FIITJEE

ALL INDIA INTERNAL TEST SERIES

CHEMISTRY, MATHEMATICS & PHYSICS

SET - A

APT - 5 | **000000** | **Ai²TS - 4**

CLASS - XI

Time Allotted : 3 Hours **Maximum Marks: 198**

INSTRUCTIONS

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.
- You are not allowed to leave the Examination Hall before the end of the test.

Caution: Question Paper CODE as given above MUST be correctly marked in the answer OMR sheet before attempting the paper. Wrong CODE or no CODE will give wrong results.

A. General Instructions

- ✓ Attempt ALL the questions. Answers have to be marked on the OMR sheets.
- ✓ This question paper contains **Three Sections**.
- ✓ **Section – I** is *"Chemistry"*, **Section – II** is *"Mathematics"* and **Section – III** is *"Physics"*.
- ✓ Each Section is further divided into Two Parts: **Part – A** & **Part – C**.
- ✓ Rough spaces are provided for rough work inside the question paper. No additional sheets will be provided for rough work.
- ✓ Blank Papers, clip boards, log tables, slide rule, calculator, cellular phones, pagers and electronic devices, in any form, are not allowed.

B. Filling of OMR Sheet

1. Ensure matching of OMR sheet with the Question paper before you start marking your answers on OMR sheet.
2. On the OMR sheet, darken the appropriate bubble with HB pencil for each character of your Enrolment No. and write in ink your Name, Test Centre and other details at the designated places.
3. OMR sheet contains alphabets, numerals & special characters for marking answers.

C. Marking Scheme For All Two Parts.

(i) **PART-A (01 – 06)** contains 6 Multiple Choice Questions which have Only One Correct answer. Each question carries **+3 marks** for correct answer and **–1 mark** for wrong answer.

(ii) **PART-A (07 – 12)** contains 6 Multiple Choice Questions which have One or More Than One Correct answer.
For each question in this part, you will be awarded
Full Marks: **+4** If only the bubble(s) corresponding to all the correct options(s) is (are) darkened.
Partial Marks: **+1** For darkening a bubble corresponding **to each correct option,** provided NO incorrect option is darkened.
Zero Marks: **0** If none of the bubbles is darkened.
Negative Marks: –2 In all other cases.
For example, if **(A), (C) and (D)** are all the correct options for a question, darkening all these three will result in **+4 marks**; darkening only **(A) and (D)** will result in **+2 marks**; and darkening **(A) and (B)** will result in **–2** marks, as a wrong option is also darkened.

(iii) **PART-C (01 – 06)** contains 6 Numerical Based questions, the answer of which may be positive or negative numbers or decimals (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30). If the numerical value has more than two decimal places, truncate/round-off the value to TWO decimal places. Each question carries **+4 marks** for correct answer. **There is no negative marking.**

Name of Candidate :

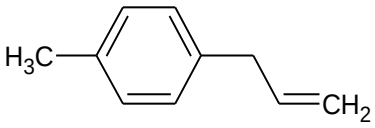
Batch ID : **Date of Examination :** / / 20

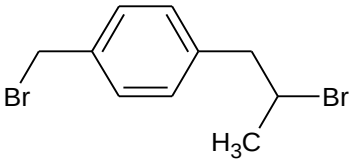
Enrolment Number :

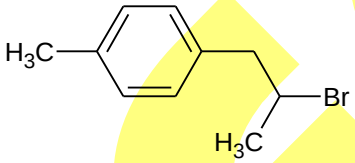
SECTION – I: CHEMISTRY

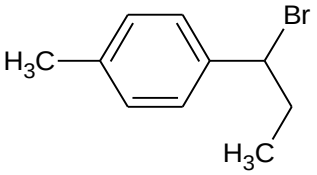
PART – A
(Single Correct Answer Type)

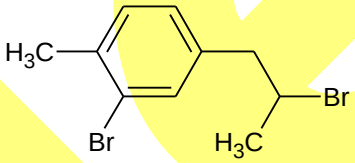
This part contains **6 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

1.  $+ HBr \longrightarrow (major)$ Product

(A) 

(B) 

(C) 

(D) 

2. A mixture of three alkyl chloride CH_3Cl, CH_3CH_2Cl, $CH_3CH_2CH_2Cl$ undergoes Wurtz coupling reaction. The product contains:
(A) Ethane, pentane, hexane only
(B) Propane, pentane, butane only
(C) Ethane, Propane, pentane, butane
(D) Ethane, Propane, butane, pentane, hexane

3. Calculate the pH of a solution having 0.1 M CH_3COOH along with 0.1 M $(CH_3COO)_2Ca$.
(Given: $K_a(CH_3COOH) = 2 \times 10^{-5}$)
(A) 4.7
(B) 5
(C) 5.3
(D) 9.3

4. Two moles of a gas expand reversibly and isothermally at temperature of 300K. Initial volume of the gas is 1 L while the final pressure is 4.926 atm. The work done by gas is
(A) –11488.28 J
(B) –15036.28 J
(C) –22488.28 J
(D) –33488.28 J

5. For the reaction: $3A_{(g)} \longrightarrow 2B_{(g)}$, the rate of formation of 'B' at 298 K is represented as $\ln\left(\frac{d[B]}{dt}\right) = -0.04 + 2\ln[A]$. The order of reaction is
(A) 0
(B) 1
(C) 2
(D) 3

6. The radial wave equation for hydrogen atom is:

$$\Psi = \frac{1}{16\sqrt{4}}\left(\frac{1}{a_o}\right)^{3/2}\left[(x-1)(x^2-8x+12)\right]e^{-x/2}$$

where $x = 2r/a_o$, a_0 = radius of first Bohr's orbit
the minimum and maximum position of radial nodes from nucleus are:
(A) $a_o, 3a_o$
(B) $0.5a_o, 3a_o$
(C) $0.5a_o, a_o$
(D) $0.5a_o, 4a_o$

PART – A
(Multiple Correct Answer Type)

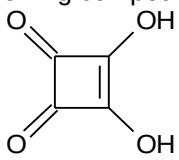
This part contains **6 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE THAN ONE** is/are correct.

7. Benzene reacts with 1 – chloro-propane in the presence of anhydrous. $AlCl_3$ to give (major as well as minor product):
(A) n – propyl benzene
(B) isopropyl benzene
(C) ethyl methyl benzne
(D) cumene

8. Which one is/are the **correct** statement(s)?
(A) Anhydrous $AlCl_3$ exists as Al_2Cl_6 (dimer)
(B) Anhydrous $AlCl_3$ is a trigonal planar molecule
(C) Anhydrous $AlCl_3$ fumes in air
(D) Anhydrous $AlCl_3$ is ionic

9. $B(OH)_3 + NaOH \rightleftharpoons NaBO_2 + Na[B(OH)_4] + H_2O$
How this reaction can be made to proceed in forward direction?
(A) Addition of cis 1,2 diol
(B) Addition of catechol
(C) Addition of trans 1,2 diol
(D) Addition od Na_2HPO_4

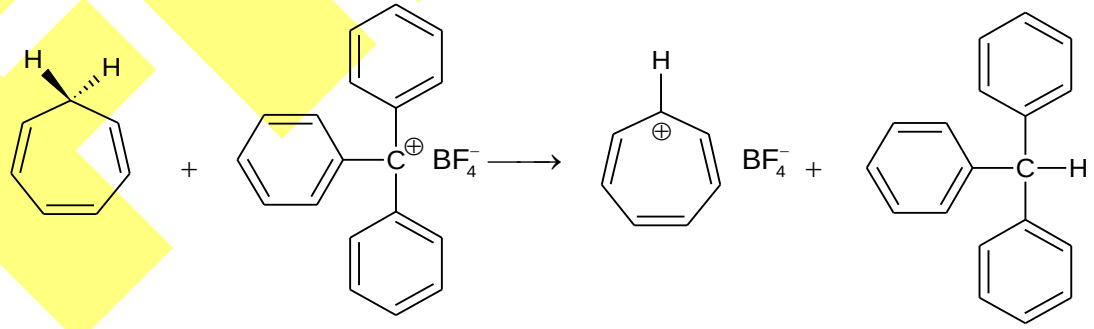
10. Consider the following compound below, select the correct statement(s):



(A) It liberate CO_2 with $NaHCO_3$
(B) It is more acidic than acetic acid.
(C) It is a dibasic
(D) all resonating structure of conjugate base are same

11. The van der Waals equation for one mole of a real gas can be written as $\left(P + \frac{a}{V^2}\right)(V - b) = RT$. For the gases H_2, NH_3 and CH_4, the value of 'a' $bar\ L^{-2}mol^{-2}$ are 0.2453, 4.170 and 2.253 respectively.
Which of the following can be inferred from the 'a' values?
(A) NH_3 can be most easily liquified
(B) H_2 can be most easily liquified
(C) value of 'a' for CH_4 is less than that of NH_3 because it has the lower molar mass
(D) greater the value of 'a' easier will be the liquefaction of gas

12. Consider the following reaction:

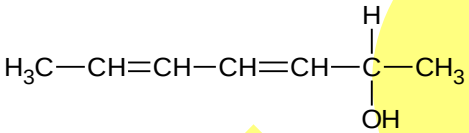


Which of the following statement(s) is (are) correct?
(A) Cation in the reactant side is approximately 10^{11} times less stable than that of product side
(B) cation in the reactant side is non–polar
(C) It is an acid–base reaction
(D) Reaction must be exothermic

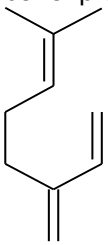
PART – C
(Numerical Answer Type)

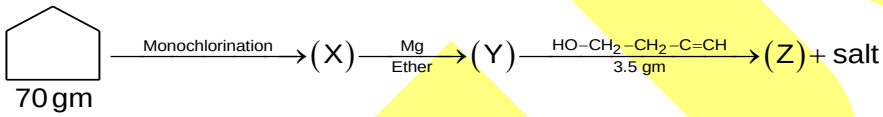
This part contains **6 numerical based questions**, the answer of which may be positive or negative numbers or decimals (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30). If the numerical value has more than two decimal places, truncate/round-off the value to TWO decimal places.

