

FIITJEE Solutions to JEE(MAIN)-2013

Time: 3 Hours

Maximum Marks: 360

PAPER – 1 : CHEMISTRY, MATHEMATICS & PHYSICS

Test Booklet Code

Q

Important Instructions:

1. Immediately fill in the particulars on this page of the Test Booklet with *Blue/Black Ball Point Pen*. Use of pencil is strictly prohibited.
2. The test is of **3 hours duration**.
3. The Test Booklet consists of **90** questions. The maximum marks are **360**.
4. There are **three** parts in the question paper A, B, C consisting of **Chemistry, Mathematics** and **Physics** having 30 questions in each part of equal weightage. Each question is allotted **4 (four)** marks for correct response.
5. *Candidates will be awarded marks as stated above in instruction No. 5 for correct response of each question. (1/4) (one fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.*
6. There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 6 above.

PART - A: CHEMISTRY

1. An unknown alcohol is treated with the "Lucas reagent" to determine whether the alcohol is primary, secondary or tertiary. Which alcohol reacts fastest and by what mechanism:

(1) tertiary alcohol by S_N1	(2) secondary alcohol by S_N2
(3) tertiary alcohol by S_N2	(4) secondary alcohol by S_N1

2. The first ionization potential of Na is 5.1 eV. The value of electron gain enthalpy of Na^+ will be:

(1) - 5.1 eV	(3) -10.2 eV
(3) + 2.55 eV	(4) - 2.55 eV

3. Stability of the species Li_2 , Li_2^- and Li_2^+ increases in the order of:

(1) $Li_2^- < Li_2^+ < Li_2$	(3) $Li_2 < Li_2^- < Li_2^+$
(3) $Li_2^- < Li_2 < Li_2^+$	(4) $Li_2 < Li_2^+ < Li_2^-$

4. The molarity of a solution obtained by mixing 750 mL of 0.5 (M) HCl with 250 mL of 2(M)HCl will be:

(1) 1.00 M	(2) 1.75 M
(3) 0.975 M	(4) 0.875 M

5. Which of the following is the wrong statement?

(1) O_3 molecule is bent	(2) Ozone is violet-black in solid state
(3) Ozone is diamagnetic gas	(4) $ONCl$ and ONO^- are not isoelectronic

6. Four successive members of the first row transition elements are listed below with atomic numbers. Which one of them is expected to have the highest $E_{M^{3+}/M^{2+}}^0$ value?

(1) Mn(Z = 25)	(2) Fe(Z = 26)
(3) Co(Z = 27)	(4) Cr(Z = 24)

7. A solution of (-) -1 - chloro -1 - phenylethane is toluene racemises slowly in the presence of a small amount of $SbCl_5$, due to the formation of :

(1) carbene	(2) carbocation
(3) free radical	(4) carbanion

8. The coagulating power of electrolytes having ions Na^+ , Al^{3+} and Ba^{2+} for arsenic sulphide sol increases in the order:

(1) $Na^+ < Ba^{2+} < Al^{3+}$	(2) $Ba^{2+} < Na^+ < Al^{3+}$
(3) $Al^{3+} < Na^+ < Ba^{2+}$	(4) $Al^{3+} < Ba^{2+} < Na^+$

9. How many litres of water must be added to 1 litre of an aqueous solution of HCl with a pH of 1 to create an aqueous solution with pH of 2?

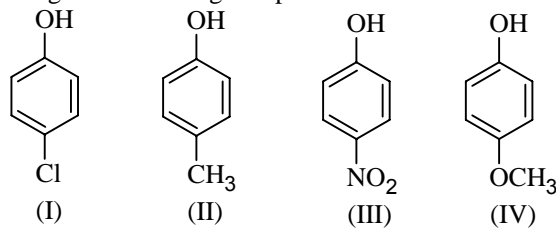
(1) 0.9 L	(2) 2.0 L
(3) 9.0 L	(4) 0.1 L

10. Which one of the following molecules is expected to exhibit diamagnetic behaviour?

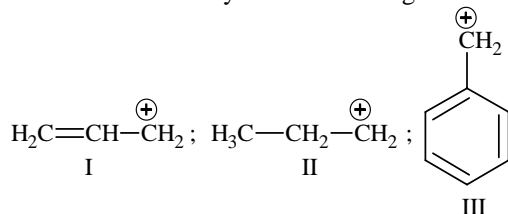
(1) N_2	(2) O_2
(3) S_2	(4) C_2

11. Which of the following arrangements does **not** represent the correct order of the property stated against it ?

(1) $Ni^{2+} < Co^{2+} < Fe^{2+} < Mn^{2+}$: ionic size
(2) $Co^{3+} < Fe^{3+} < Cr^{3+} < Sc^{3+}$: stability in aqueous solution
(3) $Sc < Ti < Cr < Mn$: number of oxidation states
(4) $V^{2+} < Cr^{2+} < Mn^{2+} < Fe^{2+}$: paramagnetic behaviour

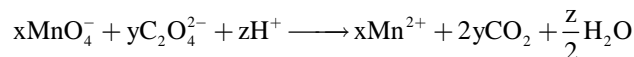
12. Experimentally it was found that a metal oxide has formula $M_{0.98}O$. Metal M, is present as M^{2+} and M^{3+} in its oxide. Fraction of the metal which exists as M^{3+} would be:
 (1) 4.08% (2) 6.05%
 (3) 5.08% (4) 7.01%
13. A compound with molecular mass 180 is acylated with CH_3COCl to get a compound with molecular mass 390. The number of amino groups present per molecule of the former compound is:
 (1) 5 (2) 4
 (3) 6 (4) 2
14. Given
 $E_{Cr^{3+}/Cr}^0 = -0.74 V$; $E_{MnO_4^-/Mn^{2+}}^0 = 1.51 V$
 $E_{Cr_2O_7^{2-}/Cr^{3+}}^0 = 1.33 V$; $E_{Cl/Cl^-}^0 = 1.36 V$
 Based on the data given above, strongest oxidising agent will be:
 (1) Cr^{3+} (2) Mn^{2+}
 (3) MnO_4^- (4) Cl^-
15. Arrange the following compounds in order of decreasing acidity:

 (1) I > II > III > IV (2) III > I > II > IV
 (3) IV > III > I > II (4) II > IV > I > III
16. The rate of a reaction doubles when its temperature changes from 300K to 310K. Activation energy of such a reaction will be:
 ($R = 8.314 JK^{-1} mol^{-1}$ and $\log 2 = 0.301$)
 (1) 48.6 kJ mol⁻¹ (2) 58.5 kJ mol⁻¹
 (3) 60.5 kJ mol⁻¹ (4) 53.6 kJ mol⁻¹
17. Synthesis of each molecule of glucose in photosynthesis involves:
 (1) 10 molecules of ATP (2) 8 molecules of ATP
 (3) 6 molecules of ATP (4) 18 molecules of ATP
18. Which of the following complex species is not expected to exhibit optical isomerism?
 (1) $[Co(en)_2 Cl_2]^+$ (2) $[Co(NH_3)_3 Cl_3]$
 (3) $[Co(en)(NH_3)_2 Cl_2]^+$ (4) $[Co(en)_3]^{3+}$
19. A piston filled with 0.04 mol of an ideal gas expands reversibly from 50.0 mL to 375 mL at a constant temperature of 37.0°C. As it does so, it absorbs 208J of heat. The values of q and w for the process will be:
 ($R = 8.314 J/mol K$) ($\ell n 7.5 = 2.01$)
 (1) q = -208 J, w = -208 J (2) q = -208 J, w = +208 J
 (3) q = +208 J, w = +208 J (4) q = +208 J, w = -208 J
20. A gaseous hydrocarbon gives upon combustion 0.72 g of water and 3.08 g of CO_2 . The empirical formula of the hydrocarbon is:
 (1) C_3H_4 (2) C_6H_5
 (3) C_7H_8 (4) C_2H_4

21. The order of stability of the following carbocations:



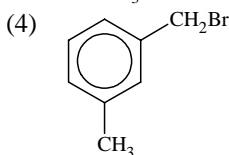
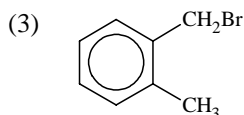
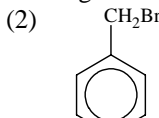
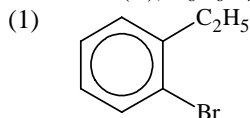
is:

- (1) II > III > I (2) I > II > III
 (3) III > I > II (4) III > II > I
22. Which of the following represents the correct order of increasing first ionization enthalpy for Ca, Ba, S, Se and Ar?
 (1) S < Se < Ca < Ba < Ar (2) Ba < Ca < Se < S < Ar
 (3) Ca < Ba < S < Se < Ar (4) Ca < S < Ba < Se < Ar
23. For gaseous state, if most probable speed is denoted by C^* , average speed by \bar{C} and mean square speed by C , then for a large number of molecules the ratios of these speeds are:
 (1) $C^* : \bar{C} : C = 1.128 : 1.225 : 1$ (2) $C^* : \bar{C} : C = 1 : 1.128 : 1.225$
 (3) $C^* : \bar{C} : C = 1 : 1.125 : 1.128$ (4) $C^* : \bar{C} : C = 1.225 : 1.128 : 1$
24. The gas leaked from a storage tank of the Union Carbide plant in Bhopal gas tragedy was:
 (1) Methylamine (2) Ammonia
 (3) Phosgene (4) Methylisocyanate
25. Consider the following reaction:



The values of x, y and z in the reaction are, respectively:

- (1) 2, 5 and 8 (2) 2, 5 and 16
 (3) 5, 2 and 8 (4) 5, 2 and 16
26. Which of the following exists as covalent crystals in the solid state?
 (1) Silicon (2) Sulphur
 (3) Phosphorous (4) Iodine
27. Compound (A), $\text{C}_8\text{H}_9\text{Br}$, gives a white precipitate when warmed with alcoholic AgNO_3 . Oxidation of (A) gives a acid (B), $\text{C}_8\text{H}_6\text{O}_4$. (B) easily forms anhydride on heating. Identify the compound (A).



