans12.** B	**Ans12. C**
Ans13. C	**Ans13.** C	**Ans13. D**
Ans14. A	**Ans14.** D	**Ans14. A**
Ans15. D	**Ans15.** B	**Ans15. B**
Ans16. C	**Ans16.** C	**Ans16. B**
Ans17. A	**Ans17.** A	**Ans17. A**
Ans18. B	**Ans18.** D	**Ans18. A**
Ans19. B	**Ans19.** C	**Ans19. B**
Ans20. C	**Ans20.** C	**Ans20. D**
Section-B	**SECTION – B**	**SECTION – B**
Ans1. 164	**Ans1.** 33	**Ans1. 3**
Ans2. 5	**Ans2.** 16	**Ans2. 406**
Ans3. 3	**Ans3.** 50	**Ans3. 16**
Ans4. 0	**Ans4.** 3	**Ans4. 2**
Ans5. 5	**Ans5.** 9	**Ans5. 4**
Ans6. 4	**Ans6.** 106	**Ans6. 4**
Ans7. 20	**Ans7.** 20	**Ans7. 1**
Ans8. 15	**Ans8.** 2	**Ans8. 766**
Ans9. 600	**Ans9.** 10	**Ans9. 36**
Ans10. 30	**Ans10.** 9	**Ans10. 1**

SOLUTION: Paper-Jee-Main-16-03-0-2021-Morning Shift
PHYSICS
SECTION – A

Sol1. $R = \frac{\rho\ell}{A} \Rightarrow I_1 = \frac{V}{R} = \frac{VA}{\rho\ell}$

When the length of the wire of the same material is doubled and the area of cross-section is halved $R' = \frac{\rho(2\ell)}{(A/2)} = \frac{4\rho\ell}{A} \Rightarrow I_2 = \frac{V}{R'} = \frac{VA}{4\rho\ell}$

Sol2. The capacitor will be repeatedly charged and discharged due to alternating source.

During charging Process

$Q = CV_0\left(1 - e^{-\frac{t}{RC}}\right) \Rightarrow$ Charge on the capacitor, and

$V_C = \frac{Q}{C} = V_0\left(1 - e^{-\frac{t}{RC}}\right) \Rightarrow$ Potential difference across capacitor

During discharging Process

$Q' = Q_0 e^{-\frac{t}{RC}} + CV_0\left(1 - e^{-\frac{t}{RC}}\right) \Rightarrow$ Charge on the capacitor,

Sol3.

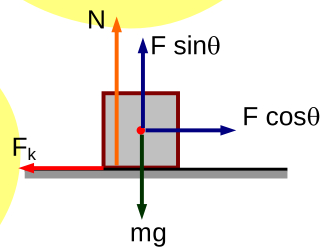
$N = mg - F\sin\theta$ ....(1)

According to Newton's second law we can write

$ma = F\cos\theta - \mu_K N$

$\Rightarrow ma = F\cos\theta - \mu_K(mg - F\sin\theta)$

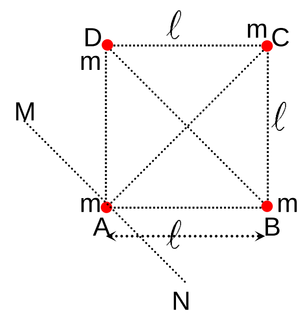
$\Rightarrow a = \frac{F}{m}\cos\theta - \mu_K\left(g - \frac{F}{m}\sin\theta\right)$



Sol4.

$I_{MN} = m(0)^2 + m\left(\sqrt{2}\,\ell\right)^2 + m\left(\frac{\ell}{\sqrt{2}}\right)^2 \times 2$

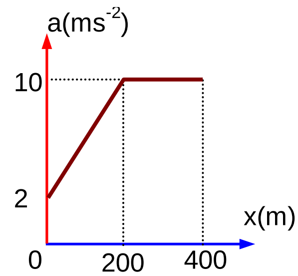
$\Rightarrow I_{MN} = 2m\ell^2 + m\ell^2 = 3m\ell^2$



Sol5. $v = \begin{cases} 10 + \frac{x}{5} & \text{if } 0 \le x \le 200\text{m} \\ 50 & \text{if } 200\text{m} \le x \le 400\text{m} \end{cases}$

$\Rightarrow a = \frac{dv}{dt} = v\frac{dv}{dx} = \begin{cases} \frac{1}{5}\left(10 + \frac{x}{5}\right) & \text{if } 0 \le x \le 200\text{m} \\ 0 & \text{if } 200\text{m} \le x \le 400\text{m} \end{cases}$

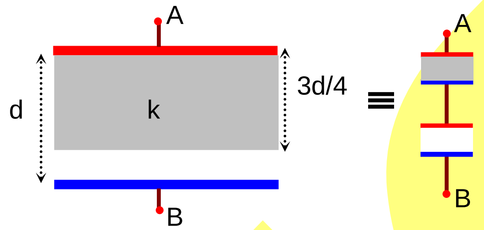
$\Rightarrow a = \begin{cases} \left(2 + \frac{x}{25}\right) & \text{if } 0 \le x \le 200\text{m} \\ 0 & \text{if } 200\text{m} \le x \le 400\text{m} \end{cases}$



Sol6.

$$C_1 = \frac{\varepsilon_0 Ak}{3d/4} = \frac{4\varepsilon_0 Ak}{3d} = \frac{4kC_0}{3}$$

$$C_2 = \frac{\varepsilon_0 A}{d/4} = \frac{4\varepsilon_0 A}{d} = 4C_0$$



Since capacitors C_1 and C_2 are in series, so

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{3}{4kC_0} + \frac{1}{4C_0} \Rightarrow \frac{1}{C'} = \frac{3+k}{4kC_0} \Rightarrow C' = \frac{4k}{k+3}C_0$$

Sol7. $c = \dfrac{E}{B} \Rightarrow E = cB = 24\,V/m$

As we know that direction of propagation of wave is $\hat{E}\times\hat{B} = \hat{c}$, so $\hat{E} = -\hat{x}$, hence

$\vec{E} = -24\hat{x}\ V/m$

Sol8. $T = 2\pi\sqrt{\dfrac{\ell}{g_{eff}}}$

When lift is stationary, then $g_{eff} = g$, so $T = 2\pi\sqrt{\dfrac{\ell}{g}}$

When lift is moving up with acceleration g/2 , then $g_{eff} = \dfrac{3g}{2}$, so $T' = 2\pi\sqrt{\dfrac{2\ell}{3g}} = \sqrt{\dfrac{2}{3}}\,T$

