FIITJEE
ALL INDIA TEST SERIES
JEE (Advanced)-2023
FULL TEST – VII
PAPER –1
TEST DATE: 23-04-2023

ANSWERS, HINTS & SOLUTIONS

Physics PART – I

Section – A

1. BCD

Sol. Using KVL

$$10-\frac{q_3}{3}=0 \Rightarrow q_3 = 30\mu C$$

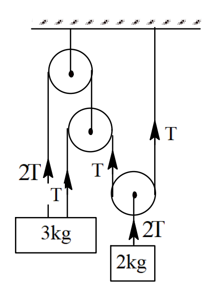
Current through the battery is zero initially as the capacitors are fully charged in steady state and not allow to flow current through it.

Current through the battery after closing the switch is $\frac{10}{2+3}=2A$.

Charge on $2\mu F$ capacitor after closing the switch is $\frac{q}{2}=2\times 2 \Rightarrow q = 8\mu C$

2. BC

Sol. $30-3T=3a_3 \;\; ....(i)$



$20-2T=2a_2 \;\; ....(ii)$

Constrained relation

$3a_3+2a_2=0 \;\; ....(iii)$

$3(10-T)+2(10-T)=0$

$30-3T+20-2T=0$

$T=10\,Newton$

So $a_3=0$

3. ACD

Sol. $w_1t - w_2t = 2pN$

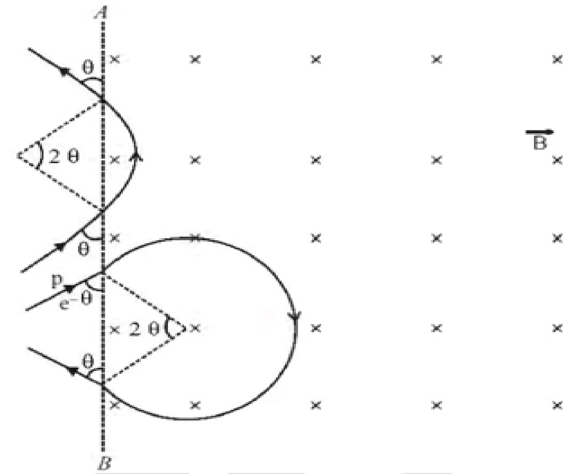
$$\frac{2\pi}{T_1}t-\frac{2\pi}{T_2}t=2\pi N \Rightarrow t=\frac{NT_1T_2}{T_2-T_1}(\text{where} N = 1,2,3,.......)$$

$t = 49.5\,\text{sec},\ 99\,\text{sec}, 198\,\text{sec},\ 297\,\text{sec},\ ....$

4. ABC

5. BD

Sol. Figure shows that the magnetic field $\vec{B}$ is present on the right-hand side of AB. The electron (e) and proton (p) moving on straight parallel paths with the same velocity enter the region of uniform magnetic field. Therefore, both will come out travelling on parallel paths.



The time taken by proton.

$$t_p = \frac{\text{distance}}{\text{speed}} = \frac{\text{arc}}{\text{speed}} = \frac{\text{angle}\times\text{radius}}{\text{speed}} = \frac{2\theta\times R_p}{v}$$

$$= \frac{2\theta}{v}\times\left(\frac{m_p v}{eB}\right) = \frac{2\theta m_p}{eB}$$

The time taken by electron is

$$t_e = \frac{(2\pi-2\theta)R_e}{v} = \frac{(2\pi-2\theta)}{v}\left(\frac{m_e v}{eB}\right) = \frac{(2\pi-2\theta)m_e}{eB}$$

6. BD

7. A

8. C

9. D

10. A

Section – B

11. 0.80

Sol. Minimum time period of a physical pendulum $T=2\pi\sqrt{\frac{2k}{g}}$, where k is radius of gyration

$\Rightarrow \frac{ma^2}{6}$ and a is side of square $k = 8\text{cm}$

$\Rightarrow$ Time=0.8sec

12. 21.00

Sol. $I_1+I_2=11I_0$ ......(i)

$\left(\sqrt{I_1}-\sqrt{I_2}\right)^2 = I_0$ ......(ii)

