FIITJEE
ALL INDIA TEST SERIES
JEE (Advanced)-2023
OPEN TEST – II
PAPER –2
TEST DATE: 16-04-2023

Time Allotted: 3 Hours **Maximum Marks: 180**

General Instructions:

- The test consists of total 54 questions.
- Each subject (PCM) has 18 questions.
- This question paper contains **Three Parts.**
- **Part-I** is Physics, **Part-II** is Chemistry and **Part-III** is Mathematics.
- Each **Part** is further divided into **Two Sections**: **Section-A & Section-B**.

Section – A *(01 – 04, 19 – 22, 37 – 40)*: This section contains **TWELVE (12)** questions. Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.

Section – A *(05 –10, 23 – 28, 41 – 46)*: This section contains **EIGHTEEN (18)** questions. Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).

Section – B *(11 – 18, 29 – 36, 47 – 54)*: This section contains **TWENTY FOUR (24)** numerical based questions. The answer to each question is a **Single Digit Integer, ranging from 0 to 9 both inclusive.**

MARKING SCHEME

Section – A (Single Correct): Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+3	If ONLY the correct option is chosen.
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	–1	In all other cases.

Section – A (One or More than One Correct): Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+4	If only (all) the correct option(s) is (are) chosen;
Partial Marks	:	+3	If all the four options are correct but ONLY three options are chosen;
Partial marks	:	+2	if three or more options are correct but ONLY two options are chosen and both of which are correct;
Partial Marks	:	+1	If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	–2	In all other cases.

Section – B: Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+3	If ONLY the correct integer is entered;
Zero Marks	:	0	Question is unanswered;
Negative Marks	:	–1	In all other cases.

Physics

PART – I

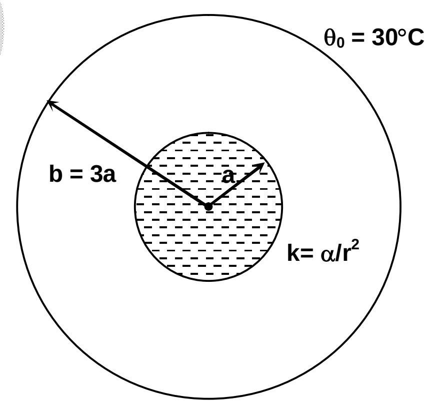
Section – A (Maximum Marks: 12)

This section contains ***FOUR (04)*** *questions. Each question has* ***FOUR*** *options.* ***ONLY ONE*** *of these four options is the correct answer.*

1. A block of mass m = 1 kg placed on a rough horizontal surface is connected to one end of a long spring of stiffness k = 100 N/m. The coefficient of kinetic friction between the horizontal surface and the block is $\mu_k = 0.50$. The free end of the spring is pulled gradually away from the block until the block begins to slide. If the block stops after sliding a distance S = 6 cm. The coefficient of static friction between the block and the horizontal surface is (Take g = 10 m/s^2)
(A) 0.60 (B) 0.70
(C) 0.80 (D) 0.90

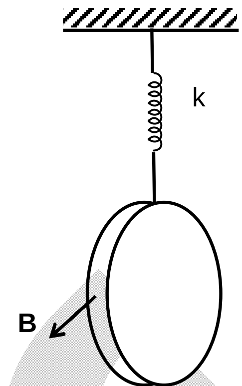
2. The air column in a pipe closed at one end is made to vibrate in its fourth overtone by a tuning fork of frequency 660 Hz. The speed of sound in air is 330 m/s. End correction may be neglected. Let P_0 denotes the mean pressure at any point in the pipe and ΔP_0 the maximum amplitude of pressure variation in the pipe. Then the maximum pressure in the pipe at a distance 18.75 cm from the open end of pipe is
(A) $\left(P_0 + \frac{\Delta P_0}{\sqrt{2}}\right)$ (B) $\left(P_0 + \frac{\Delta P_0}{2}\right)$
(C) $\left(P_0 + \frac{\sqrt{3}\Delta P_0}{2}\right)$ (D) $\left(P_0 + \Delta P_0\right)$

3. Water of mass 'm' and specific heat capacity 'S' is kept in the spherical cavity of radius 'a' of a thick spherical shell of inner and outer radii a and 3a respectively which is placed in a surrounding at temperature $\theta_0 = 30°C$ as shown in the figure. Thermal conductivity of the shell varies as $k = \frac{\alpha}{r^2}$, where α is a constant. If initial temperature of water is 70°C, then the time required to decrease the temperature of water from 70°C to 50°C is



