FIITJEE
ALL INDIA TEST SERIES
JEE (Advanced)-2023
OPEN TEST – II
PAPER –2
TEST DATE: 16-04-2023

ANSWERS, HINTS & SOLUTIONS

Physics PART – I

Section – A

1. C

Sol. $kx_0 = \mu_S mg$ …(i)

Now, $\frac{1}{2}kx_0^2 - \frac{1}{2}k(x_0 - S)^2 = \mu_k mgs$

$\frac{1}{2}k(2x_0 - S)S = \mu_k mgS$

$kx_0 - \frac{kS}{2} = \mu_k mg$

$\mu_S mg - \frac{kS}{2} = \mu_k mg$

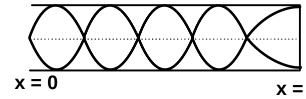
$\mu_S = \mu_k + \frac{kS}{2mg}$

$\mu_S = 0.50 + \frac{100 \times 6 \times 10^{-2}}{2 \times 10}$

$\mu_S = 0.50 + 0.30 = 0.80$

2. A

Sol. $f = \frac{9v}{4\ell} \Rightarrow \ell = \frac{9v}{4f} = \frac{9 \times 330}{4 \times 660} = \frac{9}{8} m = 112.50\,cm$



The amplitude of pressure variation in the pipe is

$a = |\Delta P_0 \sin kx|$

$a = \left|\Delta P_0 \sin\left(\frac{2\pi x}{\lambda}\right)\right|$

$a = \left|\Delta P_0 \sin\left(\frac{2\pi \times 9x}{4\ell}\right)\right|$

$$a = \left|\Delta P_0 \sin\left(\frac{9\pi x}{2\ell}\right)\right|$$

At x = 18.75 cm = $\ell/6$

$$a = \left|\Delta P_0 \sin\left(\frac{9\pi}{2\ell}\times\frac{\ell}{6}\right)\right|$$

$$a = \left|\Delta P_0 \sin\left(\frac{3\pi}{4}\right)\right| = \frac{\Delta P_0}{\sqrt{2}}$$

The maximum pressure at x = 18.75 cm is

$$P_{max} = \left(P_0 + \frac{\Delta P_0}{\sqrt{2}}\right)$$

3. B

Sol. Thermal resistance of the spherical shell,

$$R = \int_a^{3a} \frac{dr}{k4\pi r^2} = \int_a^{3a} \frac{dr}{\frac{\alpha}{r^2} 4\pi r^2} = \frac{2a}{4\pi\alpha} = \frac{a}{2\pi\alpha}$$

Now, $-mS\frac{d\theta}{dt} = \left(\frac{\theta-\theta_0}{R}\right)$

$$-mSR\int_{70}^{50} \frac{d\theta}{(\theta-\theta_0)} = \int_0^t dt$$

$$t = mSR\left[\ell n(\theta-\theta_0\right]_{50}^{70}$$

$$t = mSR\ell n\left(\frac{70-30}{50-30}\right)$$

t = mSRℓn2

$$t = \frac{mSa\ell n2}{2\pi\alpha}$$

4. A

Sol. eE = evB

E = Bv …(i)

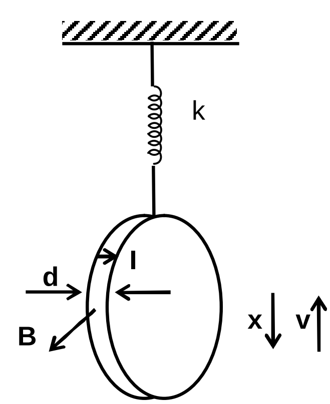
$q = \left(\frac{\varepsilon_0 A}{d}\right)V_0$ (V_0 = potential drop)

$$q = \left(\frac{\varepsilon_0 A}{d}\right)Ed$$

q = ε_0AE

q = ε_0ABv

$I = \frac{dq}{dt} = \varepsilon_0 AB\frac{dv}{dt}$ …(ii)



Now,

$$m\frac{dv}{dt} = kx - BId$$

$$m\frac{dv}{dt} = kx - Bd\left(\varepsilon_0 AB\frac{dv}{dt}\right)$$

$$m\frac{dv}{dt} = kx - \varepsilon_0 VB^2 \frac{dv}{dt}$$ (where V = Ad = volume of the disc)

$$\left(m + \varepsilon_0 VB^2\right)\frac{dv}{dt} = kx$$

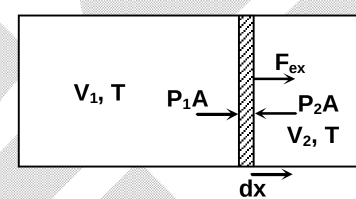
$$\frac{dv}{dt} = \frac{kx}{(m + \varepsilon_0 VB^2)}$$

$$\frac{d^2x}{dt^2} = -\frac{kx}{(m + \varepsilon_0 VB^2)}$$

Time period, $T = 2\pi\sqrt{\frac{(m + \varepsilon_0 VB^2)}{k}}$

5. A, C

Sol. $F_{ex} = (P_2 - P_1)A$ …(i)



$dV_1 = -dV_2 = Adx$

$dW_{ex} = dU$

$F_{ex}dx = 2nC_VdT$

$$(P_2 - P_1)Adx = 2\times1\times\frac{3R}{2}dT$$

$$-P_2dV_2 - P_1dV_1 = 3RdT$$

$$-nRT\frac{dV_2}{V_2} - \frac{nRT}{V_1}dV_1 = 3RdT$$

$$-\int_{V_0}^{\frac{4V_0}{3}} \frac{dV_1}{V_1} - \int_{V_0}^{\frac{2V_0}{3}} \frac{dV_2}{V_2} = 3\int_{T_0}^{T}\frac{dT}{T}$$

$$-\left[\ell n\left(\frac{4}{3}\right) + \ell n\left(\frac{2}{3}\right)\right] = 3\ell n\left(\frac{T}{T_0}\right)$$

$$\ell n\left(\frac{9}{8}\right) = \ell n\left(\frac{T}{T_0}\right)^3 \Rightarrow \frac{T}{T_0} = \left(\frac{9}{8}\right)^{1/3}$$