28. Energy of an electron is given by $E = -2.178 \times 10^{-18} \text{J} \left(\frac{Z^2}{n^2} \right)$. Wavelength of light required to excite an

electron in an hydrogen atom from level $n = 1$ to $n = 2$ will be

($h = 6.62 \times 10^{-34} \text{Js}$ and $c = 3.0 \times 10^8 \text{ms}^{-1}$)

- (1) $2.816 \times 10^{-7} \text{m}$ (2) $6.500 \times 10^{-7} \text{m}$
 (3) $8.500 \times 10^{-7} \text{m}$ (4) $1.214 \times 10^{-7} \text{m}$

29. An organic compound A upon reacting with NH_3 gives B. On heating B gives C. C in presence of KOH reacts with Br_2 to give $\text{CH}_3\text{CH}_2\text{NH}_2$. A is
- (1) $\text{CH}_3\text{CH}_2\text{CH}_2\text{COOH}$ (2) $\text{H}_3\text{C}-\underset{\text{CH}_3}{\text{CH}}-\text{COOH}$
- (3) $\text{CH}_3\text{CH}_2\text{COOH}$ (4) CH_3COOH
30. In which of the following pairs of molecules/ions, both the species are not likely to exist?
- (1) H_2^- , He_2^{2-} (2) H_2^{2+} , He_2
- (3) H_2^- , He_2^{2+} (4) H_2^+ , He_2^{2-}

PART - B: MATHEMATICS

31. The circle passing through (1, -2) and touching the axis of x at (3, 0) also passes through the point
 (1) (2, -5) (2) (5, -2)
 (3) (-2, 5) (4) (-5, 2)
32. ABCD is a trapezium such that AB and CD are parallel and $BC \perp CD$. If $\angle ADB = \theta$, $BC = p$ and $CD = q$, then AB is equal to
 (1) $\frac{p^2 + q^2 \cos \theta}{p \cos \theta + q \sin \theta}$ (2) $\frac{p^2 + q^2}{p^2 \cos \theta + q^2 \sin \theta}$
 (3) $\frac{(p^2 + q^2) \sin \theta}{(p \cos \theta + q \sin \theta)^2}$ (4) $\frac{(p^2 + q^2) \sin \theta}{p \cos \theta + q \sin \theta}$
33. Given : A circle, $2x^2 + 2y^2 = 5$ and a parabola, $y^2 = 4\sqrt{5}x$.
Statement – I : An equation of a common tangent to these curves is $y = x + \sqrt{5}$.
Statement – II : If the line, $y = mx + \frac{\sqrt{5}}{m}$ ($m \neq 0$) is their common tangent, then m satisfies $m^4 - 3m^2 + 2 = 0$.
 (1) Statement - I is True; Statement -II is true; Statement-II is **not** a correct explanation for Statement-I
 (2) Statement -I is True; Statement -II is False.
 (3) Statement -I is False; Statement -II is True
 (4) Statement -I is True; Statement -II is True; Statement-II is a **correct** explanation for Statement-I
34. A ray of light along $x + \sqrt{3}y = \sqrt{3}$ gets reflected upon reaching x-axis, the equation of the reflected rays is
 (1) $\sqrt{3}y = x - \sqrt{3}$ (2) $y = \sqrt{3}x - \sqrt{3}$
 (3) $\sqrt{3}y = x - 1$ (4) $y = x + \sqrt{3}$
35. All the students of a class performed poorly in Mathematics. The teacher decided to give grace marks of 10 to each of the students. Which of the following statistical measures will not change even after the grace marks were given ?
 (1) median (2) mode
 (3) variance (4) mean
36. If x, y, z are in A.P. and $\tan^{-1}x$, $\tan^{-1}y$ and $\tan^{-1}z$ are also in A.P., then
 (1) $2x = 3y = 6z$ (2) $6x = 3y = 2z$
 (3) $6x = 4y = 3z$ (4) $x = y = z$
37. If $\int f(x) dx = \Psi(x)$, then $\int x^5 f(x^3) dx$ is equal to
 (1) $\frac{1}{3} x^3 \Psi(x^3) - 3 \int x^3 \Psi(x^3) dx + C$ (2) $\frac{1}{3} x^3 \Psi(x^3) - \int x^2 \Psi(x^3) dx + C$
 (3) $\frac{1}{3} [x^3 \Psi(x^3) - \int x^3 \Psi(x^3) dx] + C$ (4) $\frac{1}{3} [x^3 \Psi(x^3) - \int x^2 \Psi(x^3) dx] + C$

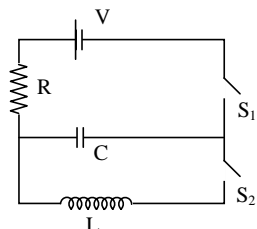
38. The equation of the circle passing through the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$, and having centre at (0, 3) is
 (1) $x^2 + y^2 - 6y + 7 = 0$ (2) $x^2 + y^2 - 6y - 5 = 0$
 (3) $x^2 + y^2 - 6y + 5 = 0$ (4) $x^2 + y^2 - 6y - 7 = 0$
39. The x-coordinate of the incentre of the triangle that has the coordinates of mid points of its sides as (0, 1), (1, 1) and (1, 0) is
 (1) $2 - \sqrt{2}$ (2) $1 + \sqrt{2}$
 (3) $1 - \sqrt{2}$ (4) $2 + \sqrt{2}$
40. The intercepts on x-axis made by tangents to the curve, $y = \int_0^x |t| dt$, $x \in \mathbb{R}$, which are parallel to the line $y = 2x$, are equal to
 (1) ± 2 (2) ± 3
 (3) ± 4 (4) ± 1
41. The sum of first 20 terms of the sequence 0.7, 0.77, 0.777, , is
 (1) $\frac{7}{9}(99 - 10^{-20})$ (2) $\frac{7}{81}(179 + 10^{-20})$
 (3) $\frac{7}{9}(99 + 10^{-20})$ (4) $\frac{7}{81}(179 - 10^{-20})$
42. Consider :
Statement – I : $(p \wedge \sim q) \wedge (\sim p \wedge q)$ is a fallacy.
Statement – II : $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$ is a tautology.
 (1) Statement - I is True; Statement -II is true; Statement-II is **not** a correct explanation for Statement-I
 (2) Statement -I is True; Statement -II is False.
 (3) Statement -I is False; Statement -II is True
 (4) Statement -I is True; Statement -II is True; Statement-II is a **correct** explanation for Statement-I
43. The area (in square units) bounded by the curves $y = \sqrt{x}$, $2y - x + 3 = 0$, x-axis, and lying in the first quadrant is
 (1) 36 (2) 18
 (3) $\frac{27}{4}$ (4) 9
44. The expression $\frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A}$ can be written as
 (1) $\sec A \operatorname{cosec} A + 1$ (2) $\tan A + \cot A$
 (3) $\sec A + \operatorname{cosec} A$ (4) $\sin A \cos A + 1$
45. The real number k for which the equation, $2x^3 + 3x + k = 0$ has two distinct real roots in [0, 1]
 (1) lies between 2 and 3 (2) lies between -1 and 0
 (3) does not exist (4) lies between 1 and 2
46. $\lim_{x \rightarrow 0} \frac{(1 - \cos 2x)(3 + \cos x)}{x \tan 4x}$ is equal to
 (1) $\frac{1}{2}$ (2) 1
 (3) 2 (4) $-\frac{1}{4}$