(A) $\frac{mSa\ell n2}{4\pi\alpha}$ (B) $\frac{mSa\ell n2}{2\pi\alpha}$
(C) $\frac{2mSa\ell n2}{3\pi\alpha}$ (D) $\frac{4mSa\ell n2}{3\pi\alpha}$

4. A conducting disc of mass 'm' and volume 'V' is suspended with the help of a light spring of force constant 'k' from a fixed support. Thickness of the disc is much smaller than its radius. A uniform magnetic field of induction 'B' parallel to the plane of the disc is established. The disc is slightly pulled down from equilibrium position and then released. The time period of small oscillations of the disc is



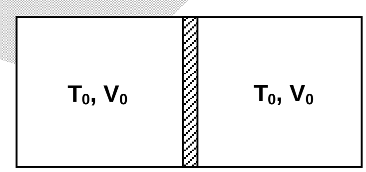
(A) $2\pi\sqrt{\frac{(m+\varepsilon_0 VB^2)}{k}}$ (B) $2\pi\sqrt{\frac{(m+\varepsilon_0 VB^2)}{2k}}$

(C) $2\pi\sqrt{\frac{(m+\varepsilon_0 VB^2)}{4k}}$ (D) $2\pi\sqrt{\frac{2(m+\varepsilon_0 VB^2)}{k}}$

Section – A (Maximum Marks: 24)

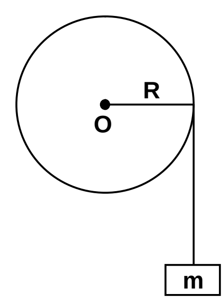
*This section contains **SIX (06)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).*

5. A heat conducting piston can freely move inside a closed thermally insulated cylinder. In equilibrium, the piston divides the cylinder into two equal parts each of which contains one mole of an ideal monoatomic gas at temperature T_0 = 500 K initially. Now, the piston is slowly displaced until their volume ratio becomes 2 : 1. Then choose the correct option(s). (Take R = 8.3 J/mol-k, $(9)^{1/3} = 2.08$)



(A) The final temeprature of the gas is 520 K.
(B) The final temperature of the gas is 550 K.
(C) The net increase in the internal energy of the gases is 498 Joule.
(D) The net increase in the internal energy of the gases is 1245 Joule.

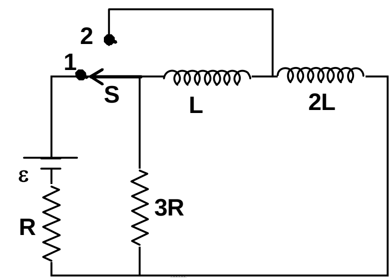
6. Charge Q is uniformly distributed over the curved surface of a long non-conducting solid cylinder of mass M, radius R and length ℓ. The cylinder can rotate without friction about a fixed horizontal axle coinciding with its own axis. Several turns of a light thin insulating string are wrapped on the cylinder and a block of mass m is suspended from the free end of the string as shown in the figure. The system is released from rest and the permeability of the medium inside the cylinder is μ_0. Then choose the correct option(s). (Take M = 4m and $\frac{\mu_0 Q^2}{4\pi\ell} = m$)



(A) The acceleration of the block is $\frac{g}{2}$ (B) The acceleration of the block is $\frac{g}{4}$

(C) The tension in the string is $\frac{3mg}{4}$ (D) The tension in the string is $\frac{3mg}{2}$

7. In the circuit shown, initially the switch 'S' is kept closed in position-1 for a long time and then it is shifted from position-1 to position-2 at the instant t = 0. Then choose the correct option(s).



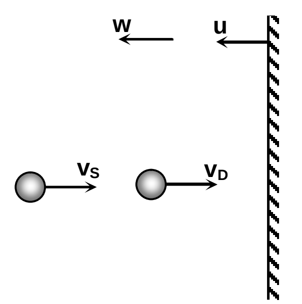
(A) The current through the switch S as function of time t after having shifted to position-2 is $\frac{\varepsilon}{R}\left[1-e^{-\left(\frac{3Rt}{2L}\right)}\right]$

(B) The current through the switch S as a function of time t after having shifted to position -2 is $\frac{\varepsilon}{R}e^{-\left(\frac{3Rt}{2L}\right)}$

(C) The total heat dissipated in the resistor 3R after having shifted the switch S to position-2 is $\frac{L\varepsilon^2}{2R^2}$

(D) The total heat dissipated in the resistor 3R after having shifted the switch S to position-2 is $\frac{L\varepsilon^2}{R^2}$

8. A source is moving with a velocity v_S = 8 m/s towards the reflector which is moving with a velocity u = 2 m/s towards the source. A detector is moving with a velocity v_D = 1 m/s towards the reflector. The wind is blowing with a velocity ω = 2 m/s away from the reflector. The frequency of sound emitted by the source is f = 320 Hz and the velocity of sound relative to air is v = 330 m/s. Then choose the correct option(s).