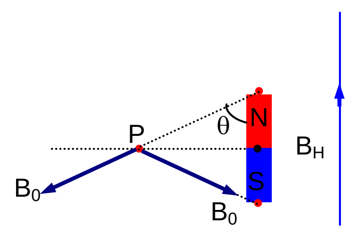
Sol9. The stopping potential in the context of photoelectric depends on frequency of incident electromagnetic radiation:

Sol10.

$$B_H = 2B_0\cos\theta = \frac{2\mu_0(m)}{4\pi\left(d^2+r^2\right)} \times \frac{r}{\sqrt{d^2+r^2}} = \frac{\mu_0 M}{4\pi\left(d^2+r^2\right)^{\frac{3}{2}}}$$

$$\Rightarrow M = \frac{4\pi\left(d^2+r^2\right)^{\frac{3}{2}} B_H}{\mu_0} = \frac{0.4\times10^{-4}\times\left(373\sqrt{373}\right)\times10^{-6}}{10^{-7}}$$

$$\Rightarrow M = 2881.5\times10^{-3}\,JT^{-1} \approx 2.880\,JT^{-1}$$



Sol11. $F = m\omega^2 R = m\left(\dfrac{2\pi}{T}\right)^2 R = \dfrac{0.2\times4\times\pi^2\times0.2}{40\times40} = 9.859\times10^{-4}\,N$

Sol12. **Using Concept of partial pressure, we can write**

$$P = P_{O_2} + P_{N_2} + P_{CO_2} = \frac{n_1RT}{V} + \frac{n_2RT}{V} + \frac{n_3RT}{V}$$

$$\Rightarrow P = \frac{RT}{V}\left(\frac{16}{32} + \frac{28}{28} + \frac{44}{44}\right) = \frac{5RT}{2V}$$

Sol13. $P_1 = P_0 + \rho gh \Rightarrow h = \dfrac{P_1 - P_0}{\rho g}$, and $P_2 = P_0 + 2\rho gh = P_0 + 2\rho g\left(\dfrac{P_1 - P_0}{\rho g}\right) = 2P_1 - P_0$

$$\Rightarrow \Delta P = P_2 - P_1 = P_1 - P_0$$

$$\Rightarrow \%\ \text{increment in pressure} = \frac{P_1 - P_0}{P_1}\times100 = \frac{200}{3}\%$$

Sol14. **According to conservation of angular momentum , we can write**

$$v_1r_1 = v_2r_2 \Rightarrow v_1 = \frac{v_2r_2}{r_1} = \frac{8\times10^{10}}{1.6\times10^{12}}\times6\times10^4 = 3\times10^3\,m/s$$

Sol15. Since magnetic flux linked with loop containing R_1 decreases with time , so

$\hat{B}_{ind} = +\hat{B}_{Source} = \left(-\hat{k}\right)$, hence current I_1 in R_1 will be clockwise

Since magnetic flux linked with loop containing R_2 increases with time , so

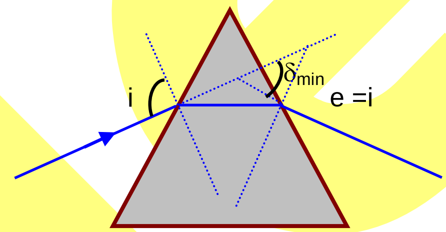
$\hat{B}_{ind} = -\hat{B}_{Source} = \left(+\hat{k}\right)$, hence current I_2 in R_2 will be anti-clockwise

Sol16. Least count of the calliper $= 1\,MSD - 1VSD = a - \frac{(n-1)a}{n} = \frac{a}{n}cm = \frac{10a}{n}mm$

Sol17. Height of Antenna $= \frac{\lambda}{4} \Rightarrow \lambda = 4h = 4\times25 = 100m$

Sol18.

Basic Fact



Sol19. $U_e = U_m = \frac{1}{2}\varepsilon_0E^2 = \frac{B^2}{2\mu_0}$ because $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = \frac{E}{B}$

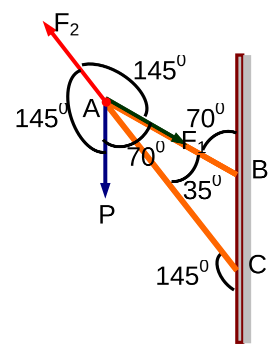
Sol20. In thermodynamics, heat and work are path function

SECTION – B

Sol1. Let the magnitude of components along AB and AC are F_1 and F_2 respectively, such that

$$\vec{P} + \vec{F}_1 + \vec{F}_2 = \vec{0}$$

Using Sing Rule we can write



$$\frac{F_1}{\sin(145^0)} = \frac{F_2}{\sin(70^0)} = \frac{P}{\sin(145^0)} \Rightarrow F_2 = \frac{P\sin(180^0 - 110^0)}{\sin(110^0 + 35^0)}$$

$$\Rightarrow F_2 = \frac{P\sin(110^0)}{\sin(110^0)\cos(35^0) + \cos(110^0)\sin(35^0)}$$

$$\Rightarrow F_2 = \frac{100\times0.939}{0.939\times0.819 - 0.342\times0.573}$$

$$\Rightarrow F_2 = \frac{100\times0.939}{0.769041 - 0.195966} = \frac{93.9}{0.573075} \approx 164\text{ N}$$

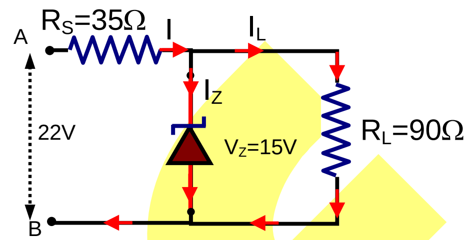
Hence components of along AB and AC are $\vec{P}\ -\vec{F}_1$ and $-\vec{F}_2$ respectively

Sol2.

$I = \frac{7}{35} = \frac{1}{5}$ A and $I_L = \frac{15}{90} = \frac{1}{6}$ A

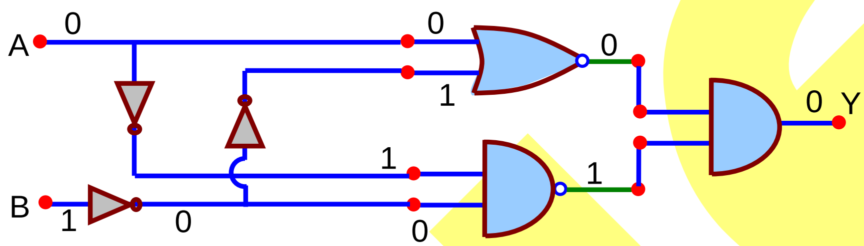
$I_Z = I - I_L = \frac{1}{5} - \frac{1}{6} = \frac{1}{30}$

$P_Z = V_z I_Z = 15 \times \frac{1}{30} = 0.5$ Watt



Sol3. $I = \frac{21}{7 \times 10^3} = 3\,mA$

Sol4.