1. A sample of gas is compressed by an average pressure of 0.50 atmosphere so as to decrease its volume from $400cm^3$ to $200cm^3$. During the process $8.00\,J$ of heat flows out to surroundings. The change in internal energy of the system (in J) is

2. Number of stereoisomers of following compound 

   is x, then find the value of x/3?

3. What is the number of products formed on reductive ozonolysis of the compound 

4. 

   The number of moles of (Z) produced would be (assuming the yield to be 100%)?

5. Heat of hydrogenation of but-1-ene and 1,3-butadiene are –30.3 K cal/mol and –56.5 K cal/mole respectively. What is the resonance energy of 1,3-butadiene (in K cal/mol)?

6. A hydrocarbon (A) C_nH_{2n-4} on ozonolysis gives $(CH_3)_2CHCH_2CHO$, $OHCCH_2CH_2CHO$ and CH_3COCH_3. The value of n/2 is

SECTION – II: MATHEMATICS

PART – A
(Single Correct Answer Type)

This part contains **6 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

1. If $\left(1+x+2x^2\right)^{20} = a_0 + a_1x + a_2x^2 + ... + a_{40}x^{40}$, then $a_0 + a_2 + a_4 + ... + a_{38}$ equals

   (A) $2^{19}\left(2^{20}+1\right)$ (B) $2^{19}\left(2^{20}-1\right)$

   (C) $2^{20}\left(2^{19}-1\right)$ (D) none of these

2. We are required to form different words with the help of the letters of the word INTEGER. Let m_1 be the number of words in which I and N are never together and m_2 be the number of words which beginning with I and end with R, then m_1/m_2 is given by

   (A) 42 (B) 30

   (C) 6 (D) 1/30

3. If $\sum_{n=1}^{k}\left[\frac{1}{3}+\frac{n}{90}\right] = 21$, where [x] denotes the integral part of x, then k is equal to

   (A) 84 (B) 80

   (C) 85 (D) none of these

4. Let $3-i$ and $2+i$, (where $i=\sqrt{-1}$) be affixes of two points A and B in the argand plane and P represents of the complex number $z = x+iy$, then the locus of P if $|z-3+i| = |z-2-i|$ is

   (A) circle on AB as diameter (B) the line AB

   (C) the perpendicular bisector of AB (D) none of the above

5. For parabola $x^2+y^2+2xy-6x-2y+3=0$, the focus is

   (A) (1, -1) (B) (-1, 1)

   (C) (3, 1) (D) none of these

6. If α, β are eccentric angles of the extremities of a focal chord of an ellipse, then eccentricity of the ellipse is

   (A) $\frac{\cos\alpha+\cos\beta}{\cos(\alpha+\beta)}$ (B) $\frac{\sin\alpha-\sin\beta}{\sin(\alpha-\beta)}$

   (B) $\sec\alpha+\sec\beta$ (D) $\frac{\sin\alpha+\sin\beta}{\sin(\alpha+\beta)}$

PART – A
(Multiple Correct Answer Type)

This part contains **6 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE THAN ONE** is/are correct.

7. Let $f_n(\theta) = \tan\frac{\theta}{2}(1+\sec\theta)(1+\sec 2\theta)(1+\sec 4\theta).....\left(1+\sec 2^n\theta\right)$, then

   (A) $f_2\left(\frac{\pi}{16}\right)=1$ (B) $f_3\left(\frac{\pi}{32}\right)=1$

   (C) $f_4\left(\frac{\pi}{64}\right)=1$ (D) $f_5\left(\frac{\pi}{128}\right)=1$

8. If $\left(\cos^2 x+\frac{1}{\cos^2 x}\right)\left(1+\tan^2 2y\right)(3+\sin 3z) = 4$, then

   (A) x may be a multiple of π (B) x cannot be an even multiple of π

   (C) z can be a multiple of π (D) y can be a multiple of $\frac{\pi}{2}$

9. If in ΔABC, CD is the angular bisector of $\angle ACB$, then CD is equal to

(A) $\left(\frac{a+b}{2ab}\right)\cos\left(\frac{C}{2}\right)$ (B) $\left(\frac{a+b}{ab}\right)\cos\left(\frac{C}{2}\right)$

(C) $\left(\frac{2ab}{a+b}\right)\cos\left(\frac{C}{2}\right)$ (D) $\frac{b\sin A}{\sin\left(B+\frac{C}{2}\right)}$

10. Which of the following is/are correct?

(A) $101^{50}-99^{50}>100^{50}$ (B) $101^{50}-100^{50}>99^{50}$

(C) $(1000)^{1000}>(1001)^{999}$ (D) $(1001)^{999}>(1000)^{1000}$

11. If n objects are arranged in a row, then the number of ways of selecting three of these objects so that no two of them are next to each other is

(A) $\frac{(n-2)(n-3)(n-4)}{6}$ (B) ${}^{n-2}C_3$

(C) ${}^{n-3}C_3+{}^{n-3}C_2$ (D) none of these

12. A line which makes an acute angle θ with the positive direction of x-axis is drawn through the point P (3,4) to meet the line $x=6$ at R and $y=8$ at S, then

(A) $PR=3\sec\theta$ (B) $PS=4\operatorname{cosec}\theta$

(C) $PR+PS=\frac{2(3\sin\theta+4\cos\theta)}{\sin 2\theta}$ (D) $\frac{9}{(PR)^2}+\frac{16}{(PS)^2}=1$

PART – C
(Numerical Answer Type)

This part contains **6 numerical based questions**, the answer of which may be positive or negative numbers or decimals (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30). If the numerical value has more than two decimal places, truncate/round-off the value to TWO decimal places.

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1. If $(1.05)^{50}=11.658$, then $\sum_{n=1}^{49}(1.05)^n$ equals

2. If α,β are the roots of the equation $8x^2-3x+27=0$, then the value of $\left[\left(\alpha^2/\beta\right)^{1/3}+\left(\beta^2/\alpha\right)^{1/3}\right]$ is

3. If the line $y-\sqrt{3}x+3=0$ cuts the parabola $y^2=x+2$ at A and B, then $PA\cdot PB$ is equal to [where $P\equiv(\sqrt{3},0)$]

4. From the point $P(1,\sqrt{3})$ on the circle $x^2+y^2=4$ a tangent is drawn to the hyperbola $\frac{x^2}{4}-\frac{y^2}{1}=1$ which meets its transverse axis at Q. From Q a line is drawn parallel to conjugate axis, which cuts the hyperbola at R above the x-axis, then PR equals

5. In a triangle ABC, $\angle B=\frac{\pi}{3}$ and $\angle C=\frac{\pi}{4}$. Let D divide BC internally in the ratio 1:3. Then $\frac{\sin(\angle BAD)}{\sin(\angle CAD)}$ is equal to

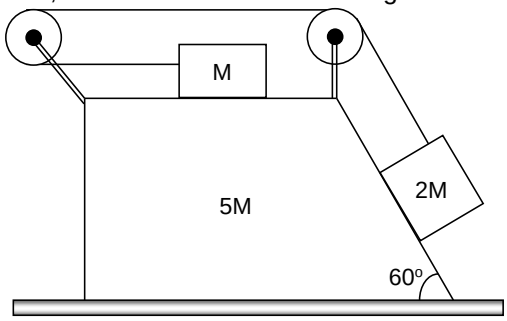
6. In a ΔABC, sides a, b, c are in AP and $\frac{2}{1!9!}+\frac{2}{3!7!}+\frac{1}{5!5!}=\frac{8^a}{(2b)!}$, then the maximum value of tan A tan B is equal to

SECTION – III: PHYSICS

PART – A
(Single Correct Answer Type)

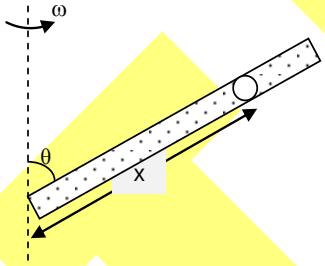
This part contains **6 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

1. In the system shown, the acceleration of the wedge of mass 5M is (there is no friction anywhere)



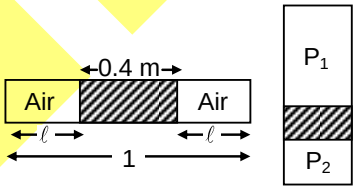
(A) zero
(B) g/2
(C) g/3
(D) g/4

2. A closed tube of length ℓ completely filled with water has a smaller air bubble trapped in it. When the tube is held at an angle θ with the vertical and rotated at a constant angular velocity ω about the vertical axis through its lower end, the bubble settles at some intermediate position in the tube at distance x form lower end. The value of x is



(A) $\frac{g\cos\theta}{\omega^2}$
(B) $\frac{g\cos\theta}{\omega^2\sin^2\theta}$
(C) $\frac{g}{\omega^2\sin^2\theta}$
(D) $\frac{\ell}{2}$

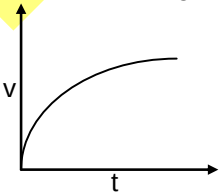
3. A long tube of length 1.2 m closed at both ends is lying horizontally. A mercury column of length 0.4 m is filled in its middle and in rest in two parts air is filled at atmospheric pressure. When the tube is turned to vertical position then displacement of mercury column will be (atmospheric pressure = 0.75 meter of Hg, density of water is 1000 kg/m³)



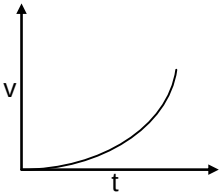
(A) 0.4 meter
(B) 0.3 meter
(C) 0.2 meter
(D) 0.1 meter

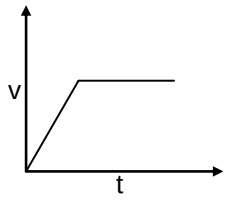
4. A piece of cork starts from rest at the bottom of a lake and floats up. Its velocity v is plotted against time t. Which of the following best represents the resulting curve?

