FIITJEE INTERNAL TEST
CBSE MOCK TEST – 1
MATHEMATICS
Class – XII
ANSWERS AND SOLUTIONS
Section A

1. $a=\frac{-35}{12}$

Sol. $\because\ A=\begin{vmatrix}0 & 5 & a\\ -3 & 0 & 1\\ 7 & -4 & 0\end{vmatrix}$

$=0-5(0-7)+a(12-0)$

$=12a+35$

$\because|A|=0$

$12a+35=0$

$a=\frac{-35}{12}$

2. Continuous

Sol. $f(x)=|x+1|$

$f(-1)=0$

$\underset{x\to-1^+}{Lt} f(x)=\underset{h\to0}{Lt}|-1+h+1|=0$

$\underset{x\to-1^-}{Lt} f(x)=\underset{h\to0}{Lt}|-1-h+1|=\underset{h\to0}{Lt}|h|=0$

$\therefore$ LHL = RHL = value of function

$\therefore$ $f(x)$ is continuous at $x=-1$

3. $\frac{\tan(3-4x)}{-4}-x+c$

Sol. $\int\tan^2(3-4x)dx$

$=\int\sec^2(3-4x)-1dx$

$=\frac{\tan(3-4x)}{-4}-x+c$

4. $2\sqrt{5}$

Sol. Distance of (2, 3, 4) from y – axis

$=\sqrt{2^2+4^2}$

$=2\sqrt{5}$

Section B

5. $\begin{bmatrix} -1 & 4 \\ -5 & 2 \end{bmatrix}$

Sol. If $A = \begin{bmatrix} 3 & 4 \\ 5 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$

$A - 2B + X = 0$

$X = 2B - A$

$= 2\begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 4 \\ 5 & 0 \end{bmatrix}$

$= \begin{bmatrix} 2 & 8 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 4 \\ 5 & 0 \end{bmatrix}$

$= \begin{bmatrix} -1 & 4 \\ -5 & 2 \end{bmatrix}$

6. $y = \tan^{-1}(a + bx)$

$\frac{dy}{dx} = \frac{1}{1 + (a + bx)^2} xb$

at $x = 0, \frac{dy}{dx} = 1$

$1 = \frac{b}{1 + a^2}$

$1 + a^2 = b$

7. $f(x) = x^2 - 5x + 1$ in $[0,5]$

$f'(x) = 2x - 5$

$f'(x) = 0$

$x = \frac{5}{2}$

$\therefore$ If $x \in \left[0, \frac{5}{2}\right), f'(x) < 0$ decreasing

If $x \in \left(\frac{5}{2}, 5\right], f'(x) > 0$ increasing

$\therefore f(x)$ is neither increasing nor decreasing in $[0,5]$

8. 10.05

Sol. $f(x) = \sqrt{x}$

$x = 100, y = 10$

$\Delta x = 1, \Delta y = ?$

$\Delta y = \frac{dy}{dx}.\Delta x = \frac{1}{2\sqrt{x}}.\Delta x$

$= \frac{1}{2\sqrt{100}} \times \Delta x$

$= \frac{1}{20} \times 1$

$\sqrt{101} = 10 + \frac{1}{20}$

$= 10.05$

9. $\frac{7}{\sqrt{5}}$

Sol. $\vec{r} = \hat{i} + \hat{j} + \alpha\left(2\hat{i} + \hat{j} + 4\hat{k}\right)$ …………..(i)

$\vec{r}.\left(-2\hat{i} + \hat{k}\right) = 5$ …………..(ii)