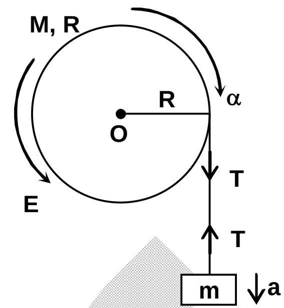
$$\Rightarrow T = 250\times(9)^{1/3} = 520 \text{ K}$$

$$\Delta U = 2nC_V\Delta T$$

$$\Delta U = 2\times1\times\frac{3R}{2}\times20$$

$$\Delta U = 60\times8.3 = 498 \text{ Joule}$$

6. B, C



Sol. $I = \dfrac{Q\omega}{2\pi}$

$B = \dfrac{\mu_0 I}{\ell} = \dfrac{\mu_0 Q\omega}{2\pi\ell}$

Induced electric field on the surface of the long cylinder,

$E = \dfrac{R}{2}\dfrac{dB}{dt} = \dfrac{R}{2}\dfrac{\mu_0 Q\alpha}{2\pi\ell}$ $(\because\ a = \alpha R)$

$E = \dfrac{\mu_0 Qa}{4\pi\ell}$ …(i)

mg – T = ma …(ii)

$RT - QER = \dfrac{MR^2\alpha}{2}$

$T - QE = \dfrac{Ma}{2}$

$T - \dfrac{\mu_0 Q^2 a}{4\pi\ell} = 2ma$ $(\because\ M = 4m)$

T = 3ma …(iii)

From equation (ii) and (iii), we get

a = g/4 and T = $\dfrac{3mg}{4}$

7. A, D

Sol. After the switch 'S' is shifted from position-1 to position-2 the current through the inductor L is $I_1 = \dfrac{\varepsilon}{R}$ and the current through the inductor 2L is $I_2 = \dfrac{\varepsilon}{R}e^{-\left(\frac{3Rt}{2L}\right)}$.

Hence the current through the switch 'S' is $I = I_1 - I_2$

$$I = \frac{\varepsilon}{R}\left[1 - e^{-\left(\frac{3Rt}{2L}\right)}\right]$$

The total heat dissipated in the resistor 3R after having shifted the switch 'S' from position-1 to position-2 is

$$H = \frac{1}{2}\times 2L\left(\frac{\varepsilon}{R}\right)^2$$

$$H = \frac{L\varepsilon^2}{R^2}$$

8. A, B, D

Sol. $f_1 = \left(\dfrac{v - w - v_D}{v - w - v_S}\right)f = \left(\dfrac{330 - 2 - 1}{330 - 2 - 8}\right)\times 320 = 327\text{ Hz}$

The frequency of sound received by the reflector,

$f' = \left(\dfrac{v - w + u}{v - w - v_S}\right) = \left(\dfrac{330 - 2 + 2}{330 - 2 - 8}\right)\times 320 = 330\text{ Hz}$

The frequency of the reflected wave received by the detector

$f_2 = \left(\dfrac{v + w + v_D}{v + w - u}\right)f' = \left(\dfrac{330 + 2 + 1}{330 + 2 - 2}\right)\times 330 = 333\text{ Hz}$

The wavelength of the reflected wave received by the detector,

$$\lambda_2 = \left(\frac{v + w + v_D}{f_2}\right) = \left(\frac{330 + 2 + 1}{333}\right) = 1\text{m}$$

The beats frequency received by the detector,

$f_b = f_2 - f_1 = 333 - 327 = 6$ Hz

9. A, D

Sol. $P = P_2 \cos\theta$

$P_1 = P_2 \sin\theta$

$P^2 + P_1^2 = P_2^2$

$2mk + 2m_1k_1 = 2m_2k_2$

$9k_2 - 4k_1 = k$

$9k_2 - 4k_1 = 13$ …(i)

Also,

$k_1 + k_2 = k - |Q|$

$k_1 + k_2 = 13 - 5.72$

$k_1 + k_2 = 7.28$

$4k_1 + 4k_2 = 29.12$ …(ii)

Solving equation (i) and (ii), we get

$13k_2 = 42.12$

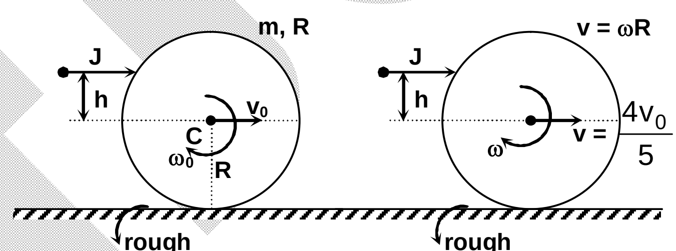
$K_2 = 3.24$ MeV

$K_1 = 4.04$ MeV

10. A, D

Sol. $J = mv_0$

Using conservation of angular momentum about contact point.



$$mv_0(R + h) = \frac{2}{3}mR^2\omega + mvR$$

$$mv_0(R + h) = \frac{5}{3}mvR$$

$$mv_0(R + h) = \frac{5}{3}mR \times \frac{4v_0}{5}$$

$$\Rightarrow h = \frac{R}{3}$$

Now, $mv_0\frac{R}{3} = \frac{2}{3}mR^2\omega_0 \Rightarrow \omega_0 = \frac{v_0}{2R}$

Total work done by the frictional force,

$$\Delta W_{fr} = \Delta k$$

$$\Delta W_{fr} = \left(\frac{1}{2}\times\frac{2}{3}mR^2\omega^2 + \frac{1}{2}mv^2\right) - \left(\frac{1}{2}\times\frac{2}{3}mR^2\omega_0^2 + \frac{1}{2}mv_0^2\right)$$

$$\Delta W_{fr} = \frac{5}{6}mv^2 - \left(\frac{mv_0^2}{12} + \frac{mv_0^2}{2}\right)$$

