FT-III-KVPY-CLASS-XII
FULL TEST – III

PART – I
MATHEMATICS

1. The lines $lx+my+n=0, mx+ny+l=0$ and $nx+ly+m=0$ (l, m, n are not all equal) are concurrent if:
(A) $l^2+m^2+n^2=1$ (B) $lm+mn+nl=1$
(C) $lm+mn+nl=0$ (D) $l+m+n=0$

Ans. D

Sol. The given lines are concurrent, if $\begin{vmatrix} l & m & n \\ m & n & l \\ n & l & m \end{vmatrix}=0 \Rightarrow \begin{vmatrix} l+m+n & m & n \\ l+m+n & n & l \\ l+m+n & l & m \end{vmatrix}=0$

$\Rightarrow (l+m+n)\left(l^2+m^2+n^2-lm-mn-nl\right)=0$

$\Rightarrow \frac{1}{2}(l+m+n)\left[(l-m)^2+(m-n)^2+(n-1)^2\right]=0$

$\Rightarrow l+m+n=0$

Since $(1-m)^2+(m-n)^2+(n-l)^2 \neq 0$ as l, m, n are not all equal.

2. If the two circles $(x-1)^2+(y-3)^2=r^2$ and $x^2+y^2-8x+2y+8=0$ intersect in two distinct points then:
(A) $2<r<8$ (B) $r<2$
(C) $r=2, r=8$ (D) $r>2$

Ans. A

Sol. $r_1=r, C_1=(1,3), r_2=3, C_2=(4,-1)$

The two circles intersect in two distinct points if $|r_1-r_2|<C_1C_2<r_1+r_2$.

$\therefore |3-r|<5<3+r$

$\therefore r>2$ and $-5<r-3<5$

$\Rightarrow 2<r<8$

3. The shortest distance between line $y-x=1$ and curve $x=y^2$ is
(A) $\frac{\sqrt{3}}{4}$ (B) $\frac{3\sqrt{2}}{8}$
(C) $\frac{8}{3\sqrt{2}}$ (D) $\frac{4}{\sqrt{3}}$

Ans. B

Sol. Shortest distance between two curves is along their common normal. Therefore tangent to $y^2 = x$ is parallel to $y - x = 1$.

$\therefore \quad \frac{dy}{dx} = \frac{1}{2y} = 1 \Rightarrow y = \frac{1}{2}$ and $x = \frac{1}{4}$

$\therefore$ Required shortest distance

= distance of $\left(\frac{1}{4}, \frac{1}{2}\right)$ from the line $x - y + 1 = 0$.

$\therefore \quad$ Distance $= \left| \frac{\frac{1}{4} - \frac{1}{2} + 1}{\sqrt{2}} \right| = \frac{3}{4\sqrt{2}} = \frac{3\sqrt{2}}{8}$

4. Tangents are drawn to the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$ at the ends of latus rectum. The area of the quadrilateral so formed is:

(A) 27
(B) $\frac{27}{2}$
(C) $\frac{27}{4}$
(D) $\frac{27}{55}$

Ans. A

Sol. Area of quadrilateral formed by tangents at the ends of latus rectum $= \frac{2a^2}{e} =$.

Here $a^2 = 9, b^2 = 5, e = \frac{2}{3}$

$\therefore \quad$ Area $= 2 \times 9 \times \frac{3}{2} = 27$.

5. If $\bar{a}, \bar{b}$ and $\bar{c}$ are unit vectors, then $|\bar{a} - \bar{b}|^2 + |\bar{b} - \bar{c}|^2 + |\bar{c} - \bar{a}|^2$ does not exceed:

(A) 4
(B) 9
(C) 8
(D) 6

Ans. B

Sol. $|\bar{a} - \bar{b}|^2 + |\bar{b} - \bar{c}|^2 + |\bar{c} - \bar{a}|^2$

$= 6 - 2\left(\bar{a}.\bar{b} + \bar{b}.\bar{c} + \bar{c}.\bar{a}\right)$

Also $|\bar{a} + \bar{b} + \bar{c}|^2 \geq 0$

$\Rightarrow \quad 3+2\left(\bar{a}.\bar{b}+\bar{b}.\bar{c}+\bar{c}.\bar{a}\right)\geq 0$

$\Rightarrow \quad -2\left(\bar{a}.\bar{b}+\bar{b}.\bar{c}+\bar{c}.\bar{a}\right)\leq 3$

$\therefore \quad \left|\bar{a}-\bar{b}\right|^2+\left|\bar{b}-\bar{c}\right|^2+\left|\bar{c}-\bar{a}\right|^2\leq 9$

6. If [x] and {x} represent integral and fractional parts of x, then the expression $[x]+\sum_{r=1}^{2000}\frac{\{x+r\}}{2000}$ is equal to:

(A) $\frac{2001x}{2}$

(B) x + 2001

(C) x

(D) $[x]+\frac{2001}{2}$

Ans. C

Sol. Let x = [x] + p where p = fractional part of x i.e. $\{x\}$

$\therefore\ \{x+r\}=\{[x]+r+p\}=p$ for all integral values of r.

$\therefore\ [x]+\sum_{r=1}^{2000}\frac{\{x+r\}}{2000}=[x]+\sum_{r=1}^{2000}\frac{p}{2000}$

$=[x]+\frac{2000p}{2000}=[x]+p=x$

7. Let $f:[1,\infty)\to[1,\infty)$ is defined by $f(x)=2^{x(x-1)}$. Then $f^{-1}(x)$:

(A) $2^{-x(x-1)}$

(B) $\frac{1}{2}\left(1+\sqrt{1+4\log_2 x}\right)$

(C) $\frac{1}{2}\left(1-\sqrt{1+4\log_2 x}\right)$

(D) None of these

Ans. B

Sol. $f:[1,\infty)\to[1,\infty):f(x)=2^{x(x-1)}$, s both one – one, onto

$y=2^{x(x-1)}\Rightarrow x^2-x=\log_2 y\,(y\geq 1)$

$\Rightarrow x^2-x-\log_2 y=0$

$\Rightarrow x=\frac{1}{2}\left(1\pm\sqrt{1+4\log_2 y}\right)$

As $\quad x\geq 1,\ \forall y\geq 1,\ x=\frac{1}{2}\left(1+\sqrt{1+4\log_2 y}\right)$

$\Rightarrow \quad f^{-1}(x)=\frac{1}{2}\left(1+\sqrt{1+4\log_2 x}\right)$

8. The lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{-k}$ and $\frac{x-1}{k}=\frac{y-4}{2}=\frac{z-5}{1}$ are co – planar if:

(A) k = 1 or –1
(B) k = 0 or –3
(C) k = 3 or –3
(D) k = 0 or –1

Ans. B

Sol. $\begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$

$1(1+2k)+1(1+k^2)-1(2-k)=0$

$1+2k+1+k^2-2+k=0$

$k^2+3k=0$

k = 0 or –3

9. $\lim_{x\to 0} \frac{\int_0^x \sin^3 x\,dx}{x^4} =$

(A) 4
(B) $\frac{3}{4}$
(C) 0
(D) $\frac{1}{4}$

Ans. D

Sol. $\lim_{x\to 0} \frac{\int_0^x \sin^3 x\,dx}{x^4} \left(\text{form } \frac{0}{0}\right)$

$= \lim_{x\to 0} \frac{\sin^3 x}{4x^3} = \lim_{x\to 0} \frac{1}{4}.\left(\frac{\sin x}{x}\right)^3 = \frac{1}{4}$

10. If $f(x)=\min\{1, x^2, x^3\}$, then:

(A) $f(x)$ is continuous $\forall x \in R$

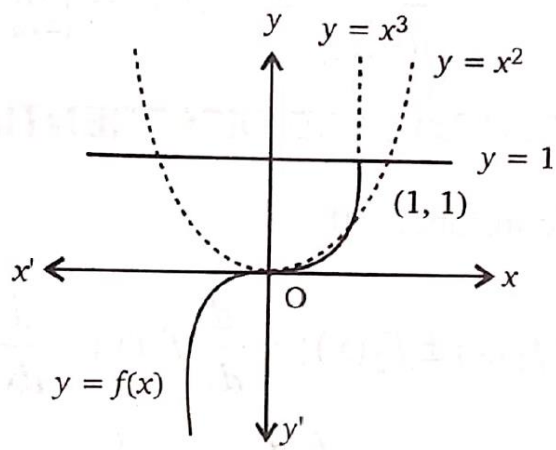
(B) $f(x) > 0, \forall x > 1$

(C) $f(x)$ is continuous but differentiable $\forall x \in R$

(D) $f(x)$ is not differentiable at two points

Ans. A

Sol. $f(x)$ is continuous but not differentiable $\forall x \in R$



11. Let $f:(-1,1) \to R$ be a differentiable function with $f(0) = -1$ and $f'(0) = 1$. Let $g(x) = \left[f(2f(x)+2)\right]^2$. Then $g'(0) =$

(A) –2
(B) 4
(C) –4
(D) 0

Ans. C

Sol. $g(x) = \left[f(2f(x)+2)\right]^2$

$\Rightarrow g'(x) = 2\left[f(2f(x)+2)\right]f'(2f(x)+2).2f'(x)$

Since $f'(0) = -1, f'(0) = 1$

$g'(0) = -2.1.2 = -4$

12. p_1 and p_2 are the length of perpendiculars from the origin on the tangents and normals to the curve $x^{2/3} + y^{2/3} = a^{2/3}$ respectively. Then $4p_1^2 + p_2^2 =$

