FIITJEE
ALL INDIA TEST SERIES
JEE (Advanced)-2023
FULL TEST – VII
PAPER –2
TEST DATE: 23-04-2023

ANSWERS, HINTS & SOLUTIONS

Physics — PART – I

Section – A

1. C

Sol. $e_1 = Bv(BF) = 5\times1\times2\sin60^\circ = 5\sqrt{3}V$

The induced emf across the loop due to change in magnetic field

$$e_2 = A\frac{dB}{dt} = \frac{\pi R^2}{3}\left(\frac{dB}{dt}\right) = \frac{\pi(1)^2}{3}\times2 = \frac{2\pi}{3}V$$

So $e = e_1 + e_2 = \left(5\sqrt{3} + \frac{2\pi}{3}\right)V$

2. C

Sol. Let the frequencies be f, f + 5, f + 10, . . . f + 5n (where n = 0, 1, 2, . . .)

Given condition f = $\frac{3}{4}(f+5n)$ where n = 100

f = $\frac{3}{4}(f+500)$ $\Rightarrow$ f = 1500 Hz

3. A

Sol. $\vec{v}_P = v_0\hat{i} + v_0\hat{j}$

$$\vec{v}_{PT} = \vec{v}_P - \vec{v}_T = \left(v_0\hat{i} + v_0\hat{j}\right) - \left(\frac{v_0}{2}\hat{i} + \frac{v_0}{2}\hat{j}\right)$$

$$\vec{v}_{PT} = \frac{v_0}{2}\hat{i} + \frac{v_0}{2}\hat{j}$$

4. A

Sol. $\left(P_0 + \frac{4T}{r}\right)V = \left(P + \frac{4T}{r/2}\right)\frac{V}{8} \Rightarrow P = 8\left[P_0 + \frac{3T}{r}\right]$

5. AB

6. BD

Sol. y' = $y\frac{1}{\mu_0(1+ay)}$

$$\frac{dy'}{dt} = \frac{1}{\mu_0}\left[\frac{dy/dt}{(1+ay)} - \frac{y\{0+a(dy/dt)\}}{(1+ay)^2}\right] \Rightarrow v' = \frac{u}{\mu_0(1+ay)^2}$$

for y = H $\Rightarrow v'_{min} = \frac{u}{\mu_0(1+aH)^2}$

y = 0 $\Rightarrow v'_{max} = \frac{u}{\mu_0}$

7. ABCD

8. CD

Sol. For constructive interference path difference = nl (n = 0, 1, 2, 3, . . .)

$$\left(SC + CP + \frac{\lambda}{2}\right) - SP = n\lambda$$

$$2SC + \frac{\lambda}{2} - SP = n\lambda$$

$$\Rightarrow\ 2\sqrt{60^2 + 25^2} - 120 = \left(n - \frac{1}{2}\right)\lambda$$

$$\Rightarrow\ 130 - 120 = \left(n - \frac{1}{2}\right)\lambda$$

$\Rightarrow\ \lambda = \frac{10}{\left(n - \frac{1}{2}\right)}$ as n = 0 is not possible

So for n = 1 $\Rightarrow\ \lambda_1 = 20$

n = 2 $\Rightarrow\ \lambda_2 = \frac{20}{3}$ and so on.

9. AD

Sol. Static friction opposes tendency of relative motion, so its acts in the direction of motion, as the tendency of feet is to slip backwards.
Friction is not doing work as the point of application of force is not moving.

10. ABD

Section – B

11. 8

Sol. At resonance $I=\frac{V}{R}\left(\omega L=\frac{1}{\omega C}\right)$ if capacitance is increased to four times

$\omega L'=\frac{1}{\omega C'} \Rightarrow L'=\frac{L}{4}(as R'=L')$

So, $I'=\frac{V}{R/4}=\frac{IR}{R/4}=4I=8A$

12. 3

Sol. Radius of curvature $R=\frac{\left[1+(dy/dx)^2\right]^{3/2}}{d^2y/dx^2}=\frac{\left[1+x^2\right]^{3/2}}{1}$

R_m = 1 (at x = 0) $R_{min}=1(at\, x=0)$

$\mu mg = mv_{max}^2/R_{min} \Rightarrow v_{max}=\sqrt{\left(\frac{9}{10}\right)(10)1}=3m/s$

