Paper-16-03-2021-Evening Shift

PHYSICS

SECTION – A

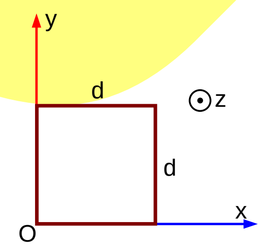
Q1. A mosquito is moving with a velocity $\vec{\upsilon} = 0.5t^2\,\hat{i} + 3t\,\hat{j} + 9\hat{k}$ m/s and accelerating in uniform conditions. What will be the direction of mosquito after 2s?

(A) $\tan^{-1}\left(\frac{2}{3}\right)$ from x – axis
(B) $\tan^{-1}\left(\frac{5}{2}\right)$ from y – axis
(C) $\tan^{-1}\left(\frac{5}{2}\right)$ from x – axis
(D) $\tan^{-1}\left(\frac{2}{3}\right)$ from y – axis

Q2. In order to determine the young's modulus of a wire of radius 0.2 cm (measured using a scale of least count = 0.001 cm) and length 1m (measured using a scale of least count = 1 mm), a weight of mass 1 kg (measured using a scale of least count = 1g) was hanged to get the elongation of 0.5 cm (measured using a scale of least count 0.001 cm.) What will be the fractional error in the value of young's modulus determined by this experiment?

(A) 9%
(B) 0.9%
(C) 0.14%
(D) 1.4%

Q3. The magnetic field in a region is given by $\vec{B} = B_0\left(\frac{x}{a}\right)\hat{k}$. A square loop of side d is placed with its edges along the x and y axes. The loop is moved with a constant velocity $\vec{\upsilon} = \upsilon_0\,\hat{i}$. The emf induced in the loop is:



(A) $\frac{B_0\upsilon_0 d^2}{a}$
(B) $\frac{B_0\upsilon_0 d}{2a}$
(C) $\frac{B_0\upsilon_0^2 d}{2a}$
(D) $\frac{B_0\upsilon_0 d^2}{2a}$

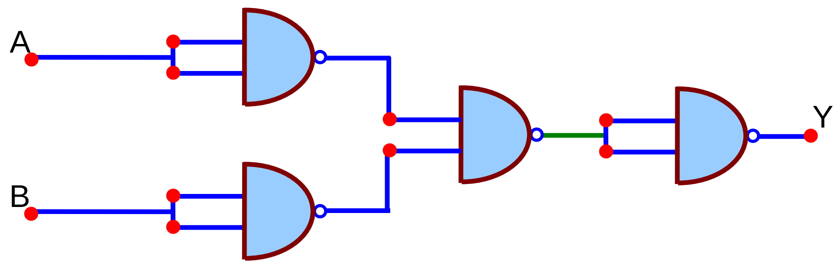
Q4. Calculate the value of mean free path (λ) for oxygen molecules at temperature 27^0C and pressure 1.01×10^5 Pa. Assume the molecular diameter 0.3 nm and the gas is ideal. $\left(k = 1.38\times10^{-23}\,JK^{-1}\right)$

(A) 58 nm
(B) 86 nm
(C) 32 nm
(D) 102 nm

Q5. Calculate the time interval between 33% decay and 67% decay if half-life of a substance is 20 minutes.

(A) 13 minutes
(B) 60 minutes
(C) 40 minutes
(D) 20 minutes

Q6. The following logic gate is equivalent to:

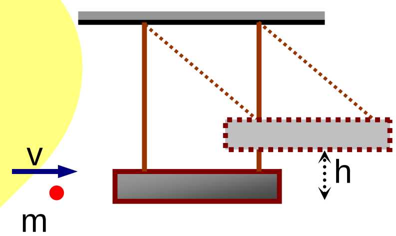


(A) NAND Gate
(B) AND Gate
(C) NOR Gate
(D) OR Gate

Q7. Red light differs from blue light as they have:
(A) Different frequencies and same wavelengths
(B) Same frequencies and same wavelengths
(C) Same frequencies and different wavelengths
(D) Different frequencies and different wavelengths

Q8. What will be the nature of flow of water form a circular tap, when its flow rate increased from 0.18L/min to 0.48L/min? The radius of the tap and viscosity of water are 0.5 cm and 10^{-3} Pa s, respectively.
(Density of water : $10^3 kg/m^3$)
(A) Remains steady flow
(B) Unsteady to steady flow
(C) Remains turbulent flow
(D) Steady flow to unsteady flow

Q9. A large block of wood of mass $M = 5.99 kg$ is hanging form two long massless cords. A bullet of mass m = 10 g is fired into the block and gets embedded in it. The (block + bullet) then swing upwards, their centre of mass rising a vertical distance $h = 9.8 cm$ before the (block + bullet) pendulum comes momentarily to rest at the end of its arc. The speed of the bullet just before collision is :
(take $g = 9.8 ms^{-2}$)



(A) 821.4 m/s
(B) 811.4 m/s
(C) 831.4 m/s
(D) 841.4 m/s

Q10. A charge Q is moving $d\vec{\ell}$ distance in the magnetic field $\vec{B}$. Find the value of work done by $\vec{B}$.
(A) 1
(B) Zero
(C) Infinite
(D) -1

Q11. Two identical antennas on identical towers are separated from each other by a distance of 45 km. What should be the minimum height of receiving antenna to receive the signals in line of sight? (Assume radius of earth is 6400 km)
(A) 79.1m
(B) 19.77m
(C) 39.55m
(D) 158.2m

Q12. Amplitude of a mass-spring system, which is executing simple harmonic motion decreases with time. If mass $= 500g$, Decay constant $= 20 g/s$ then how much time is required for the amplitude of the system to drop to half of its initial value?
$(\ln 2 = 0.693)$
(A) 34.65 s
(B) 15.01 s
(C) 17.32 s
(D) 0.034 s

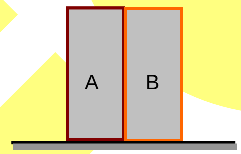
Q13. The half- life of Au^{198} is 2.7 days. The activity of 1.50 mg of Au^{198} if its atomic weight is 198 g mol^{-1} is, $\left(N_A = 6\times10^{23} / mol\right)$

(A) 240 Ci
(B) 252 Ci
(C) 535 Ci
(D) 357 Ci

Q14. The de – Broglie wavelength associated with an electron and a proton were calculated by accelerating them through same potential of 100V. What should be the ratio of their wavelengths? $\left(m_p = 1.00727u, \quad m_e = 0.00055u\right)$

(A) 43 : 1
(B) 41.4 : 1
(C) $41.4 : 1$
(D) 1860 : 1

Q15. A bimetallic strip consists of metals A and B. It is mounted rigidly as shown. The metal A has higher coefficient of expansion compared to that metal B. When bimetallic strip is placed in a cold bath, it will



(A) Neither bend nor shrink
(B) Not bend but shrink
(C) Bend towards the left
(D) Bend to wards the right

Q16. The refractive index of converging lens is 1.4. What will be the focal length of this if it is placed in a medium of same refractive index? Assume the radii of curvature of the faces of lens are R_1 and R_2 respectively.

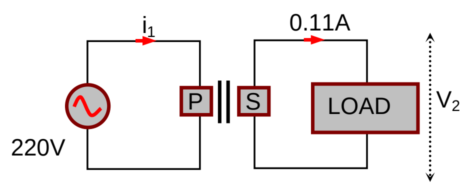
(A) 1
(B) $\dfrac{R_1R_2}{R_1 - R_2}$
(C) Infinite
(D) Zero

Q17. Find out the surface charge density at the intersection of point x = 3 m plane and x- axis, in the region of uniform line charge of 8 nC/m lying along the z-axis in free space.
(A) 0.424 nC m^{-2}
(B) 47.88 C/m
(C) 0.07 nC m^{-2}
(D) 4.0 nC m^{-2}

Q18. **Statement I :** A cyclist is moving on an un-banked road with a speed of 7 kmh^{-1} and takes a sharp circular turn along a path of radius of 2m without reducing the speed. The static friction coefficient is 0.2 The cyclist will not slip and pass the curve. $\left(g = 9.8 m/s^2\right)$

Statement II : If the road is at an angle of 45^0 , cyclist can cross of the curve of 2m radius with the speed of 18.5 kmh^{-1} without slipping
In the light of the above statements, choose the correct the answer from the options given below.
(A) **Statement I** is incorrect and **statement II** is correct
(B) Both **statement I** and **statement II** are true
(C) **Statement I** is correct and **statement II** is incorrect
(D) Both **statement I** and **statement II** are false

Q19. For the given circuit, comment on the type of transformer used.



(A) Step down transformer
(B) Auxilliary transformer
(C) Step- Up transformer
(D) Auto transformed

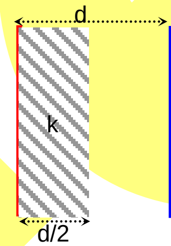
Q20. A resistor develops 500J of thermal energy in 20 s when a current of 1.5A is passed through it. If the current is increased from 1.5 A to 3 A, what will be the energy developed in 20 s.
(A) 2000 J
(B) 500 J
(C) 1000 J
(D) 1500 J

Section : B

Q1. A closed-organ pipe of length L and an open pipe contain gases of densities ρ_1 and ρ_2 respectively. The compressibility of gases are equal in both the pipes. Both the pipes are vibrating in their first overtone with same frequency. The length of the open pipe is $\frac{x}{3}L\sqrt{\frac{\rho_1}{\rho_2}}$ where x is ---------. (Round off to the Nearest Integer)

Q2. In a parallel plate capacitor set up, plate area of capacitor is 2 m^2 and the plates are separated by 1m. If the space between the plates are filled with a dielectric material of thickness 0.5 m and area 2 m^2 (See fig) the capacitance of the set- up will be -------- ε_0.

(Dielectric constant of the material = 3.2) (Round off to the Nearest Integer)

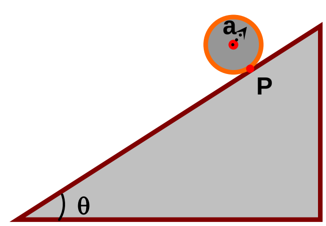


Q3. A swimmer can swim with velocity of 12 km/ h in still water. Water flowing in a river has velocity 6 km/ h. The direction with respect to the direction of flow of river water he should swim in order to reach the point on the other bank just opposite to his starting point is ---------^{0}. (Round off to the Nearest Integer)
(Find the angle in degrees)

Q4. A body of mass 2kg moves under a force of $\left(2\hat{i}+3\hat{j}+5\hat{k}\right)$N. It starts from rest and was at the origin initially. After 4 s, its new coordinates are (8, b, 20). The value of b is -------------.
(Round off the Nearest Integer)

Q5. The energy dissipated by a resistor is 10 mJ in 1 s when an electric current of 2 mA flows through it. The resistance is ----------- Ω . (Round off to the Nearest Integer)

Q6. A solid disc of radius 'a' and mass 'm' rolls down without slipping on an inclined plane making an angle θ with the horizontal. The acceleration of the will be $\frac{2}{b}g\sin\theta$ where b is --------. (Round off to the Nearest Integer)
(g = acceleration due to gravity)
θ = angle as shown in figure)



Q7. A deviation of 2^0 is produced in the yellow ray when prism of crown and flint glass are achromatically combined. Taking dispersive powers of crown and flint glass as 0.02 and 0.03 respectively and refractive index for yellow light for these glasses are 1.5 and 1.6 respectively. The refracting angles for crown glass will be ----------^{0}. (in degree)
Round off to the Nearest Integer)

Q8. A force $\vec{F}=4\hat{i}+3\hat{j}+4\hat{k}$ is applied on an intersection point of x = 2 plane and x –axis. The magnitude of torque of this force about a point (2, 3, 4) is -------------. (Round off to the Nearest Integer)

Q9. If one wants to remove all the mass of the earth to infinity in order to break it up completely. The amount of energy that needs so be supplied will be $\frac{x}{5}\frac{GM^2}{R}$ where x is ---------. (Round off to the Nearest Integer)
(M is the mass of earth, R is the radius of earth, G is the gravitational constant)

Q10. For an ideal heat engine, the temperature of the source is 127^0C. In order to have 60% efficiency the temperature of the sink should be ---------- ^0C. (Round off to the Nearest Integer)

CHEMISTRY

SECTION A

Q1. Ammonolysis of Alkyl halides followed by the treatment with NaOH solution can be used to prepare primary, secondary and tertiary amines. The purpose of NaOH in the reaction is:
(A) to activate NH_3 used in the reaction
(B) to remove acidic impurities
(C) to increase the reactivity of alkyl halide
(D) to remove basic impurities

Q2. The characteristics of elements X,Y and Z with atomic numbers, respectively, 33, 53, and 83 are:
(A) X, Y and Z are metals.
(B) X is a metalloid, Y is a non-metal and Z is a metal.
(C) X and Y are metalloids and Z is a metal.
(D) X and Z are non- metals and Y is a metalloid.

Q3. The **INCORRECT** statements below regarding colloidal solutions is:
(A) The flocculating power of Al^{3+} is more than that of Na^{+}.
(B) An ordinary filter paper can stop the flow of colloidal particles.
(C) A colloidal solution shows Brownian motion of colloidal particles.
(D) A colloidal solution shows colligative properties.