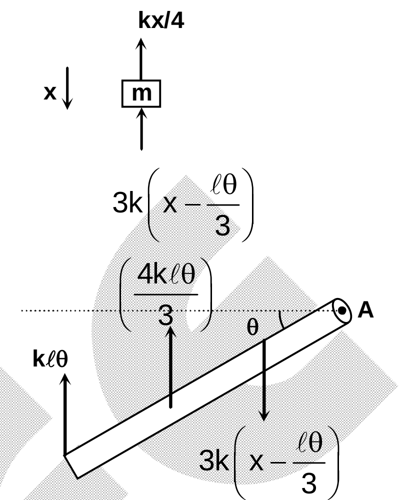
$$\Delta W_{fr} = \frac{8mv_0^2}{15} - \frac{7mv_0^2}{12} = -\frac{mv_0^2}{20}$$

Section – B

11. 8

Sol. x and θ are small

Since, the rod is light, $\tau_A = 0$



$$k\ell^2\theta + \left(\frac{4k\ell\theta}{3}\times\frac{2\ell}{3}\right) = 3k\left(x-\frac{\ell\theta}{3}\right)\frac{\ell}{3}$$

$$\frac{17k\ell^2\theta}{9} = k\ell\left(x-\frac{\ell\theta}{3}\right)$$

$$\frac{17\ell\theta}{9}+\frac{\ell\theta}{3} = x$$

$$\Rightarrow x = \frac{20\ell\theta}{9} \quad \ldots(i)$$

For the block

$$m\frac{d^2x}{dt^2} = \frac{-kx}{4} - 3k\left(x-\frac{\ell\theta}{3}\right)$$

$$= \frac{-kx}{4} - 3k\left(x-\frac{3x}{20}\right)$$

$$= \frac{-kx}{4} - \frac{51\,kx}{20}$$

$$\frac{d^2x}{dt^2} = -\left(\frac{56k}{20m}\right)x$$

Frequency of small oscillations, $f = \frac{1}{2\pi}\sqrt{\frac{56k}{20m}}$

Hence n = 8

12. 6

Sol. $$\frac{\mu_3}{v}-\frac{\mu_1}{u} = \left(\frac{\mu_2-\mu_1}{R_1}\right)+\left(\frac{\mu_3-\mu_2}{R_2}\right)$$

$$\frac{1.8}{v}-\frac{1.3}{(-40)} = \left(\frac{1.5-1.3}{+20}\right)+\left(\frac{1.8-1.5}{-20}\right)$$

$$\frac{1.8}{v}+\frac{1.3}{40} = \frac{0.2}{20}-\frac{0.3}{20}$$

$$\frac{1.8}{v} = \frac{-0.1}{20}-\frac{1.3}{40}$$

$$\frac{1.8}{v} = -\frac{1.5}{40}$$

$\Rightarrow v = -48$ cm

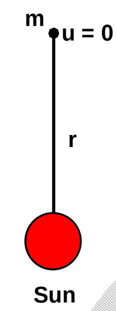
Hence, d + 48 = 60

d = 12 cm

$\Rightarrow$ d/2 = 6 cm

13. 4

Sol. $a = \frac{r+0}{2} = \frac{r}{2}$



Using Kepler's law

$$\frac{T}{T_0} = \left(\frac{r/2}{r}\right)^{3/2}$$

$$T = \frac{T_0}{2\sqrt{2}}$$

The time taken by the body to fall on the surface of sun,

$$\tau = \frac{T}{2} = \frac{T_0}{4\sqrt{2}}$$

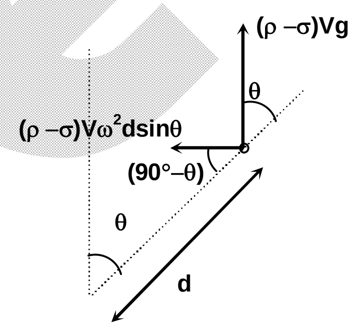
$$\tau = \frac{T_0}{4\sqrt{2}}$$

Hence, n = 4

14. 9

Sol. Let the volume of air bubble is V

For equilibrium of the air bubble,



σ = density of air
ρ = density of water

$$(\rho-\sigma)V\omega^2 d\sin^2\theta = (\rho-\sigma)Vg\cos\theta$$

$$\omega^2 d\sin^2\theta = g\cos\theta$$

$$d = \frac{g\cos\theta}{\omega^2\sin^2\theta}$$

$$d = \frac{10\times\frac{4}{5}}{100\times\frac{9}{25}} = \frac{2}{9}m$$

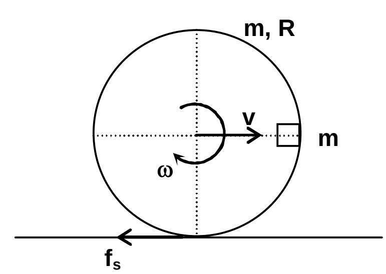
$$d = \frac{2}{9}m$$

Hence, the value of the ratio, $\frac{\ell}{d} = \frac{2}{2/9} = 9$

15. 3

Sol. For a pure rolling motion,

$v = \omega R$



$$\int f_s dt = mv_0 - 2mv \qquad \ldots(i)$$

$$\int f_s R dt = mR^2\omega$$

$$\int f_s dt = m\omega R$$

$$\int f_s dt = mv \qquad \ldots(ii)$$

From (i) and (ii), we get

$mv = mv_0 - 2mv$

$\Rightarrow 3mv = mv_0$

$\Rightarrow v_0 = 3v$

Now using COE

$$\frac{1}{2}mv_0^2 = \left(\frac{1}{2}mR^2\omega^2 + \frac{1}{2}mv^2\right) + \frac{1}{2}mv^2 + mgR$$