$\therefore$ line is parallel to plane $\vec{n}.\vec{b} = 0$

$-4 + 0 + 4 = 0$

Point on line $\Rightarrow (1,1,0)$

Plane $-2x + z = 5$

Distance $= \frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}$

$= \left|\frac{-2 + 0 - 5}{\sqrt{4 + 1}}\right|$

$= \frac{7}{\sqrt{5}}$

10. $\frac{5}{9}$

Sol. $P(A) = \frac{7}{13}$

$P(B) = \frac{9}{13}$

$P(A \cap B) = \frac{4}{13}$

$P\left(\frac{A'}{B}\right) = \frac{P(A' \cap B)}{P(B)}$

$= \frac{P(B) - P(A \cap B)}{P(B)}$

$= \frac{\frac{9}{13} - \frac{4}{13}}{\frac{9}{13}}$

$= \frac{5}{9}$

11. x = 25
y =30

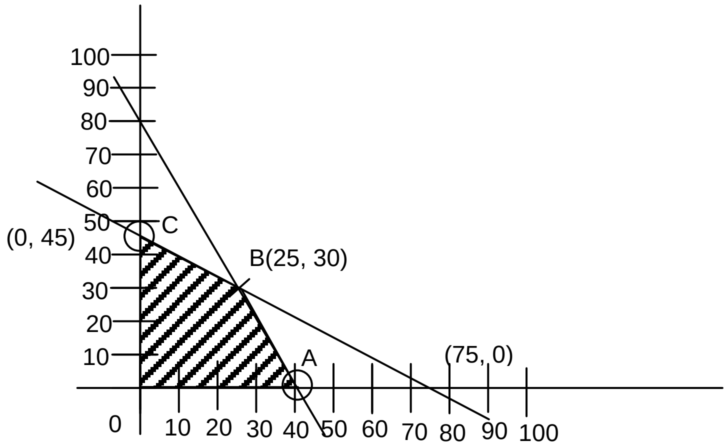
Sol. Say number of bikes of Model x = x
number of bikes of Model y = y

$x \geq 0,\ y \geq 0$

$6x + 10y \leq 450$ Man hour

$200x + 1000y \leq 80000$ cost

Profit Z = $z = 1000x + 500y$



OABC is solution

O (0, 0), A (40, 0), B (25, 30), C (0, 45)

Z at 0 $\Rightarrow 0$

Z at A $\Rightarrow 40000$

Z at $B \Rightarrow 45000$

Z at C $\Rightarrow 22500$

$\therefore$ For max Profit 45000

$x = 25$

$y = 30$

12. $\sin x + x\cos\alpha + c$

Sol. $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos\alpha} dx$

$= \int \frac{\cos^2 x - \cos^2 \alpha}{\cos x - \cos\alpha} dx$

$= \int \cos x + \cos\alpha \, dx$

$= \sin x + x\cos\alpha + C$

Section C

13. $\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right)$

Say $\sin^{-1}\frac{3}{4} = \theta \quad \sin\theta = \frac{3}{4}$

$\sin\theta = \frac{2\tan\theta/2}{1+\tan^2\theta/2}$

$\frac{3}{4} = \frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$

Put $\tan\frac{\theta}{2} = t$

$3t^2 + 3 = 8t$

$3t^2 - 8t + 3 = 0$

$t = \frac{8 \pm \sqrt{64-36}}{6}$

$= \frac{8 \pm 2\sqrt{7}}{6}$

$= \frac{4 \pm \sqrt{7}}{3}$

For $\sin\theta = \frac{3}{4} \quad \theta < 90$

$\therefore \frac{\theta}{2} < 45$

$\therefore \tan\frac{\theta}{2} < 1$

$\therefore \tan\frac{\theta}{2} \neq \frac{4+\sqrt{7}}{3}$

$\therefore \tan\frac{\theta}{2} = \frac{4-\sqrt{7}}{3}$

OR

13. $\tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right)$

$= \frac{1+\tan\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)}{1-\tan\left(\frac{1}{2}\cos^{-1}\frac{a}{b}\right)}$

Say $\theta = \cos^{-1}\frac{a}{b}$

$\therefore \cos\theta = \frac{a}{b}$

$\tan\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \sqrt{\frac{b-a}{b+a}}$

$= \frac{1+\sqrt{\frac{b-a}{b+a}}}{1-\sqrt{\frac{b-a}{b+a}}}$

$= \frac{\sqrt{b+a} - \sqrt{b-a}}{\sqrt{b+a} - \sqrt{b-a}}$

Rationalise

$= \frac{b+a+b-a+2\sqrt{b^2-a^2}}{2a}$

$= \frac{b+\sqrt{b^2-a^2}}{a}$

14. 0, –12

Sol. $\begin{vmatrix} 4-x & 4+x & 4+x \\ 4+x & 4-x & 4+x \\ 4+x & 4+x & 4-x \end{vmatrix} = 0,$