Q4. Statement I : Sodium hydride can be used as an oxidizing agent.
Statement II : The lone pair of electrons on nitrogen in pyridine makes it basic.
Choose the **CORRECT** answer from the options given below:
(A) Statement **I** is false but statement II is true
(B) Both statement **I** and statement **II** are true
(C) Statement **I** is true but statement **II** is false
(D) Both statement **I** and statement **II** are false

Q5. The green house gas/es is (are) :
(A) Carbon dioxide
(B) Oxygen
(C) Water vapour
(D) Methane
Choose the most appropriate answer from the options given below:
(A) (A) and (B) only
(B) (A), (C) and (D) only
(C) (A) only
(D) (A) and (C) only

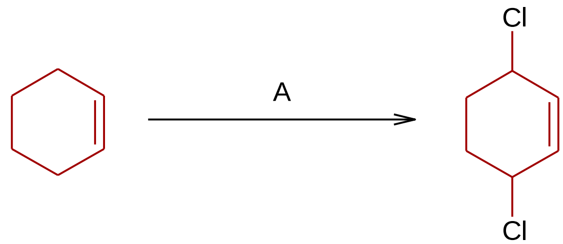
Q6. Fex_2 and Fey_3 are known when x and y are:
(A) x = F, Cl, Br, I and y=F, Cl, Br
(B) x=Cl, Br,I and y=F, Cl, Br, I
(C) x=F,Cl, Br, I and y=F, Cl, Br, I
(D) x=F, Cl, Br and y=F, Cl, Br, I

Q7. Identify the elements X and Y using the ionization energy values given below:
Ionization energy (kJ/ mol)

	1st	2nd
X	495	4563
Y	731	1450

(A) X = Mg; Y= Na
(B) X = F; Y = Mg
(C) X = Na; Y = Mg
(D) X = Mg ; Y = F

Q8.



Identify the reagent(s) 'A' and condition(s) for the reaction

(A) A = HCl ; Anhydrous $AlCl_3$
(B) A = Cl_2 ; UV light
(C) A = Cl_2 ; dark, Anhydrous $AlCl_3$
(D) A = HCl, $ZnCl_2$

Q9. Which of the following is least basic?

(A) $(CH_3CO)_2\ddot{N}H$
B) $(C_2H_5)_3\ddot{N}$
(C) $(C_2H_5)_2\ddot{N}H$
(D) $(CH_3CO)\ddot{N}HC_2H_5$

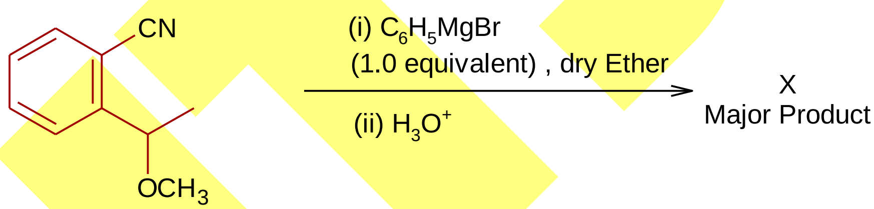
Q10. The **INCORRECT** statement regarding the structure of C_{60} is:

(A) Each carbon atom forms three sigma bonds.
(B) The five- membered rings are fused only to six-membered rings.
(C) The six-membered rings are fused to both six and five-memebered rings.
(D) It contains 12 six-membered rings and 24 five- membered rings.

Q11. Which of the following reduction reaction **CANNOT** be carried out with coke?

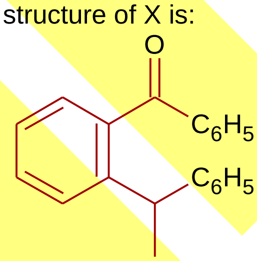
(A) $Cu_2O \rightarrow Cu$
(B) $Fe_2O_3 \rightarrow Fe$
(C) $ZnO \rightarrow Zn$
(D) $Al_2O_3 \rightarrow Al$

Q12.

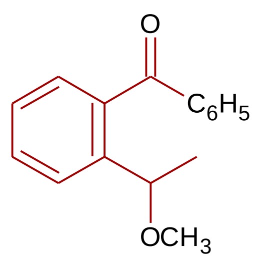


The structure of X is:

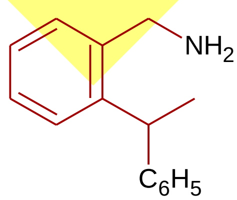
(A)



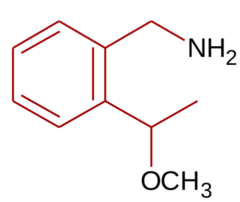
(B)



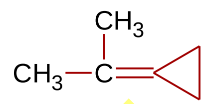
(C)



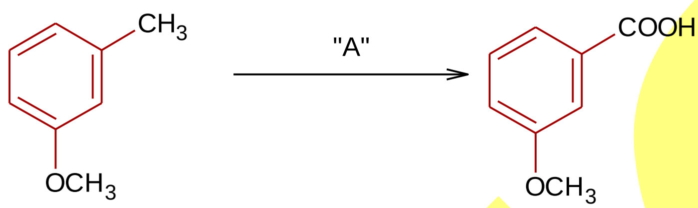
(D)



Q13. An unsaturated hydrocarbon X on ozonolysis gives A. Compound A when warmed with ammonical silver nitrate forms a bright silver mirror along the sides of the test tube. The unsaturated hydrocarbon X is:

(A) $CH_3-C(CH_3)=C(CH_3)-CH_3$
(B) $HC\equiv C-CH_2-CH_3$
(C) $H_3C-C\equiv C-CH_3$
(D) 

Q14.



In the above reaction, the reagent "A" is:
(A) $NaBH_4$, H_3O^+
(B) HCl, Zn-Hg
(C) $LiAlH_4$
(D) Alkaline $KMnO_4$, H^+

Q15. The correct statement about H_2O_2 are:
(A) Used in the treatment of effluents.
(B) Used as both oxidizing and reducing agents
(C) The two hydroxyl groups lie in the same plane.
(D) Miscible with water.
Choose the correct answer from the options given below:
(A) (B), (C) and (D) only
(B) (A), (C) and (D) only
(C) (A), (B) and (D) only
(D) (A), (B), (C) and (D)

Q16. Match **List – I** with **List – II**

List – I Test / Reagents / Observation(s)	**List – II** Species detected
(a) Lassaigne's Test	(i) Carbon
(b) Cu(II) oxide	(ii) Sulphur
(c) Silver nitrate	(iii) N, S, P, and halogen
(d) The sodium fusion extract gives black precipitate with acetic acid and lead acetate.	(iv) Halogen Specifically

The **correct** match is:
(A) (a)-(i), (b)-(ii), (c)-(iv) (d)-(iii)
(B) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)
(C) (a)-(iii), (b)-(i), (c)- (ii), (d)- (iv)
(D) (a)-(i), (b)-(iv), (c)-(iii), (d)-(ii)

Q17. Arrange the following metal complex / compounds in the increasing order of spin only magnetic moment. Presume all the three, high spin system.
(Atomic numbers Ce = 58, Gd = 64 and Eu = 63.)
(a) $(NH_4)_2[Ce(NO_3)_6]$ (b) $Gd(NO_3)_3$ and (c) $Eu(NO_3)_3$
Answer is:
(A) (a) < (c) < (b)
(B) (b)< (a) < (c)
(C) (c) < (a) < (b)
(D) (a) < (b) < (c)

Q18. The secondary structure of protein is stabilised by:
(A) Peptide bond
(B) Hydrogen bonding
(C) Glycosidic bond
(D) Van der waals forces

Q19. The exact volumes of 1 M NaOH solution required to neutralize 50 mL. of 1M H_3PO_3 solution and 100 mL of 2M H_3PO_2 solution respectively, are:
(A) 100 mL and 50 mL
(B) 100 mL and 200 mL
(C) 50 mL and 50 mL
(D) 100 mL and 100 mL

Q20. Which of the following polymer is used in the manufacture of wood laminates?
(A) cis-poly isoprene
(B) Phenol and formaldehyde resin
(C) Melamine formaldehyde resin
(D) Urea formaldehyde resin

SECTION B

Q1. Sulphurous acid (H_2SO_3) has $Ka_1 = 1.7 \times 10^{-2}$ and $Ka_2 = 6.4 \times 10^{-8}$. The pH of 0.588 M H_2SO_3 is __________. (Round off to the nearest integer).

Q2. The number of orbitals with n=5, m_l = +2 is________. (Round off to the nearest integer).

Q3. A and B decompose via first order kinetics with half- lives 54.0 min and 18.0 min respectively. Starting from an equimolar non reactive mixture of A and B, the time taken for the concentration of A to become 16 times that of B is________ min. (Round off to the nearest integer).

Q4. Ga (atomic mass 70u) crystallizes in a hexagonal close packed structure. The total number of voids in 0.581 g of Ga is___________$\times 10^{21}$. (Round off to the nearest integer). [Given : N_A = 6.023 $\times 10^{23}$]

Q5. When 35 mL of 0.15 M lead nitrate solution is mixed with 20mL of 0.12 M chromic sulphate solution, ___________$\times 10^{-5}$ moles of lead sulphate precipitate out. (Round off to the nearest integer).

Q6. In Duma's method of estimation of nitrogen, 0.1840 g of an organic compound gave 30 mL of nitrogen collected at 287 K and 758 mm of Hg pressure. The percentage composition of nitrogen in the compound is ________. (Round off to the nearest integer). [Given: Aqueous tension at 287K = 14 mm of Hg]

Q7. At 25°C, 50 g of iron reacts with HCl to form $FeCl_2$. The evolved hydrogen gas expands against a constant pressure of 1 bar. The work done by the gas during this expansion is _________J. (Round off to the nearest integer).
[Given: R= 8.314 J mol^{-1} K^{-1}. Assume hydrogen is an ideal gas]
[Atomic mass of Fe is 55.85u]

Q8. At 363 K, the vapour pressure of A is 21 kPa and that of B is 18 kPa. One mole of A and 2 moles of B are mixed. Assuming that this solution is ideal, the vapour pressure of the mixture is _______kPa. (Round off to the nearest integer).

Q9. $[Ti(H_2O)_6]^{3+}$ absorbs light of wavelength 498 nm during a d-d transition. The octahedral splitting energy for the above complex is ___ $\times 10^{-19}$J. (Round off to the nearest integer).
$h = 6.626 \times 10^{-34}$ Js; $c = 3 \times 10^{8}$ ms^{-1}

Q10. A 5.0 mmol dm^{-3} aqueous solution of KCl has a conductance of 0.55 mS when measured in a cell of cell constant 1.3 cm^{-1}. The molar conductivity of this solution is_________ mSm^2 mol^{-1}. (Round off to the nearest integer).

MATHEMATICS
SECTION A

Q1. Consider the integral

$I = \int_0^{10} \frac{[x]e^{[x]}}{e^{x-1}} dx$, where [x] denotes the greatest integer less than or equal to x. Then the value of I is equal to :

(A) $45(e+1)$
(B) $9(e-1)$
(C) $9(e+1)$
(D) $45(e-1)$

Q2. The maximum value of $f(x) = \begin{vmatrix} \sin^2 x & 1+\cos^2 x & \cos 2x \\ 1+\sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & \sin 2x \end{vmatrix}, x \in R$ is :

(A) $\frac{3}{4}$
(B) 5
(C) $\sqrt{5}$
(D) $\sqrt{7}$

Q3. If (x, y, z) be an arbitrary point lying on a plane P which passes through the points (42, 0, 0), (0, 42, 0) and (0, 0, 42), then the value of the expression

$$3+\frac{x-11}{(y-19)^2(z-12)^2}+\frac{y-9}{(x-11)^2(z-12)^2}+\frac{z-12}{(x-11)^2(y-19)^2}-\frac{x+y+z}{14(x-11)(y-19)(z-12)}$$

is equal to :

(A) 3
(B) 0
(C) –45
(D) 39

Q4. If the foot of the perpendicular from point (4, 3, 8) on the line $L_1 : \frac{x-a}{\ell} = \frac{y-2}{3} = \frac{z-b}{4}$, $\ell \neq 0$ is (3, 5, 7) then the shortest distance between the line L_1 and line $L_2 : \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ is equal to :

(A) $\frac{1}{2}$
(B) $\frac{1}{\sqrt{6}}$
(C) $\sqrt{\frac{2}{3}}$
(D) $\frac{1}{\sqrt{3}}$

Q5. Let f be a real valued function, defined on $R - \{-1, 1\}$ and given by $f(x) = 3\log_e\left|\frac{x-1}{x+1}\right| - \frac{2}{x-1}$. Then in which of the following intervals, function f(x) is increasing?