$$\frac{1}{2}mv_0^2 = \frac{3}{2}mv^2 + mgR$$

$$v_0^2 = \frac{v_0^2}{3} + 2gR$$

$$\Rightarrow \frac{2v_0^2}{3} = 2gR$$

$$v_0 = \sqrt{3gR} = \sqrt{3\times10\times0.3} = 3 \text{ m/s}$$

16. 8

Sol. $\frac{kq^2}{d_0} = 2\left(\frac{1}{2}mv_0^2 + \frac{1}{2}mv_0^2\right)$

$$\frac{kq^2}{d_0} = 2mv_0^2$$

Using COE in the CM frame,

$$\frac{1}{2}\times\frac{mv_0^2}{2} + \frac{kq^2}{d_0} = \frac{kq^2}{d}$$

$$\frac{kq^2}{8d_0} + \frac{kq^2}{d_0} = \frac{kq^2}{d}$$

$$\frac{9}{8}\frac{kq^2}{d_0} = \frac{kq^2}{d}$$

$$d = \frac{8}{9}d_0 = \frac{8}{9}\times 90 = 80 \text{ cm}$$

Hence, the distance of the closest approach,
d = 80 cm
$\therefore$ n = 8

17. 3

Sol. $q = \left(\frac{\varepsilon_0 AV}{d}\right)$ and $I = \frac{V}{R}$

$$F_e = F_m$$

$$\frac{q^2}{2\varepsilon_0 A\ell} = \frac{\mu_0 I^2}{2\pi d}$$

$$\frac{\varepsilon_0 AV^2}{2d^2\ell} = \frac{\mu_0 I^2}{2\pi d} \qquad (\because\ A = b\ell)$$

$$\frac{\varepsilon_0 bV^2}{2d^2} = \frac{\mu_0 V^2}{2\pi dR^2} \qquad \ldots(i)$$

When R′= 2R, the net force of interaction per unit length between the strips will be

$$F = \frac{\varepsilon_0 bV^2}{2d^2}\left(1-\frac{1}{4}\right)$$

$$F = \frac{3\varepsilon_0 bV^2}{8d^2}$$

Hence n = 3

18. 7

Sol. $\frac{n(n-1)}{2} = 6$

$n(n-1) = 12$

$n = 4$

Now, $13.6z^2\left(\frac{1}{4}-\frac{1}{16}\right) = 22.95$

$13.6z^2\left(\frac{3}{16}\right) = 22.95$

$z^2 = 9 \Rightarrow z = 3$

Hence, $n + z = 4 + 3 = 7$

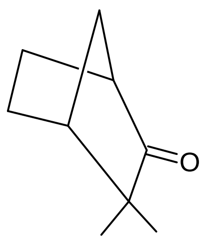
$(n + z) = 7$

Chemistry

PART – II

Section – A

19. C

Sol. In  tautomerism is not possible.

20. D

Sol. $K_2Cr_2O_7 + 14HCl \longrightarrow 2CrCl_3 + 3Cl_2 + 2KCl + 7H_2O$

n-factor of HCl $= \dfrac{6}{14} = \dfrac{3}{7}$

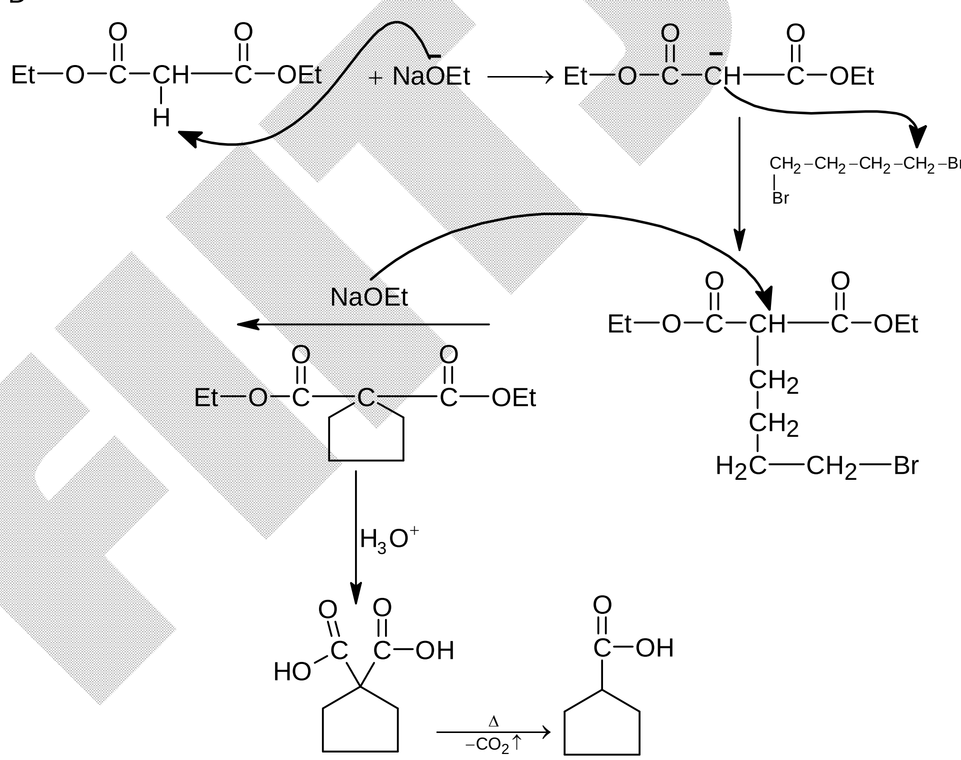
$\therefore E_{HCl} = \dfrac{M}{n.F} = \dfrac{M}{3/7} = \dfrac{7M}{3}$

21. A

Sol. Total two stereo isomers are possible.

22. D

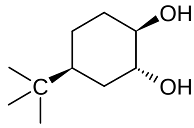
Sol.