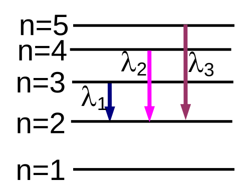
Sol5. $\frac{\Delta R}{R} = \left|\frac{\Delta V}{V}\right| + \left|-\frac{\Delta I}{I}\right| = \frac{2}{50} + \frac{0.2}{20} = \frac{5}{100} \Rightarrow \%\,\text{Error} = \frac{\Delta R}{R} \times 100 = \frac{5}{100} \times 100 = 5\%$

Sol6. Power $= \frac{(V_{rms})^2}{R} = \frac{\left(\frac{V_0}{\sqrt{2}}\right)^2}{R} = \frac{250 \times 250}{2 \times 8} = 3906.25$ watt ≈ 4kW

Sol7. $\alpha = \frac{FR}{I} = \frac{FR}{\frac{mR^2}{2}} = \frac{2F}{mR} = \frac{2 \times 20}{20 \times 0.2} = 10\ \text{rad/s}^2$

$\omega^2 = \omega_0^2 + 2\alpha\theta \Rightarrow \theta = \frac{\omega^2}{2\alpha} \Rightarrow n = \frac{\theta}{2\pi} = \frac{\omega^2}{4\pi\alpha} \Rightarrow n = \frac{\theta}{2\pi} = \frac{50 \times 50}{4 \times 3.14 \times 10} = 19.9045 \approx 20$ **Sol8.**

$\frac{\lambda_1}{\lambda_3} = \frac{R\left(\frac{1}{2^2} - \frac{1}{5^2}\right)}{R\left(\frac{1}{2^2} - \frac{1}{3^2}\right)} = \frac{21}{100} \times \frac{36}{5} = 1.512 \approx 15 \times 10^{-1}$



Sol9. $\beta = \frac{\lambda D}{d} \Rightarrow \lambda = \frac{\beta d}{D} = \frac{6 \times 10^{-3} \times 1 \times 10^{-3}}{10} = 6 \times 10^{-7}\,m = 600\,nm$

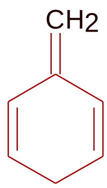
Sol10. **Using Conservation of linear momentum along X-asix , we can write**

$mv_0 = mv_2 \cos\theta \Rightarrow \cos\theta = \frac{v_0}{v_2} = \frac{10\sqrt{3}}{20} = \frac{\sqrt{3}}{2} \Rightarrow \theta = 30^0$

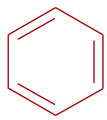
CHEMISTRY

SECTION – A

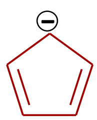
Sol1.



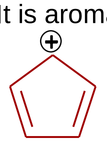
Non-aromatic because of one sp^3 hybrid carbon, also no complete delocalisation



It is aromatic; has 6π electrons

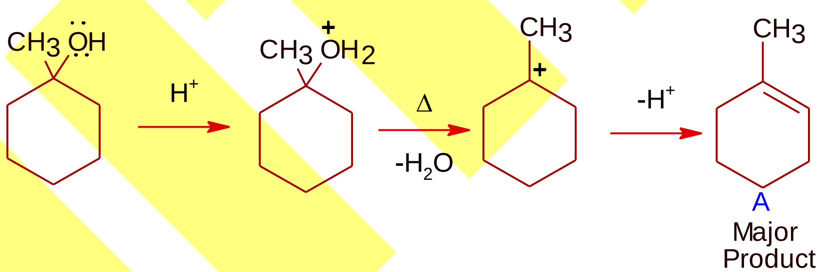


It is aromatic ; has 6π electrons

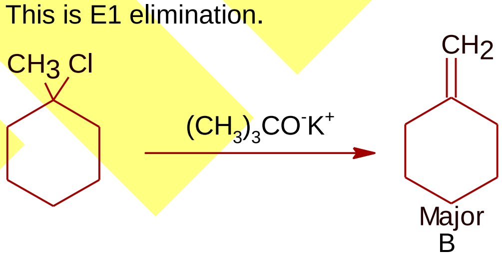


It has 4π electrons, so anti-aromatic

Sol2.

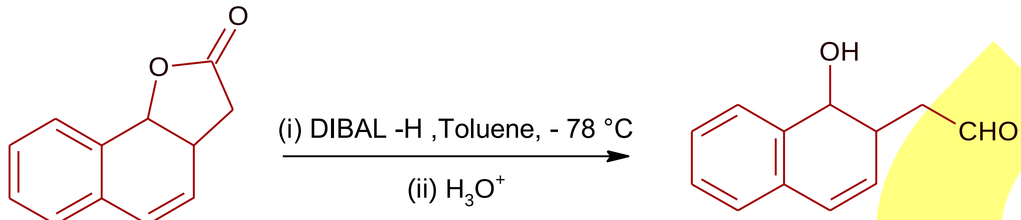


This is E1 elimination.



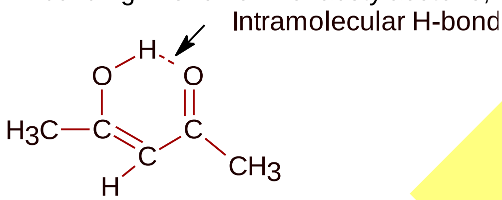
This is E2 elimination. With bulkier base like $(CH_3)_3CO^-$ Hofmann product is formed as major product.

Sol3. DIBAL – H at low temperature in non- polar solvent, followed by hydrolysis reduces esters to aldehyde and by product is alcohol.



Major Product
P

Sol4. Enol form of acetone exists in less than 0.1% quantity , since its its keto form is highly stable. But in case of acetylacetone, enol form is stabilised by intramolecular H- bonding, so its quantity increase to approximately 15%
H- bonding in enol form of acetylacetone;



Sol5. (a) Haber's process is used for synthesis of NH_3
(b) Ostwald's process is used to synthesis HNO_3
(c) Contact process is used for synthesis of H_2SO_4
(d) Hall-Heroult process is used for extraction of aluminium.

Sol6. Size of Bk^{3+} ion is less than Np^{3+} ion, since Berkelium (Bk) lie beyond Neptunium (Np) in actinoid series and the size variation here is because of actinoid contraction.

Sol7. E^o value for Ce^{4+} / Ce^{3+} is + 1.74 V which suggest that Ce^{4+} is strong oxidant reverting to its common +3 oxidation state, So Ce^{3+} is more stable than Ce^{4+}.