(A) a^3
(B) a
(C) $4a^2$
(D) a^2

Ans. D

Sol. $x^{2/3} + y^{2/3} = a^{2/3}$

$\frac{dy}{dx} = -\frac{y^{1/3}}{x^{1/3}}$

$\therefore$ Equation of tangent is $Y - y = -\frac{y^{1/3}}{x^{1/3}}(X - x)$

$\Rightarrow Xy^{1/3} + Yx^{1/3} - x^{1/3}y^{1/3}a^{2/3} = 0$

$\therefore\ p_1 = \dfrac{x^{1/3}y^{1/3}a^{2/3}}{\sqrt{y^{2/3}+x^{2/3}}} = (axy)^{1/3}$

Equation of normal is $Y - y = \dfrac{x^{1/3}}{y^{1/3}}(X - x)$

$\Rightarrow Xx^{1/3} - Yy^{1/3} + \left(y^{4/3} - x^{4/3}\right) = 0$

$\therefore\ p_2 = \dfrac{\left(y^{2/3}-x^{2/3}\right)a^{2/3}}{y^{1/3}\sqrt{x^{2/3}+y^{2/3}}} = \left(y^{2/3}-x^{2/3}\right)a^{1/3}$

$\therefore 4p_1^2 + p_2^2 = 4a^{2/3}x^{2/3}y^{2/3} + a^{2/3}\left(y^{2/3}-x^{2/3}\right)^2$

$= a^{2/3}\left\{x^{2/3}+y^{2/3}\right\}^2 = a^{2/3}.a^{4/3} = a^2$

13. A cubic $f(x)$ vanishes as x = –2 and has relative max/min. $x = \dfrac{1}{3}$ and $x = -1$. If $\int_{-1}^{1} f(x)dx = \dfrac{14}{3}$, then $f(x)$ is:

(A) $x^3 + x^2 - x + 2$
(B) $x^3 + x^2 + x + 6$
(C) $x^3 - x^2 + x + 16$
(D) None of these

Ans. A

Sol. Let $f(x) = ax^3 + bx^2 + cx + d$

$f(-2) = -8a + 4b - 2c + d = 0$

$f'(x) = 3ax^2 + 2bx + c$ ....(1)

$f'(-1) = 3a - 2b + c = 0$ ....(2)

$f'\left(\dfrac{1}{3}\right) = \dfrac{a}{3} + \dfrac{2b}{3} + c = 0$ .....(3)

Also $\int_{-1}^{1} f(x)dx = \int_{-1}^{1}\left(ax^3 + bx^2 + cx + d\right)dx$

$= \dfrac{2(b+3d)}{3} = \dfrac{14}{3}$ .....(4)

Solving (1) – (4), we get a = 1, b = 1, c = –1, d = 2

Hence $f(x) = x^3 + x^2 - x + 2$.

14. The value of $\sqrt{2}\int \frac{\sin x\,dx}{\sin\left(x-\frac{\pi}{4}\right)}$ is:

(A) $x-\log\left|\cos\left(x-\frac{\pi}{4}\right)\right|+c$

(B) $x+\log\left|\cos\left(x-\frac{\pi}{4}\right)\right|+c$

(C) $x-\log\left|\sin\left(x-\frac{\pi}{4}\right)\right|+c$

(D) $x+\log\left|\sin\left(x-\frac{\pi}{4}\right)\right|+c$

Ans. D

Sol. $I=\sqrt{2}\int \frac{\sin x}{\sin\left(x-\frac{\pi}{4}\right)}dx$, putting $x-\frac{\pi}{4}=t$

$$I=\sqrt{2}\int \frac{\sin\left(t+\frac{\pi}{4}\right)dt}{\sin t}=\sqrt{2}\int\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\cot t\right)dt$$

$$=t+\left|\log_e \sin t\right|+c'=x+\left|\log\sin\left(x-\frac{\pi}{4}\right)\right|+c$$

15. $\lim_{n\to\infty} n\left\{\frac{1}{(n+1)(n+2)}+\frac{1}{(n+2)(n+4)}+....+\frac{1}{6n^2}\right\}=$

(A) $\log\left(\frac{3}{2}\right)$

(B) $\log\left(\frac{2}{3}\right)$

(C) $\frac{1}{3}\log 2$

(D) $\frac{1}{2}\log 3$

Ans. A

Sol. $S=\lim_{n\to\infty} n\left\{\frac{1}{(n+1)(n+2)}+\frac{1}{(n+2)(n+4)}+....+\frac{1}{(n+n)(n+2n)}\right\}$

$$=\lim_{n\to\infty}\sum_{r=1}^{n}\frac{n^2}{(n+r)(n+2r)}.\frac{1}{n}=\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{\left(1+\frac{r}{n}\right)\left(1+\frac{2r}{n}\right)}.\frac{1}{n};\frac{1}{n}\to x,\ \frac{1}{n}\to dx,$$

$$=a=\lim_{n\to\infty}\left(\frac{r}{n}\right)_{r=1}=0, b=\lim\left(\frac{r}{n}\right)_{r=n}=1$$

$$\Rightarrow S=\int_0^1\frac{dx}{(1+x)(1+2x)}=\int_0^1\left(\frac{2}{1+2x}-\frac{1}{1+x}\right)dx$$

$$=\left[\log(1+2x)-\log(1+x)\right]_0^1=\log\left(\frac{3}{2}\right)$$

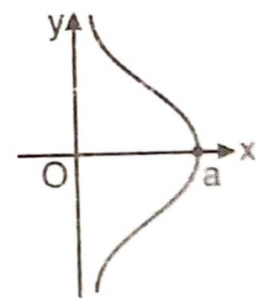
16. Area bounded by the curve $xy^2 = a^2(a-x)$ and y – axis is

(A) $\frac{\pi a^2}{2}$ (B) πa^2

(C) $3\pi a^2$ (D) $\frac{3\pi a^2}{2}$

Ans. B

Sol. The curve $xy^2 = a^2(a-x)$ is symmetrical about x – axis and lies in $(0,a]$.



$\therefore$ Area $= 2\int_0^a a\sqrt{\left(\frac{a-x}{x}\right)}dx$

$= 4a^2\int_0^{\pi/2}\cos^2\theta\, d\theta$

$\left[x = a\sin^2\theta\right]$

$= \pi a^2$

17. The solution of the differential equation $\left(1+y^2\right)+\left(x-e^{\tan^{-1}y}\right)\frac{dy}{dx}=0,$ is:

(A) $2xe^{\tan^{-1}y} = e^{2\tan^{-1}y}+k$ (B) $xe^{\tan^{-1}y} = e^{\tan^{-1}y}+k$

(C) $xe^{2\tan^{-1}y} = e^{\tan^{-1}y}+k$ (D) $(x-2) = ke^{-\tan^{-1}y}$

Ans. A

Sol. $\left(1+y^2\right)+\left(x-e^{\tan^{-1}y}\right)\frac{dy}{dx}=0$

$\Rightarrow \quad \frac{dx}{dy}+\frac{1}{1+y^2}.x = \frac{e^{\tan^{-1}y}}{1+y^2}$ [L.D.E.]

I.F. $= e^{\int\frac{1}{1+y^2}dy} = e^{\tan^{-1}y}$

$\therefore$ Solution is $xe^{\tan^{-1}y} = \int\frac{e^{\tan^{-1}y}}{1+y^2}.e^{\tan^{-1}y}\,dy + c'$

$\Rightarrow \quad xe^{\tan^{-1}y} = \frac{1}{2}e^2\tan^{-1}y + c'$

$\Rightarrow \quad 2xe^{\tan^{-1}y} = e^{2\tan^{-1}y}+k$ $[k = 2c]$

18. The value of $\prod_{n=1}^{7} \sin\frac{(2n-1)\pi}{14} =$

(A) $\frac{1}{32}$ (B) $\frac{1}{64}$

(C) $\frac{1}{128}$ (D) None

Ans. B

Sol. Given expression

$$= \sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\sin\frac{7\pi}{14}\sin\frac{9\pi}{14}\sin\frac{11\pi}{14}\sin\frac{13\pi}{14}$$

$$= \left(\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\right)^2$$

$$= \left\{\cos\left(\frac{\pi}{2}-\frac{\pi}{14}\right)\cos\left(\frac{\pi}{2}-\frac{3\pi}{14}\right)\cos\left(\frac{\pi}{2}-\frac{5\pi}{14}\right)\right\}^2$$

$$= \left(\cos\frac{3\pi}{7}\cos\frac{2\pi}{7}\cos\frac{\pi}{7}\right)^2 = \left(-\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}\right)^2$$

$$= \frac{\sin^2\left(\frac{8\pi}{7}\right)}{64\sin^2\left(\frac{\pi}{7}\right)} = \frac{1}{64}$$

19. The number of solutions of the pair of equations $2\sin^2\theta - \cos 2\theta = 0; 2\cos^2\theta - 3\sin\theta = 0$ in the interval $[0, 2\pi]$ is

(A) 0 (B) 1

(C) 2 (D) 4

Ans. C

Sol. $2\sin^2\theta = \cos 2\theta$

$\Rightarrow \sin^2\theta = \frac{1}{4}$ $\Rightarrow$ $\sin\theta = \pm\frac{1}{2}$

$2\cos^2\theta - 3\sin\theta = 0$ $\Rightarrow$ $2\sin^2\theta + 3\sin\theta - 2 = 0$

$\Rightarrow \sin\theta = \frac{1}{2}$ or -2

$\therefore \sin\theta = \frac{1}{2}$ i.e. $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

20. If $\sum_{k=1}^{\infty} \tan^{-1}\left(\frac{1}{2k^2}\right) = \theta$, then $\tan\theta =$

(A) 0 (B) 1

(C) $\sqrt{3}$ (D) None

Ans. B

Sol $T_k = \tan^{-1}\frac{1}{2k^2} = \tan^{-1}\frac{(2k+1)-(2k-1)}{1+(2k-1)(2k+1)}$

$= \tan^{-1}(2k+1) - \tan^{-1}(2k-1)$

$\therefore \sum_{k=1}^{n} \tan^{-1}\frac{1}{2k^2} = \tan^{-1}(2n+1) - \tan^{-1}1$

$\therefore \theta = \sum_{k=1}^{\infty} \tan^{-1}\frac{1}{2k^2} = \lim_{n\to\infty} \tan^{-1}(2n+1) - \frac{\pi}{4}$

$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}.$

$\therefore \tan\theta = \tan\left(\frac{\pi}{4}\right) = 1$

PHYSICS

21. Two particles move parallel to x axis about the origin with same amplitude a and frequency W. At a certain instant they are found at a distance a/3 from the origin on opposite sides but their velocities are in the same direction. What is the phase difference between the two.