13. 2

Sol. Let h be the height to which the bullet rises then, $g^1=g\left(1+\frac{h}{R}\right)^{-2}$

$\Rightarrow \frac{g}{4}=g\left(1+\frac{h}{R}\right)^{-2} \Rightarrow h=R$

We know that $v_e=\sqrt{\frac{2GM}{R}}=v\sqrt{N}$ ... (i)

Loss of kinetic energy = gain in gravitational potential energy

$\therefore \frac{1}{2}mv^2=-\frac{GMm}{2R}-\left(-\frac{GMm}{R}\right)$

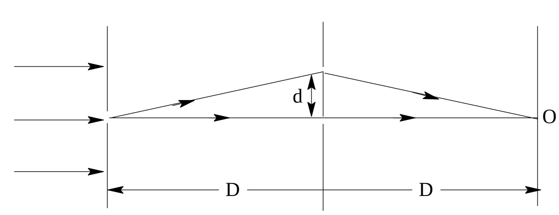
$\therefore v=\sqrt{\frac{GM}{R}}$ ...(ii)

Comparing (i) & (ii)

N = 2

14. 2

Sol. The path difference at O,



$\Delta x = 2\sqrt{D^2+d^2}-2D$

For the dark fringe at O, $\Delta x=\frac{\lambda}{2},\frac{3\lambda}{2}$

For minimum value of d,

$2\sqrt{D^2 + d^2} - 2D = \frac{\lambda}{2}$

Or $\left(D^2 + d^2\right)^{1/2} - D = \frac{\lambda}{4}$

Or $D\left(1+\frac{d^2}{D^2}\right)^{1/2} - D = \frac{\lambda}{4}$

Using binomial expansion

Or $d = \sqrt{\frac{D\lambda}{2}}$

15. 2

Sol. Given $\vec{E} = \sin 10^3 t\,\hat{i}$

$\vec{F} = m\vec{a}$

$\therefore qE = m\frac{dv}{dt}$

$\therefore dv = \frac{qEdt}{m} = \frac{q\sin 1000t}{m}dt$

$\therefore \int_0^v dv = \frac{q}{m}\int_0^{\pi/\omega} \sin 1000t\, dt$

$\therefore v = -\frac{q}{m}\left[\frac{\cos 1000t}{1000}\right]_0^{\pi/\omega}$

$= -\frac{1}{10^{-3}} \times \frac{\left[\cos 1000t\right]_0^{\pi/\omega}}{1000} = -\left[-1-1\right] = 2\,ms^{-1}$

16. 5

Sol. $\left(\frac{I - I_g}{I_g}\right)S = \frac{V}{I_g} - R$

$\frac{1.5 - 0.006}{0.006} \times \frac{2n}{249} = \frac{30}{0.006} - 4990$

$\therefore n \approx 5$

17. 7

Sol. $\frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$; $\quad \frac{p^2}{2m} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$

$\therefore \frac{1}{2m}\left(\frac{h^2}{\lambda^2}\right) = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$

$\therefore \lambda = \sqrt{\frac{4\pi\varepsilon_0 r.h^2}{q_1 q_2 (2m)}} = 7\,fm$

18. 6

Sol. Let θ_0 = room temperature

θ_1 = initial temperature of the body

If θ = temperature of the body at time t then

$$-\frac{d\theta}{dt} = k(\theta - \theta_0)$$

$$\int_{\theta_1}^{\theta} \frac{d\theta}{\theta - \theta_0} = -k\int_0^t dt$$

$$\Rightarrow \Delta\theta = \Delta\theta_0 e^{-kt}$$

Where $\Delta\theta =$ temperature difference between the body and the surrounding

$\Delta\theta_0$ = initial value of $\Delta\theta$

As per question

$$\frac{1}{2}\Delta\theta_0 = \Delta\theta_0 e^{-kt_0} \Rightarrow e^{-kt_0} = \frac{1}{2}$$

$$e^{kt_0} = 2 \qquad ...(i)$$

$$\frac{\Delta\theta_0}{4} = \Delta\theta_0 e^{-kt} \Rightarrow e^{kt} = 4 \qquad ...(ii)$$

$$\frac{kt}{kt_0} = \frac{\ln 4}{\ln 2} = 2 \Rightarrow t = 2t_0 = 6\text{minutes}$$