(A) $(-\infty, \infty) - \{-1, 1\}$
(B) $\left(-\infty, \frac{1}{2}\right] - \{-1\}$
(C) $(-\infty, -1) \cup \left(\left[\frac{1}{2}, \infty\right) - \{1\}\right)$
(D) $\left(-1, \frac{1}{2}\right]$

Q6. Consider a rectangle ABCD having 5,7,6,9 points in the interior of the segments AB, CD, BC, DA respectively. Let α be the number of triangles having these points from different sides as vertices and β be the number of quadrilaterals having these points from different sides as vertices. Then $(\beta-\alpha)$ is equal to :

(A) 1173
(B) 1890
(C) 717
(D) 795

Q7. Given that the inverse trigonometric functions take principal values only. Then, the number of real values of x which satisfy $\sin^{-1}\left(\frac{3x}{5}\right)+\sin^{-1}\left(\frac{4x}{5}\right)=\sin^{-1}x$ is equal to :

(A) 2
(B) 0
(C) 1
(D) 3

Q8. Let $\alpha \in R$ be such that the function $f(x)=\begin{cases}\dfrac{\cos^{-1}\left(1-\{x\}^2\right)\sin^{-1}\left(1-\{x\}\right)}{\{x\}-\{x\}^3} & , x\neq 0\\ \alpha & , x=0\end{cases}$ is continuous at x = 0, where $\{x\}=x-[x]$, $[x]$ is the greatest integer less than or equal to x. Then :

(A) $\alpha=\frac{\pi}{4}$
(B) $\alpha=0$
(C) no such α exists
(D) $\alpha=\frac{\pi}{\sqrt{2}}$

Q9. Let $A=\{2,3,4,5,...,30\}$ and '$\simeq$' be an equivalence relation on $A\times A$, defined by $(a,b)\simeq(c,d)$, if and only if $ad=bc$. Then the number of ordered pairs which satisfy this equivalence relation with ordered pair (4, 3) is equal to :

(A) 6
(B) 5
(C) 8
(D) 7

Q10. Let the lengths of intercepts on x-axis and y-axis made by the circle $x^2+y^2+ax+2ay+c=0, (a<0)$ be $2\sqrt{2}$ and $2\sqrt{5}$, respectively. Then the shortest distance from origin to a tangent to this circle which is perpendicular to the line $x+2y=0$, is equal to :

(A) $\sqrt{7}$
(B) $\sqrt{6}$
(C) $\sqrt{11}$
(D) $\sqrt{10}$

Q11. Let C be the locus of the mirror image of a point on the parabola $y^2=4x$ with respect to the line $y=x$. Then the equation of tangent to C at $P(2,1)$ is :

(A) $x-y=1$
(B) $2x+y=5$
(C) $x+2y=4$
(D) $x+3y=5$

Q12. The least value of $|z|$ where z is complex number which satisfies the inequality $\exp\left(\frac{(|z|+3)(|z|-1)}{||z|+1|}\log_e 2\right) \geq \log_{\sqrt{2}}|5\sqrt{7}+9i|, i=\sqrt{-1}$, is equal to :

(A) 2
(B) 3
(C) $\sqrt{5}$
(D) 8

Q13. Let A denote the event that a 6-digit integer formed by 0,1,2,3,4,5,6 without repetitions, by divisible by 3. Then probability of event A is equal to :

(A) $\frac{4}{9}$
(B) $\frac{11}{27}$
(C) $\frac{9}{56}$
(D) $\frac{3}{7}$

Q14. If $y=y(x)$ is the solution of the differential equation $\frac{dy}{dx}+(\tan x)y=\sin x, 0\leq x\leq\frac{\pi}{3}$, with $y(0)=0$, then $y\left(\frac{\pi}{4}\right)$ equal to:

(A) $\frac{1}{2}\log_e 2$
(B) $\left(\frac{1}{2\sqrt{2}}\right)\log_e 2$
(C) $\frac{1}{4}\log_e 2$
(D) $\log_e 2$

Q15. Let $P(x)=x^2+bx+c$ be a quadratic polynomial with real coefficients such that $\int_0^1 P(x)dx=1$ and $P(x)$ leaves remainder 5 when it is divided by $(x-2)$. Then the value of $9(b+c)$ is equal to :

(A) 11
(B) 15
(C) 9
(D) 7

Q16. Let $f:S\to S$ where $S=(0,\infty)$ be a twice differentiable function such that $f(x+1)=xf(x)$. If $g:S\to R$ be defined as $g(x)=\log_e f(x)$, then the value of $|g"(5)-g"(1)|$ is equal to :

(A) $\frac{205}{144}$
(B) $\frac{197}{144}$
(C) $\frac{187}{144}$
(D) 1

Q17. Let C_1 be the curve obtained by the solution of differential equation $2xy\frac{dy}{dx}=y^2-x^2, x>0$. Let the curve C_2 be the solution of $\frac{2xy}{x^2-y^2}=\frac{dy}{dx}$. If both the curves pass through $(1,1)$, then the area enclosed by the curves C_1 and C_2 is equal to :

(A) $\frac{\pi}{4}+1$
(B) $\pi-1$
(C) $\frac{\pi}{2}-1$
(D) $\pi+1$

Q18. Let $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 2\hat{i} - 3\hat{j} + 5\hat{k}$.
If $\vec{r} \times \vec{a} = \vec{b} \times \vec{r}$, $\vec{r}.\left(a\hat{i} + 2\hat{j} + \hat{k}\right) = 3$ and $\vec{r}.\left(2\hat{i} + 5\hat{j} - \alpha\hat{k}\right) = -1$, $\alpha \in R$, then the value of $\alpha + \left|\vec{r}\right|^2$ is equal to :
(A) 9 (B) 11
(C) 15 (D) 13

Q19. Let $A(-1, 1), B(3, 4)$ and $C(2, 0)$ be given three points. A line $y = mx, m > 0$, intersects lines AC and BC at point P and Q respectively. Let A_1 and A_2 be the area of ΔABC and ΔPQC respectively, such that $A_1 = 3A_2$, then the value of m is equal to :
(A) $\frac{4}{15}$ (B) 2
(C) 3 (D) 1

Q20. If the points intersections of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and the circle $x^2 + y^2 = 4b, b > 4$ lie on the curve $y^2 = 3x^2$, then b is equal to :
(A) 10 (B) 6
(C) 5 (D) 12

SECTION B

Q1. Let $S_n(x) = \log_{a^{1/2}} x + \log_{a^{1/3}} x + \log_{a^{1/6}} x + \log_{a^{1/11}} x + \log_{a^{1/18}} x + \log_{a^{1/27}} x + ...$ up to n-terms. Where a > 1. If $S_{24}(x) = 1093$ and $S_{12}(2x) = 265$, then the value of a is equal to……

Q2. For real numbers α, β, γ and δ, if $\int \frac{\left(x^2 - 1\right) + \tan^{-1}\left(\frac{x^2 + 1}{x}\right)}{\left(x^4 + 3x^2 + 1\right)\tan^{-1}\left(\frac{x^2 - 1}{x}\right)} dx$

$$= \alpha \log_e\left(\tan^{-1}\left(\frac{x^2 + 1}{x}\right)\right) + \beta\tan^{-1}\left(\frac{\gamma\left(x^2 - 1\right)}{x}\right) + \delta\tan^{-1}\left(\frac{x^2 + 1}{x}\right) + C$$

where C is an arbitrary constant, then the value of $10\left(\alpha + \beta\gamma + \delta\right)$ is equal to……..

Q3. Let $A = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ and $B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ be two 2×1 matrices with real entries such that $A = XB$, where $X = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & -1 \\ 1 & k \end{bmatrix}$, and $k \in R$. If $a_1^2 + a_2^2 = \frac{2}{3}\left(b_1^2 + b_2^2\right)$ and $\left(k^2 + 1\right)b_2^2 \neq -2b_1b_2$, then the value of k is……….

Q4. If the distance of the point $(1, -2, 3)$ from the plane $x + 2y - 3z + 10 = 0$ measured parallel to the line, $\frac{x-1}{3} = \frac{2-y}{m} = \frac{z+3}{1}$ is $\sqrt{\frac{7}{2}}$, then the value of $|m|$ is equal to……

Q5. Let n be a positive integer. Let $A=\sum_{k=0}^{n}(-1)^k\ {}^nC_k\left[\left(\frac{1}{2}\right)^k+\left(\frac{3}{4}\right)^k+\left(\frac{7}{8}\right)^k+\left(\frac{15}{16}\right)^k+\left(\frac{31}{32}\right)^k\right]$. If $63A=1-\frac{1}{2^{30}}$, then n is equal to……

Q6. Let $\frac{1}{16}$, a and b be in G.P. and $\frac{1}{a},\frac{1}{b}$, 6 be in A.P., where a, b > 0. Then $72(a+b)$ is equal to………

Q7. In ΔABC, the lengths of sides AC and AB are 12 cm and 5 cm, respectively. If the area of ΔABC is 30 cm^2 and R and r are respectively the radii of circumcircle and incircle of ΔABC, then the value of $2R+r$ (in cm) is equal to……..

Q8. Let $f:R\to R$ and $g:R\to R$ be defined as

$f(x)=\begin{cases}x+a, & x<0\\ |x-1|, & x\geq 0\end{cases}$ and $g(x)=\begin{cases}x+1, & x<0\\ (x-1)^2+b, & x\geq 0\end{cases}$, where a, b are non-negative real numbers. If $(gof)(x)$ is continuous for all $x\in R$, then a + b is equal to……….

Q9. Consider the statistics of two sets of observations as follows :

	Size	Mean	Variance
Observation I	10	2	2
Observation II	n	3	1

If the variance of the combined set of these two observations is $\frac{17}{9}$, then the value of n is equal to…….

Q10. Let $\vec{c}$ be a vector perpendicular to the vectors $\vec{a}=\hat{i}+\hat{j}-\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}+\hat{k}$. If $\vec{c}.\left(\hat{i}+\hat{j}+3\hat{k}\right)=8$ then the value of $\vec{c}.\left(\vec{a}\times\vec{b}\right)$ is equal to………

ANSWER: Paper-Jee-Main-16-03-2021-Evening Shift

PHYSICS	CHEMISTRY	MATHEMATICS
Section-A	**SECTION – A**	**SECTION – A**
Ans1. D	**Ans1.** B	**Ans1. D**
Ans2. D	**Ans2.** B	**Ans2. C**
Ans3. A	**Ans3.** B	**Ans3. A**
Ans4. D	**Ans4.** A	**Ans4. B**
Ans5. D	**Ans5.** B	**Ans5. C**
Ans6. C	**Ans6.** A	**Ans6. C**
Ans7. D	**Ans7.** C	**Ans7. D**
Ans8. D	**Ans8.** B	**Ans8. C**
Ans9. C	**Ans9.** A	**Ans9. D**
Ans10. B	**Ans10.** D	**Ans10. B**
Ans11. C	**Ans11.** D	**Ans11. A**
Ans12. A	**Ans12.** B	**Ans12. B**
Ans13. D	**Ans13.** B	**Ans13. A**
Ans14. A	**Ans14.** D	**Ans14. B**
Ans15. C	**Ans15.** C	**Ans15. D**
Ans16. C	**Ans16.** B	**Ans16. A**
Ans17. A	**Ans17.** A	**Ans17. C**
Ans18. B	**Ans18.** B	**Ans18. C**
Ans19. C	**Ans19.** B	**Ans19. D**
Ans20. A	**Ans20.** D	**Ans20. D**
Section-B	**SECTION – B**	**SECTION – B**
Ans1. 4	**Ans1.** 1	**Ans1. 16**
Ans2. 3	**Ans2.** 3	**Ans2. 6**
Ans3. 120	**Ans3.** 108	**Ans3. 1**
Ans4. 12	**Ans4.** 15	**Ans4. 2**
Ans5. 2500	**Ans5.** 525	**Ans5. 6**
Ans6. 3	**Ans6.** 19	**Ans6. 14**
Ans7. 12	**Ans7.** 2218	**Ans7. 15**
Ans8. 20	**Ans8.** 19	**Ans8. 1**
Ans9. 3	**Ans9.** 4	**Ans9. 5**
Ans10. **Dropped**	**Ans10.** 14	**Ans10. 28**

Solution: Paper-Jee-Main-16-03-2021-Evening Shift
Section : A

Sol1. $\vec{\upsilon} = 0.5t^2\,\hat{i} + 3t\,\hat{j} + 9\hat{k}\ m/s \Rightarrow \vec{a} = \frac{d\vec{\upsilon}}{dt} = \left(t\,\hat{i} + 3\,\hat{j}\right) m/s^2$

At t =2sec, $\vec{\upsilon} = 2\hat{i} + 6\hat{j} + 9\hat{k}\ m/s$ and $\vec{a} = \left(2\hat{i} + 3\hat{j}\right) m/s^2$

If we write the direction of acceleration of mosquito after 2s, then it will be $\tan^{-1}\left(\frac{2}{3}\right)$ from y – axis

Sol2. using hook's law:

$\sigma = Y\varepsilon$

$\Rightarrow \frac{f}{A} = Y\frac{x}{\ell} \Rightarrow Y = \frac{f\ell}{xA} = \frac{f\ell}{x\pi r^2}$

Using error analysis formula:

$\Rightarrow \frac{\Delta Y}{Y} = \frac{\Delta f}{f} + \frac{\Delta \ell}{\ell} + \frac{\Delta x}{x} + 2\frac{\Delta r}{r}$

$\Rightarrow \%\text{error in } Y = \left[\frac{1}{1000} + \frac{1}{1000} + \frac{0.001}{0.5} + \frac{2\times 0.001}{0.2}\right]\times 100 = 1.4\%$