Sol8. Fat soluble vitamins are stored in our body relatively for longer duration as compared to water soluble vitamins.
Vitamin B and C are water soluble.
Thiamine is vitamin B1 while ascorbic acid is vitamin C.
Vitamin A and vitamin D are fat soluble, so stored in our body relatively for longer duration.

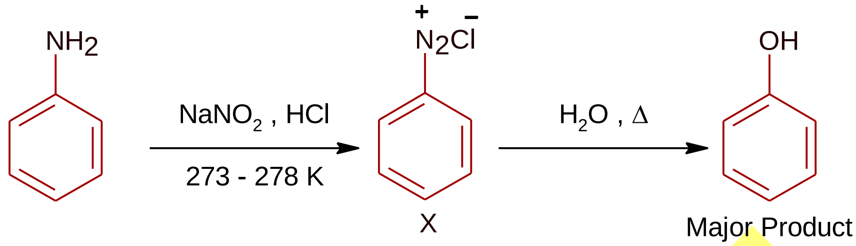
Sol9. Partially deactivated palladised charcoal is Lindlar's reagent.

Sol10. H_2O has tetrahedral geometry and bent shape with bond angle 104.5º which is smaller than 109.5º, it is because of presence of two lone pair in H_2O.

Sol11. Purification of compound is independent of physical state of the pure compound in chromatography technique.

Sol12. Formation of photochemical smog which is also known as oxidizing smog takes place in presence of sunlight and O_3.

Sol13.



Sol14.

		O.S of P
Hypophosphorous acid	– H_3PO_2	+ 1
Orthophosphoric acid	– H_3PO_4	+ 5
Hypophosphoric acid	– $H_4P_2O_6$	+ 4
Orthophosphorous acid	– H_3PO_3	+ 3

Sol15. $R-CONH_2 + Br_2 + 4NaOH \longrightarrow R-NH_2 + 2NaBr + Na_2CO_3 + 2H_2O$
This reaction is Hoffmann bromamide degradation in which amide converted to 1^o amine.
Here, option 2 does not involve Hoffmann bromamide degradation

Sol16. Hydrated halides of group 2nd Ca onward undergo dehydration on heating but hydrated halide of Mg i.e. $MgCl_2.8H_2O$ does not undergoes dehydration on heating but undergoes hydrolysis.
BeO is amphoteric while other oxides of group 2nd are basic is nature.

Sol17. Bismuth (Bi) is a metal which is group 15th element, its hydride is BiH_3 which is strongest reducing agent due to lowest thermal stability.

Sol18. Roasting is process in which sulphur is removed as SO_2 gas from sulphide ores on heating in excess of oxygen.

Sol19. H_2O_2 can act as both oxidizing & reducing agent in both acidic & basic medium.
In hydrogen economy, energy is stored & transmitted in form of dihydrogen.

Sol20. Antihistamine are antacid and antiallergic.

SECTION - B

Sol1. Edge length in bcc, $a_1 = 27\ \mathring{A}$

Let, Edge length in fcc be $a_2\ \mathring{A}$
Now, same element cystallises in bcc as well as fcc,

$$a_1 = \frac{4}{\sqrt{3}}r \Rightarrow r = \frac{\sqrt{3}}{4}a_1$$

$$a_2 = 2\sqrt{2}r \Rightarrow r = \frac{a_2}{2\sqrt{2}}$$

So; $\frac{\sqrt{3}}{4}a_1 = \frac{a_2}{2\sqrt{2}}$

$$\frac{\sqrt{3}}{4} \times 27 = \frac{a_2}{2\sqrt{2}}$$

$a_2 = 33.13\,\overset{o}{A}$

Nearest integer is 33.

Sol2. The balanced equation is

$$2MnO_4^- + 5C_2O_4^{2-} + 16H^+ \longrightarrow 2Mn^{2+} + 10CO_2 + 8H_2O$$

So, value of c = 16

Sol3. Solubility product of $A_2X = 4S_1^3$

Where , S_1 is solubility of salt A_2X

Solubility product of $MX = S_2^2$

Where , S_2 is solubility of MX,

$4S_1^3 = 4\times10^{-12}$

$S_1 = 10^{-4}M$

$S_2^2 = 4\times10^{-12}$

$S_2 = 2\times10^{-6}M$

So, $\frac{S_1}{S_2} = \frac{10^{-4}}{2\times10^{-6}} = 50$

Sol4. Moles of carbon in organic compound = Moles of carbon CO_2

$n_C = \frac{420}{44}$ moles

Mass of carbon in organic compound = $\frac{420}{44}\times12$

$= 114.54g$

Moles of hydrogen in compound = moles of hydrogen in H_2O

$n_H = 2\times\frac{210}{18}$ moles

Mass of hydrogen = $2\times\frac{210}{18}$ g

= 23.33g

% of H = $\frac{23.33}{750}\times100 = 3.11\%$

Nearest integer is 3.

Sol5. 6.5 molal solution means 6.5 moles of KOH is 1kg (1000g) of solvent (H_2O)

Moles of solute , n_B = 6.5

Mass of solute , W_B = 6.5 × 56

= 364 g

Mass of solvent, W_A = 1000g

Mass of solution = 1364g

Volume of solution = $\frac{1364}{1.89}$mL

Now; molarity = $\frac{6.5}{\frac{1364}{1.89}}\times1000M$

= 9M

Sol6. m = 10 molal
K_b = 0.5 K kg mol^{-1}.
Using : $\Delta T_b = i K_b m$
and $\alpha = \frac{i-1}{n-1}$
n for AB_2 is 3;
$\alpha = 0.1$
$0.1 = \frac{i-1}{3-1}$
$i = 1.2$
$\Delta T_b = 1.2 \times 0.5 \times 10$
$= 6^o C$
So; boiling point of solution = 100+6
= $106^o C$

Sol7. $\Delta G^o = -9.478$ kJ/ mol
Using
$\Delta G^o = -2.303 RT \log K_P$
$-9.478 \times 10^3 = -2.303 \times 8.314 \times 495 \log K_p$
$1 = \log K_p$
$K_p = 10$
Here, for given reaction
$K_p = K_c$

	A(g)	$\rightleftharpoons$	B(g)
Initial	22 mmol		–
At equilibrium	22 – x mmol		x mmol

Let V is volume of container
$$K_c = \frac{\frac{x}{V}}{\frac{22-x}{V}} = 10$$
$$\frac{x}{22-x} = 10$$
$x = 20$
So; mmol of B at equilibrium are 20.

Sol8. Complex $CoCl_3.4NH_3$ is represented as
$[CoCl_2(NH_3)_4]Cl$
Here; 4 NH_3 are neutral monodentate ligands
So,2 equivalents of ethylene diamine replaces $4NH_3$ since, ethylene diamine is didentate ligand.