Chemistry

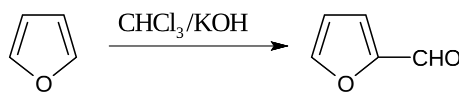
PART – II

Section – A

19. C
Sol. TiO_2

20. A
Sol. Solution as progressive hydrogenation becomes easier.

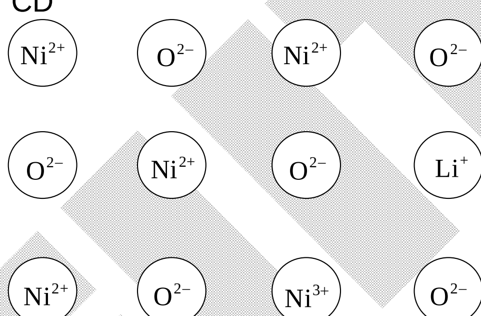
21. B
Sol. Remaining will give aromatic 6 membered substance.



22. D
Sol. Pinacol pinacolone rearrangement

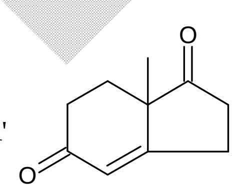
23. ABCD
Sol. For dissociative process (e.g.H_2 to 2H), more pressure is required to get the same extent of chemisorption as two species have to be chemisorbed.
In anionic surfactant negatively charged $-COO^-$ group will repeal each other so at lower concentration micelles will form.

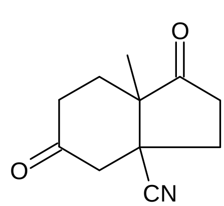
24. CD
Sol.



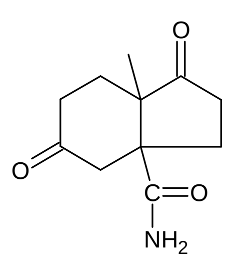
Li^+ ion occupy Ni^{2+} sites to form substitutional defects. In order to maintain the charge neutrality, every Li^+ ion is balanced by Ni^{3+} ion and it becomes a p-type semiconductor.

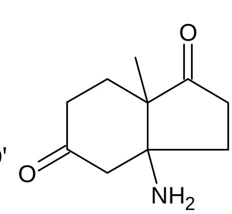
25. BC
Sol.

'A' 

'B' 

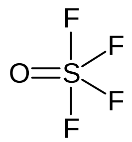
(1, 4 *addition*)

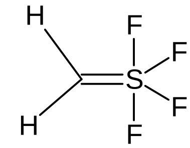
'C' 

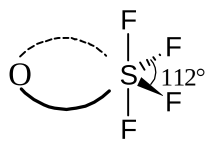
'D' 

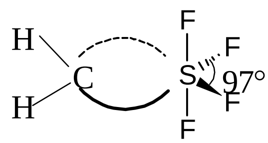
26. ABC

Sol. F-F < Cl-Cl < Br-Br < I-I (Bond length)



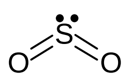


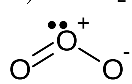




Oxygen being more EN, the electron density at S in OSF_4 will decreases so it occupies less space so bond angle $F_{eq.} - S - F_{eq.}$ will increased.

$H_2S(92°) < O_3(116.8°) < SO_2(119.5°) < NO_2(134°)$





$AsH_3 < NH_3 < SbH_3 < H_2O(BP)$

27. ABC

Sol. Near CMC, Λ_m decreases rapidly on increasing concentration.