47. Let T_n be the number of all possible triangles formed by joining vertices of an n -sided regular polygon. If $T_{n+1} - T_n = 10$, then the value of n is
 (1) 5 (2) 10
 (3) 8 (4) 7
48. At present, a firm is manufacturing 2000 items. It is estimated that the rate of change of production P w.r.t. additional number of workers x is given by $\frac{dP}{dx} = 100 - 12\sqrt{x}$. If the firm employs 25 more workers, then the new level of production of items is
 (1) 3000 (2) 3500
 (3) 4500 (4) 2500
49. **Statement – I :** The value of the integral $\int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}}$ is equal to $\frac{\pi}{6}$.
Statement – II : $\int_a^b f(x) dx = \int_a^b f(a + b - x) dx$.
 (1) Statement - I is True; Statement -II is true; Statement-II is **not** a correct explanation for Statement-I
 (2) Statement -I is True; Statement -II is False.
 (3) Statement -I is False; Statement -II is True
 (4) Statement -I is True; Statement -II is True; Statement-II is a **correct** explanation for Statement-I
50. If $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of a 3×3 matrix A and $|A| = 4$, then α is equal to
 (1) 11 (2) 5
 (3) 0 (4) 4
51. The number of values of k , for which the system of equations
 $(k + 1)x + 8y = 4k$
 $kx + (k + 3)y = 3k - 1$
 has no solution, is
 (1) 1 (2) 2
 (3) 3 (4) infinite
52. If $y = \sec(\tan^{-1}x)$, then $\frac{dy}{dx}$ at $x = 1$ is equal to
 (1) $\frac{1}{2}$ (2) 1
 (3) $\sqrt{2}$ (4) $\frac{1}{\sqrt{2}}$
53. If the lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar, then k can have
 (1) exactly one value (2) exactly two values
 (3) exactly three values (4) any value
54. Let A and B be two sets containing 2 elements and 4 elements respectively. The number of subsets of $A \times B$ having 3 or more elements is
 (1) 220 (2) 219
 (3) 211 (4) 256

55. If the vectors $\overline{AB} = 3\hat{i} + 4\hat{k}$ and $\overline{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a triangle ABC, then the length of the median through A is
- (1) $\sqrt{72}$ (2) $\sqrt{33}$
 (3) $\sqrt{45}$ (4) $\sqrt{18}$
56. A multiple choice examination has 5 questions. Each question has three alternative answers of which exactly one is correct. The probability that a student will get 4 or more correct answers just by guessing is
- (1) $\frac{13}{3^5}$ (2) $\frac{11}{3^5}$
 (3) $\frac{10}{3^5}$ (4) $\frac{17}{3^5}$
57. If z is a complex number of unit modulus and argument θ , then $\arg\left(\frac{1+z}{1+\bar{z}}\right)$ equals
- (1) $\frac{\pi}{2} - \theta$ (2) θ
 (3) $\pi - \theta$ (4) $-\theta$
58. If the equations $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0$, $a, b, c \in \mathbb{R}$, have a common root, then $a : b : c$ is
- (1) $3 : 2 : 1$ (2) $1 : 3 : 2$
 (3) $3 : 1 : 2$ (4) $1 : 2 : 3$
59. Distance between two parallel planes $2x + y + 2z = 8$ and $4x + 2y + 4z + 5 = 0$ is
- (1) $\frac{5}{2}$ (2) $\frac{7}{2}$
 (3) $\frac{9}{2}$ (4) $\frac{3}{2}$
60. The term independent of x in expansion of $\left(\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - x^{1/2}}\right)^{10}$ is
- (1) 120 (2) 210
 (3) 310 (4) 4

PART - C: PHYSICS

61. In an LCR circuit as shown below both switches are open initially. Now switch S_1 is closed, S_2 kept open. (q is charge on the capacitor and $\tau = RC$ is capacitive time constant). Which of the following statement is correct?

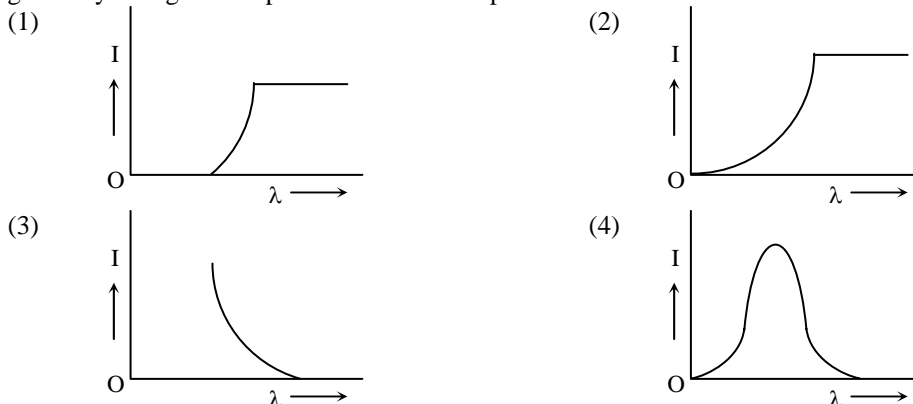


- (1) At $t = \tau$, $q = CV/2$
 (2) At $t = 2\tau$, $q = CV(1 - e^{-2})$
 (3) At $t = \frac{\tau}{2}$, $q = CV(1 - e^{-1})$
 (4) Work done by the battery is half of the energy dissipated in the resistor.
62. A diode detector is used to detect an amplitude modulated wave of 60% modulation by using a condenser of capacity 250 pico farad in parallel with a load resistance 100 kilo ohm. Find the maximum modulated frequency which could be detected by it.
- (1) 10.62 kHz (2) 5.31 MHz
 (3) 5.31 kHz (4) 10.62 MHz
63. The supply voltage to a room is 120 V. The resistance of the lead wires is 6Ω . A 60 W bulb is already switched on. What is the decrease of voltage across the bulb, when a 240 W heater is switched on in parallel to the bulb?
- (1) 2.9 Volt (2) 13.3 Volt
 (3) 10.04 Volt (4) zero volt
64. A uniform cylinder of length L and mass M having cross-sectional area A is suspended, with its length vertical, from a fixed point by a massless spring, such that it is half submerged in a liquid of density σ at equilibrium position. The extension x_0 of the spring when it is in equilibrium is:
- (1) $\frac{Mg}{k} \left(1 - \frac{LA\sigma}{M} \right)$ (2) $\frac{Mg}{k} \left(1 - \frac{LA\sigma}{2M} \right)$
 (3) $\frac{Mg}{k} \left(1 + \frac{LA\sigma}{M} \right)$ (4) $\frac{Mg}{k}$
 (Here k is spring constant)
65. Two charges, each equal to q , are kept at $x = -a$ and $x = a$ on the x -axis. A particle of mass m and charge $q_0 = \frac{q}{2}$ is placed at the origin. If charge q_0 is given a small displacement ($y \ll a$) along the y -axis, the net force acting on the particle is proportional to:
- (1) $-y$ (2) $\frac{1}{y}$
 (3) $-\frac{1}{y}$ (4) y

66. A beam of unpolarised light of intensity I_0 is passed through a polaroid A and then through another polaroid B which is oriented so that its principal plane makes an angle of 45° relative to that of A. The intensity of the emergent light is:

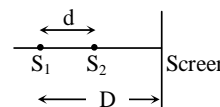
- (1) $I_0/2$ (2) $I_0/4$
 (3) $I_0/8$ (4) I_0

67. The anode voltage of a photocell is kept fixed. The wavelength λ of the light falling on the cathode is gradually changed. The plate current I of the photocell varies as follows:



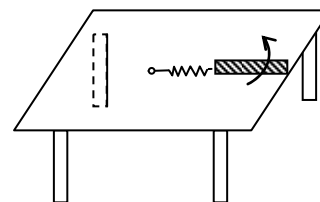
68. Two coherent point sources S_1 and S_2 are separated by a small distance 'd' as shown. The fringes obtained on the screen will be:

- (1) straight lines (2) semi-circles
 (3) concentric circles (4) points



69. A metallic rod of length ' ℓ ' is tied to a string of length 2ℓ and made to rotate with angular speed ω on a horizontal table with one end of the string fixed. If there is a vertical magnetic field 'B' in the region, the e.m.f. induced across the ends of the rod is:

- (1) $\frac{3B\omega\ell^2}{2}$ (2) $\frac{4B\omega\ell^2}{2}$
 (3) $\frac{5B\omega\ell^2}{2}$ (4) $\frac{2B\omega\ell^2}{2}$



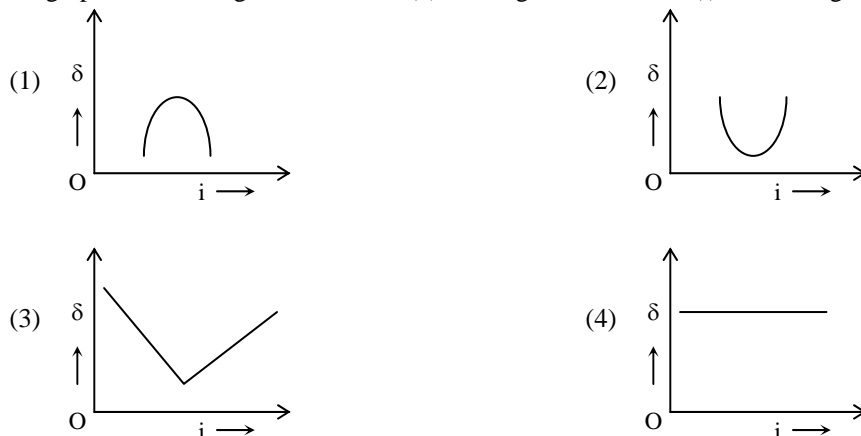
70. In a hydrogen like atom electron makes transition from an energy level with quantum number n to another with quantum number $(n - 1)$. If $n \gg 1$, the frequency of radiation emitted is proportional to

- (1) $\frac{1}{n^2}$ (2) $\frac{1}{n^{3/2}}$
 (3) $\frac{1}{n^3}$ (4) $\frac{1}{n}$

71. Assume that a drop of liquid evaporates by decrease in its surface energy, so that its temperature remains unchanged. What should be the minimum radius of the drop for this to be possible? The surface tension is T , density of liquid is ρ and L is its latent heat of vaporization.