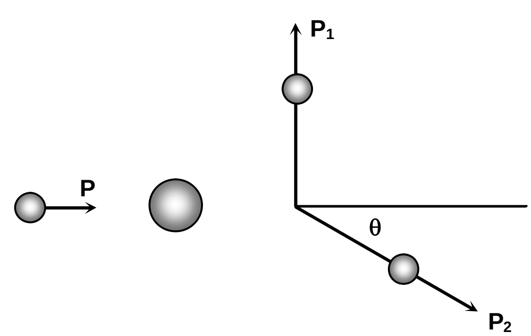
Reflector

(A) The frequency of the reflected wave received by the detector is 333 Hz.
(B) The wavelength of the reflected wave received by the detector is 1 m
(C) The beats frequency received by the detector is 4 Hz
(D) The beats frequency received by the detector is 6 Hz

9. When a neutron with kinetic energy k = 13 MeV collides with a stationary carbon nucleus, the following nuclear reaction takes place.

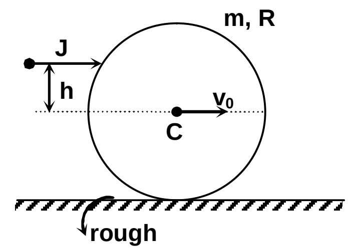
$n + {}^{12}C \rightarrow {}^{9}Be + {}^{4}He$

If Q- value of this reaction is E = –5.72 MeV and the α-particle produced is outgoing at right angle to the incoming direction of the neutron. Then choose the correct option(s).



(A) The kinetic energy of the α-particle produced is 4.04 MeV
(B) The kinetic energy of the α-particle produced is 5.12 MeV
(C) The kinetic energy of the Be-nucleus produced is 2.16 MeV
(D) The kinetic energy of the Be-nucleus produced is 3.24 MeV.

10. A uniform spherical shell of mass m and radius R is placed on a rough horizontal surface. When a horizontal impulse J is applied to the shell at a height 'h' above the centre level, it acquires an initial velocity v_0 as shown in the figure. The velocity of its centre of mass becomes $\frac{4v_0}{5}$ when it starts rolling without slipping on the horizontal surface. Then choose the correct option(s).

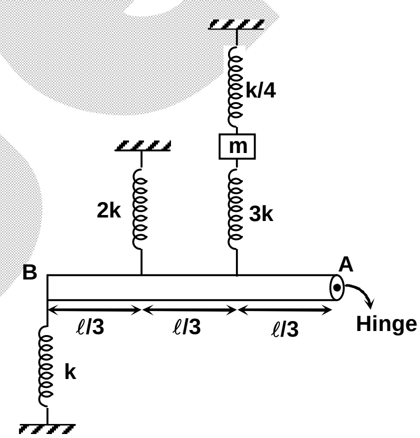


(A) The value of h is R/3.
(B) The value of h is R/2.
(C) The total work done by the frictional force is $\left(-\frac{mv_0^2}{10}\right)$
(D) The total work done by the frictional force is $\left(-\frac{mv_0^2}{20}\right)$

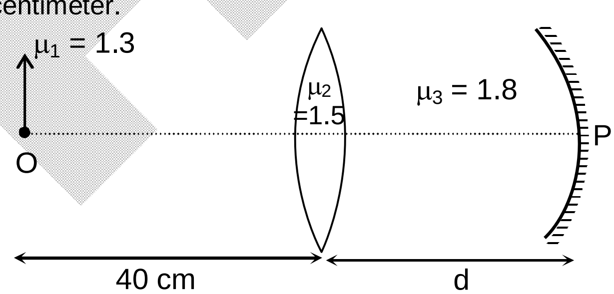
Section – B (Maximum Marks: 24)

*This section contains **EIGHT (08)** numerical based questions. The answer to each question is a **Single Digit Integer, ranging from 0 to 9 both inclusive.***

11. A thin horizontal light rod AB of length 'ℓ' hinged at end A is connected with the ideal vertical springs and a block of mass 'm' in a gravity free space as shown in the figure. The block is slightly displaced from its equilibrium position and then released. If the frequency of small oscillations of the block is $\frac{1}{2\pi}\sqrt{n\left(\frac{7k}{20m}\right)}$. Find the value of n.