$\Rightarrow \left(\sqrt{I_1}+\sqrt{I_2}\right)^2 = mI_0$ .....(iii)

Solving (i), (i) and (iii) gives

m = 21

13. 0.45

Sol. $R_{PQ} = \frac{9}{20} = 0.45\,\text{ohm}$

14. 1200.00

Sol. Speed of sound along the line wave reaching observer $= \sqrt{(260)^2 - (100)^2}$

$V = 240\,m/s$

Apparent $f = f_0\left(\frac{V}{V - V_s}\right)$

$f = 1000\left(\frac{240}{240 - 40}\right)$

$f = 1200\,Hz$

15. 35.00

Sol. $\frac{n_1\lambda/2}{n_2\lambda/2} = \frac{10}{4} \Rightarrow \frac{n_1}{n_2} = \frac{5}{2}$ so, $n_1 + n_2 = 7$

So minimum frequency $f = \frac{(n_1 + n_2)v}{2\ell} = \frac{7}{2 \times 14}\sqrt{\frac{196}{0.01}}$

f = 35 Hz

16. 7.00

Sol. If q be the charge of the metal sphere corresponding to the potential V, then $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$

Or $q = (4\pi\varepsilon_0 R)V$

$q = \left(\frac{1}{9 \times 10^9}\right) \times \frac{10^{-3}}{2} \times 1 = 5.5 \times 10^{-10}\,C$

$t = \frac{q}{0.80 \times 6.25 \times 10^{10} \times 1.6 \times 10^{-19}} \simeq 7.00$

17. 0.12

18. 50.00

Sol. As the electric field is zero inside the inner most shell so $\Delta V \text{-} \vec{E}.\Delta\vec{r} \Rightarrow \Delta V = 0$

So, the potential of inner shell will be equal to the potential at the centre irrespective of charge on shells.

Chemistry

PART – II

Section – A

19. A, C

Sol. $SiCl_4$ is hydrolyzed completely.

20. ABC

Sol. $X \Rightarrow ClO_2 \qquad Y = Cl_2O_6 \qquad Z = NaClO_2$

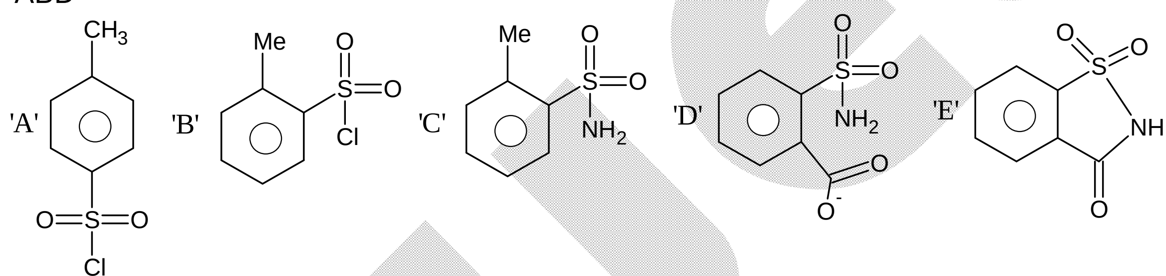
$$2HClO_3 + SO_2 \longrightarrow 2ClO_2 + H_2SO_4$$

$$ClO_2 + 3O_3 \longrightarrow Cl_2O_6 + 2O_2$$

$$2ClO_2 + H_2O_2 + 2NaOH \longrightarrow 2NaClO_2 + 2H_2O + O_2$$

21. ABD

Sol.



22. ACD

23. BD

Sol. A.O. is a single e^- wave function.

Area of the plot for r<52.9 pm is smaller than the area of the plot of r>52.9pm.

At higher atomic number every of $3d<4s$

$$\Delta E_{2\to5} = RhcZ^2\left[\frac{1}{2^2}-\frac{1}{5^2}\right] = RhcZ^2\left[\frac{21}{100}\right]$$

$$\Delta E_{2\to4} = RhcZ^2\left[\frac{1}{2^2}-\frac{1}{4^2}\right] = RhcZ^2 \times \frac{3}{16}$$

$$\Delta E_{2\to5} = \frac{28}{25}\Delta E_{2\to4}$$

24. C

Sol. From the Grahm's law d = 16 cm.

$$\frac{d}{24-d} = \sqrt{\frac{40}{10}} \Rightarrow d = 16$$

25. B

Sol. Fe^{+3} is precipitated by conc. NH_3 but Zn^{2+} is not precipitated by conc. NH_3.