Sol3. Since $\vec{B}$, $\vec{v}$ and length are perpendicular

$\varepsilon = Bv\ell$

emf will induce only in wire CD

$\varepsilon = B_0\left(\frac{d}{a}\right)v_0(d) = \frac{B_0 v_0 d^2}{a}$

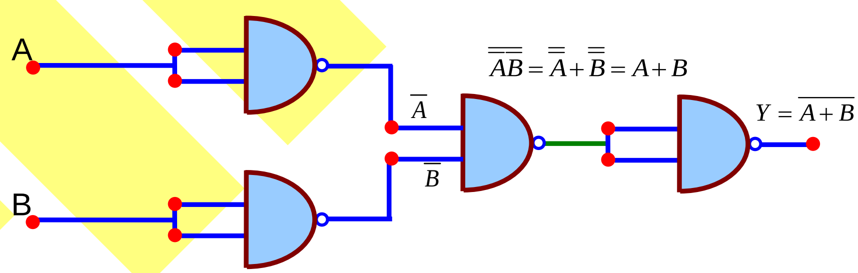
Sol4. $\lambda = \frac{kT}{\sqrt{2}\pi d^2 P} = \frac{1.38\times 10^{-23}\times 300}{1.4\times 3.14\times (0.3\times 10^{-9})^2\times 1.01\times 10^5}$ =102 nm

Sol5. $\frac{N_1}{N_0} = e^{-\lambda t} \Rightarrow \frac{2}{3} = e^{-\lambda t} \Rightarrow t_1 = \frac{1}{\lambda}\ell n\left(\frac{3}{2}\right)$

Similarly we can write $t_2 = \frac{1}{\lambda}\ell n(3)$

$$\Delta t = t_2 - t_1 = \frac{1}{\lambda}\left[\ell n(3) - \ell n\left(\frac{3}{2}\right)\right] = \frac{\ell n(2)}{\lambda} = T_{\frac{1}{2}} = 20\,\text{min}$$

Sol6.



Since output $Y = \overline{A+B}$, so it will represent NOR-Gate

Sol7. Since speed of light is constant for all colour so red colour and blue colour have different frequencies and different wavelengths

Sol8. **As we know that Reynolds's number** $R = \frac{\rho v D}{\eta}$

In First case

$$v_1 = \frac{0.18\times10^{-3}}{\pi\times\left(0.5\times10^{-2}\right)^2\times60} = \frac{0.18\times10^{-3}\times10^6\times4}{\pi\times25\times60\times4} = \frac{0.18\times4}{\pi\times6} = 0.03822 m/s$$

$$R = \frac{0.03822\times10^3\times0.1}{10^{-3}} = 3822 < 4000 \Rightarrow \text{Steady}$$

In Second case

$$v_2 = \frac{0.48\times10^{-3}}{\pi\times\left(0.5\times10^{-2}\right)^2\times60} = \frac{0.48\times10^{-3}\times10^6\times4}{\pi\times25\times60\times4} = \frac{0.48\times4}{\pi\times6} = 0.10191 m/s$$

$$R_2 = \frac{0.10191\times10^3\times0.1}{10^{-3}} = 10191 > 4000 \Rightarrow Turbulent$$

Sol9. **Using conservation of linear momentum, we can write**

$$P_i = P_f \Rightarrow mv = M(m+m)v'$$

Using conservation of Mechanical energy, we can write

$$\frac{1}{2}(M+m)(v')^2 = (M+m)gh \Rightarrow \frac{1}{2}\left(\frac{mv}{M+m}\right)^2 = gh$$

$$\Rightarrow v = \frac{M+m}{m}\sqrt{2gh} = \frac{6}{10\times10^{-3}}\sqrt{2\times9.8\times0.098} = \frac{6}{10\times10^{-3}}\sqrt{2\times\frac{98}{10}\times\frac{98}{1000}}$$

$$\Rightarrow v = \frac{6\times98\times1.414}{10\times10^{-3}\times10^2} = 831.432 \text{ m/s} \approx 831.4 \text{ m/s}$$

Sol10. As we know that magnetic force acting on a charge particle will be

$$\vec{F} = q\left(\vec{v}\times\vec{B}\right)$$

$$W = \vec{F}.d\vec{\ell}$$

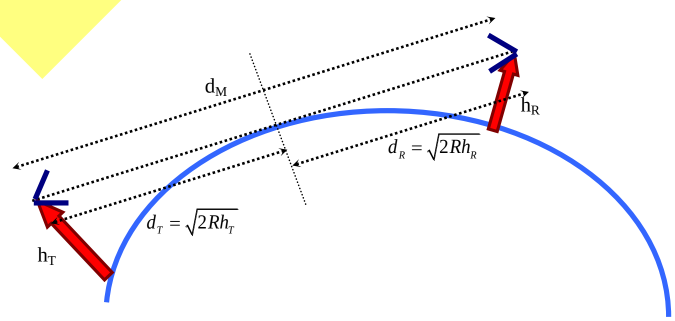
Since force and displacement will be always perpendicular so work done is always zero.

Sol11.

$$d = d_1 + d_2 = 2\sqrt{2hR}$$

$$h = \frac{d^2}{8R}$$

$$\Rightarrow h = \frac{(45\times1000)^2}{8\times6400\times1000} \approx 39.55 \text{ m}$$



Sol12. **As we know that for damping Oscillation**

$$A = A_0 e^{-\frac{b}{2m}t} \Rightarrow t_{\frac{1}{2}} = \frac{\ell n(2)}{\frac{b}{2m}} = \frac{2m\ell n(2)}{b} = \frac{2\times500\times0.693}{20} = 35.65 \text{ s}$$

Sol13. A= Activity **=** $\lambda N = \left(\frac{\ell n(2)}{t_{\frac{1}{2}}}\right)N = \left(\frac{0.693}{27\times24\times3600}\right)\times\left(\frac{1.5\times10^{-3}}{198}\times6\times10^{23}\right)$ disintegration /s

$$\Rightarrow A = \left(\frac{0.693}{27\times 24\times 3600}\right)\times\left(\frac{1.5\times 10^{-3}}{198}\times 6\times 10^{23}\right)\times\frac{1}{3.7\times 10^{10}} \approx 357 \text{ Ci}$$

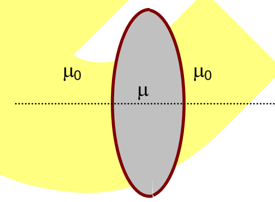
Sol14. Using de-Broglie equation:

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2Km}} = \frac{h}{\sqrt{2(eV)m}} \Rightarrow \lambda \ \alpha \ \frac{1}{\sqrt{m}}$$

$$\Rightarrow \frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_p}{m_e}} = \sqrt{1831} \approx 43:1$$

Sol15. The decrement in length is more for metal strip-A than metal strip-B, so the combined system bend towards the left

Sol16. Using lens make formula, we can write



$$\frac{1}{f} = \left(\frac{\mu}{\mu_0}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\mu = \mu_0 \Rightarrow \frac{1}{f} = 0 \Rightarrow f \Rightarrow \text{Infinite}$$

Sol17. **According to Question , we can write**

$$\frac{\sigma}{\varepsilon_0} = \frac{\lambda}{2\pi\varepsilon_0 r} \Rightarrow \sigma = \frac{\lambda}{2\pi r} = \frac{8\times 10^{-9}}{2\times 3.14\times 3} = 0.424 \ \ nCm^{-2}$$

Sol18. **Statement-I:** $F_C = \frac{mv^2}{r} \le f_\ell = \mu mg \Rightarrow v \le \sqrt{\mu g R} = \sqrt{0.2\times 10\times 2} = 2m/s$

$\Rightarrow v_{cyclist} = 7\times\frac{5}{18} = 1.94 \ m/s \le 2m/s$, so statement-I is correct

Statement-II:

$$v_{min} = \sqrt{gR\left(\frac{\tan\theta-\mu}{1+\mu\tan\theta}\right)} = \sqrt{10\times 2\left(\frac{\tan 45^0-0.2}{1+0.2\times\tan 45^0}\right)} = \sqrt{10\times 2\left(\frac{1-0.2}{1+0.2\times 1}\right)} = 3.65 \ m/s$$

$$v_{max} = \sqrt{gR\left(\frac{\tan\theta+\mu}{1-\mu\tan\theta}\right)} = \sqrt{10\times 2\left(\frac{\tan 45^0+0.2}{1-0.2\times\tan 45^0}\right)} = \sqrt{10\times 2\left(\frac{1+0.2}{1-0.2\times 1}\right)} = 5.48 \ m/s$$

$\Rightarrow v_{min} \le v_{cyclist} = 18.5\times\frac{5}{18} = 5.139 \ m/s \le v_{max}$, so statement-II is correct

Sol19. Voltage across secondary source

$$V_S = \frac{P}{i} = \frac{60}{0.11} \approx 545\,V$$

Since voltage across secondary source is more than primary source

$\Rightarrow$ Step- up transformer.

Sol20. Heat generated in the resistance

$$H = i^2 RT$$

$$H_1 = 500 = (1.5)^2 R(20)$$

$$H_2 = H = (3)^2 R(20) \Rightarrow \frac{500}{H} = \frac{1}{4} \Rightarrow H = 2000J.$$

Section : B

Sol1. **For closed- organ pipe**

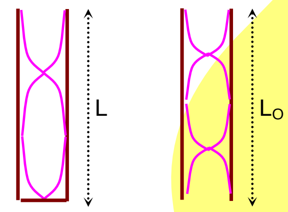
$$\frac{3\lambda_C}{4} = L \Rightarrow f_C = \frac{v}{\lambda_C} = \frac{3}{4L}\sqrt{\frac{B}{\rho_1}}$$

For open- organ pipe

$$\lambda_O = L_O \Rightarrow f_O = \frac{v}{\lambda_O} = \frac{1}{L_O}\sqrt{\frac{B}{\rho_2}}$$

According Question , we can write

$$f_C = f_O \Rightarrow L_O = \frac{4L}{3}\sqrt{\frac{\rho_1}{\rho_2}}$$



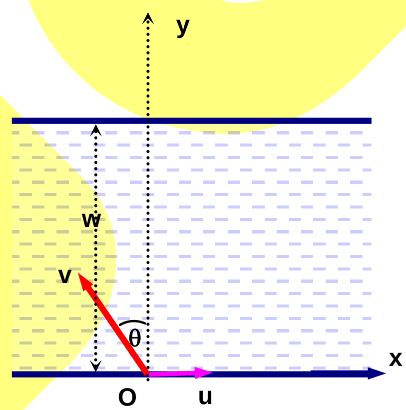
Sol2. Equivalent capacitance when dielectric is inserted between capacitor:

$$C = \frac{\varepsilon_0 A}{d - t + \frac{t}{k}} = \frac{\varepsilon_0 \times 2}{d - \frac{d}{2} + \frac{d}{2k}} = \frac{2\times 2}{d\left(1+\frac{1}{k}\right)} = \frac{64}{21}\varepsilon_0$$

Sol3.

$$\sin\theta = \frac{u}{v} = \frac{1}{2} \Rightarrow \theta = 30^\circ$$

Total angle from direction of flow $= 90^0 + 30^0 = 120^0$



Sol4. Acceleration in y-direction.

$$a_y = \frac{3}{2} m/s^2$$

Using equation of motion , we can write

$$s_y = \frac{1}{2}a_y t^2 = \frac{1}{2}\times\frac{3}{2}\times 16 = 12 \Rightarrow b = 12$$

Sol5. $H = i^2 R t$

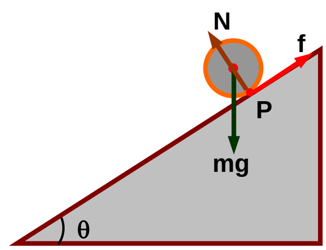
$$R = \frac{10\times 10^{-3}}{\left(2\times 10^{-3}\right)^2 \times 1} = 2500\Omega$$

Sol6.

$$mg\sin\theta - f = ma$$

$$fR = I\alpha = \frac{Ia}{R} \Rightarrow f = \frac{Ia}{R^2}$$

$$a = \frac{mg\sin\theta}{m + \frac{I}{R^2}} = \frac{g\sin\theta}{1+\frac{I}{mR^2}} = \frac{g\sin\theta}{1+\frac{1}{2}} = \frac{2g\sin\theta}{3}$$



FBD of Disc

Sol7. **Here system is made for deviation without dispersion**

$$\delta = \delta_C + \delta_F = (\mu_C - 1)A_C - (\mu_F - 1)A_F = 2^0$$

$$\theta = \theta_C + \theta_F = (\mu_{VC} - \mu_{RC})A_C - (\mu_{FC} - \mu_{FC})A_F = 0 \Rightarrow (\mu_{VC} - \mu_{RC})A_C = (\mu_{FC} - \mu_{FC})A_F \ldots(2)$$

$$\omega_C = \frac{(\mu_{VC} - \mu_{RC})A_C}{(\mu_C - 1)A_C} = 0.02 \Rightarrow (\mu_{VC} - \mu_{RC}) = 0.02(\mu_C - 1)A_C \text{ and}$$

$$\omega_F = \frac{(\mu_{FC} - \mu_{FC})A_F}{(\mu_F - 1)A_F} = 0.03 \Rightarrow (\mu_{FC} - \mu_{FC})A_F = 0.03(\mu_F - 1)A_F$$

Putting these values in equation (2), we have

$$0.03(\mu_F - 1)A_F = 0.02(\mu_C - 1)A_C \Rightarrow 3(1.6 - 1)A_F = 2(1.5 - 1)A_C \Rightarrow 1.8A_F = A_C$$

Putting this value in equation (1), we have

$$\Rightarrow 0.5A_C - 0.6\frac{A_C}{1.8} = 2^0 \Rightarrow \frac{A_C}{2} - \frac{A_C}{3} = 2^0 \Rightarrow \frac{A_C}{6} = 2^0 \Rightarrow A_C = 12^0$$

Sol8. $\vec{r} = 2\hat{i} - (2\hat{i} + 3\hat{j} + 4\hat{k}) = -(3\hat{j} + 4\hat{k})$

$$\vec{F} = 4\hat{i} + 3\hat{j} + 4\hat{k}$$

$$\vec{\tau} = \vec{r} \times \vec{F} = -(3\hat{j} + 4\hat{k}) \times 4\hat{i} + 3\hat{j} + 4\hat{k} = -\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 3 & 4 \\ 4 & 3 & 4 \end{vmatrix} = 12\hat{j} + 16\hat{k}$$

$$|\vec{\tau}| = 20$$

Sol9. Self energy of a solid sphere $= \frac{3}{5}\frac{Gm^2}{R}$

Sol10. $\eta = 1 - \frac{T_c}{T_h} \Rightarrow 0.6 = 1 - \frac{T_c}{400} \Rightarrow T_C = 0.4 \times 400 = 160K = (160 - 273) = -113^0C$

CHEMISTRY

Sol1. In ammonolysis HCl is formed as by product after substitution reaction, hence treatment of NaOH is carried out to remove acidic impurities.