- (1) $\sqrt{T/\rho L}$ (2) $T/\rho L$
 (3) $2T/\rho L$ (4) $\rho L/T$

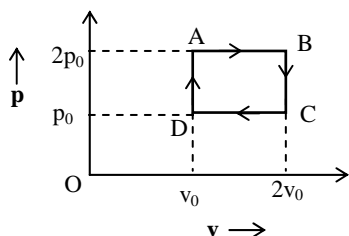
72. The graph between angle of deviation (δ) and angle of incidence (i) for a triangular prism is represented by:



73. Let $[\epsilon_0]$ denote the dimensional formula of the permittivity of vacuum. If $M =$ mass, $L =$ length, $T =$ time and $A =$ electric current, then:

- (1) $[\epsilon_0] = [M^{-1}L^{-3}T^4A^2]$ (2) $[\epsilon_0] = [M^{-1}L^2T^{-1}A^{-2}]$
 (3) $[\epsilon_0] = [M^{-1}L^2T^{-1}A]$ (4) $[\epsilon_0] = [M^{-1}L^{-3}T^2A]$

74.



The above p - v diagram represents the thermodynamic cycle of an engine, operating with an ideal monoatomic gas. The amount of heat extracted from the source in a single cycle is

- (1) $\left(\frac{13}{2}\right)p_0v_0$ (2) $\left(\frac{11}{2}\right)p_0v_0$
 (3) $4p_0v_0$ (4) p_0v_0

75. A sonometer wire of length 1.5 m is made of steel. The tension in it produces an elastic strain of 1 %. What is the fundamental frequency of steel if density and elasticity of steel are $7.7 \times 10^3 \text{ kg/m}^3$ and $2.2 \times 10^{11} \text{ N/m}^2$ respectively?

- (1) 178.2 Hz (2) 200.5 Hz
 (3) 770 Hz (4) 188.5 Hz

76. This question has statement I and statement II. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement- I: Higher the range, greater is the resistance of ammeter.

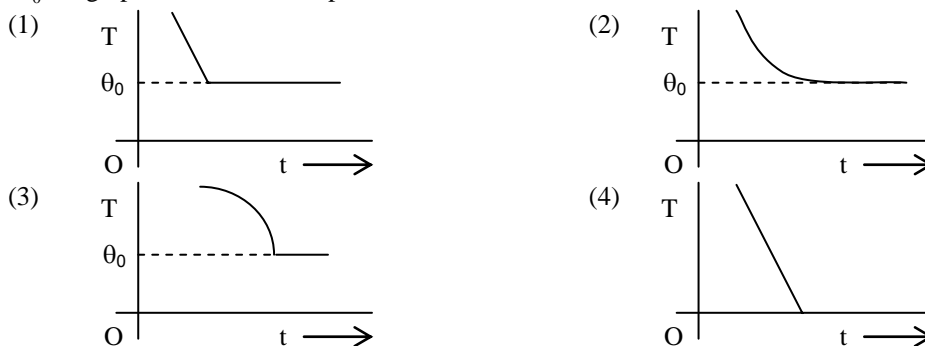
Statement- II: To increase the range of ammeter, additional shunt needs to be used across it.

- (1) Statement – I is true, Statement – II is true, Statement – II is **not** the correct explanation of Statement–I.
 (2) Statement – I is true, statement – II is false.
 (3) Statement – I is false, Statement – II is true
 (4) Statement – I is true, Statement – II is true, Statement – II is the correct explanation of statement- I .

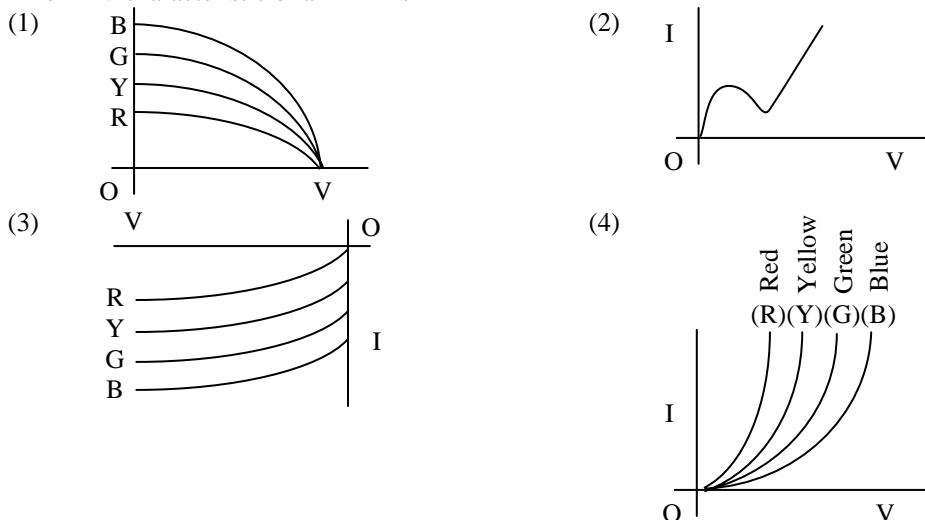
77. What is the minimum energy required to launch a satellite of mass m from the surface of a planet of mass M and radius R in a circular orbit at an altitude of $2R$?
- (1) $\frac{2GmM}{3R}$ (2) $\frac{GmM}{2R}$
 (3) $\frac{GmM}{3R}$ (4) $\frac{5GmM}{6R}$
78. A projectile is given an initial velocity of $(\hat{i} + 2\hat{j})$ m/s, where \hat{i} is along the ground and \hat{j} is along the vertical. If $g = 10 \text{ m/s}^2$, the equation of its trajectory is:
- (1) $y = 2x - 5x^2$ (2) $4y = 2x - 5x^2$
 (3) $4y = 2x - 25x^2$ (4) $y = x - 5x^2$
79. Two capacitors C_1 and C_2 are charged to 120 V and 200 V respectively. It is found that by connecting them together the potential on each one can be made zero. Then :
- (1) $3C_1 = 5C_2$ (2) $3C_1 + 5C_2 = 0$
 (3) $9C_1 = 4C_2$ (4) $5C_1 = 3C_2$
80. A hoop of radius r and mass m rotating with an angular velocity ω_0 is placed on a rough horizontal surface. The initial velocity of the centre of the hoop is zero. What will be the velocity of the centre of the hoop when it ceases to slip?
- (1) $\frac{r\omega_0}{3}$ (2) $\frac{r\omega_0}{2}$
 (3) $r\omega_0$ (4) $\frac{r\omega_0}{4}$
81. An ideal gas enclosed in a vertical cylindrical container supports a freely moving piston of mass M . The piston and cylinder have equal cross sectional area A . When the piston is in equilibrium, the volume of the gas is V_0 and its pressure is P_0 . The piston is slightly displaced from the equilibrium position and released. Assuming that the system is completely isolated from its surrounding, the piston executes a simple harmonic motion with frequency:
- (1) $\frac{1}{2\pi} \frac{V_0 M P_0}{A^2 \gamma}$ (2) $\frac{1}{2\pi} \sqrt{\frac{A^2 \gamma P_0}{M V_0}}$
 (3) $\frac{1}{2\pi} \sqrt{\frac{M V_0}{A \gamma P_0}}$ (4) $\frac{1}{2\pi} \frac{A \gamma P_0}{V_0 M}$
82. A charge Q is uniformly distributed over a long rod AB of length L as shown in the figure. The electric potential at the point O lying at a distance L from the end A is:
- O A B
 ←—— L ———→ ←—— L ———→
- (1) $\frac{3Q}{4\pi\epsilon_0 L}$ (2) $\frac{Q}{4\pi\epsilon_0 L \ln 2}$
 (3) $\frac{Q \ln 2}{4\pi\epsilon_0 L}$ (4) $\frac{Q}{8\pi\epsilon_0 L}$

83. A circular loop of radius 0.3 cm lies parallel to a much bigger circular loop of radius 20 cm. The centre of the small loop is on the axis of the bigger loop. The distance between their centres is 15 cm. If a current of 2.0 A flows through the smaller loop, then the flux linked with bigger loop is
 (1) 6×10^{-11} weber (2) 3.3×10^{-11} weber
 (3) 6.6×10^{-9} weber (4) 9.1×10^{-11} weber

84. If a piece of metal is heated to temperature θ and then allowed to cool in a room which is at temperature θ_0 the graph between the temperature T of the metal and time t will be closest to :



85. The I – V characteristic of an LED is



86. This question has Statement I and Statement II. Of the four choices given after the Statements, choose the one that best describes the two Statements.