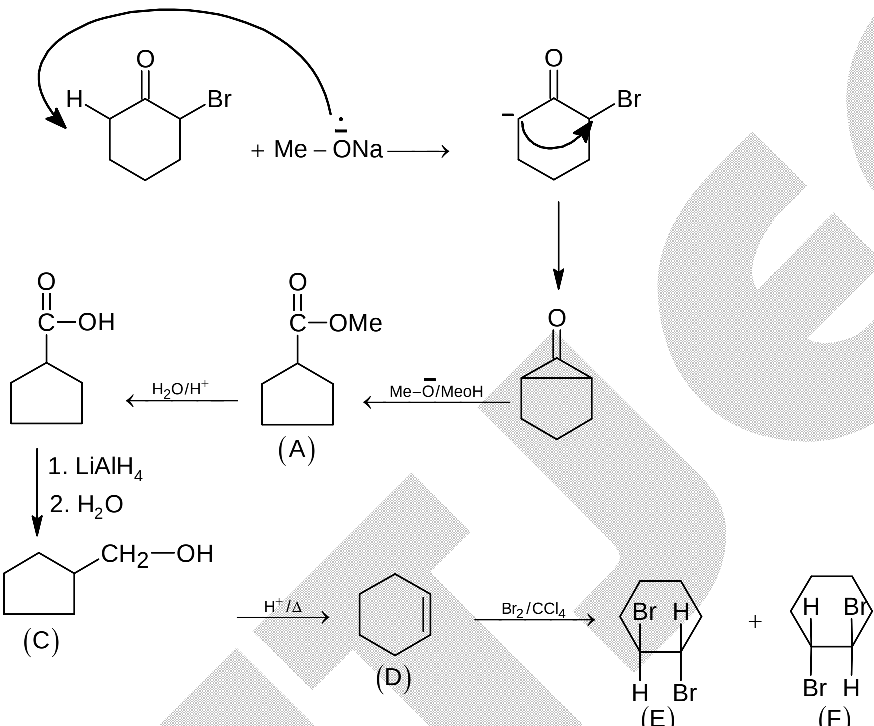
23. A, B, C

Sol. E^o_{cell}, depression in freezing point and molarity are intensive property.

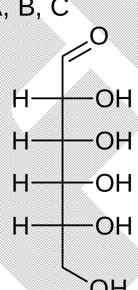
24. A, B, C, D
Sol. Factual

25. A, B, C
Sol. 

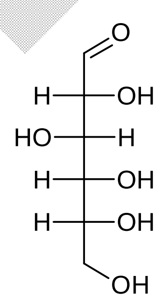
cannot be cleaved by periodic acid because both OH groups are at axial position.

26. A, C, D
Sol.


27. C, D
Sol. Complex is paramagnetic with 3 unpaired electron, Fe is in +1 oxidation state.

28. A, B, C
Sol.


D-Allose



D-Glucose

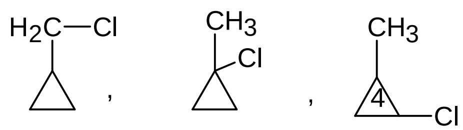
Maltose and Lactose have hemiacetal group therefore, they are reducing sugar but in sucrose hemiacetal group is absent.

Section – B

29. 2

Sol. Number of radial node = $n - \ell - 1$
$= 3 - 2 - 1 = 0$
Angular node = $\ell = 2$
$\therefore$ Total node = 2 + 0 = 2

30. 6

Sol.


Total = 6.

31. 6

Sol. $[K_{SP}]_{AgCN} = 10^{-16}$, $[K_a]_{HCN} = 10^{-10}$
As we know that,

$$S = \sqrt{K_{SP}\left\{1 + \frac{[H^+]}{K_a}\right\}}$$

$$S = \sqrt{10^{-16}\left\{1 + \frac{10^{-6}}{10^{-10}}\right\}} = \sqrt{10^{-16} + 10^{-16} \times \frac{10^{-6}}{10^{-10}}}$$

$$S = \sqrt{10^{-12}} = 10^{-6} \text{ mol/L}.$$

32. 5

Sol.

$$X(g) + Y(g) \rightleftharpoons W(g), \qquad K_C = 4 \times 10^{12}$$
$$5 - a - b \quad 5 - a \quad a$$
$$X(g) + T(g) \rightleftharpoons Q(g), \qquad K_C = 10^{12}$$
$$5 - b - a \quad 5 - b \quad b$$

Now, $4 \times 10^{12} = \frac{[W]}{[X][Y]}$ … (1)

$10^{12} = \frac{[Q]}{[T][X]}$ … (2)

$\frac{4 \times 10^{12}}{10^{12}} = \frac{[W]}{[X][Y]} \times \frac{[T][X]}{[Q]} = \frac{(a)}{(5-a)} \times \frac{(5-b)}{b}$ … (3)

And as, the K_C is very high,
$a + b = 5$ … (4)
From Eq. (3)

$$4 = \frac{a}{(5-a)} \times \frac{a}{(5-a)} = \frac{a^2}{(5-a)^2} \Rightarrow \frac{a}{5-a} = 2$$

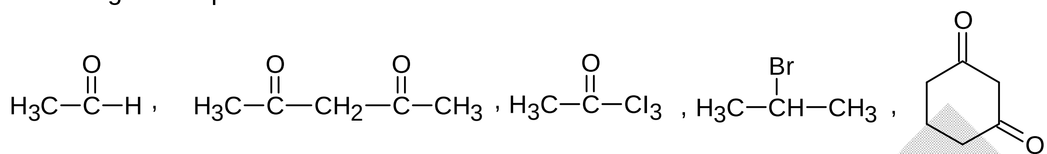
$$\therefore a = 10 - 2a \Rightarrow 3a = 10 \therefore a = \frac{10}{3}$$

$$\therefore [T] = 5 - b = 5 - \frac{5}{3} = \frac{10}{3} = x$$

The value of $\frac{3x}{2}$ is 5

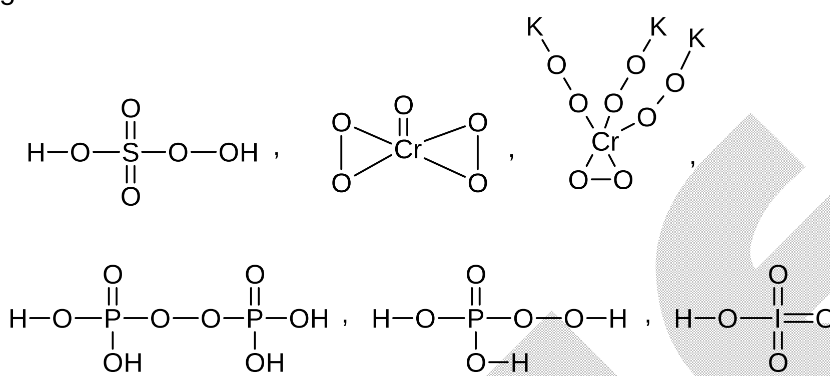
33. 5

Sol. Following will respond the Haloform test



34. 5

Sol.