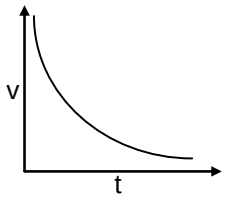
(A)



(B)



(C) 

(D) 

5. A current carrying wire heats a metal rod. The wire provides a constant power (P) to the rod. The metal rod is enclosed in an insulated container. It is observed that the temperature (T) in the metal rod changes with time (t) as

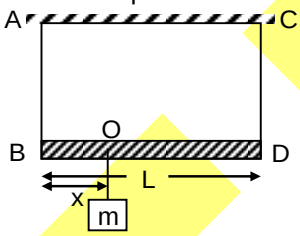
$$T(t) = T_0\left(1+\beta t^{1/4}\right)$$

where β is a constant with appropriate dimension while T_0 is a constant with dimension of temperature. The heat capacity of the metal is

(A) $\dfrac{4P\left(T(t)-T_0\right)^4}{\beta^4 T_0^5}$ (B) $\dfrac{4P\left(T(t)-T_0\right)}{\beta^4 T_0^2}$

(C) $\dfrac{4P\left(T(t)-T_0\right)^2}{\beta^4 T_0^2}$ (D) $\dfrac{4P\left(T(t)-T_0\right)^3}{\beta^4 T_0^4}$

6. A massless rod is suspended by two identical strings AB and CD of equal length. A block of mass m is suspended from point O such that BO is equal to 'x'. Further, it is observed that the frequency of 1st harmonic (fundamental frequency) in AB is equal to 2nd harmonic frequency in CD. Then, length L of BO is

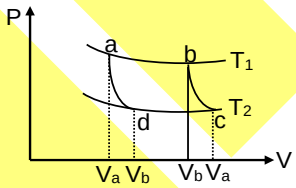


(A) L/5 (B) 4L/5
(C) 3L/4 (D) L/4

PART – A
(Multiple Correct Answer Type)

This part contains **6 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE THAN ONE** is/are correct.

7. Two adiabatic processes bc and ad for the same gas are given to intersect two isotherms at T_1 and T_2 (as shown). Then



(A) $\dfrac{V_a}{V_b}=\dfrac{T_2}{T_1}$ (B) $\dfrac{V_a}{V_d}=\dfrac{T_1}{T_2}$

(C) $\dfrac{V_b}{V_c}=\left(\dfrac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}}$ (D) $V_aV_c = V_bV_d$

8. A horizontal stretched string fixed at two ends, is vibrating in its fifth harmonic according to the equation $y(x,t)=0.01\text{m}\sin\left[\left(62.8\text{m}^{-1}\right)x\right]\cos\left[\left(628\text{s}^{-1}\right)t\right]$. Assuming $\pi = 3.14$, the correct statement(s) is (are)

(A) The number of nodes is 5.
(B) the length of the string is 0.25 m
(C) The maximum displacement of the midpoint of the string, from its equilibrium position is 0.01 m.
(D) The fundamental frequency is 100 Hz.

9. During an experiment, an ideal gas is found to obey a condition $\frac{p^2}{d} = \text{constant}$. $(d = \text{density of the gas})$. The gas is initially at temperature T, pressure p and density d. The gas expands such that density changes to $d/2$.

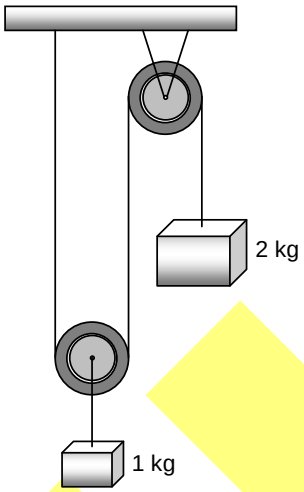
(A) The pressure of the gas changes to $\sqrt{2}p$

(B) The temperature of the gas changes to $\sqrt{2}T$

(C) The graph of the above process on $p - T$ diagram is parabola

(D) The graph of the above process on p-T diagram is hyperbola

10. In the pulley-block system shown in figure strings are light, the pulleys are light and smooth. The system is released from rest. At $t = 0.3$ s



(A) work done on 2 kg block by gravity is 6 J (B) work done on 2 kg block by string is -2 J

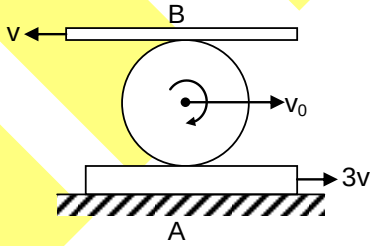
(C) work done on 1 kg block by gravity is -1.5 J (D) work done on 1 kg block by string is 2 J

11. A bag of mass M hangs by a long thread. A bullet of mass m comes horizontally with a velocity v and gets embedded in the bag. Then for the combined system, immediately after the collision, the

(A) momentum is $\frac{mvM}{M+m}$ (B) momentum is mv

(C) kinetic energy is $\frac{m^2v^2}{2(M+m)}$ (D) kinetic energy is $\frac{mv^2}{2}$

12. The disc of radius r is confined to roll without slipping at A and B. If the plates have the velocities shown, then



(A) Angular velocity of the disc is $2v/r$ (B) Linear velocity, $v_0 = v$

(C) Angular velocity of the disc is $3v/2r$ (D) None of these

PART – C
(Numerical Answer Type)

This part contains **6 numerical based questions**, the answer of which may be positive or negative numbers or decimals (e.g. 6.25, 7.00, -0.33, -.30, 30.27, -127.30). If the numerical value has more than two decimal places, truncate/round-off the value to TWO decimal places.

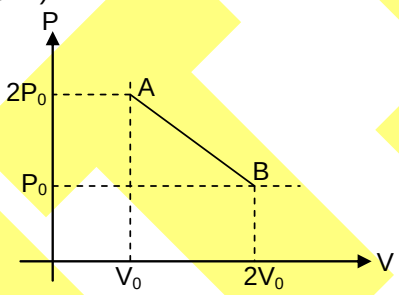
1. A hot air balloon is carrying some passengers, and a few sandbags of mass 1 kg each so that its total mass is 480 kg. Its effective volume giving the balloon its buoyancy is V. The balloon is floating at an equilibrium height of 100 m. When N number of sandbags are thrown out, the balloon rises to a new equilibrium height close to 150 m with its volume V remaining unchanged. If the variation of the density of air with height h from the ground is $\rho(h) = \rho_0 e^{-\frac{h}{h_0}}$, where $\rho_0 = 1.25\ \text{kg m}^{-3}$ and $h_0 = 6000\ \text{m}$, the value of N is_____.

2. Force acting on a particle is $F = -8x$ in SHM. The amplitude of oscillations is 2(in m) and mass of the particle is 0.5 kg. The total mechanical energy of the particle is 20 J. The potential energy of the particle in mean position is 10k Joule, find the value of k.

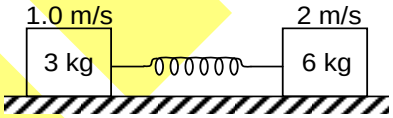
3. An object is displaced from a point $A(0, 0, 0)$ to $B(1\text{m}, 1\text{m}, 1\text{m})$ under a force $\vec{F} = (y\hat{i} + x\hat{j})$ N. Calculate the work done by this force in this process.

4. A tuning fork having frequency 256 Hz produces 4 beats per second with a wire of length 25 cm vibrating in its fundamental mode. The beat frequency decreases when the length is slightly shortened. What could be the minimum length by which the wire be shortened so that it produces no beats with the turning fork?

5. 'n' moles of an ideal gas undergoes a process $A \to B$ as shown in figure. Here $P_0 = 1\,\text{Bar}$, $V_0 = 0.02\ \text{m}^3$ and $n = 2$ mole. If the maximum temperature of the gas during the process will be 90X. Figure value of X. (Value of $R = 8\ \text{J K}^{-1}\text{mol}^{-1}$)



6. Two block of mass 3 kg and 6 kg respectively are placed on a smooth horizontal surface. They are connected by a light spring of force constant $k = 200$ N/m. Initially the spring is unstretched. The indicated velocities are imparted to the blocks. The maximum extension of the spring will be (in m):



FIITJEE ALL INDIA INTERNAL TEST SERIES - 4

CLASS - XI

CHEMISTRY, MATHEMATICS & PHYSICS

PAPER CODE: **DATE:** DD.MM.YYYY

ANSWER KEYS [SET – A]

SECTION – I (CHEMISTRY)		SECTION – II (MATHS)		SECTION– III (PHYSICS)	
PART – A		**PART - A**		**PART - A**	
1.	**C**	1.	**B**	1.	**A**
2.	**D**	2.	**B**	2.	**B**
3.	**B**	3.	**B**	3.	**D**
4.	**A**	4.	**C**	4.	**A**
5.	**C**	5.	**D**	5.	**D**
6.	**B**	6.	**D**	6.	**A**
7.	**ABD**	7.	**ABCD**	7.	**CD**
8.	**ABC**	8.	**BC**	8.	**BC**
9.	**AB**	9.	**CD**	9.	**BD**
10.	**ABCD**	10.	**ABC**	10.	**ABCD**
11.	**AD**	11.	**ABC**	11.	**BC**
12.	**ABCD**	12.	**ABCD**	12.	**AB**
PART - C		**PART - C**		**PART - C**	
1.	**2.13**	1.	**212.16**	1.	**4**
2.	**2.67**	2.	**0.25**	2.	**0.4**
3.	**3**	3.	**4.98**	3.	**1**
4.	**0.1**	4.	**3**	4.	**0.4**
5.	**4.10**	5.	**0.41**	5.	**3.12**
6.	**8**	6.	**0.33**	6.	**0.3**