(A) $\cos^{-1}\left(\frac{7}{9}\right)$ (B) $\cos^{-1}\left(\frac{5}{9}\right)$

(C) $\cos^{-1}\left(\frac{4}{9}\right)$ (D) $\cos^{-1}\left(\frac{1}{9}\right)$

Ans. A

Sol. $\cos\phi = 1 - 2\times\left(\frac{1}{3}\right)^2 = \frac{7}{9}$

22. A long string with a charge of λ per unit length passes through an imaginary cube of edge a. The maximum flux of the electric field through the cube will be

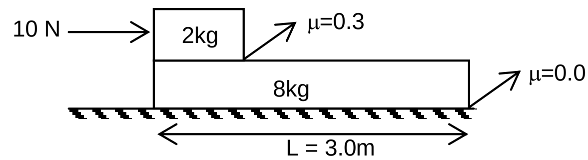
(A) $\lambda a/\varepsilon_0$ (B) $\sqrt{2}\lambda a/\varepsilon_0$

(C) $6\lambda a/\varepsilon_0$ (D) $\sqrt{3}\lambda a/\varepsilon_0$

Ans. D

Sol. $\phi = \frac{(a\sqrt{3}\,\lambda)}{\varepsilon_o}$

23. Determine the time in which the smaller block reaches other end of bigger block in the figure



(A) 4s
(B) 8
(C) 2.19s
(D) 2.13s

Ans. C

Sol. $a_r = \left(2 - \frac{3}{4}\right) = \frac{5}{4}$

$t = \sqrt{\frac{2L}{a_r}} = 2.19\,s$

24. A closed organ pipe of length *L* is vibrating in its first overtone. There is a point *Q* inside the pipe at a distance 7*L*/9 from the open end. The ratio of pressure amplitude at *Q* to the maximum pressure amplitude in the pipe is

(A) 1 : 2 (B) 2 : 1
(C) 1 : 1 (D) 2 : 3

Ans. A

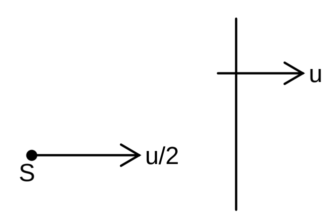
Sol. $\Delta Pm = 2\Delta P_0 \cos kx$ (assuming closed end as origin)

At point *Q*, $x = L - \frac{7L}{9} = \frac{2L}{9}$

$\Delta Pm = 2\Delta P_0 \cos\left(\frac{2\pi}{\lambda} \times \frac{2L}{9}\right) = \Delta P_0$

$\therefore$ Required ratio = 1 : 2

25. A wall is moving with velocity u and a source of sound moves with velocity u/2 in the same direction as shown in the figure. Assuming that the sound travels with velocity 10u, the ratio of incident sound wavelength on the wall to the reflected sound wavelength by the wall is equal to



(A) 9:11 (B) 11:9
(C) 4:5 (D) 5:4

Ans. A

Sol. $\lambda_r = \lambda_i \frac{11u}{9u} = \frac{11}{9}\lambda_i$

26. Velocity time equation of a particle moving in a straight line is $V = t^2 - 5t + 6$. The distance travelled by the particle in the time interval from t = 0 to t = 4 sec

(A) 0 (B) $\frac{17}{3}$

(C) 6 (D) $\frac{16}{3}$

Ans. B

Sol. $\int |\vec{V}|\, dt$ and V is negative between 2 and 3.

Distance = $\int_0^2 (t^2 - 5t + 6)dt + \int_2^3 (-t^2 + 5t - 6)dt + \int_3^4 (t^2 - 5t + 6)dt$

= $\frac{14}{3} \times 2 + \frac{-9}{2} \times 2 + \frac{16}{3}$ = $\frac{17}{3}$.

27. The period of oscillation of a simple pendulum is given by T = $2\pi\sqrt{\frac{\ell}{g}}$ where I is about 100 cm and is known to have 1mm accuracy. The period is about 2s. The time of 100 oscillations is measured by a stop watch of least count 0.1 s. The percentage error in g is

(A) 0.1% (B) 1%
(C) 0.2% (D) 0.8%

Ans. C

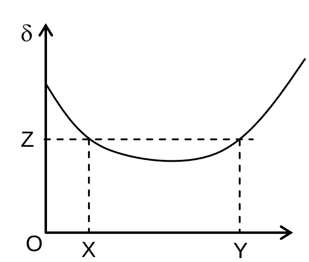
Sol. T = $2\pi\sqrt{\frac{1}{g}} \Rightarrow$ T^2 = $4\pi^2 1/g \Rightarrow g = \frac{4\pi^2 l}{T^2}$

Here % error in I = $\frac{1mm}{100cm} \times 100 = \frac{0.1}{100} \times 100 = 0.1\%$

and % error in T = $\frac{0.1}{2 \times 100} \times 100 = 0.05\%$

$\therefore$ % error in g = % error in I + 2 (% error in T) = 0.1 + 2 $\times$ 0.05 = 0.2%

28. Graph shown is between deviation (δ) and angle of incidence I then angle of prism is



(A) –x + y + z (B) x + y – z
(C) x + z – y (D) x – y + z

Ans. B

Sol. δ = x + y – z

29. In a region of space, the electric field is given by $\vec{E} = 8\hat{i} + 4\hat{j} + 3\hat{k}$. The electric flux through a surface of area of 100 units is x – y plane is

(A) 800 units (B) 300 units
(C) 400 units (D) 1500 units

Ans. B

Sol. $\phi = \vec{E} \cdot \vec{S}$
= 3 × 100 = 300

30. Two radioactive elements R and S disintegrate as

$R \longrightarrow P + \alpha$; $\lambda_R = 4.5 \times 10^{-3}$ years $^{-1}$

$S \longrightarrow Q + \beta$; $\lambda_S = 3 \times 10^{-3}$ years $^{-1}$

Starting with number of atoms of R and S in the ratio of 2 : 1, this ratio after the lapse of three half lives of R will be

(A) 3 : 2 (B) 1 : 3
(C) 1 : 1 (D) 2 : 1

Ans. C

Sol. $\frac{\lambda_R}{\lambda_S} = 1.5$

So, the rate of disintegration of R will be 1.5 times that of S. Thus, the half-life of S will be 1.5 times that of R. So, two half lives of S will be equal to the three half –lives of R.

$\frac{N_R}{N_S} = \frac{0.25}{0.25} = 1$

31. The equation of motion of a projectile is $y = 12x - \frac{3}{4}x^2$. The horizontal component of velocity is 3 ms^{-1}. What is the range of the projectile?

(A) 18 m (B) 16 m
(C) 12 m (D) 21.6 m

Ans. B

Sol. Range = $\frac{12 \times 4}{3}$

32. Critical angle of light passing from glass to air is maximum for

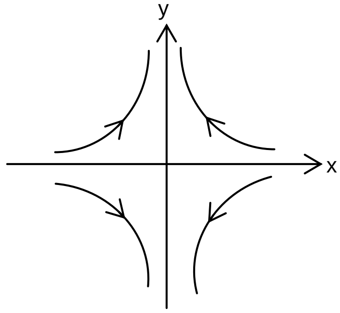
(A) red colour (B) green colour
(C) yellow colour (D) blue colour

Ans. A

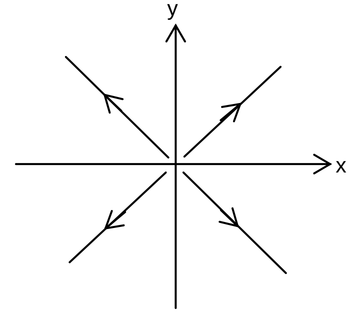
Sol. $C = \text{Sin}^{-1}\frac{1}{u}$ and $u_v > u_r$

33. The potential field depends on x – and y – coordinates as $V = x^2 - y^2$. Corresponding electric field lines in x – y plane are as

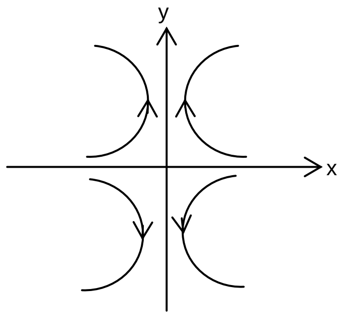
(A)



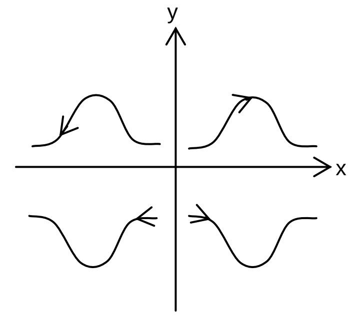
(B)



(C)



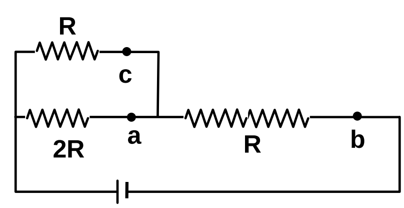
(D)



Ans. A

Sol. $\vec{E} = -2x\hat{i} + 2y\hat{j}$ & $E = 2\sqrt{x^2 + y^2}$

34. Referring to the shown circuit, the current will be minimum in



(A) a
(B) b
(C) c
(D) same in all the branches

Ans. A

Sol. $I_a = \dfrac{3V}{15R}, I_b = \dfrac{3V}{5R}$

$I_c = \dfrac{6V}{15R}$.