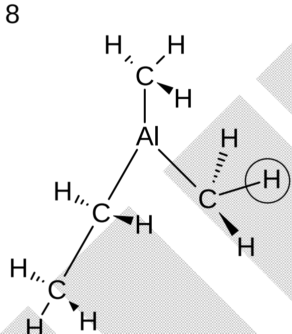
Except HIO_4 in all peroxide linkages are present.

35. 4

Sol. The species in which central atom is in intermediate oxidation state will disproportionate.

36. 8

Sol.



Total atoms = 8

Mathematics

PART – III

Section – A

37. C

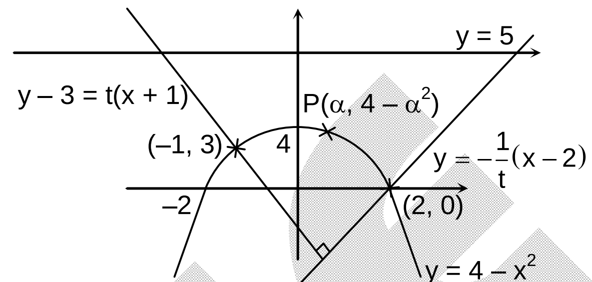
Sol. $y - tx - t - 3 = 0 \Rightarrow (y - 3) = t(x + 1)$

$ty + x - 2 = 0 \Rightarrow y = -\frac{1}{t}(x - 2)$

$y = 5$

Point $P(\alpha, 4 - \alpha^2)$ lie on curve $y = 4 - x^2$

Hence, $\alpha \in (-1, 2)$



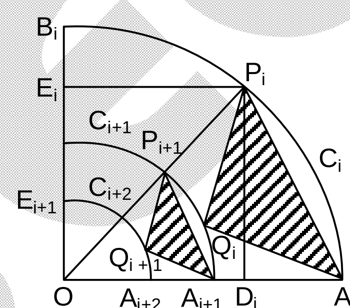
38. D

Sol. $A_iP_i = \frac{2}{\sqrt{5}} r_i$ and $r_{i+1} = \frac{r_i}{\sqrt{5}}$

And $S_i = \frac{r_i^2}{5} \Rightarrow S_{i+1} = \frac{r_{i+1}^2}{5} = \frac{r_i^2}{25}$

$\Rightarrow \frac{S_{i+1}}{S_i} = \frac{1}{5}$

$$\Rightarrow \lim_{n\to\infty}\left(S_1 + S_2 + S_3 + \ldots\ldots + S_n\right) = \lim_{n\to\infty}\left(\frac{\frac{1}{5}}{1-\frac{1}{5}}\right) = \frac{1}{4}$$



39. A

Sol. $g'(x) = f(x) + f(x) = 2f(x)$ at $g(x)$ has maximum at $x = 2 \Rightarrow g'(2) = 2f(2) = 0$

$$\Rightarrow f(x) = \begin{cases} -6x(x-1) & ; \quad x \in [0, 1] \\ -6(x-1)(x-2) & ; \quad x \in [1, 2] \\ 6(x-2)(x-3) & ; \quad x \in [2, 3] \\ 6(x-3)(x-4) & ; \quad x \in [3, 4] \end{cases}$$

$$\Rightarrow \int_{\frac{1}{2}}^{4} f(t)\,dt = -\frac{1}{2}$$

40. B

Sol. (A) Since, $|A| = 0$ hence non-invertible $\Rightarrow B \neq C$

(B) $|A| = 6ab$, $|B| = -2ab \Rightarrow \left|AB^{-1}\right| = \frac{6ab}{-2ab} = -3$

41. A, C

Sol. Equation of transverse axis, is bisector of asymptote containing point P(1, 1)

$$\Rightarrow \frac{x+y-1}{\sqrt{2}} = -\left(\frac{y-7x+3}{5\sqrt{2}}\right)$$

$\Rightarrow 5x + 5y - 5 = -y + 7x - 3$

$\Rightarrow 6y = 2x + 2 \Rightarrow 3y = x + 1 \Rightarrow$ slope $m_1 = \frac{1}{3}$

And slope of asymptote $x + y - 1 = 0$, $m_2 = -1$

$$\Rightarrow \tan\theta = \frac{b}{a} = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{\frac{1}{3}+1}{1-\frac{1}{3}}\right| \Rightarrow \frac{b}{a} = 2 \Rightarrow e = \sqrt{1+2^2} = \sqrt{5} \text{ if } b = 4, a = 2$$

42. B, D

Sol. $f(x) = x(x - 2)(3x - 7) - 2 = 0$

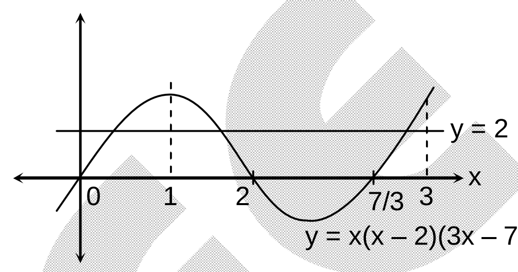
Where $f(0) < 0$, $f(1) > 0$, $f(2) < 0$, $f(3) > 0$