FIITJEE ALL INDIA INTERNAL TEST SERIES - 4

CLASS - XI

CHEMISTRY, MATHEMATICS & PHYSICS

PAPER CODE: **DATE:** DD.MM.YYYY

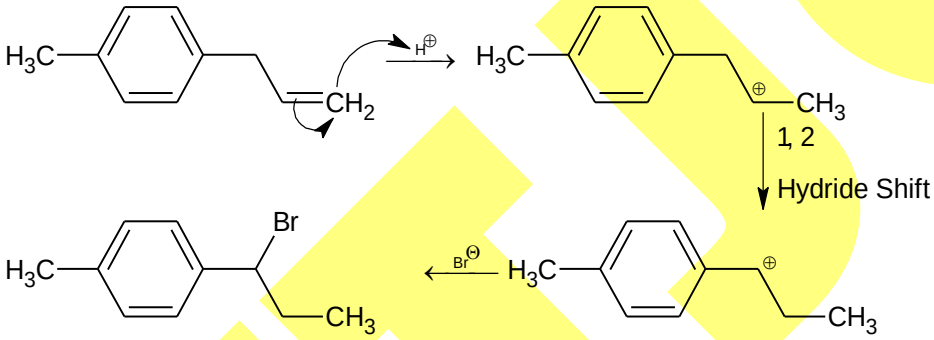
HINTS & SOLUTIONS [SET – A]

CHEMISTRY

PART – A

1. **C**

Sol.



2. **D**

Sol. Five products Ethane, Propane, butane, pentane, hexane will be formed

The possible cases will be:

(I) $2\,CH_3Cl \xrightarrow{Na} \underbrace{CH_3-CH_3}_{\text{ethane}}$

(II) $2\,CH_3CH_2Cl \xrightarrow{Na} \underbrace{CH_3CH_2-CH_2CH_3}_{\text{butane}}$

(III) $2\,CH_3CH_2CH_2Cl \xrightarrow{Na} \underbrace{CH_3CH_2CH_2-CH_2CH_2CH_3}_{\text{hexane}}$

(IV) $CH_3Cl + CH_3CH_2Cl \xrightarrow{Na} \underbrace{CH_3-CH_2CH_3}_{\text{propane}}$

(V) $CH_3CH_2Cl + CH_3CH_2CH_2Cl \xrightarrow{Na} \underbrace{CH_3CH_2-CH_2CH_2CH_3}_{\text{pentane}}$

(VI) $CH_3CH_2CH_2Cl + CH_3Cl \xrightarrow{Na} \underbrace{CH_3CH_2CH_2-CH_3}_{\text{butane}}$

In II and VI products are same so total number of products are 5

3. **B**

Sol. $pH = pK_a + \log\left(\frac{[CH_3COO^-]}{[CH_3COOH]}\right)$

$= -\log(2\times10^{-5}) + \log\left(\frac{0.1\times2}{0.1}\right)$

$= 5$

4. **A**

Sol. To calculate final volume $V_2 = \frac{2\times0.821\times300}{4.926} = 10L$

now: $V_1 = 1L,\ V_2 = 10L\ T = 300K\ n = 2mole$

$-2.303nRT\log\left(\frac{V_2}{V_1}\right) = -2.303\times2\times8.314\times300\times\log\left(\frac{10}{1}\right)$

$= -11488.28\ J$

5. **C**

Sol. $+\frac{1}{2}\frac{d[B]}{dt} = k[A]^n$

Taking ln on both side

$\ln\left(\frac{1}{2}\right) + \ln\left(\frac{d[B]}{dt}\right) = \ln k + n\ln[A]$

$\ln\left(\frac{d[B]}{dt}\right) = \left(\ln k - \ln\left(\frac{1}{2}\right)\right) + n\ln[A]$

$so\ n = 2$

6. **B**

Sol. For node $\Psi^2 = 0$, So also $\Psi = 0$

So for: $\Psi = \frac{1}{16\sqrt{4}}\left(\frac{1}{a_o}\right)^{3/2}\left[(x-1)(x^2-8x+12)\right]e^{-x/2}$

$\left[(x-1)(x^2-8x+12)\right]$ should be equal to zero

$\left[(x-1)(x^2-8x+12)\right] = 0$, solve the eq.

$(x-1)(x^2-8x+12)$

x = 1, 2, 6

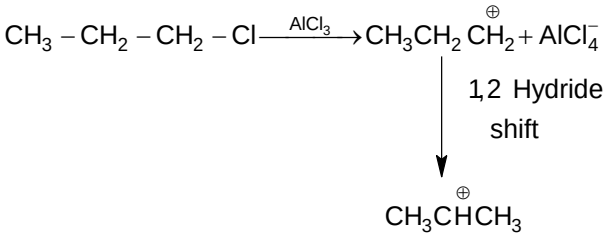
i.e. $r = 0.5a_o, a_o, 3a_o;\ \because x = 2r/a_o$

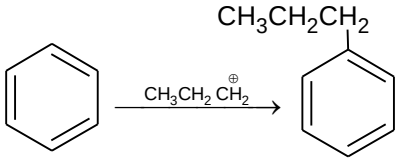
So minimum $0.5a_o$

maximum $3a_o$

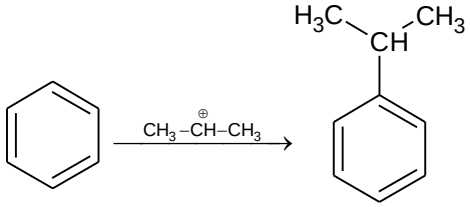
7. **ABD**

Sol.





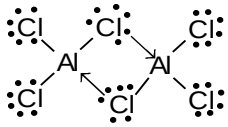
n propyl benzene



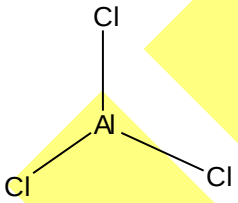
Iso propyl benzene or cumene

8. **ABC**

(A) Anhydrous $AlCl_3$ exists as Al_2Cl_6 (dimer)



(B) Anhydrous $AlCl_3$ is a trigonal planar molecule



Trigonal planar

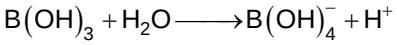
(C) White fumes are due to the presence of HCl gas.
Anhydrous aluminium chloride is hydrolysed partially with the moisture in the atmosphere to give HCl gas.
This HCl combines with the moisture in the air and appears white in colour.

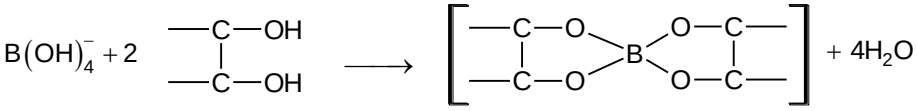
$$AlCl_3 + 3H_2O \longrightarrow Al(OH)_3 + 3HCl(\text{white fumes})$$

(D) Thus $AlCl_3$ is a covalent compound rather than ionic. Fajan's rule states that, small size of cation, higher the charge on cation and large size of anion, greater is the covalent character of the ionic bond.
So (ABC) is answer.

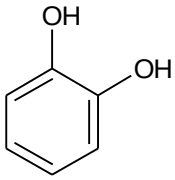
9. **AB**

Sol. $[B(OH)_4]^-$ forms compound with cis 1,2 diol or catechol i.e. it consumes the product so according to Le Chatelier principle the reaction goes in forward reaction.





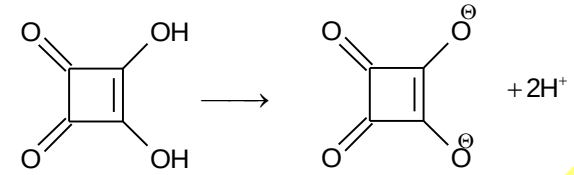
Cis – 1, 2 diols

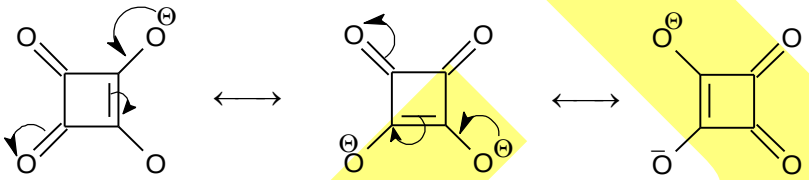


Catechol

10. **ABCD**

Sol.





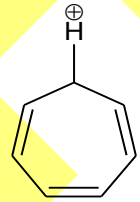
Due to resonance stabilize equivalent structures it is highly acidic

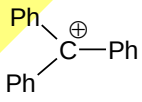
11. **AD**

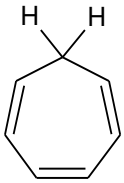
Sol. Van der Wall's constant 'a' gives information about force of attraction between gaseous molecules. Higher the value of 'a' higher will be force of attraction, and easier the liquefaction of gas. Order of intermolecular force of attraction = $H_2 < CH_4 < NH_3$

12. **ABCD**

Sol.

(A)  due to aromatic character , is more stable the $(Ph)_3\overset{\oplus}{C}$

(B)  is non polar

(C) $(Ph)_3\overset{\oplus}{C}$ takes proton from  , so is an acid base reaction

(D) As the product are more stable than reactant so reaction is exothermic

PART – C

1. **2.13**

Sol. Here $\Delta V = 200 - 400 = -200 cm^3$

A we know 1 Litre $= 1000 cm^3$

$\Rightarrow$ $\Delta V = -0.2$ Litre.