Sol2. Corresponding atomic no. are respectively $_{33}As$, $_{53}I$ and $_{83}Bi$.
As – Metalloid, I – Non-metal and Bi – Metal

Sol3. An ordinary filter can not stop the flow of colloidal particles, but specific membrane that is ultrafilter paper can stop. This happens due to specific particle size range of colloidal particles.

Sol4. In NaH, hydrogen has –1(O.S)., it only acts as a reducing agent.

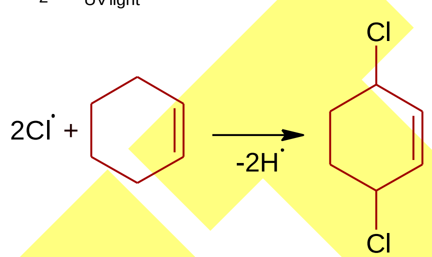
Sol5. Oxygen gas does not considered as component of green house effect.

Sol6. FeI_3 does not exist due to unstability because Fe^{3+}ions in FeI_3 easily reduced into Fe^{2+} by I^- ion.

Sol7. The difference range of 1st I.E. and 2nd I.E. of alkali metal is too much higher than corresponding difference range of alkaline earth metal hence [X= Na & Y= Mg]

Sol8. In presence of UV light, Cl_2 converted into free radical and attack on allylic carbon for substitution reaction of given compound.

$$Cl_2 \xrightarrow[\text{UV light}]{} 2Cl^{\bullet}$$

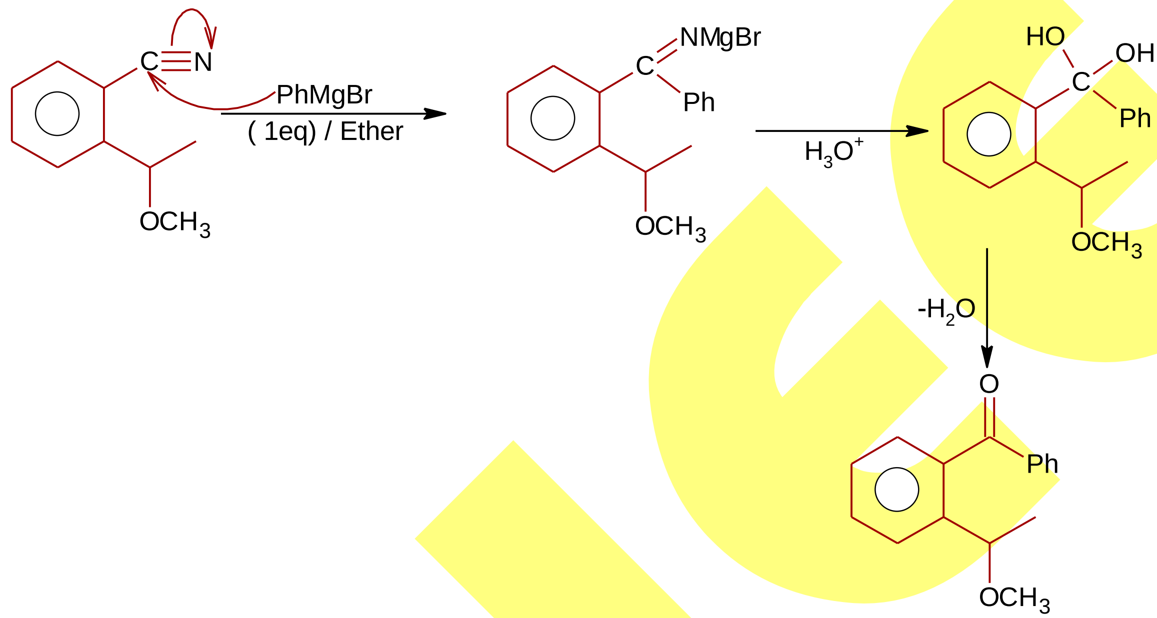


Sol9. In $(CH_3CO)_2NH$ lone pair of N- atom involve in resonance by both side of carbonyl group. So lone pair density on N decreases to great extent.

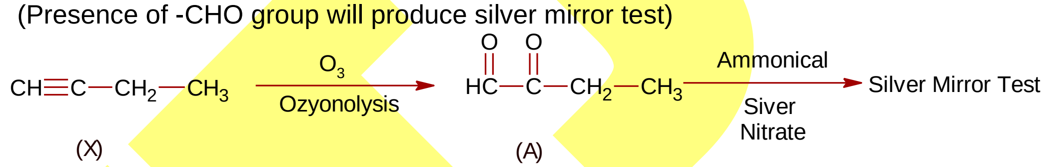
Sol10. In C_{60} molecules, all carbon atoms have identical hydrogen and three sigma bond and it contain 20 six member and 12 five membered ring.

Sol11. Reduction of Al_2O_3 is carried out by electrolytic reduction process in molten state because it is highly ionic compound.

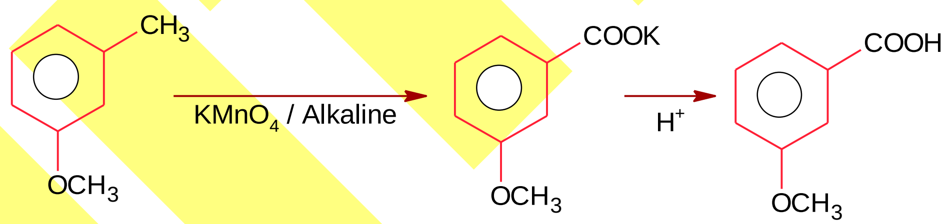
Sol12.



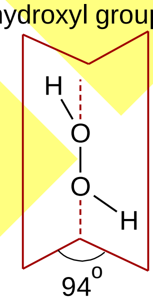
Sol13. (Presence of -CHO group will produce silver mirror test)



Sol14.



Sol15. In H_2O_2, two hydroxyl groups are in different plane, it looks like open book structure.



Sol16. Lassaign's test is useful for N,P,S and halogen.
Cu(II) oxide is useful to detect of C.
X^- precipitated by $AgNO_3$ and S^{--} ion precipitated as PbS (Black) with sodium fusion extract..

Sol17. In $(NH_4)_2[Ce(NO_3)_6] \to Ce^{4+} \Rightarrow 4f^0 \Rightarrow n = 0$ (zero unpaired e^-)

$Eu(NO_3)_3 \to Eu^{3+} \Rightarrow 4f^6 \Rightarrow n = 6$ (six unpaired e^-)

$Gd(NO_3)_3 \to Gd^{3+} \Rightarrow 4f^7 \Rightarrow n = 7$ (seven unpaired e)

Magnetic moment = $\sqrt{n(n+2)}$ BM (Higher unpaired electron have high magnetic moment)

Sol18. Hydrogen bonding developed between oxygen of carbonyl and hydrogen of N-atom is responsible for stabilization of secondary structure of protein.

Sol19. Meq of Base = Meq of Acid
For H_3PO_3 ; $1 \times V = 50 \times 1 \times 2 \quad \therefore V = 100$ml of NaOH
For H_3PO_2 ; $1 \times V = 100 \times 2 \times 1 \quad \therefore V = 200$ml of NaOH

Sol20. Due to unbreakable properties of urea – formaldehyde resin, it is used in wood laminates.

SECTION – B

Sol1.

$$\left.\begin{array}{ll} H_2SO_3 \rightleftharpoons H^+ + HSO_3^- & K_1 = 1.7\times10^{-2} \\ HSO_3^- \rightleftharpoons H^+ + SO_3^- & K_2 = 6.4\times10^{-8} \end{array}\right\} K_1 >> K_2$$

When we consider first ionization is dominating

$$\begin{array}{lccc} & H_2SO_3 & \rightleftharpoons H^+ + & HSO_3^- \\ \text{At equilibrium} & C - C\alpha & C\alpha & C\alpha \end{array}$$

Hence, $\left[H^+\right] = C\alpha = \sqrt{K_1 \times C} = \sqrt{1.7\times10^{-2}\times0.588} = 0.9998\times10^{-1}$

$\therefore$ pH = 1 (Rounded figure)

Sol2. When n=5 and m_ℓ = +2

ℓ =0, m_ℓ =0
ℓ =1 m_ℓ = -1, 0, +1
ℓ =2 m_ℓ = -2, -1, 0,+1, **(+2)**
ℓ =3 m_ℓ = -3, -2, -1,0, +1, **(+2)**, +3
ℓ =4 m_ℓ = -4, -3, -2, -1, 0, +1,**(+2)**, +3, +4

Number of orbital of n = 5 and $m\ell$ = +2 are 3.

Sol3.

$$\left.\begin{array}{l} A_0 \xrightarrow{t_{1/2}=54\min} A_t \\ B_0 \xrightarrow{t_{1/2}=18\min} B_t \end{array}\right\}$$ Both A and B have taken same time to convert into A_t and B_t form respectively.

$$\frac{0.693}{54}\times t = 2.303\log_{10}\frac{A_0}{A_t} \text{ -------------------------(i)}$$

$$\frac{0.693}{18}\times t = 2.303\log_{10}\frac{B_0}{B_t} \text{ --------------------(ii)}$$

Equation (ii) – equation (i) ; $t \times 0.693\left[\frac{1}{18} - \frac{1}{54}\right] = 2.303\log_{10}\frac{B_o \times At}{B_t \times A_0}$

Here ; $\frac{A_0}{B_0} = 1$ and $\frac{A_t}{B_t} = 16$

$\therefore t \times 0.693 \times \left[\frac{3-1}{54}\right] = 2.303\log_{10}16 = 2.303 \times 4 \times \log_{10}2$

$\therefore t = \frac{4 \times 54}{2} = 108$

Sol4. In HCP, total no. of voids are = 18 (6- octahedral and 12- tetrahedral)

No of unit cell = $\left(\frac{0.581 \times 6.023 \times 10^{23}}{70 \times 6}\right)$ (No. of atoms per unit cell = 6)

No. of voids = $\left(\frac{0.581 \times 6.023 \times 10^{23}}{70 \times 6}\right) \times 18 = 0.149 \times 10^{23}$

$= 14.9 \times 10^{21}$

$= 15 \times 10^{21}$ (Rounded 15)

Sol5.

	$3Pb(NO_3)_2$ +	$Cr_2(SO_4)_3 \longrightarrow$	$3PbSO_4$ +	$2Cr(NO_3)_3$
t= 0	35×0.15	20×0.12	0	0
t=0	5.25 mmole	2.4 mmole		
t=∞	0	(2.4-1.75) mmole	5.25mmole	$\frac{2}{3} \times 5.25$ mmole

Total moles of $PbSO_4$ precipitated = $5.25 \times 10^{-3} = 525 \times 10^{-5}$
Ans 525.