35. In a Young's double slit experiment, the slit separation is 1 mm and the screen is 1 m from the slit. For a monochromatic light of wavelength 500 nm, the distance of 3[rd] minima from the central maxima is

(A) 0.50 mm
(B) 1.25 mm
(C) 1.50 mm
(D) 1.75 mm

Ans. B

Sol. Distance of n^{th} minima from central bright fringe

$$x_n = \frac{(2n-1)\lambda D}{2d}$$

For n = 3, i.e. 3^{rd} minima

$$x_3 = \frac{(2\times 3-1)\times 500\times 10^{-9}\times 1}{2\times 1\times 10^{-3}} = 1.25\times 10^{-3}\text{ m} = 1.25\text{ mm}$$

36. Hydrogen gas absorbs radiations of wavelength λ_0 and consequently emit radiations of 6 different wavelengths of which two wavelengths are longer than λ_0. Choose the incorrect statement.

(A) The final excited state of the atom is n = 4.
(B) The initial state of the atom may be n = 2.
(C) The initial state of the atom may be n = 3.
(D) There are three transitions belonging to Lyman series.

Ans. C

Sol. Since only 6 different wavelength are excited, therefore highest excited stat is n = 4.
Two wavelengths are longer than λ_0, initially atoms were in excited state n = 2
Corresponding transitions are $4 \to 3$, $4 \to 2$, $4 \to 1$, $3 \to 2$, $3 \to 1$, $2 \to 1$.

37. Two soap bubbles of radii 2mm and 4mm are brought in contact. If the surface tension of liquid is 7×10^{-2} Nm^{-1}. Then the radius of the common surface is

(A) 2×10^{-3}m (B) 4×10^{-3}m
(C) 6×10^{-3}m (D) 8×10^{-3}m

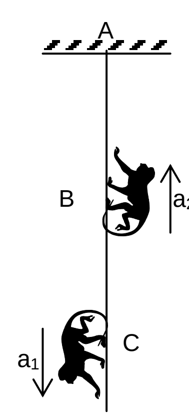
Ans. B

Sol. $P_{convex} = P_{concave} - \frac{4s}{R}$

$$P_o + \frac{4s}{R_1} = P_o + \left(\frac{4s}{R_2}\right) - \frac{4s}{R}$$

$$\Rightarrow \quad R = \frac{R_1R_2}{R_1 - R_2} = 4\text{mm}$$

38. Two monkeys each of mass m move with acceleration $a_1 = a_2 = \frac{g}{2}$ relative to the light inextensible string as shown in the figure. The ratio of tensions in the portions AB and BC of the string is

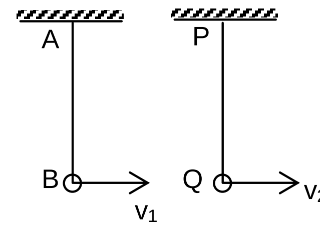


(A) 1 : 2 (B) 3 : 1
(C) 4 : 1 (D) 2 : 1

Ans. C

Sol. $\frac{T_1}{T_2} = \frac{m(g-a)}{m(g+a)}$

39. In the figure shown there are two pendulums free to move in a vertical circle about one pivoted end. The length of each is ℓ and mass of each bob is m. But AB is a light string while PQ is a light rigid rod. The ratio of minimum velocities v_1 and v_2 given to both as shown to complete the full vertical circle is



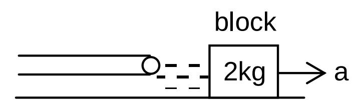
(A) 1 (B) $\frac{\sqrt{5}}{2}$

(C) 2 (D) $\frac{\sqrt{3}}{2}$

Ans. B

Sol. $\frac{v_1}{v_2} = \frac{\sqrt{5gl}}{\sqrt{4gl}}$

40. A block of metal weighing 2 kg is resting on a frictionless plane as shown in figure. It is struck by a jet releasing water at the rate of 1 kg/s and at speed of 5 m/s. The magnitude of initial acceleration of the block is



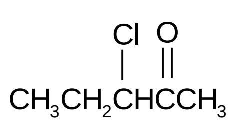
(A) 2.5 m/s^2 (B) 5 m/s^2
(C) 7.5 m/s^2 (D) 10 m/s^2

Ans. A

Sol. Force applied by water jet = rate of change in momentum of water = $1 \times 5 = 5N$
So acceleration = 5/2 m/s^2

CHEMISTRY

41.

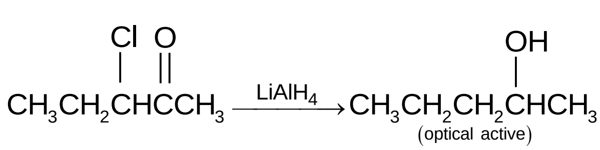


Which of the following reagent can convert the above optical active compound to an optical inactive compound?

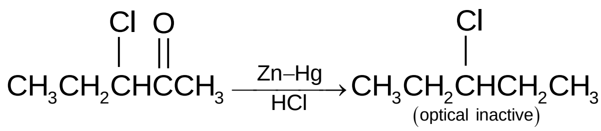
(A) $LiAlH_4$ (B) Zn/Hg/Conc.HCl
(C) KCN (D) HCN

Ans. B

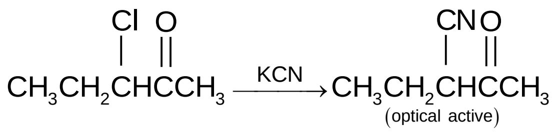
Sol. (A)



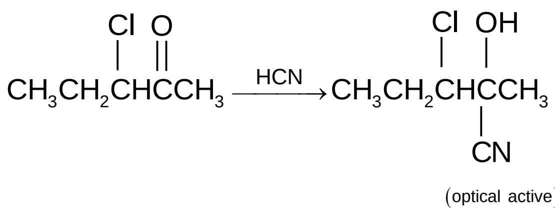
(B)



(C)



(D)



42. Which of the following reaction forms a precipitate of sulphur?

(A) $FeCl_2 + H_2S \longrightarrow$ (B) $ZnCl_2 + H_2S \longrightarrow$

(C) $FeCl_3 + H_2S \longrightarrow$ (D) $BiCl_3 + H_2S \longrightarrow$

Ans. C

Sol. (A) $FeCl_2 + H_2S \longrightarrow FeS + 2HCl$

(B) $ZnCl_2 + H_2S \longrightarrow ZnS + 2HCl$

(C) $2FeCl_3 + H_2S \longrightarrow 2FeCl_2 + S\downarrow + 2HCl$

(D) $2BiCl_3 + H_2S \longrightarrow Bi_2S_3 + 6HCl$

43. Which of the following complex ion of cobalt is diamagnetic in nature?

(A) $[Co(CN)_6]^{4-}$ (B) $[CoF_6]^{3-}$

(C) $[Co(CN)_6]^{3-}$ (D) $[CoF_6]^{4-}$

Ans. C

Sol. In (A), CN^- is a strong field ligand and the metal ion is Co^{2+}. The metal ion contains d^7 configuration. This configuration contains unpaired electrons. So, the d^7 is paramagnetic whether the ligand is strong or weak.

(B) The metal ion has d^6 configuration. For weak field ligand like F^- ion the configuration is $t_{2g}^4 e_g^2$ which contains unpaired electrons.

(C) The metal ion has d^6 configuration and the ligand is a strong field one. The configuration will be $t_{2g}^6 e_g^0$ and it is diamagnetic.

(D) The metal ion has d^7 configuration. It is paramagnetic for both strong field and weak filed ligands.

44. The Nernst equation $E = E^o - \frac{RT}{nF} \ln Q$ indicates that the reaction quotient will be equal to equilibrium constant K_C when

(A) $E = E^o$ (B) $\frac{RT}{nF} = 1$

(C) E = zero (D) $E^o = 1$

Ans. C

Sol. At equilibrium the cell e.m.f (E) becomes zero

45. A crystal is made of particles X and Y. X forms fcc packing and Y occupies all the octahedral voids. If all the particles along one body diagonal are removed then the formula of the crystal would be:

(A) X_4Y_3 (B) X_5Y_4
(C) X_4Y_5 (D) None of these

Ans. B

Sol. After removal of the particles along one body diagonal, the number of corner atoms will be six and the number of Y atoms becomes 3.

$\therefore$ No. of X particles = $6\times\frac{1}{8}+6\left(\text{face centre particles}\right)\times\frac{1}{2}=\frac{3}{4}+3=\frac{15}{4}$

No of Y particles = 3

$\therefore$ Formula = $X_{\frac{15}{4}}Y_3 = X_{15}Y_{12} = X_5Y_4$

46. What is the value of ΔH at 358 K for the reaction?

$Fe_2O_3(s)+3H_2(g)\longrightarrow 2Fe(s)+3H_2O(\ell)$

Given that, ΔH_{298} = -33.29 kJ mol^{-1} and C_p for Fe_2O_3(s), Fe(s), $H_2O(\ell)$ and H_2(g) are 103.8, 25.1, 75.3 and 28.8 J/K mole respectively.