$R_1 \to R_1 + R_2 + R_3$

$(12+x)$

$\begin{vmatrix} 1 & 1 & 1 \\ 4+x & 4-x & 4+x \\ 4+x & 4+x & 4-x \end{vmatrix} = 0$

$C_1 \to C_1 - C_3,\ C_2 \to C_2 - C_3$

$(12+x)$

$$\begin{vmatrix} 0 & 0 & 1 \\ 0 & -2x & 4+x \\ 2x & 2x & 4-x \end{vmatrix} = 0$$

$(12+x)4x^2 = 0$

$\therefore\ x = 0, -12$

OR

Sol. Use $A = \frac{A+A'}{2} + \frac{A-A'}{2}$

15. $\frac{x}{\sqrt{1-x^4}}$

Sol. $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)$

Put $x^2 = \cos\theta$

$y = \tan^{-1}\left(\frac{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}\right)$

$y = \tan^{-1}\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)$

$y = \frac{\pi}{4}-\frac{\theta}{2}$

$y = \frac{1}{4}-\frac{1}{2}\cos^{-1}x^2$

$\frac{dy}{dx} = \frac{-1}{2}\times\frac{-1}{\sqrt{1-x^4}}\times 2x = \frac{x}{\sqrt{1-x^4}}$

OR

15. $\frac{1}{2at\cos^3 2t}$

Sol. $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{a(2\cos 2t + 4\sin 2t - 2\cos 2t)}{a(-2\sin 2t + 2\sin 2t + 4t\cos 2t)}$

$\frac{dy}{dx} = \tan 2t$

$\therefore \frac{d^2y}{dx^2} = 2\sec^2 2t.\frac{dt}{dx}$

$= \frac{2\sec^2 2t}{4\cos 2t}$

$= \frac{1}{2at\cos^3 2t}$

16. $\frac{1}{2\sqrt{3}}\tan^{-1}\frac{x-\frac{1}{x}}{\sqrt{3}} + \frac{1}{4}\log\left|\frac{x+\frac{1}{x}-1}{x+\frac{1}{x}+1}\right| + C$

Sol. $\int \frac{x^2}{x^4+x^2+1}dx$

$= \frac{1}{2}\int \frac{x^2+1+x^2-1}{x^4+x^2+1}dx$

$= \frac{1}{2}\int \frac{x^2+1}{x^4+x^2+1}dx + \frac{1}{2}\int \frac{x^2-1}{x^4+x^2+1}dx$

$= \frac{1}{2}\int \frac{1+\frac{1}{x^2}dx}{x^2+\frac{1}{x^2}+1} + \frac{1}{2}\int \frac{1-\frac{1}{x^2}}{x^2+\frac{1}{x^2}+1}dx$

Say $u = x - \frac{1}{x}$ $\qquad v = x + \frac{1}{x}$

$du = 1+\frac{1}{x^2}dx \qquad dv = 1-\frac{1}{x^2}dx$

$= \frac{1}{2}\int \frac{du}{u^2+3} + \frac{1}{2}\int \frac{du}{v^2-1}$

$= \frac{1}{2}\times\frac{1}{\sqrt{3}}\tan^{-1}\frac{u}{\sqrt{3}} + \frac{1}{2}\times\frac{1}{2}\log\left|\frac{v-1}{v+1}\right|$

$= \frac{1}{2\sqrt{3}}\tan^{-1}\frac{x-\frac{1}{x}}{\sqrt{3}} + \frac{1}{4}\log\left|\frac{x+\frac{1}{x}-1}{x+\frac{1}{x}+1}\right| + C$

17. $\frac{5}{3} - \log 4$

Sol. $\int_0^1 \frac{x}{1+\sqrt{x}}dx$

$x = t^2$

$dx = 2t\,dt$

$\int_0^1 \frac{t^2.2t\,dt}{1+t} = 2\int_0^1 \frac{t^3dt}{t+1}$

$= 2\int_0^1 \frac{t^3+1-1}{t+1}dt = 2\int_0^1 t^2+1-t-\frac{1}{t+1}dt$

$= 2\left[\frac{t^3}{3}+t-\frac{t^2}{2}-\log|t+1|\right] = 2\left[\frac{1}{3}+1-\frac{1}{2}-\log 2\right]$