Sol6. $P_{N_2} = \frac{758-14}{760}$ atm

$V = \frac{30}{1000}$ lit

$T = 287K$

Mole (n) $= \frac{PV}{RT} = \frac{744}{760} \times \frac{30}{1000} \times \frac{1}{0.0821 \times 287} = 1.25 \times 10^{-3}$ (mole of N_2 gas)

Wt of N_2 gas = $1.25 \times 10^{-3} \times 28 = 0.035$gm

% Nitrogen = $\frac{0.035 \times 100}{0.1840} = 18.96\%$

Ans (Rounded) = 19

Sol7. $Fe + 2HCl \rightarrow FeCl_2 + H_2(g)$

Mole of Fe = $\frac{50}{55.85}$

Mole of H_2 = $\frac{50}{55.85}$ = 0.89 mole

Work done = $-\Delta n \times R \times T$

$= -0.89 \times 8.31 \times 298 = -2218.05$ J

Work done by the gas = 2218J (Rounded)

Ans = 2218

Sol8. $P_{Total} = P_A^o x_B + P_B^o x_B$

$P_{Total} = 21 \times \frac{1}{3} + 18 \times \frac{2}{3} = 19 kPa$

Ans = 19

Sol9. $[Ti(H_2O)_6]^{3+}$

Octahedral spiltting energy $\Delta_0 = \frac{hc}{\lambda}$

$= \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{498 \times 10^{-9}} = 0.0399 \times 10^{-17}$

$= 3.9 \times 10^{-19} J$

$= 4 \times 10^{-19} J$ (Rounded)

Ans = 4

Sol10. Specific conductance $(\kappa) = \frac{\ell}{a} \times \frac{1}{R} = \frac{\ell}{a} \times$ conductance

$\therefore \kappa = 1.3 \times 0.55 mScm^{-1}$

$\kappa = 1.3 \times 0.55 \times 100 mSm^{-1}$

Now, using $\lambda_m = \frac{\kappa}{M \times 1000} mSm^2mol^{-1}$

$= \frac{1.3 \times 0.55 \times 100}{5 \times 10^{-3} \times 1000} = 14.3 mSm^2mol^{-1}$

Ans = 14 (Rounded off)

MATHEMATICS

SECTION - A

Sol1. $I=\int_0^{10}\frac{[x].e^{[x]}}{e^{x-1}}dx$

$=\int_0^1 0.dx+\int_1^2\frac{1.e^1}{e^{x-1}}dx+\int_2^3\frac{2.e^2}{e^{x-1}}dx+------$

$+------+\int_9^{10}\frac{9.e^9}{e^{x-1}}dx$

$=\sum_{n=0}^{9}\int_n^{(n+1)}\frac{n.e^n}{e^{x-1}}dx$

$=\sum_{n=0}^{9}n.e^n\int_n^{(n+1)}\frac{1}{e^{x-1}}dx$

$=\sum_{n=0}^{9}n.e^n\int_n^{n+1}e^{(1-x)}dx$

$=\sum_{n=0}^{9}n.e^n\left(-e^{1-x}\right)\Big|_n^{n+1}$

$=\sum_{n}^{9}n.e^n\left[e^{1-n}-e^{-n}\right]$

$=0+1.e'\left(e^o-e^{-1}\right)$

$+2.e^2\left(e^{-1}-e^{-2}\right)$

$+------+9e^9\left[e^{-8}-e^{-9}\right]$

$=1(e-1)+2(e-1)+3(e-1)$

$-----+9(e-1)$

$=45(e-1)$

Sol2.

$$\begin{vmatrix}\sin^2x & 1+\cos^2x & \cos2x\\ 1+\sin^2x & \cos^2x & \cos2x\\ \sin^2x & \cos^2x & \sin2x\end{vmatrix}$$

$R_1\to R_1-R_2$

$$\begin{vmatrix}-1 & 1 & 0\\ 1+\sin^2x & \cos^2x & \cos2x\\ \sin^2x & \cos^2x & \sin2x\end{vmatrix}$$

$=-1\left(\cos^2x\sin2x-\cos2x.\cos^2x\right)\ -1\left(\left(1+\sin^2x\right)\sin2x-\sin^2x.\cos2x\right)+0$

$=\left(-\cos^2x.\sin2x+\cos2x\cos^2x-\sin2x-\sin^2x.\sin2x+\sin^2x.\cos2x\right)$

$=-\sin2x\left(\sin^2x+\cos^2x\right)+\cos2x\left(\sin^2x+\cos^2x\right)-\sin2x$

$=\cos2x-2\sin2x$

$f(x)=\cos2x-2\sin2x$

$f(x)_{max} = \sqrt{5}$

Sol3. Equation of plane

$x + y + z = 42$

$(x-11)+(y-19)+(z-12)=0$

$x+y+z=0 \Rightarrow x=y=z$

$x^3+y^3+z^3=3xyz$

$3+\frac{x}{y^2z^2}+\frac{y}{x^2z^2}+\frac{z}{x^2y^2}-\frac{42}{14(x)(y)(z)}$

$3+\frac{x^3+y^3+z^3-3xyz}{(xyz)^2}$

$3+0=3$

Sol4. $L_1 = \frac{x-a}{\ell}=\frac{y-2}{3}=\frac{z-b}{4}$

$\ell \neq 0$

$a=3-\ell, b=3$

D.R's of line joining the given points

(1,-2,-1) which is perpendicular to 4 = 0

$a_1.a_2+b_1b_2+c_1c_2=0$

$L_1 = \frac{x-1}{2}=\frac{y-2}{3}=\frac{z-4}{4}$

$L_2 = \frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$

$D=\frac{(\bar{a}_2-\bar{a}_1)(\vec{b}_1\times\vec{b}_2)}{|\vec{b}_1\times\vec{b}_2|}$

$=\frac{(i+2j+2k)(-i+2j-\hat{k})}{\sqrt{6}}=\frac{1}{\sqrt{6}}$

Sol5. $f(x)=3\log_e\left|\frac{x-1}{x+1}\right|-\frac{2}{x-1}$

$f'(x)=3\left(\frac{x+1}{x-1}\right)\left\{\frac{(x+1)-(x-1)}{(x+1)^2}\right\}+\frac{2}{(x-1)^2}$

$f'(x)=3\left(\frac{x+1}{x-1}\right)\left\{\frac{2}{(x+1)^2}\right\}+\frac{2}{(x-1)^2}$

$f'(x)=\frac{6}{(x-1)(x+1)}+\frac{2}{(x-1)^2}$

$f'(x)=\frac{6(x-1)+2(x+1)}{(x-1)^2(x+1)}$

$f'(x)=\frac{8x-4}{(x-1)^2(x+1)}$

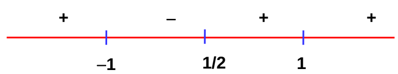
For f(x) is increasing

$$f'(x) > 0$$

$$\frac{8x-4}{(x-1)^2(x+1)} > 0$$

$$\frac{8\left(x-\frac{1}{2}\right)}{(x-1)^2(x+1)} > 0$$

$$\frac{\left(x-\frac{1}{2}\right)}{(x-1)^2(x+1)} > 0$$



$$x \in (-\infty,-1) \cup \left(\left[\frac{1}{2},\infty\right) - \{1\}\right)$$

Ans. C

Sol6. α = Number of triangle

$\alpha = 5.6.7 + 5.7.9 + 5.6.9 + 6.7.9$

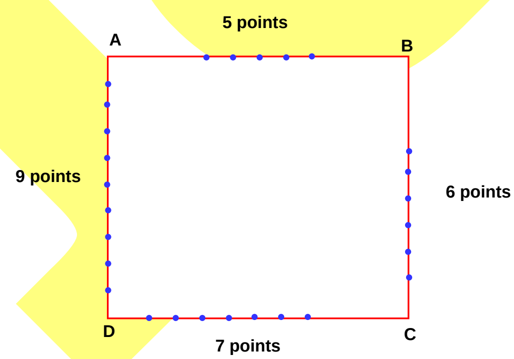
$= 210 + 315 + 270 - 378$

$= 1173$

β = Number of quadrilateral

β = 5.6.7.9 = 1896

$\beta - \alpha$ = 1890 – 1173 = 717



Sol7. $\text{Sin}^{-1}\left(\frac{3x}{5}\right) + \sin^{-1}\left(\frac{4x}{5}\right) = \sin^{-1}x$

$$\sin^{-1}\left(\frac{3x}{5}\sqrt{1-\frac{16x^2}{25}} + \frac{4x}{5}\sqrt{1-\frac{9x^2}{25}}\right) = \sin^{-1}x$$

$$\frac{3x}{5}\sqrt{\frac{25-16x^2}{25}} + \frac{4x}{5}\sqrt{\frac{25-9x^2}{25}} = x$$

$$\frac{3x}{25}\sqrt{25-16x^2} + \frac{4x}{25}\sqrt{25-9x^2} = x$$

$$3x\sqrt{25-16x^2} + 4x\sqrt{25-9x^2} = 25x$$

$$x\left(3\sqrt{25-16x^2} + 4\sqrt{25-9x^2} - 25\right) = 0$$

$\therefore x = 0$, or $\quad 3\sqrt{25-16x^2} + 4\sqrt{25-9x^2} - 25 = 0$

$$3\sqrt{25-16x^2} = 25 - 4\sqrt{25-9x^2}$$

$$9\left(25-16x^2\right) = 625 + 16\left(25-9x^2\right) - 200\sqrt{25-9x^2}$$

$$200\sqrt{25-9x^2} = 625 - 225 + 144x^2 + 400 - 144x^2$$

$$200\sqrt{25-9x^2} = 800$$

$$\sqrt{25-9x^2}=4$$
$$25-9x^2=16$$
$$x^2=1$$
$$x=\pm 1$$

Sol8. R.H.L

$$\lim_{x\to 0^+}\frac{\cos^{-1}\left(1-\{x\}^2\right).\sin^{-1}\left(1-\{x\}\right)}{\{x\}-\{x\}^3}$$

$$=\lim_{x\to 0^+}\frac{\cos^{-1}\left(1-x^2\right),\sin^{-1}\left(1-x\right)}{x\left(1-x\right)\left(1+x\right)}$$

$$=\lim_{x\to 0^+}\frac{\cos^{-1}\left(1-x^2\right)}{x.1.1}\times\frac{\pi}{2}$$

$$=\frac{\pi}{2}\lim_{x\to 0^+}\frac{\cos^{-1}\left(1-x^2\right)}{x}$$

L' Hospital Rule

$$=\frac{\pi}{2}\lim_{x\to 0^+}\frac{-1}{\sqrt{1-\left(1-x^2\right)^2}}\left(-2x\right)$$

$$=\frac{\pi}{2}\lim_{x\to 0^+}\frac{2x}{\sqrt{1-1-x^4+2x^2}}$$

$$=\frac{\pi}{2}\lim_{x\to 0^+}\frac{2x}{\sqrt{2x^2-x^4}}=\frac{\pi}{2}\lim_{x\to 0^+}\frac{2}{\sqrt{2-x^2}}$$

$$=\frac{\pi}{\sqrt{2}}$$

L.H.L

$$\lim_{x\to 0^-}\frac{\cos^{-1}\left\{1-\left(1+x\right)^2\right\}.\sin^{-1}\left(1-\left(1+x\right)\right)}{\left(1+x\right)-\{1+x)^3}$$

$$\lim_{x\to 0^-}\frac{\cos^{-1}\left\{1-1-x^2-2x\right\}.\sin^{-1}\left(-x\right)}{\left(1+x\right)\left(1-\left(1+x\right)^2\right)}$$

$$\lim_{x\to 0^-}\frac{\pi/2.-\sin^{-1}\left(x\right)}{\left(1+x\right)\left(1-\left(1+x\right)^2\right)}$$

$$\lim_{x\to 0^-}\frac{\pi/2.\sin^{-1}x}{\left(1+x\right)\left(\left(1+x\right)^2-1\right)}$$

$$=\frac{\pi}{2}\lim_{x\to 0^-}\frac{\sin^{-1}x}{\left(1+x\right)\left(1+x^2+2x-1\right)}$$

$$=\frac{\pi}{2}\lim_{x\to 0^-}\frac{\sin^{-1}x}{\left(1+x\right)\left(x+2\right).x}$$

$$=\frac{\pi}{2}\lim_{x\to 0^-}\frac{\sin^{-1}x}{x}.\frac{1}{1.2}$$

$\frac{\pi}{4}$

L.H.L $\neq$ R.H.L

Sol9. (a,b) R (c,d) $\Rightarrow$ ad = bc

(a,b) R (4,3) $\Rightarrow$ 3a = 4b

$\Rightarrow$ b= 3/4a

$A = \{2, 3, 4, 5, ----30\}$

$\therefore (a, b) = (4,3), (8,6), (12,9), (16,12), (20,15), (24,18), (28,21)$

Total 7 ordered pair satisfying $(b = 3/4a)$

Sol10. Equation of circle

$x^2+y^2+ax+2ay+c=0$;a<0

x- intercept $2\sqrt{g^2 - c} = 2\sqrt{2}$

$\sqrt{\left(\frac{a}{2}\right)^2 - c} = \sqrt{2}$

$\frac{a^2}{4} - c = 2$

$a^2 = 8 + 4c$ ………………………..(i)

y- intercept

$f^2 - c = 5$

$a^2 - c = 5$

$\therefore a^2 = 5 + c$ ……………………..(ii)

Equating (i) and (ii)