(A) 32.613 kJ mol^{-1} (B) 85.9 kJ mol^{-1}
(C) -18.29 kJ/mol (D) -28.136 kJ/mol

Ans. D

Sol. ΔC_p = [2 × 25.1 + 3 × 75.3] – [103.8 + 3 × 28.8] = 85.9 J/K mol

$\frac{\Delta H_2-\Delta H_1}{T_2-T_1}=\Delta C_p$

or, $\frac{\Delta H_{358}-(-33290)}{358-298}=85.9$

$\Rightarrow \Delta H_{358}$ = -28136 J/mol = -28.136 kJ/mol

47. 25 mL of hydrogen and 18 mL of iodine when heated in a closed container, produced 30.8 mL of HI at equilibrium. What is the degree of dissociation of HI at the same temperature?

(A) 0.615 (B) 0.245
(C) 0.831 (D) 0.198

Ans. B

Sol. The volume of the species is proportional to their concentration.

	H_2	+ I_2	$\rightleftharpoons 2HI$
Initial volume	25	18	0
Volume at equ^m	(25 - x)	(18 - x)	2x(=30.8)

$$K_C = \frac{(30.8)^2}{9.6 \times 2.6} = 38.01$$

If dissociation of HI is carried out at the same temperature, then

$$2HI \rightleftharpoons H_2 + I_2$$

$$(1-x') \quad \frac{x'}{2} \quad \frac{x'}{2} \text{ moles at equilibrium}$$

$$K'_C = \frac{1}{K_C} = \frac{1}{38.01} = \frac{\left(\frac{x'}{2}\right)\left(\frac{x'}{2}\right)}{(1-x')^2}$$

on solving $x' = 0.245$

48. A compound XY_2 has observed and normal molar masses 65.6 and 164 respectively. Calculate the apparent percentage of ionization of XY_2 which ionizes as X^{2+} and Y^-.

(A) 95% (B) 25%
(C) 75% (D) 65%

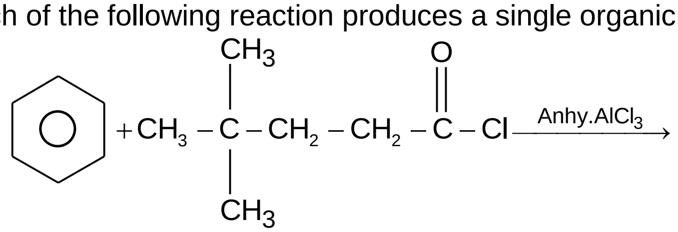
Ans. C

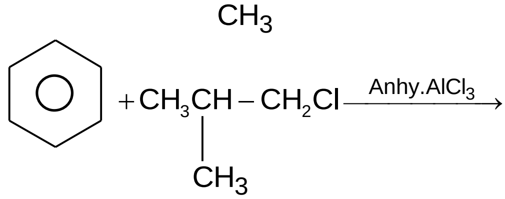
Sol. $$\text{Van't Hoff factor } (i) = \frac{\text{Normal molecular mass}}{\text{Observed molecular mass}} = \frac{164}{65.6} = 2.5$$

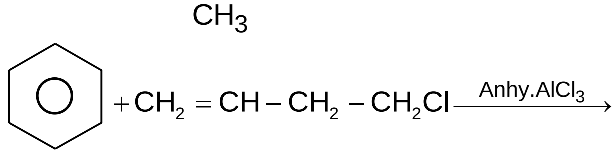
$$\alpha = \frac{i-1}{n-1} = \frac{2.5-1}{3-1} = \frac{1.5}{2} = 0.75$$

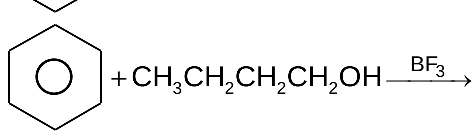
$\therefore$ % of ionization = 75%

49. Which of the following reaction produces a single organic product?

(A) 

(B) 

(C) 

(D) 

Ans. A

Sol. In (A) the initially formed carbocation $\left(CH_3\right)_3 C - CH_2 - CH_2 - \overset{+}{C}O$ does not rearrange. So one product is expected.
In (B), there takes place carbocation rearrangement, so, two products are expected.
In (C) and (D), carbocation rearrangement is also possible. So, more than one product are expected.

50. What $[H^+]$ must be maintained in a saturated H_2S (0.1M) to precipitate CdS but not ZnS if $[Cd^{2+}] = [Zn^{2+}] = 0.1M$ initially.
Given K_{sp} (CdS) = 8×10^{-27}, K_{sp}(ZnS) = 1×10^{-21}, $K_a(H_2S) = 1.1 \times 10^{-21}$

(A) 0.6 M
(B) 0.1 M
(C) 0.8 M
(D) 0.01 M

Ans. B

Sol. In order to prevent precipitation of ZnS
$[Zn^{2+}]\,[S^{2-}] < K_{sp} = 1 \times 10^{-21}$
or $(0.1)\,[S^{2-}] < 1 \times 10^{-21}$
or $[S^{2-}] < 1 \times 10^{-20}$
This is maximum value of $[S^{2-}]$ before ZnS will precipitate
Let $[H^+]$ to maintain this $[S^{2-}]$ be x.

$$H_2S \rightleftharpoons 2H^+ + S^{2-}$$

$$K_a = \frac{\left[H^+\right]^2\left[S^{2-}\right]}{\left[H_2S\right]} = \frac{x^2\left(1\times10^{-20}\right)}{0.1} = 1.1\times10^{-21}$$

or, $x = [H^+] = 0.1$ M
$\therefore$ No ZnS will precipitate at a concentration of H^+ greater than 0.1 M.

51. A diene on reductive ozonolysis produces two moles of ethanal and one mole of propan -1, 3-dial. How many geometrical isomer(s) is/are possible for the diene?

(A) 2
(B) 3
(C) 4
(D) 6

Ans. B

Sol. The diene is $CH_3CH = CH - CH_2 - CH = CHCH_3$

52. How many gram of solid KOH must be added to 100 mL of a buffer solution which is 0.1 M each with respect to acid HA and salt KA to make the pH of solution 6.
$[p^{K_a}$ of HA = 5]

(A) 0.458
(B) 0.327
(C) 5.19
(D) None of these

Ans. A

Sol. Let x millimole of KOH is added

$$pH = p^{K_a} + \log\frac{[Salt]}{[Acid]}$$

or, $6 = 5 + \log\left[\frac{s+x}{a-x}\right]$

or, $\frac{s+x}{a-x} = 10$, or $\frac{10+x}{10-x} = 10 \Rightarrow x = 8.18$

$\therefore$ W = 8.18 × 56 × 10^{-3} = 0.458 g

53. Which of the following is an incorrect expression?

(A) $\Delta H = \Delta E - P\Delta V$

(B) $\Delta G = \Delta H - T\Delta S$

(C) $\Delta G^o = -2.303 RT \log K$

(D) $\Delta S = \frac{\Delta H}{T}$

Ans. A

Sol. Correct relations is $\Delta H = \Delta E + P\Delta V$

54. For a reaction

$A(g) \longrightarrow B(g) + C(g)$

The half life period is 10 min. In what period of time would the concentration of X be reduced to 10% of original concentration.

(A) 20 min

(B) 33 min

(C) 15 min

(D) 25 min

Ans. B

Sol. $K = \frac{2.303}{t}\log\frac{N_0}{N_t}$ and $t_{1/2} = \frac{0.693}{K}$

$\frac{0.693}{10} = \frac{2.303}{t}\log\frac{100}{10}$

t = 33 min

55. State the order of the following reaction,

A + B $\rightarrow$ Product

[A]	[B]	Rate
0.1	0.1	10^{-4}
0.2	0.1	2×10^{-4}
0.1	0.2	10^{-4}

(A) 1

(B) 0

(C) 2

(D) 3

Ans. A

Sol. Let rate = $K[A]^x[B]^y$

$10^{-4} = K[0.1]^x[0.1]^y$ - (1)

$2 \times 10^{-4} = K[0.2]^x[0.1]^y$ - (2)

$10^{-4} = K[0.1]^x[0.1]^y$ - (3)

Dividing (2) by (1) we get x = 1. Dividing (3) by (1) we get y = 0

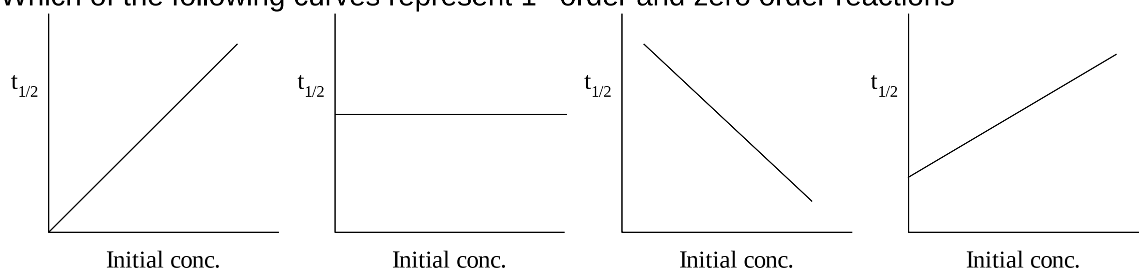
Rate = K[A]’ i.e. a first order reaction.