26. B
Sol. (A) IIIrd law states $\lim_{T\to 0} S \to 0$ for Perfectly Crystalline solid.

27 A
Sol. $C_6H_5COCH_3$ is meta-directing & deactivating

28. A
Sol. PV=nRT

Section – B

29. 0.77
Sol. $E^\circ = \frac{1.68\times 3+1.21\times 2-1.035\times 2}{7} = 0.77$

30. 100.00
Sol. $201 - x = -101$
$x = 100$

31. 18.20
Sol. $Ca_3(PO_4)_2 + \underset{0.8mole}{8Mg} \longrightarrow Ca_3P_2 + 8MgO$
$\underset{0.8mole}{MgO} + \underset{0.1mole}{P_2O_5} \longrightarrow Mg(PO_3)_2$

32. 14.14
Sol. $s = [CH_3COO^-] + [CH_3COOH]$
$s^2 = K_{sp}\left[1+\frac{[H^+]}{K_a}\right] = 2\times10^{-10}\left[1+\frac{10^{-3}}{10^{-9}}\right] = 2\times10^{-4} = 200\times10^{-6}$
$s = 14.14\times10^{-3}$

33. 0.35
Sol. $K_{eq.} = \frac{p_B}{p_A} = \frac{10}{2} = 5$
$K_{eq.}' = \frac{20}{2} = 10$
$\frac{\Delta G^\circ_{100K}}{\Delta G^\circ_{200K}} = \frac{-R\times100\ln5}{-R\times200\ln10} = \frac{1}{2}\frac{\log5}{\log10} = \frac{\log5}{2} = \frac{1-0.3}{2} = 0.35$

34. 3.81
Sol. A $\rightarrow$ 0.28 mole Benzene
B $\rightarrow$ 0.14 mole Cumene
C $\rightarrow$ 0.14 × 0.4 = 0.056 mole
D $\rightarrow$ 0.028 mole = 0.028 × 136g = 3.808g = 3.81g

35. 16.29

Sol. $\underset{\text{Solid Black}}{2As} + \frac{3}{2}O_2 \rightarrow \underset{\text{Solid}}{As_2O_3}\ \Delta U = -100\ KJ/mole$

$\Delta H = -100 - \left(\frac{3}{2} \times 8.3 \times 10^{-3} \times 298\right) = -103.7101$

$\underset{(\text{Gray})}{As} \rightarrow \underset{\text{Black}}{As} \qquad \Delta H = 60\,KJ/mole$

$\underset{\text{Gray}}{2As} + \frac{3}{2}O_2 \rightarrow \underset{\text{Solid}}{As_2O_3}\ \Delta H = 120 - 103.7101 = 16.2899\,KJ/mole$

36. 5.00

Sol. $C_3H_8 + 6H_2O \rightarrow 3CO_2 + 20H^+ + 20e$

$20H^+ + 5O_2 + 20e \rightarrow 10H_2O$

Mathematics

PART – III

Section – A

37. A, B

Sol. A, B

VC = 3 units, CO = 9 units

As per property: $AV = VN$ and $\frac{AN}{AV} = 2 = \frac{MN}{VC}$

$\Rightarrow OM = 6\tan\theta, AC = CM = 3\cot\theta \Rightarrow 9 = 3\left(2\tan\theta + \frac{1}{\tan\theta}\right)$

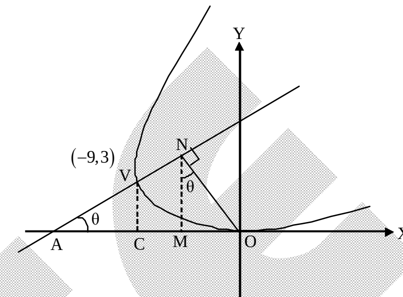
$\Rightarrow m = 1 \text{ or } m = \frac{1}{2}$