$8 + 4c = 5 + c$

$3c = -3$

$c = -1$

$a^2 = 5 + (-1)$

$a^2 = 4 \Rightarrow a = \pm 2$

as $a < 0$

$\therefore a = -2$

Now Equation of circle

$x^2 + y^2 - 2x - 4y - 1 = 0$

$(x-1)^2 + (y-2)^2 = 6$

$(x-1)^2 + (y-2)^2 = 6$

Equation of tangent perpendicular to circle $x + 2y = 0$

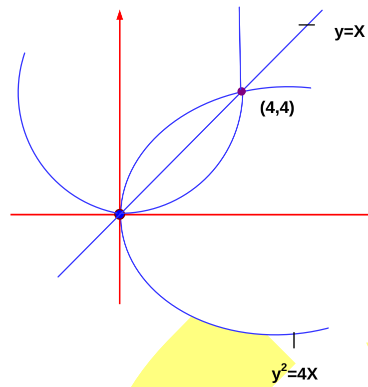
$\therefore$ slope of tangent = 2

Equation of tangent $(y-2) = 2(x-1) \pm \sqrt{6}\sqrt{1+4}$

$\Rightarrow 2x - y \pm \sqrt{30} = 0$

Shortage distance = $\frac{\sqrt{30}}{\sqrt{5}} = \sqrt{6}$

Sol11. Image of $y^2 = 4x$ w.r.t $y = x$
is $x^2 = 4y$
Equation of tangent is
$2x = 2(y+1)$
$x = y+1$



Sol12. $\exp\left(\frac{(|z|+3)(|z|-1)}{||z|+1|}\log e^2\right) \geq \log_{\sqrt{2}}\left|5\sqrt{7}+9i\right|; i=\sqrt{-1}$

$e^{\log_e 2\left(\frac{(|z|+3)(|z|-1)}{||z|+1|}\right)} \geq \log_{\sqrt{2}}(16)$

$2^{\frac{(|z|+3)(|z|-1)}{||z|+1|}} \geq 2\log_2(2)^4$

$2^{\frac{(|z|+3)(|z|-1)}{||z|+1|}} \geq 2^3$

$\frac{(|z|+3)(|z|-1)}{(|z|+1)} \geq 3$

$(|z|+3)(|z|-1) \geq 3|z|+3$

$|z|^2+2|z|-3 \geq 3|z|+3$

$|z|^2-|z|-6 \geq 0 \Rightarrow (|z|-3)(|z|+2) \geq 0$

$|z| \geq 3$

$\therefore |z|_{min} = 3$

Sol13. Possible six digit integer with
Digit 0,1,2,3,4,5,6

6	6	5	4	3	2

$\therefore$ Total = $6 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$
$= 6 \times 6!$

Favorable case

Sum f digit of no. must be divisible by 3
I (1+2+3+4+5+6) is divisible by 3
II (0+1+2+3+4+5) is divisible by 3
III (0+1+2+4+5+6) is divisible by 3
(I) Total no with digit 1,2,3,4,5,6 = 6!
(II) Total no with digit 0,1,2,3,4,5 = 5.51
(III) Total no with digit 0,1,2,4,5,6 = 5.5!
Total 5! (6+5+5)
$= 16 \times 5!$

$\therefore$ Probability $= \frac{\text{Fav.outcome}}{\text{possible outcome}}$

$$= \frac{16 \times 5!}{6 \times 6!}$$

$$= \frac{16 \times 5!}{6 \times 6 \times 5!} = \frac{4}{9}$$

Sol14. $\frac{dy}{dx} + (\tan x)y = \sin x; 0 \le x \le \frac{\pi}{3}$

y(0)=0, then y (π/4)

$I.F = e^{\int \tan x dx} = \sec x$

$y.\sec x = \int \sin x.\sec x dx$

$y \sec x = \int \tan x dx$

$y \sec x = \ln \sec x + c$

$x = 0, y = 0$

$\Rightarrow c = 0$

$y \sec x = \ln(\sec x)$

$y = \cos x \ln(\sec x)$

$y\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) \ln \sec\left(\frac{\pi}{4}\right)$

$y\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} \ln \sqrt{2}$

$y\left(\frac{\pi}{4}\right) = \frac{1}{2\sqrt{2}} \ln(2)$

Sol15. $P(x) = x^2 + bx + c$

$\int_0^1 (x^2 + bx + c) dx = 1$

$\Rightarrow \left[\frac{x^3}{3}\right]_0^1 + b\left[\frac{x^2}{2}\right]_0^1 + c[x]_0^1 = 1$

$\Rightarrow \frac{1}{3} + \frac{b}{2} + c = 1$

$3b + 6c = 6 - 2$

$3b + 6c = 4$ ---------------------(i)

$\therefore$ P(x) reminder 5 if divided by (x-2) from remainder theorem

P(2) = 5

(2)2+b(2) + c = 5

4+2b+c=5

2b+c=1…………………….(ii)

Solving (i) and (ii)

$b = \frac{2}{9}, c = \frac{5}{9}$

9(b+c)=7

By putting values

Sol16. $f(x+1) = x.f(X)$

$\ell n\{f(x+1)\} = \ell n(xf(x))$

$\ell nf(x+1) = \ell nx + \ell nf(x)$ -------(1)

As given

$g(x) = \ln f(x)$

Then

$g(x+1) = \ell nf(x+1)$

Now in equation (i)

$g(x+1) = \ell nx + g(x)$

$g(x+1) - g(x) = \ell nx$

$g'(x+1) - g'(x) = \frac{1}{x}$

$g''(x+1) - g''(x) = \frac{-1}{x^2}$

Now putting x = 1,

$g''(2) - g''(1) = \frac{-1}{1}$

$x = 2$

$g''(3) - g''(2) = \frac{-1}{4}$ ……………….(iii)

$x = 3$

$g''(4) = g''(3) = \frac{-1}{9}$ ……………..(iv)

$x = 4$

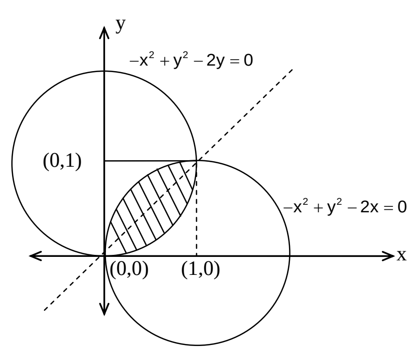
$g''(5) - g''(4) = \frac{-1}{16}$ ………….(v)

Adding (1), (iii) (iv) and (v)

$g''(5) - g''(1)' = \frac{-205}{144}$

$| g''(5) - g'(1) |= \frac{205}{144}$

Sol17. Given



$2xy\frac{dy}{dx} = y^2 - x^2, x > 0$

$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$

$y = vx$

$x\frac{dv}{dx} + v = \frac{dy}{dx}$

$\therefore x\frac{dv}{dx} + v = \frac{(vx)^2 - x^2}{2vx.x}$

$x.\frac{dv}{dx} = \frac{v^2 - 1}{2v} - v$

$x.\frac{dv}{dx} = \frac{v^2 - 1 - 2v^2}{2v}$

$$\int \frac{2v}{v^2+1}dv = \int \frac{-dx}{x}$$
$$\ell n\left(v^2+1\right) = -\ln(x) + \ln c$$
$$v^2+1 = \frac{c}{x}$$
$$\frac{y^2}{x^2}+1 = \frac{c}{x}$$
$$x^2+y^2 = cx$$
If passing through (1,1)
$$c = 2$$
$$\therefore x^2+y^2-2x = 0$$
Now
$$\frac{dy}{dx} = \frac{2xy}{x^2-y^2}$$
Put $y = vx$
We will get
$$x^2+y^2-2y = 0$$
Required area
$$= \left(\frac{1}{4}\times\pi\times1-\frac{1}{2}\right)\times 2 = \left(\frac{\pi}{2}-1\right) \text{ sq. cm}$$

Sol18. $\vec{r}\times\vec{a} = \vec{b}\times\vec{r}$
$$\therefore \vec{r} = \lambda\left(\vec{a}+\vec{b}\right)$$
$$\vec{r} = \lambda\left(\hat{i}+2\hat{j}-3k+2\hat{i}-3\hat{j}+5\hat{k}\right)$$
$$\vec{r} = \lambda\left(3\hat{i}-\hat{j}+2\hat{k}\right)\text{..........................(I)}$$
Given
$$\vec{r}.\left(a\hat{i}+2\hat{j}+\hat{k}\right) = 3$$
From (I)
$$\lambda\left(3\hat{i}-\hat{j}+2\hat{k}\right)\left(\alpha\hat{i}+2\hat{j}+\hat{k}\right) = 3$$
$$\lambda\left(3\alpha-2+2\right) = 3$$
$$\lambda\alpha = 1\text{..(II)}$$
Again
$$\vec{r}\left(2\hat{i}+5\hat{j}-\alpha\hat{k}\right) = -1$$
From (I)
$$\lambda\left(3\hat{i}-\hat{j}+2\hat{k}\right)\left(2\hat{i}+5\hat{j}-\alpha\hat{k}\right) = -1$$
$$\lambda\left(6-5-2\alpha\right) = -1$$
$$\lambda\left(+1-2\alpha\right) = -1$$
$$-1+2\alpha = \frac{1}{\lambda}$$
$$2\lambda\alpha-\lambda = 1$$
From (II)
$$\lambda\alpha = 1$$

$\therefore 2-\lambda=1$

$\lambda=1$

$\therefore \alpha=1$

$\vec{r}=\left(3\hat{i}-\hat{j}+2\hat{k}\right)$

$|\vec{r}|=\sqrt{9+1+4}$

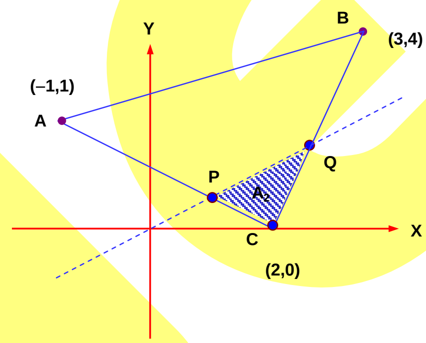
$|\vec{r}|=\sqrt{14}$

$\alpha=1$

$\alpha+|\vec{r}|=1+\left(\sqrt{14}\right)^2$

$=1+14=15$

Sol19. $\therefore$ Area of $\Delta ABC=\frac{1}{2}\left\|\begin{matrix}-1 & 1 & 1\\ 2 & 0 & 1\\ 3 & 4 & 1\end{matrix}\right\|$



$A_1=\frac{1}{2}\left|-1(0-4)-1(2-3)+1(8-0)\right|$

$A_1=\frac{13}{2}$

Now Equation of AC

$\frac{y-1}{x+1}=\frac{0-1}{2-(-1)}$

$(y-1)=\frac{-1}{3}(x+1)$

Equation of BC

$\frac{y-0}{x-2}=\frac{4-0}{3-2}$

$y=4(x-2)$

Solving Equation of AC and y = mx

$(mx-1)=\frac{-x-1}{3}$

$3mx-3=-x-1$

$x(3m+1)=2$

$\left(x=\frac{2}{3m+1}, y=\frac{2m}{3m+1}\right)$

Solving with BC

$y=mx$

$y=4(x-2)$

$mx=4x-8$

$\left(x=\frac{-8}{m-4}; y=\frac{-8m}{m-4}\right)$

$\therefore$ Area of $\Delta PQC = \frac{1}{2}\left\|\begin{matrix} 2 & 0 & 1 \\ \frac{2}{3m+1} & \frac{2m}{3m+1} & 1 \\ \frac{-8}{m-4} & \frac{-8m}{m-4} & 1 \end{matrix}\right\|$

$= \frac{26m^2}{(3m+1)(m-4)}$

$A_1 = \frac{13}{2}$

$A_2 = \left|\frac{26m^2}{(3m+1)(8m-4)}\right|$

$\frac{A_1}{A_2} = 3$ (given)

$\frac{13}{2A_2} = 3$

$A_2 = \frac{13}{6}$

$\frac{26m^2}{(3m+1)(m-4)} = \frac{\pm 13}{6}$

$\frac{2m^2}{(3m+1)(m-4)} = \pm\frac{1}{6}$

Taking +ve
$9m^2 + 11m + 4 = 0$

Taking -ve
$15m^2 - 11m - 4 = 0$
$m = 1, \frac{-4}{15}$

Sol20. $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ lie on the curve $y^2 = 3x^2$ then b is equal to

$\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ ……………………….(I)

$x^2 + y^2 = 4b$ ……………………..(II)

$y^2 = 3x^2$ …………………………..(III)

From (II) and (III)

$x^2 + 3x^2 = 4b$

$4x^2 = 4b$

$x^2 = b$

$\frac{b}{16} + \frac{3b^2}{b^2} = 1$

$b^2 + 48 = 16b$

$b^2 - 16b + 48 = 0$

$b = 12$, b = 4 rejected as b > 4.