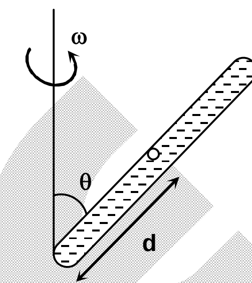


12. A thin equiconvex lens (μ_2= 1.5) of focal length 20 cm relative to air and a concave mirror of focal length 30 cm are separated by a distance 'd'. The refractive indices of the medium on the two sides of the lens are μ_1 = 1.3 and μ_3 = 1.8 as shown in the figure. The optic axes of the lens and the mirror are coinciding. When a point object 'O' is placed at a distance 40 cm from the lens on its optic axis, the final image coincides with the point object 'O'. Find the value of $\left(\frac{d}{2}\right)$ in centimeter.

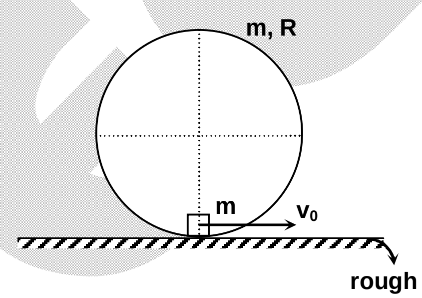


13. A small body starts falling on to the sun from a distance equal to the radius of the earth's orbit around the sun. The initial velocity of the body is zero relative to the heliocentric reference frame. If the time period of revolution of earth around the sun is T_0 and the time taken by the body to fall on the surface of sun is $\left(\frac{nT_0}{16\sqrt{2}}\right)$. Find the value of n.

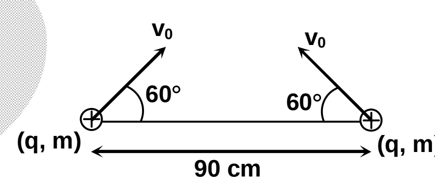
14. A closed tube of length ℓ = 2m completely filled with water has a small air bubble trapped in it. When the tube is held at an angle $\theta = 37°$ with the vertical and rotated with a constant angular velocity ω = 10 rad/s about a vertical axis passing through its lower end, the bubble settles at a distance 'd' from the lower end of the tube as shown in the figure. Find the value of the ratio $\left(\frac{\ell}{d}\right)$. (Take g = 10 m/s^2)



15. A small block of mass 'm' is initially getting at the lowest point of a uniform circular hoop of same mass m and radius R which is placed on a rough horizontal surface at rest. When an initial horizontal velocity v_0 is imparted to the small block, the hoop starts rolling without slipping on the horizontal surface as shown in the figure. If the block is just able to reach the end of the horizontal diameter of the hoop. Find the value of v_0 (in m/s). (there is no friction between the block and the hoop) (Take g = 10 m/s^2, R = 30 cm)

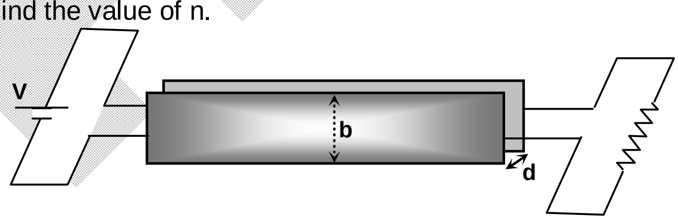


16. Two identical point charges are moving in the free space. When the two particles are getting at a separation 90 cm, their velocity vectors are equal in modulus and making angle 60° with the line joining them as shown in the figure. At this instant, their electric potential energy is two times their total kinetic energy. If the distance of closest approach between them is (10n) cm. Find the value of n.



17. A load resistance is connected across the terminals of a battery of voltage V with the help of two long conducting strips each of width 'b' and connecting wires. The strips are arranged parallel to each other at a separation d(<<b). For a certain value of load resistance, the net force of electrostatic and magnetic interactions between the strips vanishes. When the load resistance is made two times of the value at which the net interaction force vanishes, the net force of electrostatic and magnetic interaction per unit length between the strips becomes $\left(\frac{n\varepsilon_0 bV^2}{8d^2}\right)$.

Find the value of n.