$\Rightarrow [\alpha] = 0, [\beta] = 1, [\gamma] = 2$

$\Rightarrow g(x) = x(x - 1)(x - 2)$

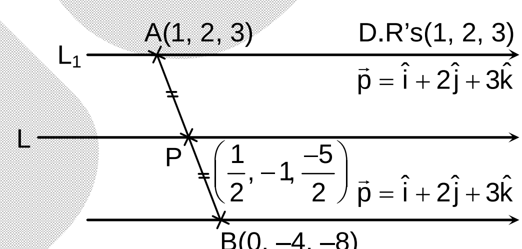
$$\Rightarrow \lim_{x\to\infty}\left(\frac{x(x-2)(3x-7)}{x(x-1)(x-2)}\right) = \lim_{x\to\infty}\left(\frac{3-\frac{7}{x}}{1-\frac{1}{x}}\right) = 3$$

$$y = \frac{f(x)+2}{g(x)}$$



43. A, B, D

Sol. Since, line L_1 II L_2. Hence, there will be infinite planes cutting line L and parallel to given line.



Equation of line $L : \frac{x-\frac{1}{2}}{1} = \frac{y+1}{2} = \frac{z+\frac{5}{2}}{3}$

$\Rightarrow$ Plane $P_1 : 2x - y - 2 = 0$

Plane $P_2 : 3x - z - 4 = 0$

$\Rightarrow$ Required plane $p : p_1 + \lambda p_2 = 0$

$(2x - y - 2) + \lambda(3x - z - 4) = 0, \lambda \in R$

Another is system of planes parallel to plane containing L_1 and L_2

Normal to lines L_1 and L_2 : $\vec{x} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 6 & 11 \\ 1 & 2 & 3 \end{vmatrix} = -4\hat{i} + 8\hat{j} - 4\hat{k}$

$\Rightarrow$ One of D.R's normal = (1, –2, 1)

$\Rightarrow$ Equation of plane containing line's L_1 and L_2 $x - 2y + z = 0$

$\Rightarrow$ Any plane parallel to the plane is $x - 2y + z = d, d \in R$

44. B, C

Sol. $h'(x) = f(x)\cdot f'''(x) - f'(x)\cdot f''(x) \Rightarrow h''(x) = -(f''(x))^2 \le 0 \ \forall \ x \in R$

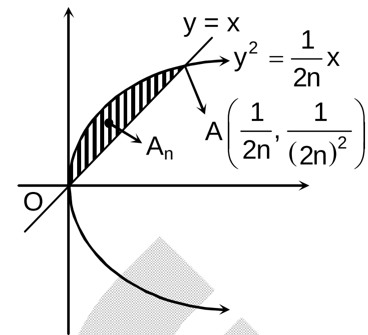
$\Rightarrow h'(x)$ is decreasing function let $h'(x_0) = 0, x_0 \in (\alpha, \gamma)$

$\Rightarrow x_0$ is point of local maximum

$h(x_0) = f(x_0)\cdot f''(x_0) - (f'(x_0))^2 < 0$ if $|a| > 1 \Rightarrow h(x) < 0 \ \forall \ x \in (\alpha, \gamma)$ if $|a| > 1$

45. C, D

Sol. For point A: $x^2 = \frac{1}{2n}x \Rightarrow x = \frac{1}{2n}$



$\Rightarrow A_n = \int_0^{\frac{1}{2n}} \left(\frac{\sqrt{x}}{\sqrt{2n}} - x\right)dx = \left(\frac{1}{\sqrt{2n}} \cdot \frac{2}{3}x^{\frac{3}{2}} - \frac{x^2}{2}\right)\Bigg|_0^{\frac{1}{2n}}$

$= \frac{2}{3} \cdot \frac{1}{(2n)^2} - \frac{1}{2(2n)^2} = \frac{1}{6(2n)^2}$

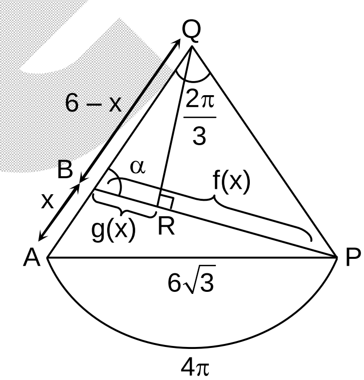
$\Rightarrow \sqrt{A_n} = \frac{1}{2n\sqrt{6}}$

$\Rightarrow \sqrt{A_r \cdot A_{r+1} \cdot A_{r+2}} = \left(\frac{1}{48\sqrt{6}}\right)\left(\frac{1}{r(r+1)(r+2)}\right)$

$= \frac{1}{2 \times 48\sqrt{6}}\left(\frac{1}{r(r+1)} - \frac{1}{(r+1)(r+2)}\right) \Rightarrow \sum_{r=1}^{n} \sqrt{A_r\ A_{r+1}\ A_{r+2}} = \frac{1}{96\sqrt{6}}\left(\frac{1}{1.2} - \frac{1}{n(n+1)}\right)$

46. A, B, D

Sol. In ΔPQB



$\cos\frac{2\pi}{3} = \frac{6^2 + (6-x)^2 - (f(x))^2}{12(6-x)} = -\frac{1}{2}$

$\Rightarrow f(x) = \sqrt{x^2 - 18x + 108}\ \ \forall\ x \in [0, 6]$

In ΔPQB

$\cos\alpha = \frac{g(x)}{6-x} = \frac{(6-x)^2 + (f(x))^2 - 6^2}{2f(x)\cdot(6-x)}$

$\Rightarrow g(x) = \frac{x^2 - 15x + 54}{\sqrt{x^2 - 18x + 108}}$

R is a point from where down-hill road will begin

$\Rightarrow$ g(x) will be maximum where x = 0 i.e. when QR is maximum

$\Rightarrow g(0) = \frac{54}{\sqrt{2.54}} = 3\sqrt{3}$ and $f(0) = 6\sqrt{3}$

Section – B

47. 3

Sol. N = $(2021)^{2025} + (2025)^{2021}$ = $(289 \times 7 - 2)^{2025} + (289 \times 7 + 2)^{2021}$