56. The heats of neutralization of four acids A, B, C and D when neutralized against a common base are 13.7, 9.4, 11.2 and 12.4 kCal respectively. The weakest among these acids is
(A) A (B) B
(C) C (D) D

Ans. B

Sol. B is correct as the heat released on neutralization is minimum.

57. Which of the following curves represent 1st order and zero order reactions



(i) (ii) (iii) (iv)

(A) (i) and (iii) (B) (ii) and (i)
(C) (ii) and (iii) (D) (i) and (iv)

Ans. B

Sol. For 1st order reaction $t_{1/2} = \frac{0.693}{K}$ is independent of initial concentration so (ii) is correct

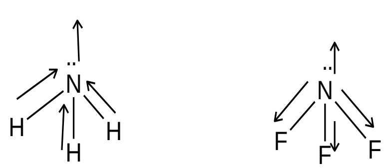
For zero order reaction $t_{1/2} = \frac{[A_0]}{2K}$ i.e. $t_{1/2} \propto$ initial concentration

So (i) is correct.

58. Which of the following is correct (regarding dipole moment) :
(A) dipole moment of $NF_3 > NH_3$ (B) dipole moment of $NH_3 > NF_3$
(C) dipole moment of $NF_3 = NH_3$ (D) None of these

Ans. B

Sol.

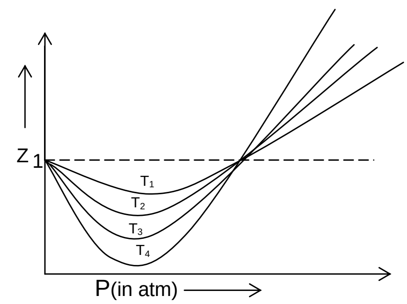


59. For potassium the slope of the line in a graph of K.E. Vs. υ (frequency of incident light) will be (work function = 2.3 eV.)
(A) 6.627×10^{-34} JS (B) 1.602×10^{-19} J
(C) 1.0×10^{15} J S (D) 1.5×10^{-16} J

Ans. A

Sol. $h\upsilon - h\upsilon_0 = K.E.$
$mx + c = y$
Slope $m = h$

60. Above curves are given for N_2 gas at different temperature. The highest temperature observed in the above figure is:



(A) T_1 (B) T_2
(C) T_3. (D) T_4

Ans. A

Sol. As temperature increases, deviation from ideal behavior decreases.

PART – II

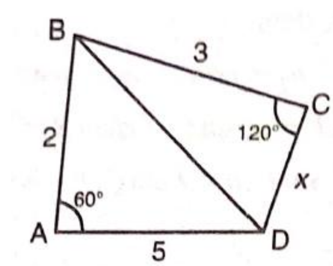
MATHEMATICS

61. The two adjacent sides of a cyclic quadrilateral are 2 and 5 and the angle between them is 60^o. If the third side is 3, the remaining fourth side is:

(A) 2 (B) 3
(C) 4 (D) 5

Ans. A

Sol. In cyclic quadrilateral ABCD,



Let AB = 2, AD = 5
$\angle BAD = 60^o$, BC = 3
Let CD = x
$\angle BCD = 180^o - \angle BAD = 120^o$ K

In ΔABD, $\cos 60^o = \dfrac{AB^2 + AD^2 - BD^2}{2AB.AD}$

$\Rightarrow \quad BD = \sqrt{19}$

In $\Delta BCD, \cos 120^o = \dfrac{BC^2 + CD^2 - BD^2}{2BC.CD}$

$\Rightarrow \quad x^2 + 3x - 10 = 0 \Rightarrow (x-2)(x+5) = 0$

$\Rightarrow \quad x = CD = 2$

$\because x \neq -5$

62. Let (x_0, y_0) be the solution of the following equation

$(2x)^{\ln 2} = (3y)^{\ln 3}$

$3^{\ln x} = 2^{\ln y}$

Then x_0 is:

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$
(C) $\frac{1}{2}$ (D) 6

Ans. C

Sol. $3^{\ln x} = 2^{\ln y} \Rightarrow \ln x \ln 3 = \ln y \ln 2$ ……(1)

$(2x)^{\ln 2} = (3y)^{\ln 3} \Rightarrow \ln 2(\ln 2 + \ln x) = \ln 3(\ln 3 + \ln y)$

$\Rightarrow \ln 2 \ln x - \ln 3 \ln y = (\ln 3)^2 - (\ln 2)^2$ ……(2)

Eliminating in y in (1) and (2), we get

$$\ln x\left[(\ln 2)^2-(\ln 3)^2\right]=\ln(2)\left[(\ln 3)^2-(\ln 2)^2\right]$$
$$\Rightarrow \ln x=-\ln 2 \Rightarrow x=\frac{1}{2}$$

63. If the roots of the equation $10x^3-cx^2-54x-27=0$ are in H.P. then c =

(A) 7 (B) $\frac{1}{9}$

(C) 9 (D) None

Ans. C

Sol. Roots of $10x^3-cx^2-54x-27=0$ are in H.P.
Then roots of $27x^3+54x^2+cx-10=0$ are in A.P.
Let roots be $\alpha-\beta,\alpha,\alpha+\beta$. Then $3\alpha=-2$
$$\Rightarrow\ 27\left(-\frac{8}{27}\right)+54\left(\frac{4}{9}\right)-\frac{2c}{3}-10=0$$
$$\Rightarrow\ -8+24-\frac{2C}{3}-10=0\Rightarrow C=9$$

64. Let $a,b,c\in R^{+}$. Then minimum value of $\frac{a+3c}{a+2b+c}+\frac{4b}{a+b+2c}-\frac{8c}{a+b+3c}$ is:

(A) $12\sqrt{2}-17$ (B) $12\sqrt{2}+17$

(C) $17\sqrt{2}-12$ (D) does not exist

Ans. A

Sol. Let $x=\frac{a+3c}{a+2b+c}+\frac{4b}{a+b+2c}-\frac{8c}{a+b+3c}$
Putting $a+2b+c=p,\ a+b+2c=q,\ a+b+3c=r$
Then $c=r-q,\ q-p=c-b=r-q-b$
$\therefore b=r-q-q+p=r-2q+p,$ and
$$a+3c=p-2b-c+3c=p-2b+2c$$
$$=p-2(r+p-2q)+2(r-q)$$
$$=2q-p$$
$$\therefore\qquad x=\frac{2q-p}{p}+\frac{4(p+r-2q)}{q}-\frac{8(r-q)}{r}$$
$$=2\frac{q}{p}+4\frac{p}{q}+4\frac{r}{q}+8\frac{q}{r}-(1+8+8)$$
$$=-17+\left(2\frac{q}{p}+4\frac{p}{q}\right)+\left(4\frac{r}{q}+8\frac{q}{r}\right)$$

Using $AM \ge GM$ of positive numbers,
$x \ge -17 + 2\sqrt{8} + 2\sqrt{32} = -17 + 12\sqrt{2}$
Equality also hold when $\frac{2q}{p} = \frac{4p}{q}, \frac{4r}{q} = \frac{8q}{r}$
i.e. $4p^2 = 2q^2 = r^2$
Hence, min. $x = 12\sqrt{2} - 17$ exists.

65. Let ω be an imaginary cube root of unity and $1, \alpha_1, \alpha_2, ..., \alpha_{2010}$ be the roots of $x^{2011} = \omega^{2013}$, then $\prod_{r=1}^{2010} \left(\frac{\alpha_r - \omega}{\alpha_r - \omega^2} \right) =$

(A) 1
(B) ω
(C) ω^2
(D) None of these

Ans. A

Sol. $\omega^{2013} = 1$ gives $1, \alpha_1, \alpha_2, ..., \alpha_{2010}$ are the roots of $x^{2011} = 1$
$\therefore x^{2011} - 1 = (x-1)(x-\alpha_1)(x-\alpha_2)......(x-\alpha_{2010})$
Putting $x = \omega$ and ω^2, we get
$\omega^{2011} - 1 = (\omega - 1)(\omega - \alpha_1)(\omega - \alpha_2)...(\omega - \alpha_{2010})$ ...(1)
$(\omega^2)^{2011} - 1 = (\omega^2 - 1)(\omega^2 - \alpha_1)(\omega^2 - \alpha_2)...(\omega^2 - \alpha_{2010})$ ....(2)
On dividing equation (1) by equation (2), we get
$\frac{\omega^{2011} - 1}{\omega^{4022} - 1} = \frac{(\omega - 1)}{\omega^2 - 1} . \frac{\omega - \alpha_1}{\omega^2 - \alpha_1} . \frac{\omega - \alpha_2}{\omega^2 - \alpha_2} ... \frac{\omega - \alpha_{2010}}{\omega^2 - \alpha_{2010}}$
$\Rightarrow \prod_{r=1}^{2010} \left(\frac{\omega - \alpha_r}{\omega^2 - \alpha_2} \right) = 1$

66. The value of $\left\{ \frac{3^{2003}}{28} \right\}$, where $\{.\}$ denotes the fractional part is equal to:

(A) $\frac{15}{28}$
(B) $\frac{5}{28}$
(C) $\frac{19}{28}$
(D) $\frac{9}{28}$

Ans. C

Sol. $3^{2003} = 3^2.3^{2001} = 9.27^{667} = 9\{28-1\}^{667}$
$= 9\{28^{667} - {}^{667}C_1 28^{666} + ... + 667.28 - 1\}$

$= 9 \times 28 \times \text{integer} - 28 + 19$

$\therefore$ Remainder when 3^{2003} is divided by 28 is 19.

$$\therefore \left\{\frac{3^{2003}}{28}\right\} = \frac{19}{28}$$

67. The number of positive integral solution of $2x_1 + 3x_2 + 4x_3 + 5x_4 = 25$ is:

(A) 20 (B) 22
(C) 23 (D) None

Ans. D

Sol. Equation is $2x_1 + 3x_2 + 4x_3 + 5x_4 = 25, x_i \in N$.