External Pressure $(P) = 0.50$ then

Work done $= -P\Delta V = +0.50(0.2)$

$= +0.1$ atm Litre

and 1 litre- atmosphere $= 101.3$ Joule

$\therefore$ $W = +10.13$ Joule

According to F.L.O.T.

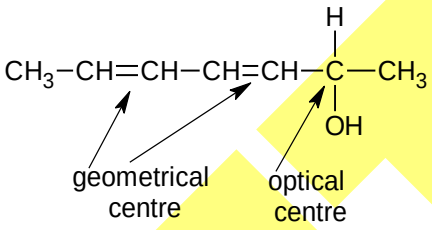
$q = \Delta E - W$

then $\Delta E = q + W$

Given $q = -8.00 J$

$\Rightarrow$ $\Delta E = -8 + 10.13 = 2.13$ Joule

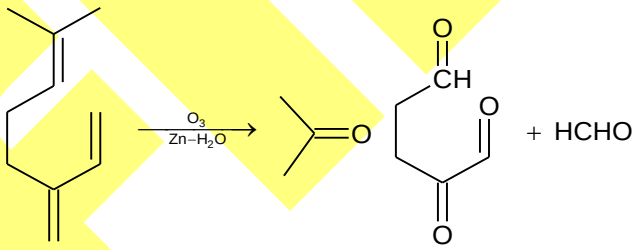
2. **2.67**

Sol. 

So total product will be $2^3 = 8$

$\frac{8}{3} = 2.67$

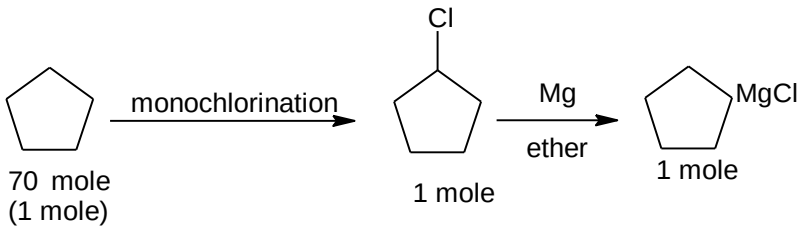
3. **3**

Sol. 

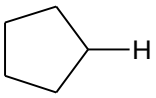
3 product

4. **0.1**

Sol.



$HO\text{-}CH_2CH_2C \equiv CH$
3.5 gm
= (0.05 mole)
acidic 'H'
= 0.1 mole acidic 'H'



5. **4.10**

Sol. $CH_3CH_2CH=CH_2 \xrightarrow{1\text{ eq }H_2} CH_3CH_2CH_2CH_3 \quad \Delta H = -30.3\text{kj/mole}$

assume $CH_2=CH-CH=CH_2$ does not have resonance

$CH_2=CH-CH=CH_2 \xrightarrow{2\text{ eq }H_2} CH_3CH_2CH_2CH_3 \quad \Delta H = -30.3\times 2 = -60.6\text{ kj/mole}$

Resonance energy = 60.6 – 56.5 = 4.1 kj/mol

6. **8**

Sol. Based on the formula of hydrocarbon C_nH_{2n-4} the degree of unsaturation is 3.

So, there is two case:

(I) 3 double bonds

(II) 1 double bond and 1 triple bond

Second case can be easily rejected because products of ozonolysis are aldehyde or ketone rather than carboxylic acid.

So the alkene would be:

$(CH_3)_2CHCH_2CH=CHCH_2CH_2CH=CHCH_2CH_2CH=C(CH_3)_2$

Which on ozonolysis will give products as:

$(CH_3)_2CHCH_2CH=CHCH_2CH_2CH=CHCH_2CH_2CH=C(CH_3)_2$

$\xrightarrow{\text{Ozonolysis}} (CH_3)_2CHCH_2CHO + 2\ OHCCH_2CH_2CHO + CH_3COCH_3$

MATHEMATICS

PART – A

1. **B**

Sol. Putting $x=1$ and -1,

Then, $4^{20} = a_0 + a_1 + a_2 + ... + a_{40}$

and $2^{20} = a_0 - a_1 + a_2 - .... + a_{40}$

$\therefore \quad 4^{20} + 2^{20} = 2\left(a_0 + a_2 + a_4 + ... + a_{40}\right)$

$\therefore \quad a_0 + a_2 + a_4 + ... + a_{38} + a_{40} = \dfrac{4^{20}+2^{20}}{2}$

$\Rightarrow \quad a_0 + a_2 + a_4 + ... + a_{38} = \dfrac{4^{20}+2^{20}}{2} - a_{40}$

$T_{R+1} = \text{in}\left(1+x+2x^2\right)^{20}$

$= {}^{20}C_R\left(x+2x^2\right)^{20}$

$a_{40} = \text{coefficient of } x^{40}$

$= 2^{20}$

Then, $a_0 + a_2 + a_4 + ... + a_{38} = \frac{4^{20} + 2^{20}}{2} - 2^{20}$

$= \frac{2^{40} - 2^{20}}{2}$

$= 2^{19}\left(2^{20} - 1\right)$

2. **B**

Sol. [IN] TEGER

Number of ways $= \frac{6!}{2!} \times 2! = 6! = 720$

$m_1 = \frac{7!}{2!} - 720 = 1800$

and $\boxed{I}$ NTEGE $\boxed{R}$

$m_2 = \frac{5!}{2!} = 60$

$\therefore \quad m_1 / m_2 = 30$

3. **B**

Sol. If $\left[\frac{1}{3} + \frac{n}{90}\right] = 0 \Rightarrow 0 \le \frac{1}{3} + \frac{n}{90} < 1$

$\Rightarrow \quad 1 \le n < 60$

and if $\left[\frac{1}{3} + \frac{n}{90}\right] = 1 \Rightarrow 1 \le \frac{1}{3} + \frac{n}{90} < 2$

$\Rightarrow \quad 60 \le n < 150$

$\because \quad \sum_{n=1}^{k}\left[\frac{1}{3} + \frac{n}{90}\right] = 21$

$\Rightarrow \quad \sum_{n=1}^{59}\left[\frac{1}{3} + \frac{n}{90}\right] + \sum_{n=60}^{k}\left[\frac{1}{3} + \frac{n}{90}\right] = 21$

$\Rightarrow \quad 0 + \sum_{n=60}^{k}\left[\frac{1}{3} + \frac{n}{90}\right] = 21$

which is possible

only when $k = 80 \left(\begin{matrix}\because \ 60 \le n < 150 \\ \therefore \ \left[\frac{1}{3} + \frac{n}{90}\right] = 1\end{matrix}\right)$

4. **C**

Sol. We have

$|z - 3 + i| = |z - 2 - i|$

$\Rightarrow \quad |z - (3 - i)| = |z - (2 + i)|$

$\Rightarrow \quad PA = PB$

Hence locus of P is the perpendicular bisector of AB.

5. **D**

Sol. $\because \ x^2 + y^2 + 2xy - 6x - 2y + 3 = 0$

$\Rightarrow \quad (x + y)^2 = (6x + 2y - 3)$

or $(x + y + \lambda)^2 = \lambda^2 + 2\lambda(x + y) + 6x + 2y - 3 \qquad \ldots(i)$

where λ is such that

(slope of $x + y + \lambda = 0$) $\times$ (slope of $\lambda^2 + 2\lambda(x + y) + 6x + 2y - 3 = 0$)

$= -1$

$\Rightarrow \quad (-1)\times -\frac{(2\lambda+6)}{(2\lambda+2)} = -1$

$\Rightarrow \quad 2\lambda+6 = -2\lambda-2$

$\therefore \quad 4\lambda = -8$

$\Rightarrow \quad \lambda = -2$

From Eq. (i),

$$(x+y-2)^2 = 4-4(x+y)+6x+2y-3$$
$$= (2x-2y+1)$$

or $$2\left(\frac{x+y-2}{\sqrt{2}}\right)^2 = \sqrt{8}\left(\frac{2x-2y+1}{\sqrt{4+4}}\right)$$

$\Rightarrow$ $$\left(\frac{x+y-2}{\sqrt{2}}\right)^2 = \sqrt{2}\left(\frac{2x-2y+1}{\sqrt{8}}\right)$$

Let $Y = \frac{x+y-2}{\sqrt{2}}$ and $X = \frac{2x-2y+1}{\sqrt{8}}$

and $4a = \sqrt{2}$

then, focus $X = a$ and $Y = 0$

$$\frac{2x-2y+1}{\sqrt{8}} = \frac{\sqrt{2}}{4} \text{ and } \frac{x+y-2}{\sqrt{2}} = 0$$

$\Rightarrow \quad x = y$ and $x+y=2$

then, $\quad x = 1, y = 1$

$\therefore$ focus is (1, 1)

6. **D**

Sol. Let P $(a\cos\alpha, b\sin\alpha)$ and Q $(a\cos\beta, b\sin\beta)$ are the ends of focal chord of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Then equation of PQ is

$$\frac{x}{a}\cos\left(\frac{\alpha+\beta}{2}\right)+\frac{y}{b}\sin\left(\frac{\alpha+\beta}{2}\right) = \cos\left(\frac{\alpha-\beta}{2}\right)$$

It passes through $(\pm ae, 0)$

$$\pm e\cos\left(\frac{\alpha+\beta}{2}\right) = \cos\left(\frac{\alpha-\beta}{2}\right)$$

Taking +ve sign, then

$$e = \frac{\cos\left(\frac{\alpha-\beta}{2}\right)}{\cos\left(\frac{\alpha+\beta}{2}\right)}$$
$$= \frac{2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)}{2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)}$$
$$= \frac{\sin\alpha+\sin\beta}{\sin(\alpha+\beta)}$$