Sol9. T_1 = 300 K $K_1 = 1\times10^{-3}$ s^{-1}
T_2 = 200 K K_2 = ?
E_a = 11.488 kJ / mol
Using Arrhenius equation
$$\log\frac{K_2}{K_1} = \frac{Ea}{2.303R}\left[\frac{T_2 - T_1}{T_1T_2}\right]$$

$$\log\frac{K_2}{10^{-3}} = \frac{11.488\times10^3}{2.303\times8.314}\times\left[\frac{-100}{6\times10^4}\right]$$

$$\log\frac{K_2}{10^{-3}} = -1$$

$$\frac{K_2}{10^{-3}} = 10^{-1}$$

$$K_2 = 10^{-4}s^{-1}$$

$$K_2 = 10\times10^{-5}s^{-1}$$

Sol10. $K.E = \phi - \phi_0$

$$\phi_0 = 3eV = 3\times1.6\times10^{-19}J = 4.8\times10^{-19}J$$

$$\phi = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\times3\times10^8}{248\times10^{-9}}J = 0.08\times10^{-17}J = 8\times10^{-19}J$$

$$K.E = 8\times10^{-19} - 4.8\times10^{-19} = 3.2\times10^{-19}J$$

Now using,

$$\lambda = \frac{h}{\sqrt{2K.Em}}$$

$$\lambda = \frac{6.63\times10^{-34}}{\sqrt{2\times3.2\times10^{-19}\times9.1\times10^{-31}}}m = \frac{6.63\times10^{-34}}{7.63\times10^{-25}}m = 0.87\times10^{-9}m = 8.7\,\mathring{A}$$

So; nearest integer is 9.

MATHEMATICS
SECTION – A

Sol1. $\log_{1/\sqrt{2}}\left(\frac{|z|+11}{(|z|-1)^2}\right) \le 2$

$\Rightarrow \frac{|z|+11}{(|z|-1)^2} \ge \frac{1}{2}$ $\qquad |z| \ne 1$

$\Rightarrow |z|^2 - 4|z| - 21 \le 0$ $\qquad \Rightarrow (|z|-7)(|z|+3) \le 0$

$\Rightarrow |z| - 7 \le 0$

$\therefore\ |z|\max = 7$

Sol2. $b = a + c$ - (i)

Variance of a, b, c & a+2, b+2, c+2, are same

$\therefore\ d^2 = \frac{1}{3}(a^2+b^2+c^2) - \left(\frac{a+b+c}{3}\right)^2$

$= \frac{1}{3}(a^2+b^2+c^2) - \left(\frac{2b}{3}\right)^2$ as $a + c = b$

$9d^2 = 3(a^2+b^2+c^2) - 4b^2$

$\Rightarrow b^2 = 3(a^2+c^2) - 9d^2$

Sol3. $(|x|-3)|x+4| = 6$

Case(i) $x < -4$

$(-1)^2(x+3)(x+4) = 6 \quad \Rightarrow\ x = -6$

Case(ii) $-4 \le x < 0$

$\Rightarrow\ x^2 + 7x + 18 = 0$ $\qquad$ no real solution

Case(iii) $x \ge 0 \quad (x-3)(x+4) = 6$

$\Rightarrow\ x^2 + x - 18 = 0$

$x = \frac{-1 \pm \sqrt{73}}{2}$

$x \ge 0 \Rightarrow x = \frac{\sqrt{73}-1}{2}$

Only two solution .

Sol4. $f(x) = (4a-3)(x + \ell n5) + 2(a-7)\cot\frac{x}{2}\sin^2\frac{x}{2}$

$= (4a-3)(x+\ell n5) + (a-7)\sin x$

$f'(x) = (4a-3) + (a-7)\cos x$

For critical points $f'(x) = 0$

$\Rightarrow\ \cos x = \frac{3-4a}{a-7}$ $\qquad \Rightarrow -1 \le \frac{3-4a}{a-7} \le 1$

$\Rightarrow a \in \left[-\frac{4}{3}, 2\right]$

Sol5. $\log_{10}^{\sin x} + \log_{10}^{\cos x} = -1$

$\Rightarrow \sin x \cos x = \frac{1}{10}$ - (1)

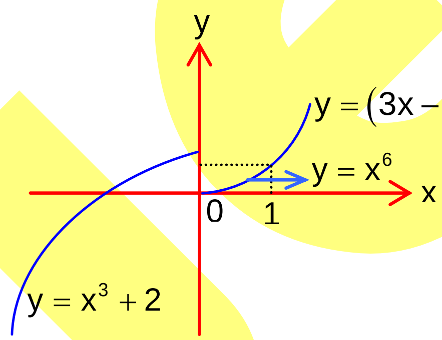
$\log_{10}(\sin x + \cos x) = \frac{1}{2}(\log_{10}^{n} - 1) = \frac{1}{2}\log\frac{n}{10}$

$\Rightarrow \sin x + \cos x = \sqrt{\frac{n}{10}}$

$\Rightarrow 1 + 2\sin x \cos x = \frac{n}{10}$ (squaring both side)

$\Rightarrow 1 + \frac{2}{10} = \frac{n}{10} \Rightarrow n = 12$

Sol6. $f\{g(x)\} = \begin{cases} x^3 + 2 & ; x^3 < 0 \,\&\, x < 1 \\ x^6 & ; x^3 \geq 0 \,\&\, x < 1 \\ 3x & ; 3x - 2 < 0 \,\&\, x \geq 0 \\ (3x-2)^2 & ; 3x - 2 \geq 0 \,\&\, x \geq 1 \end{cases}$



$\Rightarrow fog(x) = \begin{cases} x^3 + 2 & ; x < 0 \\ x^6 & ; 0 \leq x < 1 \\ (3x-2)^2 & ; x \geq 1 \end{cases}$

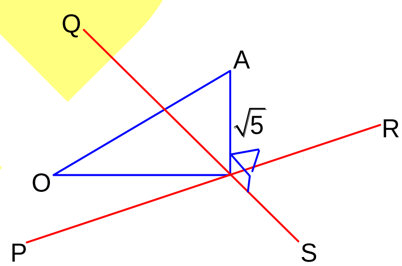
fog(x) is discontinuous at $x = 0$. $\therefore$ non differentiable
fog (x) is not differentiable at x = 0

Sol7. PR (line)

$\vec{r} = 3i - j + 2k + \lambda\left(4\hat{i} - \hat{j} + 2\hat{k}\right)$ - (I)

QS (line)

$\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \mu\left(-2\hat{i} + \hat{j} - 2\hat{k}\right)$ - (II)



If they interest at T then

$\left.\begin{matrix} 3 + 4\lambda = 1 - 2\mu \\ -1 - \lambda = 2 + \mu \\ 2 + 2\lambda = -4 - 2\mu \end{matrix}\right\} \Rightarrow \lambda = 2 \,\&\, \mu = -5$

$\therefore T(11, -3, 6)$

Also, OT is coplanar with lines PR and QS.