18. A hydrogen like atom in the first excited state absorbs a photon of energy 22.95 eV and gets excited to a higher energy level of quantum number n. Now the excited atom can emit six different photons, some of higher and some of lower energies than the absorbed photon. If the atomic number of the atom is Z then find the value of (n + z).

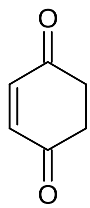
Chemistry

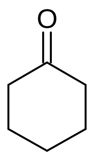
PART – II

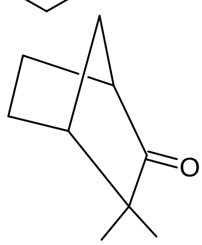
Section – A (Maximum Marks: 12)

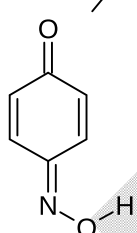
*This section contains **FOUR (04)** questions. Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.*

19. In which of the following tautomerism is not possible

(A)


(B)


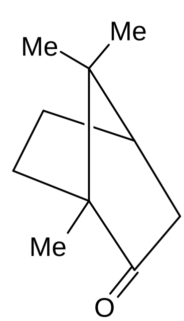
(C)


(D)


20. The equivalent mass of HCl in given balanced reaction (if molar mass of HCl is M)

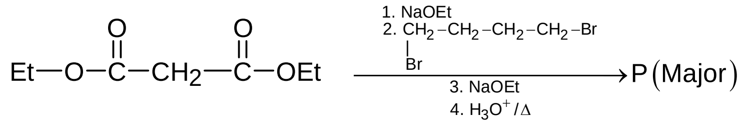
$K_2Cr_2O_7 + 14HCl \longrightarrow 2CrCl_3 + 3Cl_2 + 2KCl + 7H_2O$

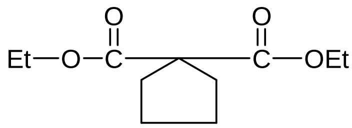
(A) $\frac{M}{1}$
(B) $\frac{M}{6}$
(C) $\frac{M}{4}$
(D) $\frac{7M}{3}$

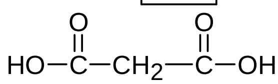
21. Number of stereo isomers possible for


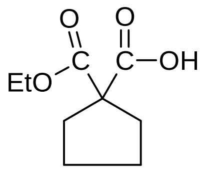
is

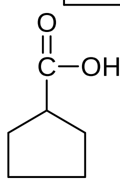
(A) 2
(B) Stereo isomerism is not possible
(C) 4
(D) 6

22.  , P is

(A) 

(B) 

(C) 

(D) 

Section – A (Maximum Marks: 24)

*This section contains **SIX (06)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).*

23. Among the following, the intensive properties is/are

(A) E^{o}_{cell} (B) Depression in freezing point

(C) Molarity (D) Entropy

24. Among the following, the correct statemen(s) about polymers is/are

(A) Vinyl chloride is monomers of PVC

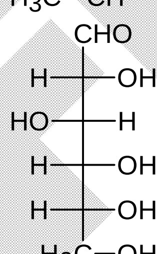
(B) 1,3-Butadiene is monomer of Buna rubber

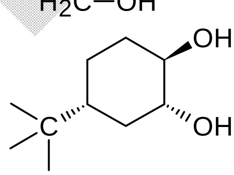
(C) Nylon-6,6 is a condensation polymer

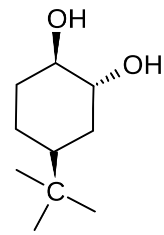
(D) Ethylene is monomer of polyethene

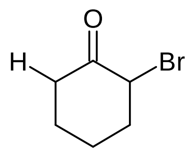
25. Of the following compounds, which can be cleaved by periodic acid is/are

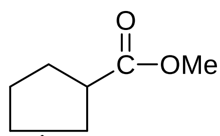
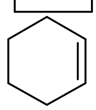
(A) 

(B) 

(C) 

(D) 

26.  $\xrightarrow{MeOH/MeO^-Na^+ (2\ eq)} (A) \xrightarrow{H_2O/H^+} (B) \xrightarrow[2.\ H_2O]{1.\ LiAlH_4} (C) \xrightarrow{H^+/\Delta} (D) \xrightarrow[CCl_4]{Br_2} (E+F)$

Choose the correct option is/are
(A) E and F are enantiomers
(B) E and F are diastereomers
(C) A is 
(D) D is 

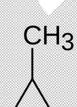
27. Choose the correct statement regarding complex $\left[Fe(H_2O)_5(NO)\right]^{2+}$ is/are
(A) Complex is paramagnetic with 2 unpaired electron.
(B) Complex is diamagnetic.
(C) Complex is paramagnetic with 3 unpaired electron.
(D) In complex iron is +1 oxidation state.