7. **ABCD**

Sol. $f_n(\theta) = \tan(\theta/2)\prod_{r=0}^{n}\left(1+\sec 2^r\theta\right)$

$$= \tan(\theta/2)\prod_{r=0}^{n}\left\{\frac{1+\cos\left(2^r\theta\right)}{\cos\left(2^r\theta\right)}\right\}$$

$$= \tan(\theta/2)\prod_{r=0}^{n}\frac{2\cos^2\left(2^{r-1}\theta\right)}{\cos\left(2^r\theta\right)}$$

$$= 2^{n+1}\cdot\tan(\theta/2)\prod_{r=0}^{n}\frac{\cos^2\left(2^{r-1}\theta\right)}{\cos\left(2^r\theta\right)}$$

$$= 2^{n+1}\cdot\tan(\theta/2)\cos^2(\theta/2)\cdot\prod_{r=0}^{n}\frac{\cos\left(2^r\theta\right)}{\cos\left(2^n\theta\right)}$$

$$= 2^n\cdot\sin\theta\cdot\frac{\sin\left(2^n\theta\right)}{2^n\cdot\sin\theta\cdot\cos\left(2^n\theta\right)}$$

$$= \tan\left(2^n\theta\right)$$

$\therefore$ Alternate (A): $f_2\left(\frac{\pi}{16}\right) = \tan\left(\frac{\pi}{4}\right) = 1$

Alternate (B): $f_3\left(\frac{\pi}{32}\right) = \tan\left(\frac{\pi}{4}\right) = 1$

Alternate (C): $f_4\left(\frac{\pi}{64}\right) = \tan\left(\frac{\pi}{4}\right) = 1$

Alternate (D): $f_5\left(\frac{\pi}{128}\right) = \tan\left(\frac{\pi}{4}\right) = 1$

8. **BC**

Sol. $\left(\cos^2 x + \frac{1}{\cos^2 x}\right)\left(1+\tan^2 2y\right)\left(3+\sin 3z\right) = 4$

$\cos^2 x + \frac{1}{\cos^2 x} \geq 2$

$1+\tan^2 2y \geq 1$

$2 \leq 3+\sin 3z \leq 4$

So, the only possibility is

$\cos^2 x + \frac{1}{\cos^2 x} = 2$

$1+\tan^2 2y = 1$

$3+\sin 3z = 2$

$\therefore \cos x = \pm 1$

$\therefore x = m\pi;\ m \in I$

$\tan 2y = 0$

$\therefore y = \frac{n\pi}{2};\ n \in I$

$\sin 3z = -1$

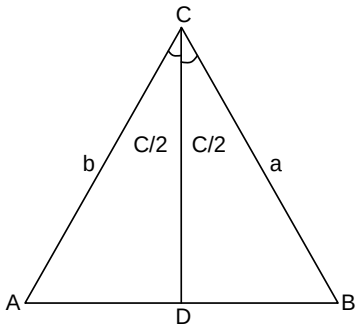
$\therefore z = (4p-1)\frac{\pi}{6};\ p \in I$

9. **CD**

Sol. $\Delta CAB = \Delta CAD + \Delta CDB$

$\Rightarrow \quad \frac{1}{2}ab\sin C = \frac{1}{2}b\cdot CD\cdot\sin\frac{C}{2} + \frac{1}{2}\alpha\cdot CD\sin\frac{C}{2}$

$\Rightarrow \quad CD = \frac{2ab}{a+b}\cos\frac{C}{2}$



In $\Delta CAD, \frac{CD}{\sin A} = \frac{b}{\sin \angle CDA}$

or $CD = \frac{b \sin A}{\sin\left(B + \frac{C}{2}\right)}$

10. **ABC**

Sol. $\because \ (101)^{50} - (99)^{50} = (100+1)^{50} - (100-1)^{50}$

$= 2\left\{{}^{50}C_1 (100)^{49} + {}^{50}C_3 (100)^{47} + {}^{50}C_5 (100)^{45} + ...\right\}$

$= (100)^{50} + 2\left\{{}^{50}C_3 (100)^{47} + {}^{50}C_5 (100)^{45} + ...\right\} > (100)^{50}$

$\Rightarrow \quad (101)^{50} - (99)^{50} > (100)^{50}$

or $\quad (101)^{50} - (100)^{50} > (99)^{50}$

Also, $\left(\frac{1001}{1000}\right)^{999} = \left(1 + \frac{1}{1000}\right)^{999}$

$= 1 + {}^{999}C_1\left(\frac{1}{1000}\right) + {}^{999}C_2\left(\frac{1}{1000}\right)^2 + ... < 1+1+1+1+.....+1$

$= 1000$

$\therefore \quad \left(\frac{1001}{1000}\right)^{999} < (1000)$

$\Rightarrow \quad (1001)^{999} < (1000)^{1000}$

11. **ABC**

Sol. Let a_0 be the number of objects to the left of the first object chosen, a_1 be the number of object between first and second, the number of object between the second and the third is a_2 and number of object right to the third object is a_3.

$\therefore \quad a_0, a_3 \geq 0$ and $a_1, a_2 \geq 1$

Also, $\quad a_0 + a_1 + a_2 + a_3 = n - 3 \qquad$ ....(i)

$\because \quad 1 + a_0, 1 + a_3 \geq 1$ and $a_1, a_2 \geq 1$

$\Rightarrow \quad (1 + a_0) + a_1 + a_2 + (1 + a_3) = n - 1$

$\therefore$ Required number of ways $= {}^{n-1-1}C_{4-1} = {}^{n-2}C_3$

$= \frac{(n-2)(n-3)(n-4)}{6}$

Also $\quad {}^{n-2}C_3 = {}^{n-3}C_3 + {}^{n-3}C_2$

12. **ABCD**

Sol. Equation of any line through $P(3,4)$ making an angle θ with the positive direction of x-axis is

$\frac{x-3}{\cos\theta} = \frac{y-4}{\sin\theta} = r \qquad$ ....(i)

Where r is the distance of any point on the line from P.

Therefore, coordinates of any point on the line (i) are

$(3+r\cos\theta, 4+r\sin\theta)$ ....(ii)

If Eq. (ii) represents R, then

$$3+r\cos\theta=6 \Rightarrow r=\frac{3}{\cos\theta}=PR$$

If Eq. (ii) represents S, $4+r\sin\theta=8$

$\Rightarrow r=4/\sin\theta=PS$

Hence, $PR=3\sec\theta, PS=4\text{cosec}\theta$

$$PR+PS=\frac{3\sin\theta+4\cos\theta}{\sin\theta\cos\theta}=\frac{2(3\sin\theta+4\cos\theta)}{\sin 2\theta}$$

and $$(3/PR)^2+(4/PS)^2=\cos^2\theta+\sin^2\theta=1$$

$\Rightarrow$ $$\frac{9}{(PR)^2}+\frac{16}{(PS)^2}=1$$

PART – C

1. **212.16**

Sol. $$\sum_{n=1}^{49}(1.05)^n=(1.05)+(1.05)^2+....+(1.05)^{49}$$

$$=\frac{(1.05)\left\{(1.05)^{49}-1\right\}}{(1.05-1)}$$

$$=\frac{(1.05)^{50}-(1.05)}{0.05}=\frac{11.658-1.05}{0.05}$$

$$=212.16$$

2. **0.25**

Sol. $\alpha+\beta=3/8, \alpha\beta=27/8$

$\therefore$ $$\left(\frac{\alpha^2}{\beta}\right)^{1/3}+\left(\frac{\beta^2}{\alpha}\right)^{1/3}=\left(\frac{\alpha^3}{\alpha\beta}\right)^{1/3}+\left(\frac{\beta^3}{\alpha\beta}\right)^{1/3}$$

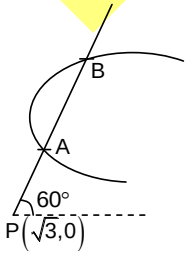
$$=\frac{\alpha+\beta}{(\alpha\beta)^{1/3}}$$

$$=\frac{3/8}{(27/8)^{1/3}}=\frac{3/8}{3/2}=\frac{1}{4}$$

3. **4.98**

Sol. $\because$ $P\equiv(\sqrt{3},0)$

$$\frac{x-\sqrt{3}}{\cos 60^\circ}=\frac{y-0}{\sin 60^\circ}=r$$



or $$x=\sqrt{3}+\frac{r}{2}, y=\frac{r\sqrt{3}}{2}$$

or $\left(\sqrt{3}+\frac{r}{2}, \frac{r\sqrt{3}}{2}\right)$ lie on

$y^2 = x + 2$, then

$$\frac{3r^2}{4} = \sqrt{3} + \frac{r}{2} + 2$$

$$\Rightarrow \quad \frac{3r^2}{4} - \frac{r}{2} - \left(2 + \sqrt{3}\right) = 0$$

$$\therefore \ PA \cdot PB = r_1 r_2 = \left| -\frac{\left(2+\sqrt{3}\right)}{\frac{3}{4}} \right| = \frac{4}{3}\left(2+\sqrt{3}\right)$$

4. **3**

Sol. Let $P \equiv \left(1, \sqrt{3}\right) \equiv \left(2\cos\pi/3, 2\sin\pi/3\right)$

Recall the definition of parametric point on hyperbola,
We have

$$R = \left(2\sec\frac{\pi}{3}, 1\cdot\tan\frac{\pi}{3}\right) = \left(4, \sqrt{3}\right)$$