$\Rightarrow TA \perp^{r} OT$

$\left.\begin{matrix} |\overrightarrow{OT}| = \sqrt{166} \\ |\overrightarrow{TA}| = \sqrt{5} \end{matrix}\right\} \Rightarrow |\overrightarrow{OA}| = \sqrt{|\overrightarrow{OT}|^2 + |\overrightarrow{TA}|^2} = \sqrt{171}$

Sol8. $\frac{dy}{dx} + 2y\tan x = \sin x.$

$\Rightarrow I.F. = e^{2\int \tan x\, dx} = e^{2\ell n \sec x.} = \sec^2 x.$

$\therefore$ solution is $y\sec^2 x = \int \sin x \sec^2 x\, dx = \int \sec x \tan x\, dx$

$\Rightarrow y\sec^2 x = \sec x + c.$

$y\left(\pi/3\right) = 0 \Rightarrow 0 = \sec\frac{\pi}{3} + c \Rightarrow c = -2.$

$\therefore y = \frac{\sec x - 2}{\sec^2 x}$

Now let $g(t) = \frac{1}{t} - \frac{2}{t^2} \quad |t| \ge 1$

$\therefore g'(t) = -\frac{1}{t^2} + \frac{4}{t^3}$

$g''(t) = \frac{2}{t^3} - \frac{12}{t^4}$

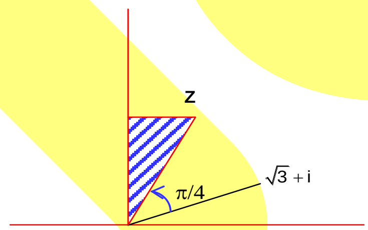
g(t) is maximum for $t = 4.$

$\therefore g(t)\max = \frac{1}{8}$

Sol9. Let $\alpha i + \beta j = z$ be a complex number

& $\sqrt{3}i + j = \left(\sqrt{3} + i\right)$ where $i = \sqrt{-1}$

$\frac{z}{\sqrt{3} + i} = \frac{|z|}{\left|\sqrt{3} + i\right|} e^{i\frac{\pi}{4}}$ and $|z| = \left|\sqrt{3} + i\right| = 2$



$\therefore z = \left(\sqrt{3} + i\right)\left(\frac{1+i}{\sqrt{2}}\right)$

$= \frac{1}{\sqrt{2}}\left[\left(\sqrt{3} - 1\right) + i\left(\sqrt{3} + 1\right)\right]$

$\therefore \alpha = \frac{\sqrt{3} - 1}{\sqrt{2}}, \beta = \frac{\sqrt{3} + 1}{\sqrt{2}}.$

$\therefore$ area of required triangle

$\Delta = \frac{1}{2}\left(\frac{\sqrt{3} - 1}{\sqrt{2}}\right)\left(\frac{\sqrt{3} + 1}{\sqrt{2}}\right) = \frac{1}{2}$

Sol10. $\left(3^{\frac{1}{4}} + 5^{\frac{1}{8}}\right)^{60}$

General term $= {}^{60}C_r\, 3^{\frac{r}{4}}\, 5^{\frac{60-r}{8}}$

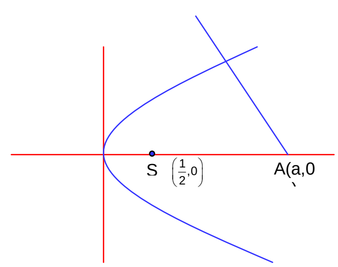
Terms are rational for r = 4, 12, 20, 28, 36, 44, 52, 60

$\therefore$ number of irrational terms 61- 8 = 53 = n

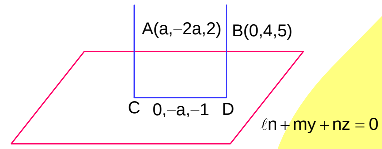
$\therefore$ n – 1 = 52

Sol11. $y^2 = 2x = 4\left(\frac{1}{2}\right)x.$

$a > 2 \times \frac{1}{2} \Rightarrow a > 1$



Sol12. AC is collinear with l, m, n.



$\therefore \; \frac{\ell}{a} = \frac{m}{-a} = \frac{n}{4} = \lambda$

Again (0, - a, -1) lies on lx+ my + nz = 0

$\Rightarrow a = -\frac{n}{m}. = -\frac{4\lambda}{-a\lambda} \Rightarrow a^2 = 4,\; a = \pm 2$

for a > 0 $\underline{a = 2}$ direction ratios of BD are 2, - 2, 4.

$\therefore$ Equation of BD

$\frac{x}{2} = \frac{y-4}{-2} = \frac{z-5}{4} = r$

Any point on it $(2r, 4-2r, 5+4r)$

lies on plane $lx + my + nz \equiv 2x - 2y + 4z = 0$

$\Rightarrow 4r - 2(4-2r) + 4(5+4r) = 0 \Rightarrow r = -\frac{1}{2}$

$\therefore D(-1,5,3),\; C(0,-2,-1)$

$\therefore CD = \sqrt{1+49+16} = \sqrt{66}$

Sol13. $(81)^{\sin^2 x} + (81)^{\cos^2 x} = 30.$

Put $\Rightarrow y + \frac{81}{y} = 30$ where $81^{\sin^2 x} = y$

$\Rightarrow y = 3, 27$

Either $\sin^2 x = \frac{1}{4}. \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}.$

OR, $\sin^2 x = \frac{3}{4} \Rightarrow x = \frac{\pi}{3}, \frac{2\pi}{3}.$

$(\text{as } 0 \le x \le \pi)$

Total possible solution $= 4.$

Sol14.

p	q	$p \wedge q$	$p \to q$	$(p \wedge q) \to (p \to q)$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

Sol15. Equation of chord of $x^2 + y^2 = 25$ with mid point (h, k) is $xh + yk = h^2 + k^2$.