28. Choose the correct statement is/are
(A) D-Allose and D-Glucose are C_3 epimers.
(B) Maltose is reducing sugar.
(C) Lactose is reducing sugar.
(D) Sucrose is reducing sugar.

Section – B (Maximum Marks: 24)

*This section contains **EIGHT (08)** numerical based questions. The answer to each question is a **Single Digit Integer, ranging from 0 to 9 both inclusive.***

29. Total number of radial and angular node in 3d is

30.  $+ Cl_2 \xrightarrow{h\nu} x$[Monohalogen derivative including stereo isomers], the value of 'x' is

31. Solubility of AgCN in a buffer solution whose pH is 6 is 10^{-x} mol / L. The value of x?

$$\begin{bmatrix} \text{if } [K_{SP}]_{AgCN} = 10^{-16} \\ [K_a]_{HCN} = 10^{-10} \end{bmatrix}$$

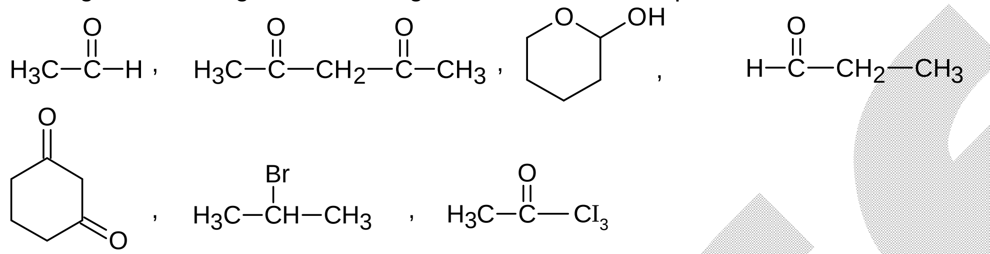
32. An amount of 5 moles each X, Y and T is added to a 1 L closed container.

$$X(g) + Y(g) \rightleftharpoons W(g), \qquad K_C = 4\times10^{12}$$

$$X(g) + T(g) \rightleftharpoons Q(g), \qquad K_C = 10^{12}$$

If at equilibrium, moles of T is x, then the value of $\frac{3x}{2}$ is

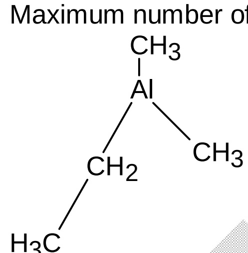
33. Among the following number of organic molecule will respond Haloform test is



34. Number of compounds having peroxy linkages is
H_2SO_5, CrO_5, K_3CrO_8, $H_4P_2O_8$, H_3PO_5, HIO_4

35. The number of species which would undergo disproportionation of heating
H_3PO_4, H_2SO_3, HOCl, H_3PO_2, HNO_3, H_3BO_3, HNO_2

36. Maximum number of atoms lie in one plane for the molecule.



Mathematics PART – III

Section – A (Maximum Marks: 12)

*This section contains **FOUR (04)** questions. Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.*

37. If $\alpha \in R$ be such that the point $P(\alpha, 4 - \alpha^2)$ lies inside the triangle equation of whose three sides are $y - tx - t - 3 = 0$; $ty + x - 2 = 0$ and $y = 5$, where t is a parameter and $t \in (-\infty, -3]$, then
(A) $\alpha \in (0, 3)$
(B) $\alpha \in (-\infty, t)\ \forall\ t \in (-\alpha, -3]$
(C) $\alpha \in (-1, 2)$
(D) $\alpha \in (t, \alpha)\ \forall\ t \in (-\alpha, -3]$

38. Let $C_1, C_2, C_3, ...$ be the circles with radius r_i such that equation of circle $C_i : x^2 + y^2 = r_i^2$. Circle C_i intersects positive x-axis and y-axis at A_i and B_i respectively. There exist a point P_i, perpendicular from P_i on OA_i and OB_i are points D_i and E_i respectively such that $OD_i : OE_i = 3 : 4$ and there exist a point Q_i on circle C_{i+1}, if $A_iQ_i = P_iQ_i$ and $\angle A_iQ_iP_i = \frac{\pi}{2}$. If S_i be the area of triangle $A_iQ_iP_i$ and $r_1 > r_2 > r_3 > ..... > r_n$, where $r_1 = 1$, then $\lim_{n\to\infty}\left(\sum_{i=1}^{n} S_i\right)$ is
(A) $\frac{5}{8}$
(B) $\frac{3}{4}$
(C) $\frac{5}{8}$
(D) $\frac{1}{4}$