$\Rightarrow Eq^n$ of Axis can be

$(y-3) = 1(x+9)$

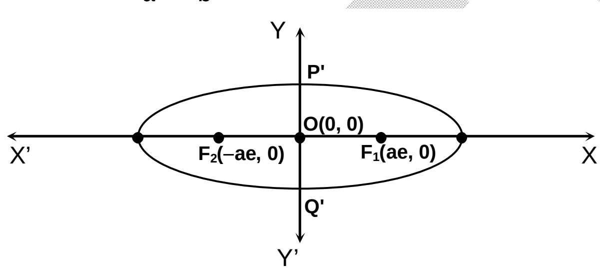
or

$(y-3) = \frac{1}{2}(x+9)$



38. A, B, C

Sol. Ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (a > b)



$F_1Q = a + ae = 3$

$F_1P = a - ae = 2$

$2a = 5 \Rightarrow b^2 = 6$

$b^2 = a^2(1 - e^2) \Rightarrow a^2e^2 = a^2 - b^2$

Ellipse is $\frac{4x^2}{25} + \frac{y^2}{6} = 1$

Similarly Ellipse (s) in other case ax $\frac{x^2}{4} + \frac{y^2}{3} = 1$

$\Rightarrow \frac{x^2}{9} + \frac{y^2}{8} = 1$ here PQ will be 5, $\sqrt{7}$, $\sqrt{17}$

39. C, D

Sol. $\log_2(f(x)) - \log_2(f(y)) \le (x - y)^2$

Suppose y = x + h and Limit $h \to 0$

$$\frac{d}{dx}\left(\log_2\left(f(x)\right)\right) \le 0 \Rightarrow \frac{f'(x)}{f(x)} \le 0 \qquad ...(1)$$

If x = y + h and Limit $h \to 0$

$$\frac{d}{dx}\left(\log_2\left(f(x)\right)\right) \geq 0 \Rightarrow \frac{f'(x)}{f(x)} \geq 0 \qquad ...(2)$$

From (1) to (2) we get

$f'(x) = 0$

40. B, D

Sol. Using the condition in the problem

$$P_n = \frac{1}{2}(P_{n-1} + P_{n-2}) n \geq 3$$

$$\Rightarrow \; P_n = \frac{2}{3} + \frac{1}{3}\left(-\frac{1}{2}\right)^n$$

41. A, B

Sol. $T = |\vec{a}||\vec{b}||\vec{c}||\cos\alpha||\sin\theta_1|$

$= |\vec{a}||\vec{b}||\vec{c}||\cos\alpha||\sin\theta_2|$

$= |\vec{a}||\vec{b}||\vec{c}||\cos\alpha||\sin\theta_3|$

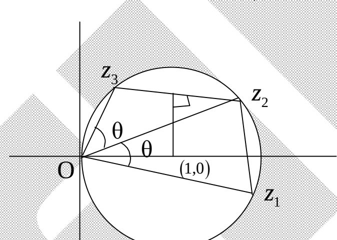
$Y = 2\left(|\vec{a}\times\vec{b}| + |\vec{b}\times\vec{c}| + |\vec{c}\times\vec{a}|\right)$

$= 2\left(|\vec{a}||\vec{b}||\sin\theta_1| + |\vec{b}||\vec{c}||\sin\theta_2| + |\vec{c}||\vec{a}||\sin\theta_3|\right)$

$$\frac{Y}{T} = \frac{2}{|\cos\alpha|}\left(\frac{1}{|\vec{a}|} + \frac{1}{|\vec{b}|} + \frac{1}{|\vec{c}|}\right) = 4\left(\frac{1}{|\vec{a}|} + \frac{1}{|\vec{b}|} + \frac{1}{|\vec{c}|}\right) \Rightarrow |\cos\alpha| = \frac{1}{2}$$

42. A, B, D

Sol. 1st option – chord $(Z_3 - Z_2)$ is $\perp$ to line joining $\frac{z_2 + z_3}{2}$ and 1.