Statement – I : A point particle of mass m moving with speed v collides with stationary point particle of mass M . If the maximum energy loss possible is given as $f\left(\frac{1}{2}mv^2\right)$ then $f = \left(\frac{m}{M+m}\right)$.

Statement – II : Maximum energy loss occurs when the particles get stuck together as a result of the collision.

- (1) Statement – I is true, Statement – II is true, Statement – II is not a correct explanation of Statement – I.
 (2) Statement – I is true, Statement – II is false.
 (3) Statement – I is false, Statement – II is true
 (4) Statement – I is true, Statement – II is true, Statement – II is a correct explanation of Statement – I.

87. The amplitude of a damped oscillator decreases to 0.9 times its original magnitude in 5s. In another 10s it will decrease to α times its original magnitude, where α equals.

- (1) 0.81 (2) 0.729
 (3) 0.6 (4) 0.7

88. Diameter of plano-convex lens is 6 cm and thickness at the centre is 3 mm. If speed of light in material of lens is 2×10^8 m/s, the focal length of the lens is :
- (1) 20 cm (2) 30 cm
(3) 10 cm (4) 15 cm
89. The magnetic field in a travelling electromagnetic wave has a peak value of 20 nT. The peak value of electric field strength is :
- (1) 6 V/m (2) 9 V/m
(3) 12 V/m (4) 3 V/m
90. Two short bar magnets of length 1 cm each have magnetic moments 1.20 Am^2 and 1.00 Am^2 respectively. They are placed on a horizontal table parallel to each other with their N poles pointing towards the South. They have a common magnetic equator and are separated by a distance of 20.0 cm. The value of the resultant horizontal magnetic induction at the mid - point O of the line joining their centres is close to (Horizontal component of earth's magnetic induction is $3.6 \times 10^{-5} \text{ Wb/m}^2$)
- (1) $2.56 \times 10^{-4} \text{ Wb/m}^2$ (2) $3.50 \times 10^{-4} \text{ Wb/m}^2$
(3) $5.80 \times 10^{-4} \text{ Wb/m}^2$ (4) $3.6 \times 10^{-5} \text{ Wb/m}^2$

[Code – Q]
JEE(MAIN) 2013
ANSWERS

PART - A: CHEMISTRY

1.	1	2.	1	3.	1	4.	4
5.	Incorrect question			6.	3	7.	2
8.	1	9.	3	10.	1 or 4	11.	2 or 4
12.	1	13.	1	14.	3	15.	2
16.	4	17.	4	18.	2	19.	4
20.	3	21.	3	22.	2	23.	2
24.	4	25.	2	26.	1	27.	3
28.	4	29.	3	30.	2		

PART - B: MATHEMATICS

31.	2	32.	4	33.	1	34.	1
35.	3	36.	4	37.	2	38.	4
39.	1	40.	4	41.	2	42.	1
43.	4	44.	1	45.	3	46.	3
47.	1	48.	2	49.	3	50.	1
51.	1	52.	4	53.	2	54.	2
55.	2	56.	2	57.	2	58.	4
59.	2	60.	2				

PART - C: PHYSICS

61.	2	62.	3	63.	3	64.	2
65.	4	66.	2	67.	3	68.	3
69.	3	70.	3	71.	3	72.	2
73.	1	74.	2	75.	1	76.	3
77.	4	78.	1	79.	1	80.	2
81.	2	82.	3	83.	4	84.	2
85.	4	86.	3	87.	2	88.	2
89.	1	90.	1				

HINTS AND SOLUTIONS

PART - A: CHEMISTRY

1. Reaction proceeds through carbocation formation as 3° carbocation is highly stable, hence reaction proceeds through S_N1 with 3° alcohol.

2. $\text{Na} \xrightleftharpoons[\Delta H = -5.1\text{eV}]{\Delta H = +5.1\text{eV}} \text{Na}^+ + e^-$, here the backward reaction releases same amount of energy and known as Electron gain enthalpy.


3. $\text{Li}_2(6) = \sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2$
 B.O. = $\frac{4-2}{2} = 1$

$\text{Li}_2^+(5) = \sigma 1s^2 \sigma^* 1s^2 \sigma 2s^1$
 B. O. = $\frac{3-2}{2} = 0.5$

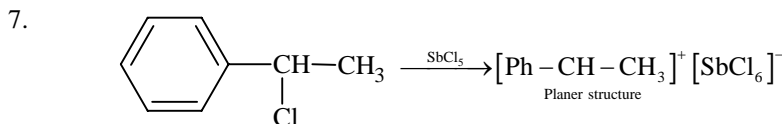
$\text{Li}_2^-(7) = \sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^1$
 B.O. = $\frac{4-3}{2} = 0.5$

Li_2^+ is more stable than Li_2^- because Li_2^- has more numbers of antibonding electrons.

4. $M_1V_1 + M_2V_2 = MV$
 $M = \frac{M_1V_1 + M_2V_2}{V} = \frac{0.5 \times 750 + 2 \times 250}{1000}$
 $M = 0.875$

5. (1) Correct, as  is bent.
 (2) Correct, as ozone is violet-black solid.
 (3) Correct, as ozone is diamagnetic.
 (4) Correct, as $\text{ONCl} = 32$ electrons and $\text{ONO}^- = 24$ electron hence are not isoelectronic.
 All options are correct statements.

6. $E_{\text{Mn}^{+3}/\text{Mn}^{+2}}^0 = 1.57 \text{ V}$
 $E_{\text{Fe}^{+3}/\text{Fe}^{+2}}^0 = 0.77 \text{ V}$
 $E_{\text{Co}^{+3}/\text{Co}^{+2}}^0 = 1.97 \text{ V}$
 $E_{\text{Cr}^{+3}/\text{Cr}^{+2}}^0 = -0.41 \text{ V}$



8. As_2S_3 is an anionic sol (negative sol) hence coagulation will depend upon coagulating power of cation, which is directly proportional to the valency of cation (Hardy-Schulze rule).

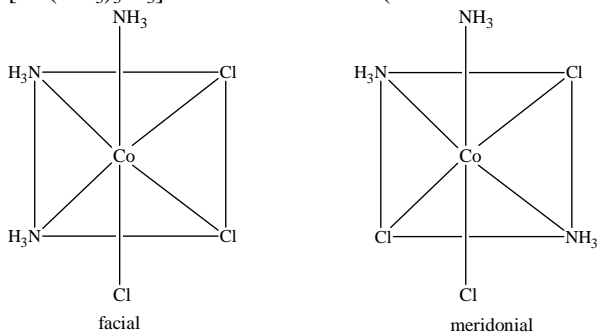
9. Initial pH = 1, i.e. $[H^+] = 0.1$ mole/litre
 New pH = 2, i.e. $[H^+] = 0.01$ mole/litre
 In case of dilution: $M_1V_1 = M_2V_2$
 $0.1 \times 1 = 0.01 \times V_2$
 $V_2 = 10$ litre.
 Volume of water added = 9 litre.
10. $N_2 \rightarrow$ Diamagnetic
 $O_2 \rightarrow$ Paramagnetic
 $S_2 \rightarrow$ Paramagnetic
 $C_2 \rightarrow$ Diamagnetic
11. The exothermic hydration enthalpies of the given trivalent cations are:
 $Sc^{+3} = 3960$ kJ/mole
 $Fe^{+3} = 4429$ kJ/mole
 $Co^{+3} = 4653$ kJ/mole
 $Cr^{+3} = 4563$ kJ/mole
 Hence Sc^{+3} is least hydrated; so least stable (not most stable)
 Fe^{+2} contains 4 unpaired electrons where as Mn^{+2} contains 5 unpaired electrons hence (4) is incorrect.
12. Metal oxide = $M_{0.98}O$
 If 'x' ions of M are in +3 state, then
 $3x + (0.98 - x) \times 2 = 2$
 $x = 0.04$
 So the percentage of metal in +3 state would be $\frac{0.04}{0.98} \times 100 = 4.08\%$
13. $R - NH_2 + CH_3 - \overset{\overset{O}{\parallel}}{C} - Cl \longrightarrow R - NH - \overset{\overset{O}{\parallel}}{C} - CH_3 + HCl$
 Each $CH_3 - \overset{\overset{O}{\parallel}}{C}$ addition increases the molecular wt. by 42.
 Total increase in m.wt. = $390 - 180 = 210$
 Then number of NH_2 groups = $\frac{210}{42} = 5$
14. As per data mentioned
 MnO_4^- is strongest oxidising agent as it has maximum SRP value.
15. Correct order of acidic strength is III > I > II > IV
16. As per Arrhenius equation:

$$\ln \frac{k_2}{k_1} = -\frac{E_a}{R} \left(\frac{1}{T_2} - \frac{1}{T_1} \right)$$

$$2.303 \log 2 = -\frac{E_a}{8.314} \left(\frac{1}{310} - \frac{1}{300} \right)$$

$$\Rightarrow E_a = 53.6 \text{ kJ/mole}$$
17. $12H_2O + 12NADP + 18ADP \xrightarrow{\text{Light reaction}} 6O_2 + 18ATP + 12NADPH$
 $6CO_2 + 12NADPH + 18ATP \xrightarrow{\text{Dark reaction}} C_6H_{12}O_6 + 12NADP + 18ADP + 6H_2O$
 Net reaction: $6CO_2 + 6H_2O \longrightarrow C_6H_{12}O_6 + 6O_2$

18. $[\text{Co}(\text{NH}_3)_3\text{Cl}_3]$ exists in two forms (facial and meridional)



Both of these forms are achiral. Hence, $[\text{Co}(\text{NH}_3)_3\text{Cl}_3]$ does not show optical isomerism.