39. Let f: $[0, 4] \to R$ is continuous function such that $|f(x)| = 6|(x - n + 1)(x - n)|$ where $n \in N\ \forall\ x \in [n - 1, n]$ and $g(x) = \int_0^x f(t)dt - \int_x^4 f(t)dt$ has maximum at x = 2, then $\int_{\frac{1}{2}}^{4} f(t)dt$ is/are
(A) $-\frac{1}{2}$
(B) 0
(C) $\frac{1}{2}$
(D) 1

40. Which of the following statement is/are **CORRECT?**
(A) If A, B and C are square matrix of order n such that AB = AC and |A| = 0, then B = C
(B) If A = dia. (a, b, 6), B = dia. (–b, 2, a) where $a, b \in R^+$, then $|AB^{-1}| = -3$
(C) If $A = [a_{ij}]_{3\times 3}$, where $a_{ij} = 1\ \forall$ i, j = 1, 2, 3, then $A^3 = 6A$
(D) If $A = [a_{ij}]_{3\times 3}$,where $a_{ij} = 1\ \forall$ i, j = 1, 2, 3, then $A^3 = 27A$

Section – A (Maximum Marks: 24)

*This section contains **SIX (06)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).*

41. If $x + y - 1 = 0$ and $y - 7x + 3 = 0$ be the asymptotes to the hyperbola H. Point P(1, 1) lie on the hyperbola, then
(A) equation of transverse axis is $3y - x = 1$
(B) equation of transverse axis is $y + 3x = 2$
(C) eccentricity of hyperbola H is $\sqrt{5}$
(D) length of transverse axis is 2 if length of conjugate axis is 6

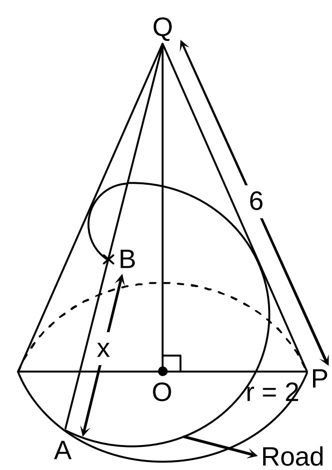
42. Let α, β, γ be the root of equation $f(x) = 3x^3 - 13x^2 + 14x - 2 = 0$. If $[\alpha]$, $[\beta]$, $[\gamma]$ be the root of cubic polynomial equation $g(x) = 0$, then for $h(x) = \left(\frac{f(x)+2}{g(x)}\right)$ (where [.] represents greatest integer function)
(A) $\lim_{x\to\infty} h(x)$ does not exist
(B) range of h(x) is R – {–1, 3, 7}
(C) range of h(x) is R – {0 1, 2}
(D) $\lim_{x\to\infty} h(x) = 3$

43. The equation of plane equidistant from lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and $2x - y - 4 = 0 = 3x - z - 8$ is/are
(A) $x + y - z - 2 = 0$
(B) $x - 2y + z = 5$
(C) $(2 + 3\lambda)x - y - \lambda z - 2(2 + \lambda) = 0\ \forall\ \lambda \in R$
(D) $2x - 4y + 2z = 3p\ \forall\ p \in R$

44. Let a, b, c $\in$ R, $f(x) = ax^3 + bx^2 + cx + d = 0$ has three real roots $\alpha < \beta < \gamma$ and $f(\beta - x) + f(\beta + x) = 0\ \forall\ x \in R$. If $h(x) = f(x)\cdot f''(x) - (f'(x))^2$, then
(A) h(x) = 0 has exactly two real roots for $x \in (\alpha, \gamma)$
(B) h(x) = 0 has no real root for $x \in (\alpha, \gamma)$
(C) $h(x) < 0\ \forall\ x \in (\alpha, \gamma)$ if $a > 1$
(D) $h(x) > 0\ \forall\ x \in (\alpha, \gamma)$ if $a < -1$