2nd option – angle by chord $(Z_3 - Z_2)$ at $(1,0)$ is double of angle at $(0,0)$

4th option – Ptolemy's theorem

43. C

44. B

45. A

Sol. Clearly $I_1 + I_2 = \sqrt{2}\, e^{\pi/4} - 1$ and $I_3 = 1/2$.

46. B

Sol. (I) $A^{-1} + B^{-1} = (A + B)^{-1} \Rightarrow (A^{-1} + B^{-1})(A + B) = 1$

$\Rightarrow$ $I + A^{-1}B + B^{-1}A = 0$ $\Rightarrow$ $P = A^{-1}B$

$I + P + P^{1} = 0 \Rightarrow P + P^2 + 1 = 0$

$\Rightarrow$ $P^2 + P^3 + P = 0$

$\Rightarrow$ $P^3 = I \Rightarrow |P| = 1$

$\Rightarrow$ $|A^{-1}B| = 1 \Rightarrow |A| = |B|$

(II) $B^2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

(III) In $A^2, A^3, \ldots A^{20}$, and $A^{20} + B$

$C_{22}, C_{23}, C_{32}, C_{33}$ remains same

(IV) $a_{22}b_{18} + a_{22}b_{28} + \ldots + a_{28}b_{58} = \frac{2.8}{1.4} + \frac{2.8}{4.7} + \ldots + \frac{2.8}{13.16} = \frac{2.8}{3}\left[1 - \frac{1}{4} + \frac{1}{4} + \frac{1}{7} + \ldots + \frac{1}{3} - \frac{1}{16}\right]$

$= \frac{16}{3} \times \frac{15}{16} = 5$

Section – B

47. 7400.00

Sol. Apparently $2f = 2\left(a_1^2 + a_2^2 + a_3^2 + \ldots\ldots + a_{2019}^2\right) - (2a_1a_3 + 2a_2a_4 + 2a_3a_5 + \ldots\ldots + 2a_{2017}a_{2019})$

$\Rightarrow$ $2f = \left(a_1^2 + a_2^2 + a_{2018}^2 + a_{2019}^2\right) + \sum_{i=1}^{2017}\left(a_{i+2} - a_i\right)^2$ ...(1)

Since, a_1, a_2 and $a_{i+2} - a_i$) are all non-negative integers

$\Rightarrow a_1^2 \ge a_1,\ a_2^2 \ge a_2\ \ (a_{i+1} - a_i)^2 \ge a_{i+2} - a_i$ for i = 1, 2, 3, ... 2016

$\Rightarrow$ $a_1^2 + a_2^2 + \sum_{i=1}^{2016}\left(a_{i+2} - a_i\right)^2 \ge a_1 + a_2 + \sum_{i=1}^{2016}\left(a_{i+2} - a_i\right)^2 = a_{2017} + a_{2018}$ ...(2)

$\Rightarrow$ $2f \ge a_{2017} + a_{2018} + (a_{2019} - a_{2017})^2 + a_{2018}^2 + a_{2019}^2$

As $a_{2019} = 99$ and $a_{2018} \ge a_{2017} > 0$

$\Rightarrow$ $f \ge \frac{1}{2}\left(2a_{2017} + \left(99 - a_{2017}\right)^2 + a_{2017}^2 + 99^2\right)$

$\Rightarrow$ $f \ge (a_{2017} - 49)^2 + 7400$

$\Rightarrow$ $f_{minimum} = 7400$

48. 4.93

Sol. We know that

$\sum_{r=0}^{n}(-1)^r\, {}^nC_r\,(n - r)^k = \begin{cases} 0 & \text{if } k < n \\ \underline{|n} & \text{if } k = n \end{cases}$