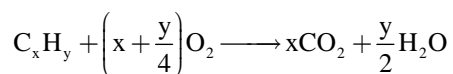
19. Process is isothermal reversible expansion, hence $\Delta U = 0$.

$$\therefore q = -W$$

$$\text{As } q = +208 \text{ J}$$

$$\text{Hence } W = -208 \text{ J}$$

- 20.



$$\text{Weight (g)} \quad \quad \quad 3.08 \text{ g} \quad 0.72 \text{ g}$$

$$\text{moles} \quad \quad \quad 0.07 \quad 0.04$$

$$\frac{x}{y/2} = \frac{0.07}{0.04}$$

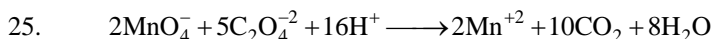
$$\Rightarrow \frac{x}{y} = \frac{7}{8}$$

21. Order of stability is $\text{III} > \text{I} > \text{II}$.
(Stability \propto extent of delocalization)

22. Increasing order of first ionization enthalpy is
 $\text{Ba} < \text{Ca} < \text{Se} < \text{S} < \text{Ar}$

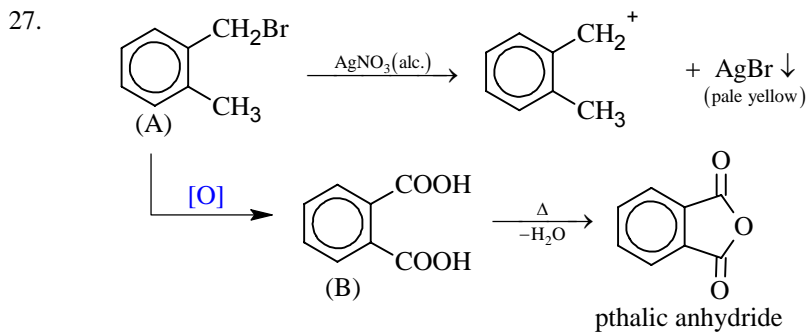
$$23. \quad C^* = \sqrt{\frac{2RT}{M}}, \quad \bar{C} = \sqrt{\frac{8RT}{\pi M}}, \quad C = \sqrt{\frac{3RT}{M}}$$

24. It was methyl isocyanate (CH_3NCO)



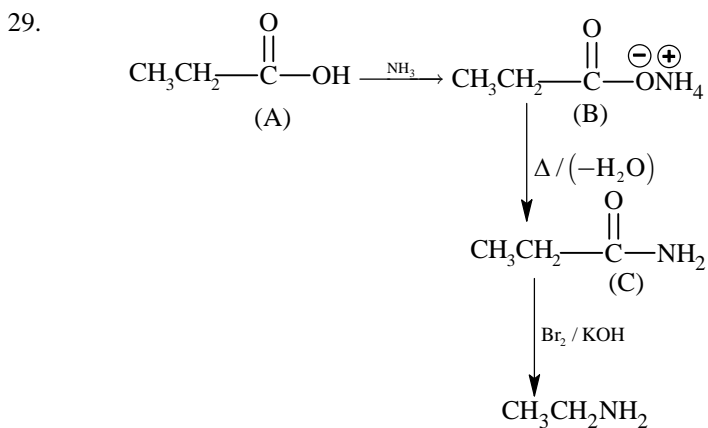
$$x = 2, y = 5, z = 16$$

26. Silicon (Si) – covalent solid
Sulphur (S_8) – molecular solid
Phosphorous (P_4) – Molecular solid
Iodine (I_2) – Molecular solid



28.
$$E = \frac{hc}{\lambda} = 2.178 \times 10^{-18} \times Z^2 \left[\frac{1}{1^2} - \frac{1}{2^2} \right]$$

$\Rightarrow \lambda = 1.214 \times 10^{-7} \text{ m}$



30. Bond order of H_2^+ and He_2 is zero, thus their existence is not possible.

PART - B: MATHEMATICS

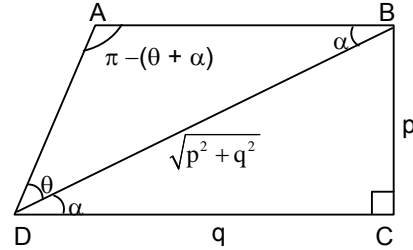
31. $(x - 3)^2 + y^2 + \lambda y = 0$
 The circle passes through $(1, -2)$
 $\Rightarrow 4 + 4 - 2\lambda = 0 \Rightarrow \lambda = 4$
 $(x - 3)^2 + y^2 + 4y = 0 \Rightarrow$ Clearly $(5, -2)$ satisfies.

32. Using sine rule in triangle ABD

$$\frac{AB}{\sin \theta} = \frac{BD}{\sin(\theta + \alpha)}$$

$$\Rightarrow AB = \frac{\sqrt{p^2 + q^2} \sin \theta}{\sin \theta \cos \alpha + \cos \theta \sin \alpha} = \frac{\sqrt{p^2 + q^2} \sin \theta}{\frac{\sin \theta \cdot q}{\sqrt{p^2 + q^2}} + \frac{\cos \theta \cdot p}{\sqrt{p^2 + q^2}}}$$

$$\Rightarrow AB = \frac{(p^2 + q^2) \sin \theta}{(p \cos \theta + q \sin \theta)}$$



33. Let the tangent to the parabola be $y = mx + \frac{\sqrt{5}}{m}$ ($m \neq 0$).

Now, its distance from the centre of the circle must be equal to the radius of the circle.

$$\text{So, } \left| \frac{\sqrt{5}}{m} \right| = \frac{\sqrt{5}}{\sqrt{2}} \sqrt{1 + m^2} \Rightarrow (1 + m^2) m^2 = 2 \Rightarrow m^4 + m^2 - 2 = 0.$$

$$\Rightarrow (m^2 - 1)(m^2 + 2) = 0 \Rightarrow m = \pm 1$$

So, the common tangents are $y = x + \sqrt{5}$ and $y = -x - \sqrt{5}$.

34. Slope of the incident ray is $-\frac{1}{\sqrt{3}}$.

So, the slope of the reflected ray must be $\frac{1}{\sqrt{3}}$.

The point of incidence is $(\sqrt{3}, 0)$. So, the equation of reflected ray is $y = \frac{1}{\sqrt{3}}(x - \sqrt{3})$.

35. Variance is not changed by the change of origin.

Alternate Solution:

$$\sigma = \sqrt{\frac{\sum |x - \bar{x}|^2}{n}} \text{ for } y = x + 10 \Rightarrow \bar{y} = \bar{x} + 10$$

$$\sigma_1 = \sqrt{\frac{\sum |y + 10 - \bar{y} - 10|^2}{n}} = \sqrt{\frac{\sum |y - \bar{y}|^2}{n}} = \sigma.$$

36. If x, y, z are in A.P.

$$2y = x + z$$

and $\tan^{-1}x, \tan^{-1}y, \tan^{-1}z$ are in A.P.

$$2 \tan^{-1}y = \tan^{-1}x + \tan^{-1}z \Rightarrow x = y = z.$$

Note: If $y = 0$, then none of the options is appropriate.