28. ABD

Sol. $E^0_{Cr^{3+}/Cr^{2+}} < E^0_{Fe^{3+}/Fe^{2+}} < E^0_{Co^{3+}/Co^{2+}}$

Section – B

29. 6

Sol. $H_2S_4O_6 + 6Al \longrightarrow 4H_2S + 6Al^{3+}$

n=18 n=3

30. 4

Sol. Be_2H_4, Be_2Cl_4,, I_2Cl_6,Fe_2Cl_4,

31. 4

Sol. $H_2S_2O_8 + 2H_2O \rightarrow 2H_2SO_4 + H_2O_2$

$XeOF_4 + 2H_2O \rightarrow XeO_3 + 4HF$

32. 2

Sol. $1+5-(2+2)=2$

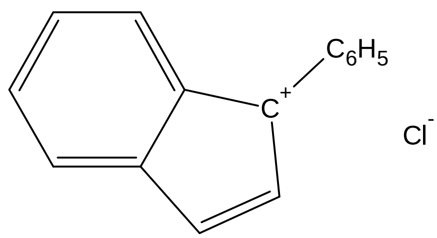
33. 7

Sol. iii and vi are wrong

34. 0
Sol. No 1° amine

35. 3
Sol. Number of Fe atoms = $\frac{50148}{16716}=3$

36. 3
Sol. Three aromatic rings.



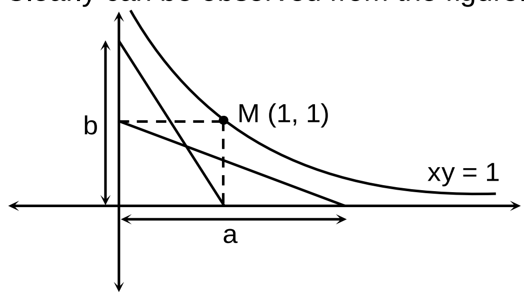
Mathematics

PART – III

Section – A

37. C

Sol. Clearly can be observed from the figure.



38. A

Sol. $f(x)[(f(x)^6 + 1] = x$

$f(0)[(f(0))^6 + 1] = 0 \Rightarrow f(0) = 0$

and $7(f(x))^6.f'(x) + f'(x) = 1$

$\Rightarrow f'(x)[7(f(x))^6 + 1] = 1$

$\Rightarrow f'(x) > 0 \ \forall\ x \in R$

Hence f(x) is increasing function $\forall\ x \in R$

so there exists an inverse of f(x) such that $f^{-1}(0) = 0$

$\Rightarrow x^7 + x = f^{-1}(x)$.

$\Rightarrow \int_0^{\sqrt{2}} (x^7 + x)dx = \left.\frac{x^8}{8} + \frac{x^2}{2}\right|_0^{\sqrt{2}} = 2 + 1 = 3$

Now, we know that $\int_0^a f(x)dx + \int_0^{f(a)} f^{-1}(x)dx = af(a)$

Hence $\int_0^{\sqrt{3}} f(x)dx + \int_0^{f(\sqrt{3})} f^{-1}(x)dx = \sqrt{3}f(\sqrt{3})$

$\int_0^{\sqrt{3}} f(x)dx = \sqrt{3}f(\sqrt{3}) - \int_0^{f(\sqrt{3})} (x^7 + x)dx$

$\int_0^{\sqrt{3}} f(x)dx = \frac{f(\sqrt{3})}{8}\left[8\sqrt{3} - (f(\sqrt{3}))^7 - 4f(\sqrt{3})\right]$

39. D

Sol. $\int_{-1}^{1} \frac{dx}{(x^2 + x + 1) + \sqrt{x^4 + 3x^2 + 1}} = \frac{\pi}{4}$

40. C 51

Sol. $\left(x + 2\sqrt{y} - 1\right)^n = \sum_{r=0}^{n} {}^nC_r\, x^{(n-r)} \left(2\sqrt{y} - 1\right)^r$

For r = 4 $(-1)^4\ {}^nC_4 x^{n-4} = \frac{n(n-1)(n-2)(n-3)}{24}$

Also, coefficient of xy is ${}^nC_{n-1}\,{}^{n-1}C_2 4(-1)^{n-3} = (-1)^{n-3}2n(n-1)(n-2)$

$\Rightarrow \frac{n(n-1)(n-2)(n-3)}{24} = (-1)^{n-3}2n(n-1)(n-2)$

As $n > 4 \Rightarrow n-3 = (-1)^{n-3}\, 48$

Here n = 51

41. B, C

Sol. Using concept of rotation $\frac{Z_a - Z_d}{Z_m - Z_d} = e^{i\frac{2\pi}{7}} \Rightarrow Z_d = \frac{Z_m e^{i\frac{2\pi}{7}} - Z_a}{\left(e^{i\frac{2\pi}{7}} - 1\right)}$