SECTION- B

Sol1. $S_n(x) = \log_{a^{1/2}} x + \log_{a^{1/3}} x + \log_{a^{1/11}} x + -----$

$= \log_a x^{(2+3+6+11+18.........)}$

Let

$S' = 2+3+6+11+18+....+t_n$

$S' = \quad 2+3+6+11+.....+t_{n-1}+t_n$

$(-) \Rightarrow t_n = 2+1+3+5+7+......(n-1)\text{term}$

$= 2+(n-1)^2$

$S_n(x) = \log_a x\left\{\sum_{n=1}^{n}(n-1)^2 + 2\right\}$

$S_n(x) = \log_a x\left\{2n + \frac{(n-1)n(2n-1)}{6}\right\}$

$\Rightarrow S_{24}(x) = \left\{2\times 24 + \frac{23.24.47}{6}\right\}\log_a x = 1093$

$\Rightarrow \log_a x = \frac{1}{4}$

$\Rightarrow x = a^{1/4}$

$S_{12}(2x) = 265$

$\log_a(2x)\left\{2\times 12\frac{11.12.21}{6}\right\} = 265$

$\log_a(2x) = \frac{1}{2}$

$2x = a^{1/2}$

$2.a^{1/4} = a^{1/2}$

$2 = \frac{a^{1/2}}{a^{1/4}}$

$a = 16$

Sol2. $\int \frac{(x^2-1)+\tan^{-1}\left(\frac{x^2+1}{x}\right)}{(x^4+3x^2+1)\tan^{-1}\left(\frac{x^2+1}{x}\right)}dx$

$\int \frac{(x^2-1)}{(x^4+3x^2+1)\tan^{-1}\left(\frac{x^2+1}{x}\right)}dx + \int \frac{1}{(x^4+3x^2+1)}dx$

$\int \frac{\left(1-\frac{1}{x^2}\right)}{\left(x^2+\frac{1}{x^2}+3\right)\tan^{-1}\left(x+\frac{1}{x}\right)}dx + \frac{1}{2}\int \frac{2}{x^4+3x^2+1}dx$

$$I=\int\frac{\left(1-\frac{1}{x^2}\right)dx}{\left(x^2+\frac{1}{x^2}+3\right)\tan^{-1}\left(x+\frac{1}{x}\right)}+\frac{1}{2}\int\frac{\left(x^2+1\right)-\left(x^2-1\right)}{x^4+3x^2+1}dx$$

I_1 $\qquad\qquad$ I_2

$$I_1=\int\frac{\left(1-\frac{1}{x^2}\right)}{\left(x^2+\frac{1}{x^2}+3\right)\tan^{-1}\left(x+\frac{1}{x}\right)}dx$$

Let $\tan^{-1}\left(x+\frac{1}{x}\right)=t$

$$\left(1-\frac{1}{x^2}\right)\frac{1}{\left(x+\frac{1}{x}\right)^2+1}=\frac{dt}{dx}$$

$$\frac{\left(1-\frac{1}{x^2}\right)}{\left(x^2+\frac{1}{x^2}+3\right)}dx=dt$$

$$\therefore I_1=\int\frac{dt}{t}$$

$$I_1=\ln(t)$$

$$I_1=\ln\left\{\tan^{-1}\left(x+\frac{1}{x}\right)\right\}$$

$$I_2=\left\{\frac{1}{2}\int\frac{\left(1+\frac{1}{x^2}\right)}{x^2+\frac{1}{x^2}+3}dx-\frac{1}{2}\int\frac{\left(1-\frac{1}{x^2}\right)}{\left(x^2+\frac{1}{x^2}+3\right)}dx\right.$$

$$I_2=\left\{\frac{1}{2}\int\frac{\left(1+\frac{1}{x^2}\right)}{\left(x-\frac{1}{x}\right)^2+5}dx-\frac{1}{2}\int\frac{\left(1-\frac{1}{x^2}\right)}{\left(x+\frac{1}{x}\right)^2+1}dx\right\}$$

$$I_2=\left\{\frac{1}{2\sqrt{5}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{5}x}\right)-\frac{1}{2}\tan^{-1}\left(\frac{x^2+1}{x}\right)\right\}+C$$

$$I=I_1+I_2$$

$$I=\ln\left\{\tan^{-1}\left(x+\frac{1}{x}\right)\right\}+\frac{1}{2\sqrt{5}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{5}x}\right)-\frac{1}{2}\tan^{-1}\left(\frac{x^2+1}{x}\right)$$

On comparision with given question

$$\alpha=1,\beta=\frac{1}{2\sqrt{5}},\delta=\frac{-1}{2},r=\frac{1}{\sqrt{5}}$$

$$\therefore 10(2+\beta\gamma+\delta)=10\left(1+\frac{1}{10}-\frac{1}{2}\right)$$

$$=\left(\frac{10+1-5}{10}\right)10=6$$

Sol3. $\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & -1 \\ 1 & k \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$

$\begin{bmatrix} \sqrt{3}\,a_1 \\ \sqrt{3}\,a_2 \end{bmatrix} = \begin{bmatrix} b_1 - b_2 \\ b_1 + kb_2 \end{bmatrix}$

$b_1 - b_2 = \sqrt{3}a_1$ ……………………………………..(I)

$b_1 + kb_2 = \sqrt{3}\,a_2$ ……………………………………(II)

From given

$a_1^2 + a_2^2 = \frac{2}{3}\left(b_1^2 + b_2^2\right)$ ………………………………(III)

squaring and adding (II) and (I)

$b_1^2 + b_2^2 - 2b_1b_2 + b_1^2 + k^2b_2^2 + 2kb_1, b_2 = 3\left(a_1^2 + a_2^2\right)$

$\frac{2}{3}b_1^2 + \frac{1}{3}\left(1+k^2\right)b_2^2 + \frac{2}{3}(k-1)b_1b_2 = a_1^2 + a_2^2$ ……………………..(IV)

On comparing (III) and (IV) we get

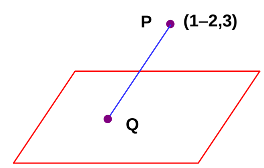
$\frac{2}{3}(k-1) = 0$ at $\left(\frac{1+k^2}{3}\right) = \frac{2}{3}$

$k = 1$

$k = \pm 1$

$\therefore k = 1$

Sol4. $\frac{x-1}{3} = \frac{y-2}{-m} = \frac{z+3}{1}$



line parallel to given line and passing through (1, –2,3)

$\frac{x-1}{3} = \frac{y+2}{-m} = \frac{z-3}{1} = \lambda$

$Q \equiv (x,y,z) \equiv \left((1+3\lambda),(-m\lambda-2),(\lambda+3)\right)$

$\left((1+3\lambda),(-m\lambda-2),(\lambda+3)\right)$ satisfying plane $x+2y-3z+10=0$ for some value of λ:

$\therefore (1+3\lambda)-4-2m\lambda-3\lambda-9+10=0$

$\lambda m = 1 \quad \lambda = \frac{1}{m}$

$Q = \left(1+\frac{3}{m}, -3, \frac{1}{m}+3\right)$

$PQ = \sqrt{\frac{9}{m^2}+\frac{1}{m^2}+1} = \sqrt{\frac{7}{2}}$

$|m| = 2$

Sol5. $A = \sum_{k=0}^{n}(-1)^K\, n_{C_K}\left[\left(\frac{1}{2}\right)^k + \left(\frac{3}{4}\right)^K + \left(\frac{7}{8}\right)^k + \left(\frac{15}{16}\right)^K + \left(\frac{31}{32}\right)^K\right]$

$= \sum_{k=0}^{n}(-1)^K\, n_{C_K}\left(\frac{1}{2}\right)^k + - - - - - \sum_{k=0}^{n}(-1)^K\, n_{C_K}\left(\frac{31}{32}\right)^K$

$= \left(1-\frac{1}{2}\right)^n + \left(1-\frac{3}{4}\right)^n - - - - - - - + \left(1-\frac{31}{32}\right)^n$

$$=\left(\frac{1}{2}\right)^n+\left(\frac{1}{4}\right)^n+\left(\frac{1}{8}\right)^n+\left(\frac{1}{16}\right)^n+\left(\frac{1}{32}\right)^n$$

$$A=\left(\frac{1}{2}\right)^n\frac{\left\{\left(1-\left(\frac{1}{2^n}\right)\right)^5\right\}}{\left(1-\frac{1}{2^n}\right)}=\frac{1-\frac{1}{2^{5n}}}{\left(2^n-1\right)}\Rightarrow\left(2^n-1\right)A=1-\frac{1}{2^{5n}}$$

$$63A=1-\frac{1}{2^{30}} \text{ on comparing}$$

$$\therefore n=6$$

Sol6. $\frac{1}{16}$, a and b are in G.P

$$\therefore a^2=\frac{b}{16}$$

$$b=16a^2$$

$$\frac{1}{a},\frac{1}{b},6$$

$$\frac{2}{b}=\frac{1}{a}+6$$

$$\frac{1}{8a^2}=\frac{1}{a}-1+6$$

$$\frac{1}{a^2}=\frac{8}{a}+48$$

$$\frac{1}{a^2}-\frac{8}{a}-48=0$$

$$\frac{1}{a}=12,-4$$

$$a=\frac{1}{12},\frac{-1}{4}$$

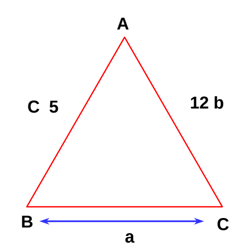
$$a=\frac{1}{12},a>0$$

$$b=16a^2$$

$$b=\frac{1}{9}$$

$$72(a+b)=6+8=14$$

Sol7. $\Delta=\frac{1}{2}\times 5.12.\sin A=30$



$$\sin A=1$$

$$A=90^o$$

$$BC=\sqrt{5^2+(12)^2}$$

$$BC=13$$

$$r=\frac{\Delta}{s}=\frac{30}{15}=2$$

$$2R+r=13+2=15$$

Sol8. $f:R\longrightarrow R \quad g:R\longrightarrow R$

$$f(x)=\begin{cases}(x+a); & x<0\\ |x-1|; & x\geq 0\end{cases}$$

$$g(x)=\begin{cases}(x+1); & x<0\\ (x-1)^2+b; & x\geq 0\end{cases}$$

$$\left[g\left(f(x)\right)\right]=\begin{cases}f(x)+1 & f(x)<0\\ (f(x)-1)^2+b & f(x)\geq 0\end{cases}$$

$$\left[g\left(f(x)\right)\right]=\begin{cases}(x+a)+1 & x+a<0; \quad x<0\\ |x-1|+1 & |x-1|<0; \quad x\geq 0\\ (x+a-1)^2 b & x+a\geq 0; \quad x<0\\ (|x-1|-1)^2+b & |x-1|\geq 0, \quad x>0\end{cases}$$

$$g\left(f(x)\right)=\begin{cases}x+a+1 & x\in(-\infty,-a)\ \&\in(-\infty,0)\\ |x-1|+1 & x\in\phi\\ (x+a-1)^2+b & x\in[-a,\infty]\ \&\ x\in(-\infty,0)\\ (|x-1|-1)^2+b & x\in R\ \&\ x\in[0,\infty)\end{cases}$$

$$g[f(x)]=\begin{cases}x+a+1 & x\in(-\infty,-a)\\ (x+a-1)^2+b & x\in[-a,0)\\ (|x-1|-1)^2+b & x\in[0,\infty)\end{cases}$$

$g[f(x)]$ is continuous at x = -a

$\therefore 1+b=1$

$b=0$

$g\left(f(x)\right)$ is continuous at x = 0

$b=(a-1)^2+b$

$a-1=0$

$a=1$

$\therefore a+b=1+0=1$

Sol9. For 1st observation

Mean = $\frac{\sum x_i}{10} = 2$ $\frac{\sum x_i^2}{10} - (2)^2 = 2$ (Variance)

$\sum x_i = 20$ $\sum x_i^2 = 60$

2nd observation

$\frac{\sum y_i}{n} = 3$

$\sum y_i = 3n$

$\frac{\sum y_i^2}{n} - 3^2 = 1$ (Variance)

$\sum y_i^2 = 10n$

Combined variance

$$\sigma^2 = \frac{\sum\left(x_i^2 + y_i^2\right)}{10+n} - \left(\frac{\sum x_i + y_i}{10+n}\right)^2$$

$$\frac{17}{9} = \frac{60+10n}{10+n} - \frac{(20+3n)^2}{(10+n)^2}$$

$$17\left(n^2 + 20n + 100\right) = 9\left(n^2 + 40n + 200\right)$$

$$8n^2 - 2n - 100 = 0$$

$$2n^2 - 5n - 25 = 0$$

$$n = 5$$

Sol10. $\vec{C} = \lambda\left(\vec{a} \times \vec{b}\right)$

$$= \lambda\begin{vmatrix} i & j & k \\ 1 & 1 & -1 \\ 1 & 2 & 1 \end{vmatrix}$$

$$= \lambda\left[\hat{i}(1+2) - \hat{j}(1+1) + \hat{k}(2-1)\right]$$

$$= \lambda\left[3\hat{i} - 2\hat{j} + \hat{k}\right]$$

Given $\vec{C}.\left(\hat{i} + \hat{j} + 3\hat{k}\right) = 8$

$$\lambda\left(3\hat{i} - 2\hat{j} + \hat{k}\right)\left(\hat{i} + \hat{j} + 3\hat{k}\right) = 8$$

$$\lambda = 2$$

$$\vec{C} = 2\left(\vec{a} \times \vec{b}\right)$$

$$\vec{C}.\left(\vec{a} \times \vec{b}\right) = 2\left|\vec{a} \times \vec{b}\right|^2 = 28$$