Number of solutions

= coefficient of x^{25} in $\left(x^2 + x^4 + x^6 + x^8 + x^{10} + ...\right)$

$\left(x^3 + x^6 + x^9 + ...\right)\left(x^4 + x^8 + ...\right)\left(x^5 + x^{10} + ...\right)$

= coefficient of x^{25} in $x^{14}\left(1 + x^2 + x^4 + x^6 + x^8 + x^{10} + ...\right)$

$.\left(1 + x^3 + x^6 + x^9 + ...\right)\left(1 + x^4 + x^8 + ...\right)\left(1 + x^5 + x^{10} + ...\right)$

= Coefficient of x^{11} in

$\left(1 + x^2 + 2x^4 + 2x^6 + 3x^8 + 3x^{10}\right) \times \left(1 + x^3 + x^5 + x^6 + x^8 + x^9 + x^{10} + x^{11}\right)$

$= 1 + 1 + 2 + 3 = 7$

68. The sum of all 4 digits numbers that can be formed by using the digits 2, 4, 6, 8 (repetition of digits not allowed) is:

(A) 133320 (B) 533280
(C) 53328 (D) None

Ans. A

Sol. Sum of all 4 digit numbers formed by 2, 4, 6, 8 without repetition

$= 3!\left(2 + 4 + 6 + 8\right).\left(1111\right) = 133320$

69. The probability of three persons having the same date and month for birthday is:

(A) $\frac{1}{365}$ (B) $\frac{1}{\left(365\right)^2}$

(C) $\frac{1}{\left(365\right)^3}$ (D) None

Ans. B

Sol. Probability $=\frac{\left({}^{365}C_1\right)}{(365)^3}=\frac{1}{(365)^2}$

70. Consider the system of linear equations:

$$x_1+2x_2+x_3=3$$
$$2x_1+3x_2+x_3=3$$
$$3x_1+5x_2+2x_3=1$$

The system has:

(A) no solution
(B) infinite number of solutions
(C) exactly 3 solutions
(D) a unique solutions

Ans. A

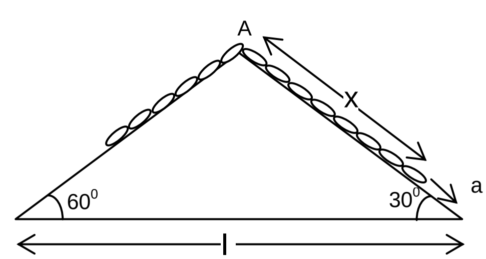
Sol. $\Delta=\begin{vmatrix}1 & 2 & 1\\ 2 & 3 & 1\\ 3 & 5 & 2\end{vmatrix}=0$

$$\Delta_x=\begin{vmatrix}3 & 2 & 1\\ 3 & 3 & 1\\ 1 & 5 & 2\end{vmatrix}=5\neq 0$$

$\therefore$ The system of equations has no solution.

PHYSICS

71. A uniform chain of mass m is placed on a smooth inclined plane ABC as shown in the figure. If the acceleration of the chain is a = $\frac{g}{10}$, then the length x is (length chain is ℓ)



(A) $\frac{\ell\ \ 1+5\sqrt{3}}{5\ \ \sqrt{3}+1}$

(B) $\frac{\ell}{2}$

(C) $\frac{\ell\ \ 1+\sqrt{3}}{5\sqrt{3}+1}$

(D) none of these

Ans. A

Sol. $\frac{m}{\ell}xg\sin 30^0\ \frac{-m}{u}\ \ \ell-x\ \ g\sin 60^0=m\left(\frac{g}{u}\right)$

$$x=\frac{\ell\ \ 1+5\sqrt{3}}{5\ \ \sqrt{3}+1}$$

72. A charge particle enters into a region containing uniform electric field (E) and uniform magnetic field (B) along x-axis and y-axis respectively. If it passes the region undeviated, the velocity of charge particle is given by

(A) $2\hat{i} + \frac{E}{B}\hat{k}$ (B) $2\hat{j} + \frac{E}{B}\hat{k}$

(C) $2\hat{i} - \frac{E}{B}\hat{k}$ (D) none of these

Ans. B

Sol. If $\vec{V} = V_x\hat{i} + V_y\hat{j} + V_z\hat{k}$, $\vec{E} = E\hat{i}$, $\vec{B} = B\hat{j}$

$\therefore \vec{F} = q\left[E\hat{i} + (V_x\hat{i} + V_y\hat{j} + V_z\hat{k}) \times B\hat{j}\right]$

$= qE\hat{i} + qV_xB\hat{k} + 0 - qV_zB\hat{i}$

$= (E - V_zB)\,\hat{i} + V_xB\,\hat{k}$

For no deviation net force should either be zero or in the direction of velocity of particle.

$\therefore$ For $F = 0$, $V_Z = E/B$, $V_x = 0$, $V_y \to$ has any value

73. A certain sample of monoatomic ideal gas is subject to a thermodynamic process in which V and T are related as $V^2 = kT$ (k is constant). The molar specific heat of the gas in this process is

(A) $\frac{3R}{2}$ (B) 2R

(C) $\frac{5R}{2}$ (D) $\frac{7R}{2}$

Ans. B

Sol. $v^2 = kT$

$2V\,dV = kdT$

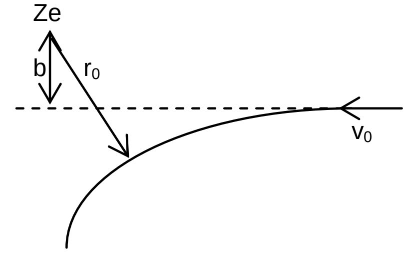
$\Delta Q = \Delta U + \Delta W$

$c = c_v + \frac{Pdv}{ndT}$

Pv = nRT

Pdv + vdP = nRdT

74. An α - particle is projected with velocity v_0 towards a very heavy nucleus of charge +Ze (where Z is the atomic number) from a very large distance as shown in the figure. If the distance of nearest approach is r_0 then the velocity at this point is



(A) $\frac{v_0 b}{r_0}$ (B) $\frac{v_0 r_0}{b}$

(C) $\frac{4v_0}{4+A}$ (D) $\frac{4v_0}{4-A}$

Ans. A

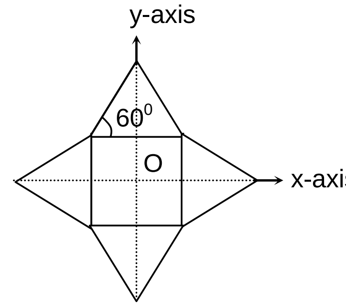
Sol. $mv_0b = mvr_0$

$v = \frac{v_0 b}{r_0}$

75. Moment of inertia of a uniform symmetric plate as shown in figure about x-axis is I. Moment of inertia of this plate about an axis passing through centre of plate O and perpendicular to the plane of plate is

(A) 2I (B) I

(C) I/2 (D) I/4



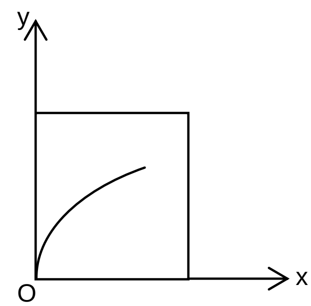
Ans. A

Sol. From perpendicular axis theorem $I_z = I_x + I_y = 2I$

76. A beam of light enters a rectangular slab at a nearly grazing incidence at the point (0,0). The refractive index μ_x of the slab depends only on the x coordinate. The slope of trajectory followed by the beam in the slab at a general x coordinate will be proportional to

(A) $\frac{1}{\mu_x}$ (B) μ_x

(C) $\frac{1}{\sqrt{\mu_r^2 - 1}}$ (D) $\sqrt{\mu_r^2 - 1}$



Ans. C

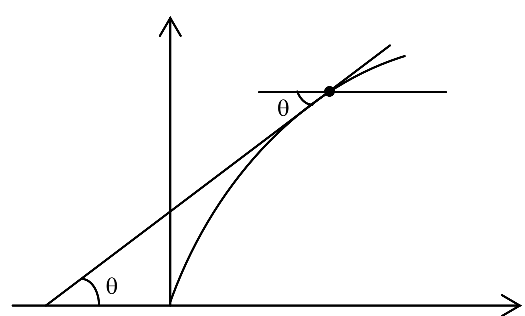
Sol. Slope of trajectory

$\tan\theta = \frac{dy}{dx}$

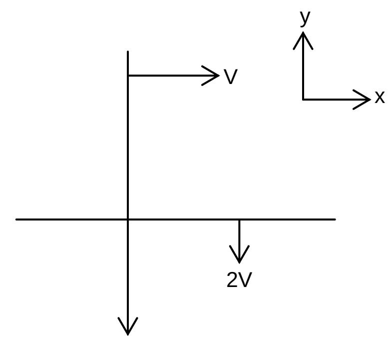
$\mu_x \sin\theta = \text{const} = C(\text{say})$

$\sin\theta = \frac{C}{\mu_x} = \frac{1}{\mu}$

$\tan\theta = \frac{1}{\sqrt{\mu_x^2 - 1}}$



77. Two thin rods are moving perpendiculy as shown in the figure. If the friction acting between them is F_R then unit vector in the direction of friction force on the rod lying along x axis is