$= 2\left[\frac{5}{6}-\log 2\right] = \frac{5}{3}-\log 4$

OR

17. $\frac{-\pi}{8}\log 2$

Sol. $\int_0^1 \frac{\log(17x)}{1+x^2}dx$

$x = \tan\theta$

$\int_0^{\pi/4} \log(1+\tan\theta)\,d\theta = \frac{-\pi}{8}\log 2$

18. $\left(1+x^2\right)\tan y = 2$

Sol. Particular solution of $\sec^2 y\left(1+x^2\right)dy + 2x\tan y\,dx = 0$

$$\frac{\sec^2 y\,dy}{\tan y} + \frac{2x\,dx}{1+x^2} = 0$$

$$\log\left|\tan y\right| + \log\left|1+x^2\right| = \log c$$

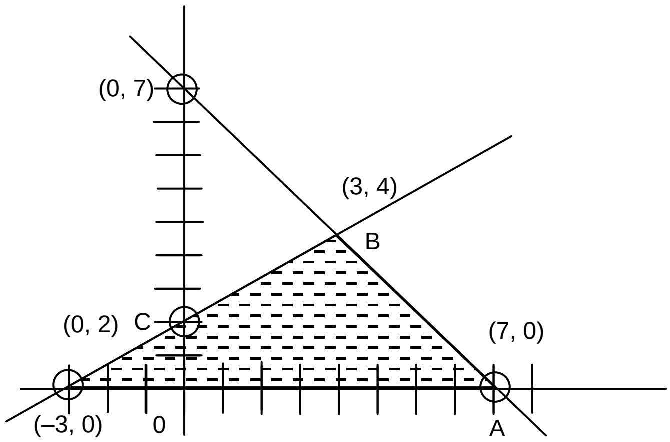
$$y = \frac{\pi}{4},\ x = 1$$

$c = t$

$\left(1+x^2\right)\tan y = 2$

19. $x = 0,\ y = 2$

Sol.



OABC is solution set

Z at $\theta \to 0$

Z at $A(7, 0) \Rightarrow 13\times 7 - 0 = 91$

Z at $C(0,2) \Rightarrow 0 - 15\times 2 = -30$

Z at B (3, 4) $\Rightarrow 13\times 3 - 15\times 4 = 33 - 60 = -21$

$\therefore$Z min at C = – 30 x = 0, y = 2

20. $\frac{1}{2}$

Sol. $\vec{a} = 2\hat{i} - 3\hat{j} - \hat{k}$

$\vec{b} = -\hat{i} + \hat{k}$

$\vec{c} = 2\hat{j} - \hat{k}$

$\vec{a} + \vec{b} = \hat{i} - 3\hat{j}$

$\vec{b} + \vec{c} = -\hat{i} + 2\hat{j}$

Area $= \frac{1}{2}\left|\left(\vec{a}+\vec{b}\right)\times\left(\vec{b}+\vec{c}\right)\right|$

$$= \frac{1}{2}\left\|\begin{matrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -3 & 0\\ -1 & 2 & 0\end{matrix}\right\|$$

$$= \frac{1}{2}\left|-\hat{k}\right| = \frac{1}{2}$$

21. $64\hat{i}-2\hat{j}-28\hat{k}$

Sol. $\vec{a}=\hat{i}+4\hat{j}+2\hat{k}$

$\vec{b}=3\hat{i}-2\hat{j}+7\hat{k}$

$\vec{c}=2\hat{i}-\hat{j}+4\hat{k}$

Say $\vec{p}=\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$

$2\alpha-\beta+4\gamma=18$ (i) $\vec{p}.\vec{c}=18$

$\alpha+4\beta+2\gamma=0$ $\vec{p}.\vec{a}=0$

$3\alpha-2\beta+7\gamma=0$ $\vec{p}.\vec{b}=0$

$$\frac{\alpha}{28+4}=\frac{-\beta}{7-6}=\frac{\gamma}{-2-12}$$

$$\frac{\alpha}{32}=\frac{-\beta}{1}=\frac{\gamma}{-14}=\alpha$$

Put in (1)