$\therefore \quad PR = 4 - 1 = 3$

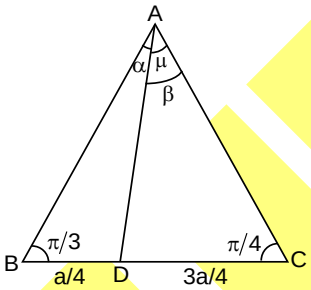
5. **0.41**

Sol. $\because BD : DC = 1 : 3$

$\therefore BD = \frac{a}{4}$ and $DC = \frac{3a}{4}$

In ΔABD sine rule and in ΔADC,

$$\frac{AD}{\sin\frac{\pi}{3}} = \frac{\frac{a}{4}}{\sin\alpha}, \quad \frac{\frac{3a}{4}}{\sin\beta} = \frac{AD}{\sin\frac{\pi}{4}}$$



$$\therefore \sin\alpha = \frac{\frac{a}{4}\cdot\frac{\sqrt{3}}{2}}{AD} \text{ and } \sin\beta = \frac{3a}{4\sqrt{2}AD}$$

$$\therefore \frac{\sin\alpha}{\sin\beta} = \frac{a\sqrt{3}}{8AD}\cdot\frac{4\sqrt{2}AD}{3a} = \frac{1}{\sqrt{6}}$$

6. **0.33**

Sol. $\because \quad \frac{2}{1!9!} + \frac{2}{3!7!} + \frac{1}{5!5!} = \frac{8^a}{(2b)!}$

$$\Rightarrow \frac{1}{1!9!} + \frac{1}{3!7!} + \frac{1}{5!5!} + \frac{1}{3!7!} + \frac{1}{9!1!} = \frac{8^a}{(2b)!}$$

$$\Rightarrow \frac{1}{10!}\left(\frac{10!}{1!9!} + \frac{10!}{3!7!} + \frac{10!}{5!5!} + \frac{10!}{7!3!} + \frac{10!}{9!1!}\right) = \frac{8^a}{(2b)!}$$

$$\Rightarrow \quad \frac{1}{10!}\left({}^{10}C_1 + {}^{10}C_3 + {}^{10}C_5 + {}^{10}C_7 + {}^{10}C_9\right) = \frac{8^a}{(2b)!}$$

$\Rightarrow \quad \frac{2^9}{10!} = \frac{8^a}{(2b)!} = \frac{2^{3a}}{(2b)!}$

$\Rightarrow \quad a = 3,\ b = 5$

Also, $2b = a + c \Rightarrow 10 = 3 + c \Rightarrow c = 7$

$\therefore \quad a = 3,\ b = 5,\ c = 7$

$\because \quad \frac{\tan A + \tan B}{2} \geq \sqrt{\tan A \tan B}$ …(i)

Also, $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

$= \frac{9 + 25 - 49}{30} = -\frac{1}{2}$

$\therefore \quad C = 120°$ and $A, B < 60°$

$\tan A + \tan B + \tan C = \tan A \tan B \tan C$

$\Rightarrow \quad \tan A + \tan B - \sqrt{3} = -\sqrt{3} \tan A \tan B$ …(ii)

Also, $\tan A + \tan B > 0$

$\Rightarrow \quad \sqrt{3}(1 - \tan A \tan B) > 0$

$\Rightarrow \quad \tan A \tan B < 1$ …(iii)

From Eqs. (i) and (ii),

$\frac{\sqrt{3}(1 - \tan A \tan B)}{2} \geq \sqrt{(\tan A \tan B)}$

Let $\tan A \tan B = \lambda$

$\therefore \quad \sqrt{3}(1 - \lambda) \geq 2\sqrt{\lambda}$

$\Rightarrow \quad 3\lambda^2 - 10\lambda + 3 \geq 0$

$\Rightarrow \quad (3\lambda - 1)(\lambda - 3) \geq 0$

$\because \quad \lambda - 3 < 0$ [from Eq. (iii)]

$\therefore \quad 3\lambda - 1 \leq 0$

$\Rightarrow \quad \lambda \leq \frac{1}{3}$

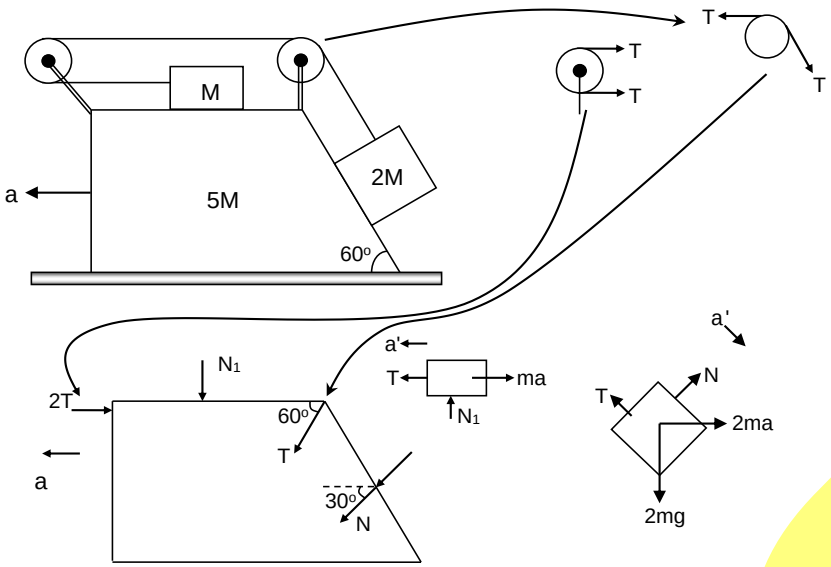
$\tan A \tan B \leq \frac{1}{3}$

PHYSICS

PART – A

1. **A**

Sol. Let acceleration of the wedge in a in left direction



$N = 2mg\sin 30 - 2ma\sin 60$ ….(i)

$(5M)a = N\cos 30 + T\cos 60 - 2T$ ….(ii)

$2Mg\cos 30 + 2Ma\cos 60 - T = 2Ma'$ …..(iii)

$T - ma = ma'$ ….(iv)

$$2Mg\cos 30 + 2Ma\cos 60 - T = 2M\left[\frac{T - Ma}{M}\right]$$

$2Mg\cos 30 + 2Ma\cos 60 - T = 2T - 2Ma$

$\sqrt{3}Mg + 3Ma = 3T$ ….(v)

Put values in equation (ii)

$$5Ma = \left[Mg - Ma\sqrt{3}\right]\frac{\sqrt{3}}{2} + \frac{1}{2}\left[\frac{Mg}{\sqrt{3}} + Ma\right] - 2\left[\frac{Mg}{\sqrt{3}} + Ma\right]$$

$$= \frac{\sqrt{3}}{2}Mg - \frac{3Ma}{2} + \frac{Mg}{2\sqrt{3}} + \frac{Ma}{2} - \frac{2Mg}{\sqrt{3}} - 2Ma$$

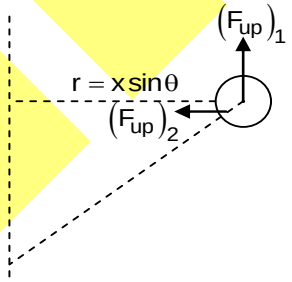
$$8Ma = \frac{\sqrt{3}}{2}Mg + \frac{Mg}{2\sqrt{3}} - \frac{2Mg}{\sqrt{3}}$$

$$= \frac{3Mg + Mg - 4Mg}{2\sqrt{3}} = 0$$

$a = 0$

2. **B**

Sol.



$(F_{up})_1 = V\rho_{liq}g$

$(F_{up})_2 = V\rho_{liq}\left[r\omega^2\right]$

Balancing component along tube-

$V\rho_{liq}\left[x\sin\theta\right]\omega^2\sin\theta = V\rho_{liq}g\cos\theta$

$$x = \frac{g\cos\theta}{\omega^2\sin^2\theta}$$

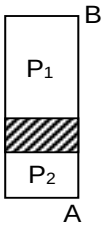
3. **D**

Sol. Let the length of air column on each side = L

Then, $2L + 0.4 = 1.2$ m

$\therefore \quad L = 0.4$m or 40 cm

Let the mercury column, when the tube is held vertical be displaced by 'y' to attain equilibrium if P_1 & P_2 be the pressure at two ends.



$P_1 + 0.4 = P_2$ ..... (i)

Since moles on two sides are same.

For end A,

$$\frac{P \times (L \times a)}{RT} = \frac{P \times ((L - y) \times a)}{RT}$$

$$P_2 = \frac{LP}{L - y} \quad \text{..... (ii)}$$

For end B,

$$\frac{P \times (L \times a)}{RT} = \frac{P_1 \times ((L + y) \times a)}{RT}$$

$$P_1 = \frac{LP}{L + y} \quad \text{..... (iii)}$$

Using equation (i), (ii) and (iii),

$$\frac{LP}{L - y} - \frac{LP}{L + y} = 0.4$$

Since L = 0.4 m, P = 0.75 m of Hg

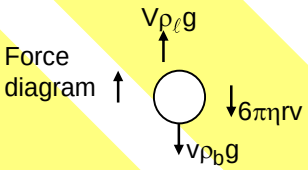
we get,

$y^2 + 1.5y - 0.16 = 0$

So, $y = 0.1$ m

4. **A**

Sol. Suppose density of fluid is ρ_ℓ and density of body is ρ_b.



$ma = V\rho_\ell g - V\rho_b g - 6\pi\eta rv$

Put $V\rho_\ell g - V\rho_b g = A$ and $6\pi\eta r = B$

$$m\frac{dv}{dt} = A - Bv$$

$$\int_0^v \frac{dv}{A - Bv} = \int_0^t \frac{dt}{m} \left\{ \begin{matrix} A - Bv = X \\ -Bdv = dX \end{matrix} \right\}$$

$$-\frac{1}{B}\int_A^{A-Bv} \frac{dX}{X} = \int_0^t \frac{dt}{m}$$

$$-\frac{1}{B}\ln\left|\frac{A - Bv}{A}\right| = \frac{t}{m} \Rightarrow 1 - \left(\frac{B}{A}\right)v = e^{-Bt/m}$$

$$v = \frac{A}{B}\left[1 - e^{-Bt/m}\right]$$

Hence correct option will be (A).