Similarly, $Z_e = \frac{Z_d e^{i\frac{2\pi}{7}} - Z_a}{\left(e^{i\frac{2\pi}{7}} - 1\right)} \Rightarrow \left|\text{Arg}\left(\frac{Z_d - Z_e}{Z_m - Z_p}\right)\right| = \frac{5\pi}{14}$

42. A, C

Sol. $I_{n+k} = \int_{\frac{n+k}{2}}^{\frac{n+k+1}{2}} \frac{\sin\left(\pi\sin^2 \pi x\right)}{\left(\sqrt{2}\right)^x} dx$

Let $x = \frac{k}{2} + t \Rightarrow I_{n+k} = \int_{\frac{n}{2}}^{\frac{n+1}{2}} \frac{\sin\left(\pi\sin^2 \pi\left(\frac{k}{2} + t\right)\right)}{\left(\sqrt{2}\right)^{\frac{k}{2}+t}} dt$

$I_{n+k} = \frac{1}{2^{\frac{k}{4}}} \int_{\frac{n}{2}}^{\frac{n+1}{2}} \frac{\sin\left(\pi\sin^2 \pi t\right)}{\left(\sqrt{2}\right)^t} dt$

$I_{n+k} = \frac{I_n}{2^{\frac{k}{4}}}$

43. A, B, D

Sol. Let t = –x $\Rightarrow t \in \left(0, \frac{1}{2}\right)$

A = cos(sinπt) > 0, B = sin(cosπt) > 0, C = cos ($\pi - \pi$t) < 0

Now sinπt + cosπt = $\sqrt{2}\sin\left(t + \frac{1}{4}\right)\pi \le \sqrt{2} < \frac{\pi}{2}$

Now $0 < \cos \pi t < \frac{\pi}{2} - \sin \pi t < \frac{\pi}{2}$

$\Rightarrow \sin(\cos\pi t) < \sin\left(\frac{\pi}{2} - \sin\pi t\right)$

$\sin(\cos\pi t) < \cos(\sin\pi t)$

44. A, C

Sol. Apparently $a > \frac{\pi}{2}$ then maximum value of f(x) in [0, a] is 1, so

$^2K_\pi + \frac{5\pi}{6} \le a < 2a \le x_a\pi - + \frac{13\pi}{6}$, where K = 0, a = $\frac{5\pi}{6}$, or 2a = $\frac{13\pi}{12}$

Not possible for $K \ge 1$

Hence a = $\frac{5\pi}{6}$ or $\frac{13\pi}{12}$

45. B, C, D

Sol. Clearly f(x) is strictly decreasing in (0, 3]
and f(x) is strictly increasing in (3, 9]
and f(x) is strictly decreasing in (9, ∞]
f(3) = 0, f(9) = 1

46. A, C

Sol. adj(A) = 2B, adj(B) = A
Adj(A) = 2B

$|A|A^{-1} = 2B \Rightarrow |B|B^{-1} = A$

$\Rightarrow \quad 4A^{-1} = 2B \quad \Rightarrow 2B^{-1} = A$

$4I = 2BA \Rightarrow BA = 2I$

Adj(AAB) + adj(ABB) = adj(2A) + adj(2B)

$|B|=2 \Rightarrow |A| = 4$

Section – B

47. 4

Sol. $(x_0 - r - 1)^2 + y_0^2 = r^2$

$\Rightarrow 4x_0 \le y_0^2 = r^2 - (x_0 - r - 1)^2$

$\Rightarrow 4x_0 \le 2r(x_0 - 1) - (x_0 - 1)^2$

$\Rightarrow x_0^2 - 2(r-1)x_0 + (2r+1) \le 0$

Hence $(2(r - 1)^2 - 4(2r + 1) \ge 0$

$\Rightarrow |r| \ge 4$

48. 5

Sol. Assuming common difference of $[a_n]$ is d where $d \in I$

$\Rightarrow a_1 + a_2 = a_k$ for some positive integer k

$\Rightarrow 2a_1 + d = a_1 + (k - 1)d \Rightarrow a_1 = (k - 2)\,d$

$\Rightarrow k \ne 2$ and d = $\frac{a_1}{(k-2)}$ as $a_n = a_1 + (n - 1)d \Rightarrow a_n = a_1 + \left(\frac{n-1}{n-2}\right)a_1$