Or, $y = \left(-\frac{h}{k}\right)x + \frac{h^2+k^2}{k}$

If this touches $\frac{x^2}{9} - \frac{y^2}{16} = 1$

Then $\left(\frac{h^2+k^2}{k}\right)^2 = 9\left(-\frac{h}{k}\right)^2 - 16$

$\Rightarrow (h^2 + k^2)^2 = 9h^2 - 16k^2$

$\therefore$ Required locus $\left(x^2+y^2\right)^2 = 9x^2-16y^2$

Sol16. $A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$ $\quad i=\sqrt{-1}$

$A^2 = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}\begin{bmatrix} i & -i \\ -i & i \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix}$

$A^4 = \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} = \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}$

$A^8 = \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}\begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix} = \begin{bmatrix} 128 & -128 \\ -128 & 128 \end{bmatrix}$

$\begin{bmatrix} 128 & -128 \\ -128 & 128 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$

$128x-128y = 8 \Rightarrow 16x-16y = 1$ - (I)

$-128x+128y = 64 \Rightarrow -2x+2y = 1$ - (II)

$(1) \Rightarrow \quad x-y = \frac{1}{16}$

$(2) \Rightarrow x-y = -\frac{1}{2}$

System is inconsistent hence No solution

Sol17. $\left(1-x+x^3\right)^n = \sum_{J=0}^{3n} a_3 x^3.$

$= a_0 + a_1x + a_2x^2 + a_3x^3 + ... + a_{3n}x^{3n}$ - (I)

Let $A = a_0 + a_2 + a_4 + ...$

$B = a_1 + a_3 + a_5 + ...$

In (I) put $x=1$, $1 = a_0 + a_1 + a_2 + a_3 ....$

In (I) put $x=-1$, $1 = a_0 - a_1 + a_2 - a_3 + ...$

$\therefore$ $\quad$ A = 1 $\quad$ & B = 0.

Now $\sum_{J=0}^{\left[\frac{3n}{2}\right]} a_{2J} + 4\sum_{J=0}^{\left[\frac{3n-1}{2}\right]} a_{2J+1} = A + 4B = 1$

Sol18. $s_k = \sum_{r=l}^{k} \tan^{-1}\left(\frac{6^r}{2^{2r+1}+3^{2r+1}}\right)$

$t_r = \tan^{-1}\frac{\frac{6^r}{2^{2r+1}}}{1+\left(\frac{3}{2}\right)^{2r+1}}$

$= \tan^{-1}\frac{\frac{3^r}{2^{r+1}}}{1+\left(\frac{3}{2}\right)^{r+1} \times \left(\frac{3}{2}\right)^r}$

$$= \tan^{-1}\frac{\left(\frac{3}{2}\right)^{r+1}-\left(\frac{3}{2}\right)^{r}}{1+\left(\frac{3}{2}\right)^{r+1}\left(\frac{3}{2}\right)^{r}}$$

$$= \tan^{-1}\left(\frac{3}{2}\right)^{r+1} - \tan^{-1}\left(\frac{3}{2}\right)^{r}$$

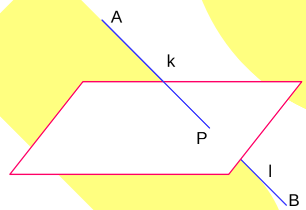
$$\therefore\ s_k = \sum_{r=1}^{k} t_r = \left\{\tan^{-1}\left(3/2\right)^2 - \tan^{-1}\left(\frac{3}{2}\right)\right\} + \left\{\tan^{-1}\left(\frac{3}{2}\right)^3 - \tan^{-1}\left(3/2\right)^2\right\}$$

$$+\left\{\tan^{-1}\left(3/2\right)^4 - \tan^{-1}\left(3/2\right)^3\right\} + ... + \tan^{-1}(\infty)$$

$$= \tan^{-1}\infty - \tan^{-1}3/2 = \frac{\pi}{2} - \tan^{-1}3/2$$

$$= \cot^{-1} 3/2$$

Sol19. $A(-3,-6,1)$ $\quad B(2,4,-3)$



$P\left(\frac{2k-3}{k+1}, \frac{4k-6}{k+1}, \frac{-3k+1}{k+1}\right)$ lies on

$lx + my + nz = 0$

$\Rightarrow K(2\ell + 4m - 3n) = 3\ell + 6m - n.$

$\Rightarrow k = \frac{3\ell + 6m - n}{2\ell + 4m - 3n}.$

Now place P contains the line

$\frac{x-1}{-1} = \frac{y+4}{2} = \frac{z+2}{3}$

$$\left.\begin{array}{l}\Rightarrow \ell - 4m - 2n = 0\\ \& -\ell + 2m + 3n = 0\end{array}\right\} \Rightarrow \frac{\ell}{-8} = \frac{m}{-1} = \frac{n}{-2}$$

$\therefore\ k = \frac{-24-6+2}{-16-4+6} = 2.$

Sol20. Two drawn cards are spades

$\therefore$ probability that missing card is spade $= \frac{11}{50}$

$\therefore$ Probability that the missing card is not spade $= 1 - \frac{11}{50} = \frac{39}{50}$

SECTION – B

Sol1. set $= S\{11, 8, 21, 16, 26, 32, 4\}.$

$S \equiv \{4, 8, 11, 16, 21, 26, 32\}$

$GP \equiv \{4, 8, 16, 32\}$

$AP \equiv \{11, 16, 21, 26\}$

GP series up to maximum possible 4 digit number

$\{4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096, 8192\}.$

AP series in extended form

$\{11,16,21,26,31.....\}$
The last dight in AP series is either 1 or 6
$\therefore$ the common terms between the two series are of last digit 6.
hence there are 3 terms in common.

Sol2. $y(x)=\int_0^x (2t^2-15t+10)dt.$

$\frac{dy}{dx}=2x^2-15x+10.$

$-\frac{dx}{dy}$ for $(a,b)=\frac{-1}{2a^2-15a+10}=-\frac{1}{3}$

$\therefore 2a^2-15a+7=0 \qquad (2a-1)(a-7)=0$

$\therefore\ a=\frac{1}{2}$ or $a=7.$ $\qquad a=1/2$ Rejected as a >1

$b=\int_0^a (2t^2-15t+10)dt=\frac{2}{3}a^3-\frac{15}{2}a^2+10a.$

$6b=4a^3-45a^2+60a$

$6b=4(7)^3-45(7)^2+60\times 7=-413$

$\therefore\ |a+6b|=|7-413|=406$

Sol3. $f(x)+f(x+1)=2$ - (1)
replace x lay x+1
$f(x+1)+f(x+2)=2$ - (2)
(2) – (1) $\Rightarrow\ f(x+2)=f(x)$
$\therefore$ f(x) is periodic with period 2

$I_1=\int_0^8 f(x)dx=4\int_0^2 f(x)\,dx.$

$I_2=\int_{-1}^3 f(x)dx \quad =\int_0^4 f(x-1)dx.$

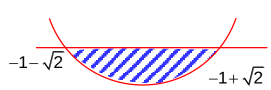
$f(x-1)+f(x)=2.$

Now $I_1+2I_2=4\int_0^2 f(x)dx+2\int_0^4 f(x-1)dx$

$=2\int_0^4 f(x)dx+2\int_0^4 f(x-1)dx$

$=2\int_0^4 [f(x)+f(x-1)]dx=2\int_0^4 2dx=16$

Sol4. $\frac{dy}{dx}=2(x+1)$



$dy=2(x+1)dx.$

$\Rightarrow y=(x+1)^2-c$ - (I)