5. **D**

Sol. Rate of heat transfer through metal rod is:

$$\frac{dQ}{dt} = C\frac{dT}{dt} = P(\text{constant}) \qquad \text{.....(i)}$$

Also temperature variation is given as

$$T = T_0\left(1+\beta t^{1/4}\right) \qquad \text{.....(ii)}$$

$$\therefore \quad \frac{dT}{dt} = \frac{T_0\beta}{4}t^{-3/4}$$

By equation (i)

$$C = \frac{P}{\left(\frac{dT}{dt}\right)} = \frac{4P}{\beta T_0}t^{3/4}$$

Substituting the value of t from equation (ii), we get

$$C = \frac{4P(T-T_0)^3}{(\beta T_0)^4}$$

6. **A**

Sol. We know that,

$f \propto v \propto \sqrt{T}$

From the data given in the question,

$f_{AB} = 2f_{CD} \Rightarrow f_{AB}^2 = 4f_{CD}^2 \Rightarrow T_{AB} = 4T_{CD}$

Since, rod BD is in equilibrium, Net torque about O will be zero.

$\tau_{net} = 0$

$\Rightarrow T_{AB} \times x = T_{CD} \times (L-x)$

$\Rightarrow 4T_{CD} \times x = T_{CD} \times (L-x)$

$\Rightarrow 4x = L - x$

$\Rightarrow x = \frac{L}{5}$

7. **CD**

Sol. Now, using equation of adiabatic process,

$T_a V_a^{\gamma-1} = T_d V_d^{\gamma-1}$

$T_1 V_a^{\gamma-1} = T_2 V_d^{\gamma-1}$

$$\frac{T_2}{T_1} = \left(\frac{V_a}{V_d}\right)^{\gamma-1} \quad \text{..... (i)}$$

Same way, during bc process

$$\frac{T_2}{T_1} = \left(\frac{V_b}{V_c}\right)^{\gamma-1} \quad \text{..... (ii)}$$

From equation (i) and (ii)

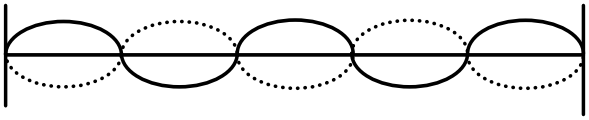
$$\frac{V_a}{V_d} = \frac{V_b}{V_c} \Rightarrow V_a V_c = V_b V_d$$

From equation (ii)

$$\frac{V_b}{V_c} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}}$$

8. **BC**

Sol. $y = 0.01\,m\sin(20\pi x)\cos 200\pi t$



No. of nodes is 6

$20\pi = k = \frac{2\pi}{\lambda}$

$\therefore \quad \lambda = \frac{1}{10}m = 0.1\,m$

Length of the spring $= \frac{5\lambda}{2} = 0.25$ m

Mid point is the antinode

Frequency at this mode is $f = \frac{200\pi}{2\pi} = 100$ Hz

$\therefore$ Fundamental frequency $= \frac{100}{5} = 20$ Hz.

9. **BD**

Sol. $p \propto \sqrt{d}$ as per process. If d becomes half, then p will become $\frac{1}{\sqrt{2}}$ times.

Further, $d \propto \frac{1}{V}$

$\therefore \quad \frac{p^2}{(1/V)} = \text{constant}$

or $(pV)p = \text{constant}$

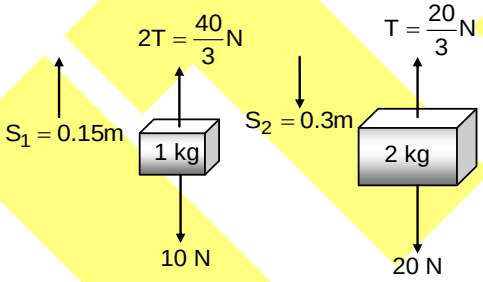
Hence, $Tp = \text{constant} \qquad (\text{as } pV \propto T)$

$\therefore \quad p \propto \frac{1}{T}$

or $p - T$ graph is a rectangular hyperbola.

10. **ABCD**

Sol.



Hence, (A), (B), (C) and (D) are correct.

11. **BC**

Sol. By law of Conservation of Momentum

$mv = (M+m)V$

Momentum of combined system is $(M+m)V = mv$

$KE = \frac{1}{2}(M+m)V^2$

$\Rightarrow \quad KE = \frac{m^2v^2}{2(M+m)}$

Hence, (B) and (C) are correct.

12. **AB**

Sol. Suppose angular velocity of disc is ω_0.

$v_0 - r\omega_0 = 3v$ ....(i)

$v_0 + r\omega_0 = -v$ ....(ii)

From Eqs. (i) and (ii), we get

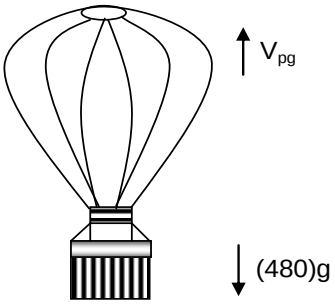
$2v_0 = 2v \Rightarrow v_0 = v$

$$v_0 + v_0 = -r\omega_0 \Rightarrow \omega_0 = \frac{2v}{r}$$

PART – C

1. **4**

Sol.



Using equilibrium equation before and after removal of sandbags,

$V\rho_1 g = 480g$ ....(i)

$V\rho_2 g = (480-n)g$ ....(ii)

$$\frac{\rho_0 e^{-h_1/h_0}}{\rho_0 e^{-h_2/h_0}} = \frac{480}{480-n}$$

$$e^{\left[\frac{-h_1}{h_0}+\frac{h_2}{h_0}\right]} = \frac{480}{480-n}$$

$$e^{\left[-\frac{100}{6000}+\frac{150}{6000}\right]} = \frac{480}{480-n}$$

$$e^{50/6000} = \frac{480}{480-n}$$

We know that

$e^x = 1+x$

$$1+\frac{50}{6000} = \frac{480}{480-n}$$

$$\frac{480-n}{480} = \frac{6000}{6050}$$

$\Rightarrow \quad n = 4$

2. **0.4**

Sol. $k = 8$ N/m, $a = \frac{F}{m} = \frac{8x}{0.5} = 16x$

$\omega = \sqrt{16} = 4$ rad/sec

$$\frac{1}{2}kA^2 = \frac{1}{2}(8)(2)^2 = 16J$$

Potential energy at mean position $= E - \frac{1}{2}kA^2 = 20 - 16 = 4J$

3. **1**

Sol. $W = \int_A^B \vec{F}\cdot d\vec{r}$, where $d\vec{r} = \hat{i}dx + \hat{j}dy + \hat{k}dz$

$$\Rightarrow \quad W\int_A^B \left(y\hat{i}+x\hat{j}\right)\cdot\left(\hat{i}dx+\hat{j}dy+\hat{k}dz\right)$$

$$\Rightarrow \quad W = \int_A^B (ydx+xdy) = \int_A^B d(xy)$$

$$\Rightarrow \quad W = (xy)\Big|_{(0,\ 0,\ 0)}^{(1,\ 1,\ 1)}$$

$$\Rightarrow \quad W = 1\,J$$

4. **0.4**

Sol. $f \propto \frac{1}{l}$

By decreasing the length, frequency of wire will increase. But beat frequency is decreasing. Hence, its original frequency was (256-4) Hz or 252 Hz, Now, we have to make it 256 Hz for no beats.

$$\frac{f_1}{f_2} = \frac{l_2}{l_1}$$

$$\therefore \quad l_2 = \left(\frac{f_1}{f_2}\right) l_1$$

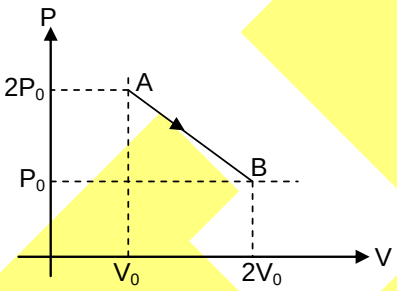
$$= \left(\frac{252}{256}\right)(25)$$

$$= 24.6 \text{ cm}$$

$$\therefore \quad \Delta l = l_1 - l_2 = 0.4 \text{ cm}$$

5. **3.12**

Sol.



Equation of AB

$$P = mV + C$$

$$= \left[\frac{P_0 - 2P_0}{2V_0 - V_0}\right] V + (3P_0)$$

$$P = \frac{-P_0}{V_0} V + 3P_0$$

Put $P = \frac{nRT}{V}$

$$nRT = \frac{-P_0}{V_0} V^2 + 3P_0 V$$

For maximum value of temperature-

Differentiate and put $\frac{dT}{dV} = 0$

$$nRdT = \frac{-P_0}{V_0} \times 2V\ dV + 3P_0\ dV$$

$$= \left(\frac{-P_0}{V_0} \times 2V + 3P_0\right) dV$$

$$\frac{dT}{dV} = \frac{1}{nR}\left[\frac{-P_0}{V_0} \times 2V + 3P_0\right] = 0$$

$$\frac{-2V}{V_0} + 3 = 0$$

$$V = \frac{3V_0}{2}$$

Put this in equation of straight line-

$$P = \frac{3P_0}{2}$$

$$T_{max} = \frac{PV}{nR} = \frac{9P_0V_0}{4nR}$$

6. **0.3**

Sol. w.r.t. COM frame

$$W_{ext} + W_{int} = (\Delta KE)_{COM}$$

$$= (KE_f)_{COM} - (KE_i)_{COM}$$

$$-\frac{1}{2}kx^2 = 0 - \frac{1}{2}\left(\frac{m_1m_2}{m_1+m_2}\right)[v_1 - v_2]^2$$

$$\frac{1}{2} \times 200 \times x^2 = \frac{1}{2} \times \frac{18}{9} \times [3]^2$$

$$x^2 = \frac{2 \times 9}{200} \Rightarrow x^2 = \frac{9}{100}$$

$$x = \frac{3}{10}m = 30 \text{ cm}$$