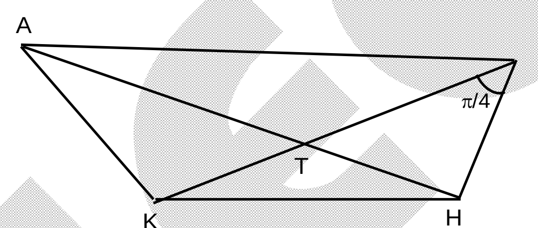
$\Rightarrow$ $a_1 + a_2 + a_3 + \ldots + a_{n-2} + a_{n-1} + a_n = a_1 n + \frac{n(n-1)}{2}d$

$\Rightarrow$ $\sum_{i=1}^{n} a_i = a_1 + \left((n-1)(k-2) + \frac{n(n-1)}{2}\right)d$

As 2019 is product of points (3) and (673)
Hence $k - 2 = -1, 1, 3, 673, 2019$

49. 2
Sol. $f^{2022}(x) \geq x\ \forall\ x \in R$
and $h^{2025}(x) = x$
Hence domain of function $\phi(x)$ is $(-\infty, 0)\cup(0, 1)\cup(1, \infty)$

50. 3



Sol. We have $\frac{AK + HI + \frac{KI}{\sqrt{2}}}{3} \geq \left(AK \cdot HI \cdot \frac{KI}{\sqrt{2}}\right)^{1/3}$

$AK + HI + \frac{KI}{\sqrt{2}} \geq 3.$

Clearly, AK $\perp$ to face KIH
$\therefore AI^2 = AK^2 + KI^2$
$\Rightarrow AI = \sqrt{3}$.

51. 1
Sol. Substituting
x = y = 1, we get
$f^2(3) + f^2(1) = 2f(1)$
$(f(1) - 1)^2 = 0 \Rightarrow f(1) = 1$
Putting y = 1, we get

$\Rightarrow$ $f(x)\, f(1) + f\left(\frac{3}{x}\right) f(x) = 2f(3)$

$\Rightarrow$ $f(x) f\left(\frac{3}{x}\right) = 1$ ...(1)

Also $f(x)\, f(1) + f\left(\frac{3}{x}\right) f(3) = 2f(x)$

$f(x) = f\left(\frac{3}{x}\right)\ \forall\ x > 0$ ....(2)

Conclusively for (1) and (2)
$f^2(x) = 1\ \forall\ n > 0$
Substituting $x = y = \sqrt{t}$

$f^2\left(\sqrt{t}\right) + f^2\left(\frac{3}{\sqrt{t}}\right) = 2f(t)$ as $f(t) > 0$ $\Rightarrow$ $f(x) = 1\ \forall\ x > 0$

52. 5
Sol. Let us calculate the number of ordered set pairs (X, Y) satisfying max. X $\leq$ min.Y. Let m = max.(X); So subsets of X are taken from the set {1, 2, 3, . . . (m – 1)} and {m} in 2^{m-1} ways

Similarly min(Y) $\geq$ m. $\Rightarrow$ Y is non-empty subset of {m, m+1, m + 2, . . . , n)} $\Rightarrow$ $(2^{1+n-m} - 1)$ ways

Hence (X, Y) for max.(X) $\leq$ min.(Y) = $\sum_{m=1}^{n} 2^{m-1}\left(2^{n+1-m} - 1\right) =$ = $n.2^n - 2^n + 1$

Hence total ways = $(2^n - 1)^2 - n.2^n + 2^n - 1 = 2^{2n} - 2^n(n+1)$

53. 2

Sol. $\ln(a)|_{maximum} = \dfrac{\ln\left(\frac{1}{e}\right)_{max}}{\left(\frac{1}{e}\right)_{min}} = \dfrac{1}{e}$

$\ln(a)|_{minimum} = \dfrac{\ln\left(0^+\right)}{(1)} = -\infty$

Hence intervals of a is $(0, e^{-e})$

54. 4

Sol. Let $I_n = \int_0^{\pi/2} \sin^n x dx$

$1 < \dfrac{I_{2n}}{I_{2n+1}} < \dfrac{2n+1}{2n}$

$\lim_{h\to\infty} = \dfrac{I_{2n}}{I_{2n+1}} = 1$, Hence $\left[\dfrac{\pi^2}{2}\right] = 4$