= $7x - 2^{2025} + 7y + 2^{2021} = 7m + 2^{2021} - 2^{2025}$

= $7m + 4(7+1)^{673} - (7+1)^{675} = 7m + 4.7\lambda + 4 - 7k - 1$

= 7p + 3 $\Rightarrow$ Remainder is 3

48. 6

Sol. Given $A^2 = I$, $A = A' = A^{-1}$ and $X^{2023} = \underbrace{(APA')(APA').....(APA')}_{2023\text{ times}} = AP^{2023}A'$

Let P = I + B where $B = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow B^2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}$, $B^3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\Rightarrow P^{2023} = {}^{2023}C_0 I^{2023} + {}^{2023}C_1 I^{2022} B + {}^{2023}C_2 I^{2021} B^2$

$\Rightarrow A'X^{2023}A^{-1} = A'AP^{2023}A'A = P^{2023}$

$\Rightarrow Tr\left(A'X^{2023}A^{-1}\right) = 3$

49. 0

Sol. $\alpha^6 = -1 \Rightarrow \left(\alpha^6\right)^{2n+1} = -1,\ \left(\alpha^6\right)^{2n} = 1$

50. 2

Sol. $f(x) = \frac{1}{2}\left(x^2 - 1\right)$

$$\Rightarrow I = \int_0^2 \left(\frac{\frac{1}{2}\left(x^2-1\right)}{x^2-2x-1}\right)dx = \frac{1}{2}\int_0^2\left(\frac{\left(x^2-2x-1\right)+2x-2+2}{\left(x^2-2x-1\right)}\right)dx$$

$$= \frac{1}{2}\int_0^2\left(1+\frac{2(x-1)}{x^2-2x-1}+2\frac{1}{(x-1)^2-\left(\sqrt{2}\right)^2}\right)dx$$

$$= \frac{1}{2}\left(x+\ln\left(x^2-2x-1\right)+\frac{2}{2\sqrt{2}}\ln\left(\frac{x-1-\sqrt{2}}{x-1+\sqrt{2}}\right)\right)\Bigg|_0^2$$

$$= \frac{1}{2}\left(2+0+\frac{1}{\sqrt{2}}\ln\left(\left|\frac{1-\sqrt{2}}{1+\sqrt{2}}\right|\right)-\frac{1}{\sqrt{2}}\ln\left(\frac{1+\sqrt{2}}{\sqrt{2}-1}\right)\right)$$

$$= 1-\frac{2}{2\sqrt{2}}\left(\ln\left(\frac{1+\sqrt{2}}{\sqrt{2}-1}\right)\right) = 1+\frac{1}{\sqrt{2}}\ln\left(\frac{\left(\sqrt{2}-1\right)}{\left(\sqrt{2}+1\right)}\right)$$

$$= 1-\sqrt{2}\ln\left(\sqrt{2}+1\right) = 1+\sqrt{2}\ln\left(\frac{1}{\sqrt{2}+1}\right) = 1+\sqrt{2}\ln\left(\sqrt{2}-1\right)$$

51. 1

Sol. $(n + 2)a_n - (n + 1)a_{n+1} = 3^{-n}(n + 1)(n + 2)$

$$\Rightarrow \frac{a_n}{n+1} - \frac{a_{n+1}}{n+2} = 3^{-n}$$

$$\Rightarrow \frac{a_1}{2} - \frac{a_1}{3} = 3^{-1}$$

$$\frac{a_2}{3} - \frac{a_3}{4} = 3^{-2}$$

$$\frac{a_3}{4} - \frac{a_4}{5} = 3^{-3}$$

$$\vdots$$

$$\frac{a_{n-1}}{n} - \frac{a_n}{n+1} = 3^{-(n-1)}$$

$$\frac{3}{2} - \frac{a_n}{n+1} = \frac{1}{3}+\frac{1}{3^2}+.....+\frac{1}{3^{n-1}} = \frac{\frac{1}{3}\left(1-\frac{1}{3^{n-1}}\right)}{\frac{2}{3}}$$

$$\frac{a_n}{n+1} = \frac{3}{2}-\frac{1}{2}\left(1-\frac{1}{3^{n-1}}\right) \Rightarrow \lim_{n\to\infty}\left(\frac{a_n}{n+1}\right) = \frac{3}{2}-\frac{1}{2} = 1$$

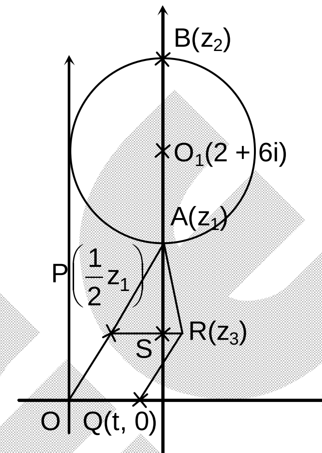
52. 3

Sol. $AA' = I \Rightarrow \vec{a}, \vec{b}, \vec{c}$ are orthogonal unit vectors $\Rightarrow \vec{c} = \pm\frac{(\vec{a}\times\vec{b})}{|\vec{a}\times\vec{b}|}$

53. 4

Sol. Min AR = AS

$\Rightarrow t = 1$



54. 2

Sol. $P\left(\frac{A}{B}\right) = P(A) = x\ (\text{say}) \Rightarrow P\left(\frac{\bar{A}}{B}\right) = P(\bar{A}) = 1 - x$

$\Rightarrow a\sqrt{x} + b\left(\sqrt{1-x}\right) = \frac{2}{3}$

Let $\sqrt{x} = \cos\theta \Rightarrow \sqrt{1-x} = \sin\theta \Rightarrow a\cos\theta + b\sin\theta = \frac{2}{3}$

w.k.t. $a\cos\theta + b\sin\theta \le \sqrt{a^2 + b^2}$

$a^2 + b^2 \ge \frac{4}{9} = k$