For $y=0\quad x=-1-\sqrt{c},-1+\sqrt{c}$

$A=\frac{4\sqrt{8}}{3}=\int_{-1-\sqrt{c}}^{-1+\sqrt{c}}\{(x+1)^2-c\}dx.$

$$\Rightarrow \frac{4\sqrt{8}}{3} = \left|\left[\frac{(x+1)^3}{3} - cx\right]_{-1-\sqrt{c}}^{-1+\sqrt{c}}\right|$$

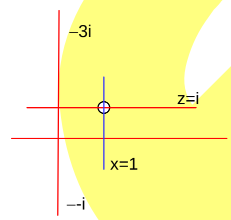
$$= \frac{4}{3}c\sqrt{c}.$$

$$\Rightarrow c = 2$$

$$\therefore y = (x+1)^2 - 2.$$

$$\therefore y(1) = (1+1)^2 - 2 = 2$$

Sol5. $\left|\frac{z+i}{z-3i}\right| = 1 \Rightarrow |z+i| = |z-3i| \Rightarrow z = x+i$



$w = z\bar{z} - 2z + 2$

Let $z = x + iy$

$\Rightarrow w = x^2 + y^2 - 2(x+iy) + 2$

$= (x-1)^2 + y^2 + 1 - 2iy$

$\text{Re}(w) = (x-1)^2 + y^2 + 1$

$\text{Re}(w)$ is minimum for $x = 1$

$\therefore$ The common z satisfying both equation is $z = 1+i$

$\therefore\ w = 2 - 2(1+i) + 2 = 2 - 2i$

$w^2 = 4(1-1-2i) = -8i$

$w^4 = 64i^2 = -64 \in R$

$\therefore$ least n $\in$ N for which $w^n \in R$ is n = 4

Sol6. $\lim\limits_{x\to 0}\frac{ae^x - b\cos x + ce^{-x}}{x\sin x} = 2$

$$\lim_{x\to 0}\frac{a\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+....\right) - b\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}.....\right) + C\left(1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+.....\right)}{x^2\left(\frac{\sin x}{x}\right)} = 2$$

$$\Rightarrow \lim_{x\to 0}\frac{(a-b+c) + x(a-c) + \frac{x^2}{2}(a+b+c) + \text{terms with higher power of } x}{x^2} = 2$$

$$\Rightarrow a-b+c = 0,\ a-c = 0 \text{ and } \frac{a+b+c}{2} = 2 \qquad \Rightarrow a+b+c = 4$$

Sol7. $A = \lim\limits_{x\to\infty}\frac{2}{n}\left[f\left(\frac{1}{n}\right) + f\left(\frac{2}{n}\right) + ... + f\left(\frac{n-1}{n}\right) + f\left(\frac{n}{n}\right)\right]$

$$= \lim_{x\to\infty}\frac{2}{n}\sum_{r=1}^{n} f\left(\frac{r}{n}\right) = 2\int_0^1 f(x)\,dx = 2\int_0^1 \log_2\left(1+\tan\frac{\pi x}{4}\right)dx.$$

put $\frac{\pi x}{4} = t$.

$$\Rightarrow A = \frac{2}{\log 2} \times \frac{4}{\pi}\int_0^{\pi/4} \log(1+\tan t)dt = \frac{8}{\pi \log 2} \times \frac{\pi}{8}\log 2$$
$$= 1$$

Sol8. Let $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$

$$A.A^T = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}\begin{pmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{pmatrix}$$

Sum of diagonal elements of AA^T

$a_{11}^2 + a_{12}^2 + a_{13}^2 + a_{21}^2 + a_{22}^2 + a_{23}^2 + a_{31}^2 + a_{32}^2 + a_{32}^2 = 9$

where each $a_{iJ} \in \{0,1,2,3\}$

Case I one of a_{ij} = 3 and rest are 0 in ${}^9C_1 = 9$ ways

Case II two of a_{ij} are 2 and one $a_{ij} = 1$ and rest are zero in $9c_2 \times 7c_1 = 252$ ways

Case III one of a_{ij} = 2, 5 of a_{ij} = 1 and rest are zero in $9 \times 8c_5 = 504$ ways

Case IV all aij = 1 in 1 ways $\therefore$ Total = $9 + 252 + 504 + 1 = 766$

Sol9. $\left(P^{-1}AP - I\right)^2$

$= \left(P^{-1}AP - I\right)\left(P^{-1}AP - I\right)$

$= P^{-1}A^2P - 2P^{-1}AP + I$

$= P^{-1}(A-I)^2 P$

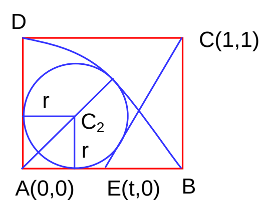
$\left|P^{-1}AP - I\right|^2 = \left|P^{-1}\right|\left|A-I\right|^2\left|P\right| = \left|A-I\right|^2$

$$|A-I| = \begin{bmatrix} 1 & 7 & w^2 \\ -1 & w^2 & 1 \\ 0 & -w & -w \end{bmatrix}$$

$= -6w$

$|A-I|^2 = 36w^2$

Sol10. $AC_2 = \sqrt{2}r = 1 - r$



$\Rightarrow r = \frac{1}{\sqrt{2}+1} = \sqrt{2} - 1$ - (I)

Equation CE $y - 1 = \frac{1}{1-t}(x-1)$

$\Rightarrow x + (t-1)y - t = 0$

as this line is tangent to circle C_2

$\therefore \ r = \left|\dfrac{r + r(t-1) - t}{\sqrt{1+(t-1)^2}}\right|$

$\Rightarrow r^2 + r^2(t-1)^2 = t^2(r-1)^2$

$\Rightarrow \left(3-2\sqrt{2}\right) + \left(3-2\sqrt{2}\right)(t-1)^2 = t^2\left(6-4\sqrt{2}\right)$

$\Rightarrow \ 1+(t-1)^2 = 2t^2 \Rightarrow t^2 + 2t - 2 = 0$

$\Rightarrow t = -1 \pm \sqrt{3}$

$t > 0 \Rightarrow t = \sqrt{3} - 1$

$\therefore BE = 1 - \left(\sqrt{3}-1\right) = 2 - \sqrt{3} = \alpha + \sqrt{3}\beta$

Comparing $\alpha = 2, \beta = -1$

$\therefore \alpha + \beta = 2 - 1 = 1$