$2(32\alpha)+\alpha-56\alpha=18$

$9\alpha=18$

$\alpha=2$

$\therefore \vec{p}=64\hat{i}-2\hat{j}-28\hat{k}$

22. $\frac{1}{22}$

Sol. $E_1 \Rightarrow$ Perks is cyclist

$E_2 \Rightarrow$ Perks is scooter driver

$E_3 \Rightarrow$ Perks is Car driver

$A \Rightarrow$ Person meet with an accident

$$P(E_1)=\frac{4000}{24000}=\frac{1}{6}$$

$$P(E_2)=\frac{8000}{24000}=\frac{1}{3}$$

$$P(E_3)=\frac{12000}{24000}=\frac{1}{2}$$

$$P\left(\frac{E_1}{A}\right)=\frac{P(E_1).P\left(\frac{A}{E_1}\right)}{P(E_1).P\left(\frac{A}{E_1}\right)+P(E_2).P\left(\frac{A}{E_2}\right)+)P(E_3).P\left(\frac{A}{E_3}\right)}$$

$$=\frac{\frac{1}{6}\times 0.02}{\frac{1}{6}\times 0.02+\frac{1}{3}\times 0.06+\frac{1}{2}\times 0.3}$$

$$=\frac{\frac{1}{3}\times 0.01}{\frac{1}{3}\times 0.01+\frac{1}{3}\times 0.06+\frac{1}{2}\times 0.3}$$

$$=\frac{.01}{.01+0.01+\frac{1}{3}\times 0.06+\frac{1}{2}\times 0.3}$$

$$= \frac{.01}{.01 + .06 + .15} = \frac{1}{22}$$

23. $^5C_2 \times \left(\frac{1}{6}\right)^2 \times \left(\frac{5}{6}\right)^3 \times \frac{1}{6}$

Sol. Upto 5 toss there should be two sixes

$$\Rightarrow {}^5C_2 \times \left(\frac{1}{6}\right)^2 \times \left(\frac{5}{6}\right)^3 \times \frac{1}{6}$$

Section D

24. $x = -1, y = 2, z = 3$

Sol. $A = \begin{bmatrix} 2 & 1 & -2 \\ 1 & 3 & 1 \\ 3 & -1 & 1 \end{bmatrix}$

$$|A| = \begin{bmatrix} 2 & 1 & -2 \\ 1 & 3 & 1 \\ 3 & -1 & 1 \end{bmatrix}$$

$$= 2(3+1) - 1(1-3) - 2(-1-9)$$

$$= 8 + 2 + 20 = 30$$

$$\text{adj} A = \begin{bmatrix} 4 & 1 & 7 \\ 2 & 8 & -4 \\ -10 & 5 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{30}\begin{bmatrix} 4 & 1 & 7 \\ 2 & 8 & -4 \\ -10 & 5 & 5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{30}\begin{bmatrix} 4 & 2 & -10 \\ 1 & 8 & 5 \\ 7 & -4 & 5 \end{bmatrix}\begin{bmatrix} 9 \\ 2 \\ 7 \end{bmatrix}$$

$$= \frac{1}{30}\begin{bmatrix} -30 \\ 60 \\ 90 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$$