$\frac{\underline{|2024} \times 10^4}{2025} = \frac{10^4}{2025}$ = 4.938

49. 498.00

Sol. Identifying A $\to$ All permutations except 0 at first place = 600

B $\to$ not 0 after 2 and not 4 after 3

C $\to$ 0 after 2 and not 4 after 3

D $\to$ not 0 after 2 and 4 after 3

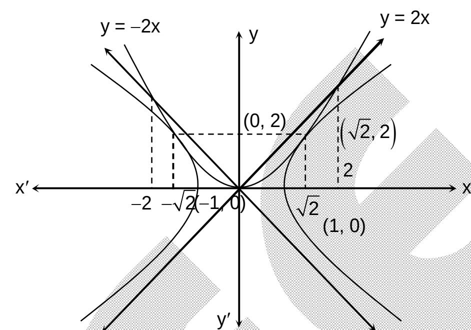
No. of such permutations = $n(A)-\frac{3}{4}n(B)-\frac{n(C)}{2}-\frac{n(D)}{2}$

$=600-\frac{3}{4}\times 24-\frac{96}{2}-\frac{72}{2}$ = 498

50. 9.65

Sol. Analyzing the given figure

$m \in \left\{2,-2,2\sqrt{2},-2\sqrt{2}\right\}$



51. $\frac{4\sqrt{3}}{9}$, 0.76 or 0.77

Sol. Let radius of S = 0 be r where r > 0

$S \equiv (x-1)^2 + (y-r)^2 = r^2$

$\left(\frac{y^2}{4}-1\right)^2 + y^2 - 2ry = 0$

$\Rightarrow$ r = $\frac{\left(y^2+4\right)^2}{32y}$ . . . (1)

Equ. (1) must have only one positive solution

Suppose f(y) = $\frac{\left(y^2+4\right)^2}{32y}$ $y > 0$

$\Rightarrow$ $y^2 + 4 = y^2\ \frac{4}{3}+\frac{4}{3}+\frac{4}{3} \ge 4\sqrt[4]{y^2-\left(\frac{4}{3}\right)^3}$

$\Rightarrow$ f(y) = $\frac{1}{32y}\times 16\sqrt{y^2.\left(\frac{4}{3}\right)^3} = \frac{4\sqrt{3}}{9}$

$\Rightarrow$ $r \ge \frac{4\sqrt{3}}{9}$ Assuming r > $\frac{4\sqrt{3}}{9}$

If r > $\frac{4\sqrt{3}}{9}$ then, f(y) changes continuously with y and becomes arbitrary when $y \to 0^+$ or $y \to \infty$

$\Rightarrow$ Equ. (1) has both solutions $\left(0, \frac{2\sqrt{3}}{3}\right)$ and $\left(\frac{2\sqrt{3}}{3}, 0\right)$

Hence $r \ge \frac{4\sqrt{3}}{9}$ is only consider solution and $\left(\frac{1}{3}, \frac{2\sqrt{3}}{3}\right)$ is unique common point.

52. 5.33

Sol. Suppose ln(x) = a, ln(y) = b, ln(z) = c
Since x, y, z > 1
Hence a + (c/b) = 3, b + (c/a) = 4
ab + c = 3b = 4a
Suppose a = 3u, b = 4u (u > 0) then
$c = 4a - ab = 12u - 12u^2$
$\ln x \ln z = ac = 3u.12u(1 - u) = 18u^2(u - 2u) \le 18\left(\frac{u+u+(2-2u)^3}{3}\right) = 18\left(\frac{2}{3}\right)^3 = \frac{16}{3}$

53. $\frac{1}{\sqrt{3}}$ 0.57 or 0.58

Sol. Apparently angle S_1PS_2 is greatest if P is point of contact of tangency of circle S_1, S_2, P on the lie
$y = -2x$.
Suppose A is origin (0, 0).
$\angle APS_1 = \angle AS_2P$ and $\Delta APS_1 \sim \Delta AS_2P$
$\frac{S_1P}{S_2P} = \frac{AP}{S_2P}$ also $AP^2 = AS_1.AS_2$
$\frac{PS_1}{PS_2} = \sqrt{\frac{S_1A}{S_2A}} = \sqrt{\frac{3}{9}} = \frac{1}{\sqrt{3}}$

54. 48.31

Sol. f(x) + f(x + 1) + … + f(x + 10)
$= |e^x - 1| + |e^x - 2| + \dots |e^x - 11|$
$\Rightarrow \int_0^{11} f(x)\,dx = 65 + 4\ln 2 - 7e$