37. $\int f(x) dx = \psi(x)$

Let $x^3 = t$
 $3x^2 dx = dt$

then $\int x^5 f(x^3) dx = \frac{1}{3} \int t f(t) dt$

$= \frac{1}{3} \left[t \int f(t) dt - \int \{1 \cdot \int f(t) dt\} dt \right] = \frac{1}{3} x^3 \psi(x^3) - \int x^2 \psi(x^3) dx + C.$

38. foci $\equiv (\pm ae, 0)$

We have $a^2 e^2 = a^2 - b^2 = 7$

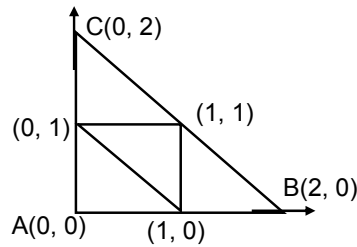
Equation of circle $(x - 0)^2 + (y - 3)^2 = (\sqrt{7} - 0)^2 + (0 - 3)^2$

$\Rightarrow x^2 + y^2 - 6y - 7 = 0.$

39. x-coordinate = $\frac{ax_1 + bx_2 + cx_3}{a + b + c}$

$= \frac{2 \times 2 + 2\sqrt{2} \times 0 + 2 \times 0}{2 + 2 + 2\sqrt{2}}$

$= \frac{4}{4 + 2\sqrt{2}} = \frac{2}{2 + \sqrt{2}} = 2 - \sqrt{2}.$



Alternate Solution:

x-coordinate = $r = (s - a) \tan A/2$

$= \left(\frac{4 + 2\sqrt{2}}{2} - 2\sqrt{2} \right) \tan \frac{\pi}{4} = 2 - \sqrt{2}.$

40. $\frac{dy}{dx} = |x| = 2 \Rightarrow x = \pm 2 \Rightarrow y = \int_0^2 |t| dt = 2$ for $x = 2$

and $y = \int_0^{-2} |t| dt = -2$ for $x = -2$

\therefore tangents are $y - 2 = 2(x - 2) \Rightarrow y = 2x - 2$

and $y + 2 = 2(x + 2) \Rightarrow y = 2x + 2$

Putting $y = 0$, we get $x = 1$ and -1 .

41. $t_r = 0.777 \dots r$ times
 $= 7(10^{-1} + 10^{-2} + 10^{-3} + \dots + 10^{-r})$

$= \frac{7}{9}(1 - 10^{-r})$

$S_{20} = \sum_{r=1}^{20} t_r = \frac{7}{9} \left(20 - \sum_{r=1}^{20} 10^{-r} \right) = \frac{7}{9} \left(20 - \frac{1}{9}(1 - 10^{-20}) \right) = \frac{7}{81}(179 + 10^{-20})$

42. S1:

p	q	$\sim p$	$\sim q$	$p \wedge \sim q$	$\sim p \wedge q$	$(p \wedge \sim q) \wedge (\sim p \wedge q)$
T	T	F	F	F	F	F
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	F	T	T	F	F	F

Fallacy

S2:

p	q	~ p	~ q	p ⇒ q	~ q ⇒ ~ p	(p ⇒ q) ⇔ (~ q ⇒ ~ p)
T	T	F	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

Tautology

S2 is not an explanation of S1

43.

$$2\sqrt{x} = x - 3$$

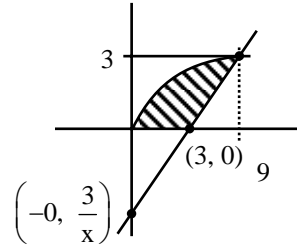
$$4x = x^2 - 6x + 9$$

$$x^2 - 10x + 9$$

$$x = 9, x = 1$$

$$\int_0^3 (2y + 3) - y^2 dy$$

$$\left[y^2 + 3y - \frac{y^3}{3} \right]_0^3 = 9 + 9 - 9 = 9$$



44.

$$\frac{1}{\cot A(1 - \cot A)} - \frac{\cot^2 A}{(1 - \cot A)} = \frac{1 - \cot^3 A}{\cot A(1 - \cot A)} = \frac{\cos^3 A + \cot A}{\cot A} = 1 + \sec A \operatorname{cosec} A$$

45.

If $2x^3 + 3x + k = 0$ has 2 distinct real roots in $[0, 1]$, then $f'(x)$ will change sign

$$\text{but } f'(x) = 6x^2 + 3 > 0$$

So no value of k exists.

46.

$$\lim_{x \rightarrow 0} \frac{(1 - \cos 2x)}{x(\tan 4x)} (3 + \cos x)$$

$$\lim_{x \rightarrow 0} 2 \left(\frac{\sin x}{x} \right)^2 \cdot \frac{1}{4} \left(\frac{4x}{\tan 4x} \right) (3 + \cos x) = 2 \times 1 \times \frac{1}{4} \times 1 \times (3 + 1) = 2.$$

47.

$${}^{n+1}C_3 - {}^nC_3 = 10 \Rightarrow {}^nC_2 = 10 \Rightarrow n = 5.$$

48.

$$\int_{2000}^P dP = \int_0^{25} (100 - 12\sqrt{x}) dx$$

$$(P - 2000) = 25 \times 100 - \frac{12 \times 2}{3} (25)^{3/2}$$

$$P = 3500.$$

49.

$$I = \int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}}$$

$$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\tan x}}{1 + \sqrt{\tan x}} dx$$

$$2I = \frac{\pi}{6}$$

$$I = \frac{\pi}{12}.$$

50.
$$P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$$

$|\text{Adj } A| = |A|^2$

$|\text{Adj } A| = 16$

$1(12 - 12) - \alpha(4 - 6) + 3(4 - 6) = 16.$

$2\alpha - 6 = 16.$

$2\alpha = 22.$

$\alpha = 11.$

51. For no solution

$$\frac{k+1}{k} = \frac{8}{k+3} \neq \frac{4k}{3k-1} \quad \dots (1)$$

$\Rightarrow (k+1)(k+3) - 8k = 0$

or $k^2 - 4k + 3 = 0 \Rightarrow k = 1, 3$

But for $k = 1$, equation (1) is not satisfied

Hence $k = 3.$

52. $y = \sec(\tan^{-1}x)$

$$\frac{dy}{dx} = \sec(\tan^{-1}x) \tan(\tan^{-1}x) \cdot \frac{1}{1+x^2}$$

$$\left. \frac{dy}{dx} \right|_{x=1} = \sqrt{2} \times 1 \times \frac{1}{2} = \frac{1}{\sqrt{2}}.$$

53.
$$\begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$$

$1(1 + 2k) + 1(1 + k^2) - 1(2 - k) = 0$

$k^2 + 1 + 2k + 1 - 2 + k = 0$

$k^2 + 3k = 0$

$(k)(k + 3) = 0$

2 values of $k.$

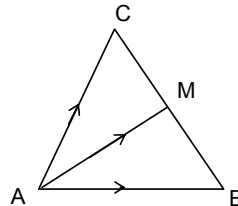
54. $A \times B$ will have 8 elements.

$2^8 - {}^8C_0 - {}^8C_1 - {}^8C_2 = 256 - 1 - 8 - 28 = 219.$

55.
$$\overline{AM} = \frac{\overline{AB} + \overline{AC}}{2}$$

$\overline{AM} = 4\hat{i} - \hat{j} + 4\hat{k}$

$|\overline{AM}| = \sqrt{16 + 16 + 16} = \sqrt{33}$



56. P (correct answer) = $1/3$

$${}^5C_4 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)^1 + {}^5C_5 \left(\frac{1}{3}\right)^5$$

$$\frac{5 \times 2}{(3)^5} + \frac{1}{(3)^5} = \frac{11}{3^5}.$$

57. $|z| = 1 \Rightarrow z\bar{z} = 1$
 $\frac{1+z}{1+\bar{z}} = \frac{1+z}{1+\frac{1}{z}} = z.$

58. For equation $x^2 + 2x + 3 = 0$
 both roots are imaginary.
 Since $a, b, c \in \mathbb{R}$.
 If one root is common then both roots are common
 Hence, $\frac{a}{1} = \frac{b}{2} = \frac{c}{3}$
 $a : b : c = 1 : 2 : 3.$

59. $4x + 2y + 4z = 16$
 $4x + 2y + 4z = -5$
 $d_{\min} = \frac{21}{\sqrt{36}} = \frac{21}{6} = \frac{7}{2}.$

60. $\left(\frac{(x^{1/3} + 1)(x^{2/3} - x^{1/3} + 1)}{x^{2/3} - x^{1/3} + 1} - \frac{1}{\sqrt{x}} \cdot \frac{(\sqrt{x} + 1)(\sqrt{x} - 1)}{(\sqrt{x} - 1)} \right)^{10} = (x^{1/3} - x^{-1/2})^{10}$
 $T_{r+1} = (-1)^r {}^{10}C_r x^{\frac{20-5r}{6}} \Rightarrow r = 4$
 ${}^{10}C_4 = 210.$

PART - C: PHYSICS

61. Charge on the capacitor at any time 't' is

$$q = CV(1 - e^{-t/\tau})$$

at $t = 2\tau$

$$q = CV(1 - e^{-2})$$

62. $f_c = \frac{1}{2\pi RC} = \frac{1}{2 \times 3.14 \times 100 \times 10^3 \times 250 \times 10^{-12}} = 6.37 \text{ kHz}$

f_c = cut off frequency

As we know that $f_m \ll f_c$

\therefore (3) is correct

Note: The maximum frequency of modulation must be less than f_m , where

$$f_m = f_c \frac{\sqrt{1 - m^2}}{m}$$

$m \Rightarrow$ modulation index

63. Resistance of bulb = $\frac{120 \times 120}{60} = 240\Omega$

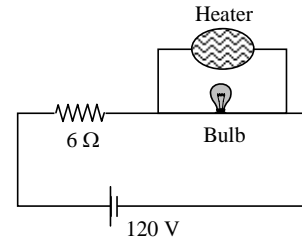
Resistance of Heater = $\frac{120 \times 120}{240} = 60\Omega$

Voltage across bulb before heater is switched on, $V_1 = \frac{120}{246} \times 240$

Voltage across bulb after heater is switched on, $V_2 = \frac{120}{54} \times 48$

Decrease in the voltage is $V_1 - V_2 = 10.04$ (approximately)

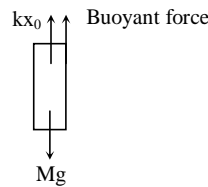
Note: Here supply voltage is taken as rated voltage.