$x = -1, y = 2, z = 3$

25. $f(x)$ given all numbers

26. $\ell = r\sqrt{2}$ $b = \frac{r}{\sqrt{2}}$

Sol. $A = 2xy$

$= 2r\cos\theta . r\sin\theta$

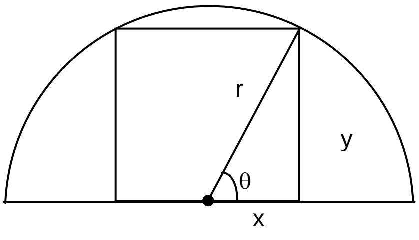
$= r^2 \sin 2\theta$

For are a $\sin 2\theta = 1$

$\theta = 45^o$

$\ell = 2r\cos 45 = r\sqrt{2}$

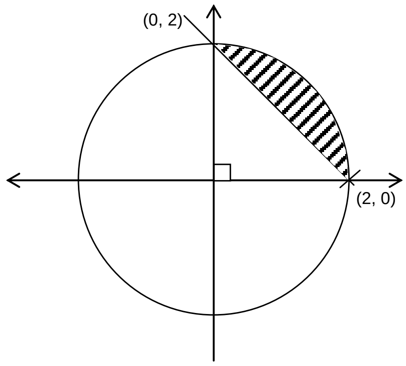
$b = r\sin 45 = \frac{r}{\sqrt{2}}$



27. $\pi - 2$

Sol. Area $= \frac{\pi \times 2^2}{4} - \frac{1}{2} \times 2 \times 2$

$= \pi - 2$



28. $\frac{1}{2}\log\left(x^2 + y^2\right) + \tan^{-1}\frac{y}{x} = \frac{1}{2}\log 2 + \frac{\pi}{4}$

Sol. $\left(x + y\right)dy + \left(x - y\right)dx = 0$

All terms of degree 1

$\therefore$ Homogeneous

$$\frac{dy}{dx} = \frac{-x + y}{y + x} = \frac{\frac{y}{x} - 1}{\frac{y}{x} + 1}$$

$$v + x\frac{dv}{dx} = \frac{v - 1}{v + 1}$$

$$x\frac{dv}{dx} = \frac{v - 1}{v + 1} - b = \frac{v - 1 - b^2 - b}{v + 1}$$

$$\frac{v + 1}{v^2 + 1}dv = -\frac{dx}{x}$$

$$\frac{1}{2}\int \frac{2v\,dx}{v^2 + 1} + \frac{dv}{v^2 + 1} = -\int \frac{dx}{x}$$

$$\frac{1}{2}\log\left(b^2 + 1\right) + \tan^{-1}\theta = -\log x + c$$

$$\frac{1}{2}\log\left(y^2 + x^2\right) - \log x + \tan^{-1}\frac{y}{x} = -\log x + c$$

$$\log\sqrt{x^2 + y^2} + \tan^{-1}\frac{y}{x} = C$$

$x = 1,\ y = 1$

$$\log\sqrt{2} + \frac{\pi}{4} = c$$

$$\frac{1}{2}\log\left(x^2 + y^2\right) + \tan^{-1}\frac{y}{x} = \frac{1}{2}\log 2 + \frac{x}{y}$$

29. $\frac{2}{\sqrt{5}}$

Sol. $\vec{r} = \left(\hat{i} - 2\hat{j} + 3\hat{k}\right) + t\left(-\hat{i} + \hat{j} - 2\hat{k}\right)$

$\vec{r} = \left(\hat{i} - \hat{j} + \hat{k}\right) + s\left(\hat{i} - 2\hat{j} + 2\hat{k}\right)$

S.D. $= \frac{\left(\vec{c} - \vec{a}\right).\left(\vec{b} \times \vec{d}\right)}{\left|\vec{b} \times \vec{d}\right|}$

$$\vec{b} \times \vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 1 & -2 & 2 \end{vmatrix}$$

$= -2\hat{i} + \hat{k}$

$\vec{c} - \vec{a} = \hat{j} - 2\hat{k}$

$(\vec{c} - \vec{a})(\vec{b} \times \vec{d}) = 0 + 0 - 2$

$$\therefore \text{ S.D. } = \left| \frac{-2}{\sqrt{4+1}} \right| = \frac{2}{\sqrt{5}}$$

OR

29. 1

Sol. Coplanar vectors

$$\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$$

$C_1 \to C_1 - C_2 \quad C_2 \to C_2 - C_3$

$$\begin{vmatrix} a-1 & 0 & 1 \\ 1-b & b-1 & 1 \\ 0 & 1-c & c \end{vmatrix}$$

$(a-1)((b-1)c - (1-c)) + (1-b)(1-c) = 0$

$(a-1)(b-1)c + (1-a)(1-c) + (1-b)(1-c) = 0$

$$\frac{c}{1-c} + \frac{1}{1-b} + \frac{1}{1-a} = 0$$

$$-\frac{-c+1-1}{1-c} + \frac{1}{1-b} + \frac{1}{1-a} = 0$$

$$\therefore \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1$$