45. Let A_n be the area of region bounded by the curve $(2ny^2 - x)(y - x) = 0$; $n \in N$, then
(A) $A_1, A_2, A_3, ...,$ are in H.P.
(B) $\sqrt{A_1}, \sqrt{A_2}, \sqrt{A_3}, ...,$ are in G.P.
(C) $\lim_{n\to\infty}\left(\sum_{r=1}^{n}\sqrt{A_r \cdot A_{r+1} \cdot A_{r+2}}\right) = \frac{1}{192\sqrt{6}}$
(D) $\lim_{n\to\infty}\left(\sum_{r=1}^{n} e^{-\frac{1}{\sqrt{A_r}}}\right) = \left(\frac{1}{e^{2\sqrt{6}} - 1}\right)$

46. The figure illustrate a right circular cone shaped mountain. If a shortest distance road is built for sight-seeing around the mountain, in which road starts at point A and ends at B (A, B and Q are collinear). Let f(x) be the total length of road. The road will go up-hill for some distance L(x) and down-hill for the distance of g(x). Given PQ = 6 ; OP = 2 ; AB = x, then



(A) $f(x) = \sqrt{x^2 - 18x + 108}\ \forall\ x \in [0, 6]$
(B) Maximum value of g(x) is $3\sqrt{3}$
(C) $g(x) = \frac{f(x)}{\sqrt{x^2 - 15x + 54}}\ \forall\ x \in [0, 6]$
(D) If at $x = x_0$, g(x) take a maximum value, then $f(x_0) = 6\sqrt{3}$

Section – B (Maximum Marks: 24)

*This section contains **EIGHT (08)** numerical based questions. The answer to each question is a **Single Digit Integer, ranging from 0 to 9 both inclusive.***

47. The remainder when a number $(2021)^{2025} + (2025)^{2021}$ is divided by 7 is

48. Let A be 3 × 3 involuntary symmetric matrix, such that X = APA′ where $P = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ and |A| ≠ 0, then $\text{Tr}\left(A'X^{2023}A^{-1} + I\right)$ is

49. Let α is non-real root of equation $z^6 + 1 = 0$, then the value of $\left|\alpha^{30} + \alpha^{33} + \alpha^{63} + \alpha^{83} + \alpha^{101}\right| + \left(\alpha^{30} + \alpha^{33} + \alpha^{63} + \alpha^{83} + \alpha^{101}\right)$ is

50. Let f(x) be an even function such that $f(x) + f(x-2) = x^2 + (1-2x)\ \forall\ x \in R$ and $\int_0^2 \left(\frac{f(x)}{x^2 - 2x - 1}\right) dx = 1 + a\ln(b)$, then the value of $[a^2] + [b]$
(where [.] represents greatest integer function)

51. Let $a_1, a_2, a_3, \ldots\ a_n$ be a sequence of numbers satisfying $(n+1)(a_n - a_{n+1}) = \left(\frac{n^2 + 3n + 2}{3^n}\right) - a_n\ \forall$ $n \in N$. If $a_1 = 3$, then $\lim_{n\to\infty}\left(\frac{a_n}{n+1}\right)$ is

52. Let $\vec{a} = \frac{1}{7}\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$; $\vec{b} = \frac{1}{7}\left(6\hat{i} + 2\hat{j} - 3\hat{k}\right)$ and $\hat{c}$ be a unit vector and $A = \begin{bmatrix} \frac{2}{7} & \frac{3}{7} & \frac{6}{7} \\ \frac{6}{7} & \frac{2}{7} & -\frac{3}{7} \\ \hat{c}\cdot\hat{i} & \hat{c}\cdot\hat{j} & \hat{c}\cdot\hat{k} \end{bmatrix}$ and $AA' = I$, then $\left[3\left|\hat{c}\cdot\hat{i}\right| + 2\left|\hat{c}\cdot\hat{j}\right| + \left|\hat{c}\cdot\hat{k}\right|\right]$ is _____ (where [.] represents greatest integer function)

53. Let point's $A(z_1)$ and $B(z_2)$ satisfy the inequation $|z - 2 - 6i| \le 2$. If $z_3 = \frac{1}{2}z_1 + t,\ t \in R$ where $z_1 = 2 + 4i$, then for least $|z_3 - z_1|$ the value of 4t is/are

54. Let A and B be two independent events. If a, b ∈ R such that $a\left(\sqrt{P\left(\frac{A}{B}\right)}\right) + b\left(\sqrt{P\left(\frac{\bar{A}}{B}\right)}\right) = \frac{2}{3}$. If least value of $a^2 + b^2 = k$, then $\left[\frac{1}{k}\right]$ is _____ (where [.] represents greatest integer function)