64. At equilibrium $\Sigma F = 0$

$$kx_0 + \left(\frac{AL}{2} \sigma g\right) - Mg = 0$$

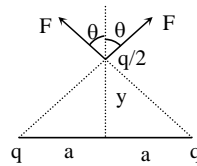
$$x_0 = Mg \left[1 - \frac{LA\sigma}{2M}\right]$$



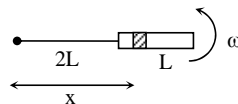
65. $F_{net} = 2F \cos \theta$

$$= 2 \frac{k \cdot q \cdot q / 2}{(\sqrt{a^2 + y^2})^2} \cdot \frac{y}{\sqrt{a^2 + y^2}}$$

$$= \frac{kq^2 y}{a^3} \quad (y \ll a)$$



69. $de = B(\omega x) \cdot dx$



$$e = B\omega \int_{2L}^{3L} x dx$$

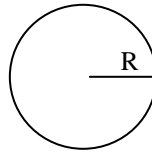
$$= \frac{5B\omega L^2}{2}$$

70. $v \propto \left[\frac{1}{(n-1)^2} - \frac{1}{n^2} \right]$

$$\propto \frac{(2n-1)}{n^2(n-1)^2}$$

$$\propto \frac{1}{n^3} \quad (\text{since } n \gg 1)$$

71. $\rho 4\pi R^2 \Delta R L = T 4\pi [R^2 - (R - \Delta R)^2]$
 $\rho R^2 \Delta R L = T [R^2 - R^2 + 2R\Delta R - \Delta R^2]$
 $\rho R^2 \Delta R L = T 2R\Delta R \quad (\Delta R \text{ is very small})$
 $R = \frac{2T}{\rho L}$



73. $\frac{1}{4\pi\epsilon_0} \frac{q^2}{r^2} = F$

$$\epsilon_0 = \frac{[A^2 T^2]}{[M L T^{-2} L^2]} = [M^{-1} L^{-3} A^2 T^4]$$

74. Heat is extracted from the source in path DA and AB is

$$\Delta Q = \frac{3}{2} R \left(\frac{P_0 V_0}{R} \right) + \frac{5}{2} R \left(\frac{2P_0 V_0}{R} \right) = \frac{13}{2} P_0 V_0$$

75. Fundamental frequency $f = \frac{1}{2\ell} \sqrt{\frac{T}{\mu}}$

$$= \frac{1}{2\ell} \sqrt{\frac{T}{A\rho}}$$

$$= \frac{1}{2\ell} \sqrt{\frac{\text{stress}}{\rho}} = \frac{1}{2 \times 1.5} \sqrt{\frac{2.2 \times 10^{11} \times 10^{-2}}{7.7 \times 10^3}}$$

76. For Ammeter, $S = \frac{I_g G}{I - I_g}$

So for I to increase, S should decrease, so additional S can be connected across it.

77. $T.E_f = -\frac{GMm}{6R}$

$$T.E_i = -\frac{GMm}{R}$$

$$\Delta W = T.E_f - T.E_i = \frac{5GMm}{6R}$$

78. $x = t$
 $y = 2t - 5t^2$

Equation of trajectory is $y = 2x - 5x^2$

79. $120C_1 = 200C_2$
 $6C_1 = 10C_2$
 $3C_1 = 5C_2$

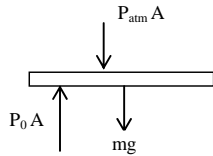
80. From conservation of angular momentum about any fix point on the surface

$$mr^2\omega_0 = 2mr^2\omega$$

$$\therefore \omega = \frac{\omega_0}{2}$$

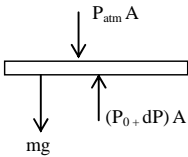
$$\therefore V_{CM} = \frac{\omega_0 r}{2}$$

81.



FBD of piston at equilibrium

$$\Rightarrow P_{atm} A + mg = P_0 A \quad \dots(1)$$



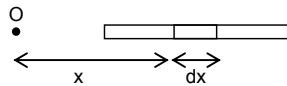
FBD of piston when piston is pushed down a distance x

$$P_{atm} + mg - (P_0 + dP) A = m \frac{d^2 x}{dt^2} \quad \dots(2)$$

Process is adiabatic $\Rightarrow PV^\gamma = C \Rightarrow -dP = \frac{\gamma P dV}{V}$

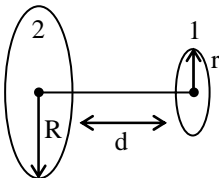
Using 1, 2, 3 we get $f = \frac{1}{2\pi} \sqrt{\frac{A^2 \gamma P_0}{MV_0}}$

82.



$$V = \int_{x=L}^{x=2L} \frac{k}{x} \left(\frac{Q}{L} \right) dx = \frac{Q \ln 2}{4\pi\epsilon_0 L}$$

83.



Let M_{12} be the coefficient of mutual induction between loops

$$\phi_1 = M_{12} i_2$$

$$\Rightarrow \frac{\mu_0 i_2 R^2}{2(d^2 + R^2)^{3/2}} \pi r^2 = M_{12} i_2$$

$$\Rightarrow M_{12} = \frac{\mu_0 R^2 \pi r^2}{2(d^2 + R^2)^{3/2}}$$

$$\phi_2 = M_{12} i_1 \Rightarrow \phi_2 = 9.1 \times 10^{-11} \text{ weber}$$

84. The temperature goes on decreasing with time (non-linearly) The rate of decrease will be more initially which is depicted in the second graph.

85. For LED, in forward bias, intensity increases with voltage.

86. Loss of energy is maximum when collision is inelastic as in an inelastic collision there will be maximum deformation.

$$\text{KE in COM frame is } \frac{1}{2} \left(\frac{Mm}{M+m} \right) V_{rel}^2$$

$$\text{KE}_i = \frac{1}{2} \left(\frac{Mm}{M+m} \right) V^2 \quad \text{KE}_f = 0 \quad (\because V_{rel} = 0)$$

$$\text{Hence loss in energy is } \frac{1}{2} \left(\frac{Mm}{M+m} \right) V^2$$

$$\Rightarrow f = \frac{M}{M+m}$$

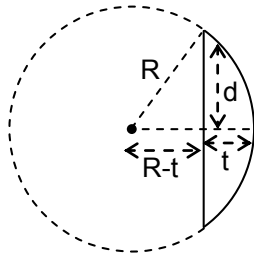
87. $A = A_0 e^{-kt}$

$$\Rightarrow 0.9A_0 = A_0 e^{-5k}$$

$$\text{and } \alpha A_0 = A_0 e^{-15k}$$

$$\text{solving } \Rightarrow \alpha = 0.729$$

88.



$$R^2 = d^2 + (R-t)^2$$

$$R^2 - d^2 = R^2 \left\{ 1 - \frac{t}{R} \right\}^2$$

$$1 - \frac{d^2}{R^2} = 1 - \frac{2t}{R}$$

$$R = \frac{(3)^2}{2 \times (0.3)} = \frac{90}{6} = 15 \text{ cm}$$

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\frac{1}{f} = \left(\frac{3}{2} - 1 \right) \left(\frac{1}{15} \right)$$

$$f = 30 \text{ cm}$$

89. $E_0 = CB_0$
 $= 3 \times 10^8 \times 20 \times 10^{-9}$
 $= 6 \text{ V/m}$

90. $B_{net} = B_{M_1} + B_{M_2} + B_H$
 $= \frac{\mu_0 M_1}{4\pi x^3} + \frac{\mu_0 M_2}{4\pi x^3} + B_H$
 $= \frac{\mu_0}{4\pi x^3} (M_1 + M_2) + B_H$
 $= \frac{10^{-7}}{10^{-3}} \times 2.2 + 3.6 \times 10^{-5}$
 $= 2.56 \times 10^{-4} \text{ Wb/m}^2$