(A) $\frac{-\hat{i}-2\hat{j}}{\sqrt{5}}$

(B) $\frac{\hat{i}+2\hat{j}}{\sqrt{5}}$

(C) $\frac{3\hat{i}+2\hat{j}}{\sqrt{5}}$

(D) none of these

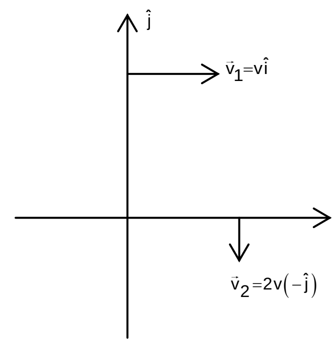
Ans. B

Sol. $\vec{v}_{21} = \vec{v}_2 - \vec{v}_1 = -v\hat{i} - 2v\hat{j}$

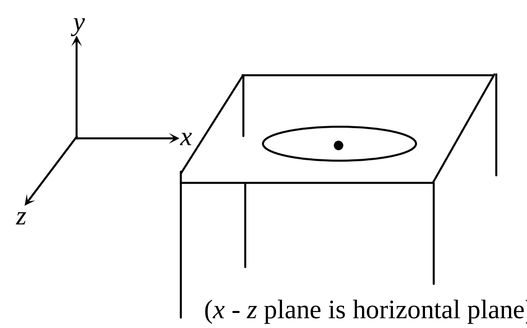
$\vec{v}_{21} = -\frac{\hat{i}-2\hat{j}}{\sqrt{5}}$

So friction on 2[nd] rod will act opposite to $\hat{v}_{21}$

$\Rightarrow \hat{F}_r = \frac{\hat{i}+2\hat{j}}{\sqrt{5}}$



78. A uniform conducting ring of mass π kg and radius 1 m is kept on smooth horizontal table. A uniform but time varying magnetic field $B = (\hat{i} + t^2\hat{j})$ Tesla is present in the region. (Where t is time in seconds). Resistance of ring is 2Ω. Then, ($g = 10$ m/s^2). Time (in second) at which ring start toppling is



(x - z plane is horizontal plane)

(A) $\frac{10}{\pi}$

(B) $\frac{20}{\pi}$

(C) $\frac{5}{\pi}$

(D) $\frac{25}{\pi}$

Ans. A

Sol. $i = \frac{\frac{d\phi}{dt}}{R} = \frac{(\pi \cdot 1^2)\cdot(0+2t)}{2} = \pi t$

At the time of toppling

$T_{ng} = |\vec{\mu} \times \vec{B}|$

$= \pi \times g \times 1 = |(\pi \cdot 1^2 \times \Lambda t)(-\hat{j}) \times (i + t^2\hat{j})|$

$= \pi g = |\pi^2 + \hat{k}| = \pi^2 t$

$\therefore \quad t = \frac{g}{\pi} = \frac{10}{\pi}$

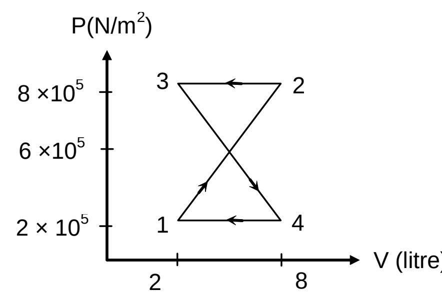
79. The work done in the process 1 – 2– 3–4 shown on P-V diagram is

(A) 300 J

(B) 600 J

(C) 900 J

(D) 1200J



Ans. C

Sol. $V_4 - V_1 = 6\text{litre}$

From geometry $V_2 - V_3 = 3\text{litre}$

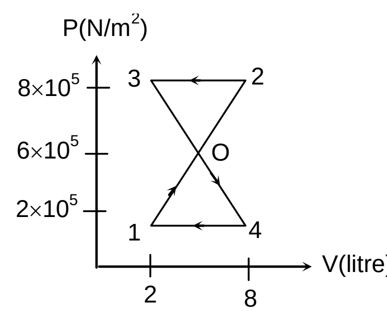
$W_{104} = \frac{1}{2} \times 4 \times 10^5 \times 6 \times 10^{-3}$

$= 1200\text{J}$

$W_{230} = -\frac{1}{2} \times 2 \times 10^5 \times 3 \times 10^{-3}$

$= -300\text{J}$

$W = 900 \text{ J}$



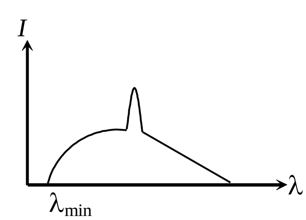
80. An X-ray tube has three main controls.

(i) the target material (its atomic number Z)

(ii) the filament current (I_f) and

(iii) the accelerating voltage (V)

Figure shows a typical intensity distribution against wavelength. Which of the following is incorrect?



(A) The limit λ_{min} is proportional to V^{-1}

(B) The sharp peak shifts to the right as Z is increased

(C) The penetrating power of X ray increases if V is increased

(D) The intensity everywhere increases if filament current I_f is increased

Ans. B

Sol. As accelerating voltage is increased, energy increases; the λ_{min} decreases and X-rays are getting harder (less wavelength) and penetrating power increases if filament current increases, more electrons are emitted.

CHEMISTRY

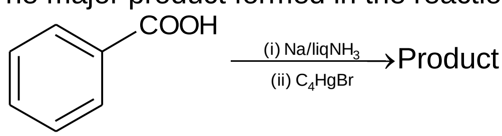
81. The molar conductivity of 0.009 M aqueous solution of a weak acid (HA) is 0.005 Sm^2 mol^{-1} and the limiting molar conductivity of HA is 0.05 Sm^2 mol^{-1} at 298 K. Assuming activity coefficient to be unity, the acid dissociation constant(K_a) of HA at this temperature is

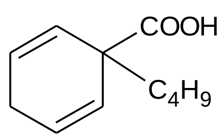
(A) 1×10^{-4} (B) 0.1
(C) 9×10^{-4} (D) 1.1×10^{-5}

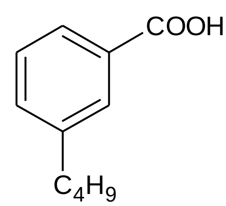
Ans. C

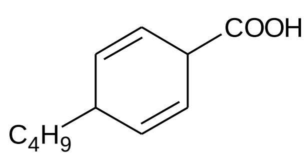
Sol. Degree of dissociation $'\alpha' = \frac{\Lambda_m}{\Lambda^{\infty}}; \Lambda_m = \frac{\kappa}{C}$

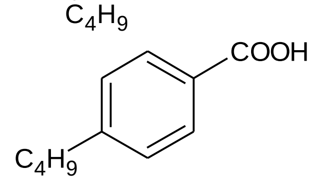
82. The major product formed in the reaction is



(A)


(B)


(C)


(D)


Ans. C

Sol. Birch reduction

83. An electron is found in an orbital with a radial and two angular nodes which orbital the electron is in

(A) 1s (B) 2p
(C) 3d (D) 4d

Ans. D

Sol. Number of radial node = n - ℓ - 1

No. of angular node = ℓ

84. The decreasing order of the first ionization energy of the following element is

(A) $Xe > Be > As > Al$ (B) $Xe > As > Al > Be$
(C) $Xe > As > Be > Al$ (D) $Xe > Be > Al > As$

Ans. A

Sol. Noble gases have high ionization energies.

85. For H^- like atoms, the ground sale energy is proportional to (where u is reduced mass)

(A) $\frac{\mu}{z^2}$ (B) $\frac{z^2}{\mu}$

(C) μz^2 (D) $\frac{1}{\mu z^2}$

Ans. B

Sol. $E \alpha = \frac{z^2}{n^2}$

86. Two total number way in which two non-identical spin 1/2 portions can be oriented relative to a constant magnetic field is

(A) 1 (B) 2

(C) 3 (D) 4

Ans. B

Sol. Total spin 'S' = 2n + 1, hence two ways.

87. If the electron were spin 3/2 particles, instead of spin 1/2 then the number of electron that can be accommodated in a level are

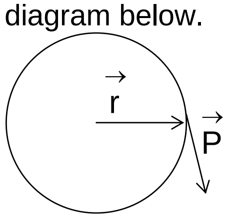
(A) 2 (B) 3

(C) 4 (D) 5

Ans. C

Sol. S = 2n + 1

88. An electron moves around the nucleus in a circular orbit, according to the Bohr model. The radial vector $\vec{r}$ and the instantaneous linear moment vector $\vec{P}$ are shown in the diagram below.



The direction of the angular momentum vector is

(A) along $\vec{r}$ (B) along $\vec{P}$

(C) opposite to $\vec{P}$ (D) perpendicular to both $\vec{r}$ and $\vec{P}$

Ans. D

Sol. Get the projection.

89. The compound which has L $\leftarrow$ M charge transfer is

(A) $Ni(CO)_4$
(B) $K_2Cr_2O_7$
(C) HgO
(D) $[Ni(H_2O)_6]^{2+}$

Ans. A

Sol. CO is π-acceptor ligand hence there will be charge transfer from Ni to CO(ligand).

90. The electrons identified by quantum numbers n and ℓ, (i) n = 4, ℓ = 1, (ii) n = 4, ℓ = 0, (iii) n = 3, ℓ = 2, (iv) n = 3, ℓ = 1 can be placed in order of increasing energy from lowest to highest as

(A) (iv) < (ii) < (iii) < (i)
(B) (ii) < (iv) < (i) < (iii)
(C) (i) < (iii) < (ii) < (iv)
(D) (iii) < (i) < (iv) < (ii)

Ans. A

Sol. (i) 4p (ii) 4s (iii) 3d (iv) 3p