FIITJEE
ALL INDIA TEST SERIES
JEE (Advanced)-2023
OPEN TEST – II
PAPER –1
TEST DATE: 16-04-2023

Time Allotted: 3 Hours **Maximum Marks: 180**

General Instructions:

- The test consists of total 54 questions.
- Each subject (PCM) has 18 questions.
- This question paper contains **Three Parts.**
- **Part-I** is Physics, **Part-II** is Chemistry and **Part-III** is Mathematics.
- Each **Part** is further divided into **Two Sections**: **Section-A & Section-B**.

Section – A *(01 –06, 19 – 24, 37 – 42):* This section contains **EIGHTEEN (18)** questions. Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).

Section – A *(07 – 10, 25 – 28, 43 – 46):* This section contains **TWELVE (12)** Matching List Type Questions. Each question has **FOUR** statements in **List-I** entries (I), (II), (III) and (IV) and **FIVE** statements in **List-II** entries (P), (Q), (R), (S) and (T). The codes for lists have choices (A), (B), (C), (D) out of which, **ONLY ONE** of these four options is correct answer.

Section – B *(11 – 18, 29 – 36, 47 – 54):* This section contains **TWENTY FOUR (24)** numerical based questions. The answer to each question is a **NUMERICAL VALUE.** If the numerical value has more than two decimal places, truncate/round-off the value to **TWO** decimal places.

MARKING SCHEME

Section – A (One or More than One Correct): Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+4	If only (all) the correct option(s) is (are) chosen;
Partial Marks	:	+3	If all the four options are correct but ONLY three options are chosen;
Partial marks	:	+2	if three or more options are correct but ONLY two options are chosen and both of which are correct;
Partial Marks	:	+1	If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	–2	In all other cases.

Section – A (Single Correct): Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+3	If ONLY the correct option is chosen.
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	–1	In all other cases.

Section – B: Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+3	If ONLY the correct numerical value is entered at the designated place;
Zero Marks	:	0	In all other cases.

Physics

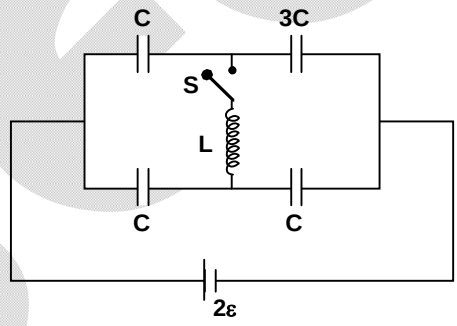
PART – I

Section – A (Maximum Marks: 24)

*This section contains **SIX (06)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).*

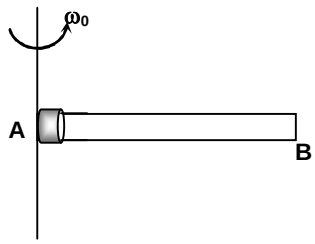
1. An almost massless metal sheet is placed on a horizontal tabletop lubricated with oil. The metal sheet has a square shape of side length ℓ = 1m and the oil layer has thickness h = 1 mm. Initially one edge of the sheet coincides with one edge of the table. The sheet is pulled outwards without rotation with a constant horizontal force F = 25 N. If coefficient of viscosity of the oil is η = 0.4 N-s/m^2, then choose the correct option(s).
(A) The time required to pull half of the sheet out of the table is 3 sec.
(B) The time required to pull half of the sheet out of the table is 6 sec.
(C) The velocity of the sheet at the moment when half of the sheet is pulled out will be 12.5 cm/s.
(D) The velocity of the sheet at the moment when half of the sheet is pulled out will be 25 cm/s.

2. The circuit shown consists of four capacitors, an inductance L, an ideal battery of emf 2ε and a switch 'S'. The switch 'S' is kept open for a long time and then it is closed. Then choose the correct option(s).



(A) The charge $\frac{C\varepsilon}{3}$ will flow through the battery after having closed the switch 'S'.
(B) The charge $\frac{C\varepsilon}{6}$ will flow through the battery after having closed the switch 'S'.
(C) The maximum current $2\varepsilon\sqrt{\frac{C}{3L}}$ will flow through the inductor after having closed the switch 'S'.
(D) The maximum current $\varepsilon\sqrt{\frac{C}{3L}}$ will flow through the inductor after having closed the switch 'S'.

3. A smooth horizontal rod of length ℓ = 1 m and mass M = 3 kg with a sleeve of mass m = 1 kg initially getting at end A is rotated with an initial angular velocity ω_0 = 10 rad/s. When the sleeve reaches the other end B, which of the following options is/are correct. (Take g = 10 m/s^2)



(A) The velocity of sleeve relative to the rod is $5\sqrt{2}$ m/s.
(B) The velocity of sleeve relative to the ground is $5\sqrt{3}$ m/s
(C) The normal force acting on the sleeve is $25\sqrt{2}$ newton
(D) The normal force acting on the sleeve is $15\sqrt{6}$ newton.

4. A particle starts moving in xy plane with a constant tangential acceleration $a_t = 4\ m/s^2$ and normal acceleration $a_n = 2t^4$, where t is in second. If the particle was at rest at t = 0. Then choose the correct option(s).

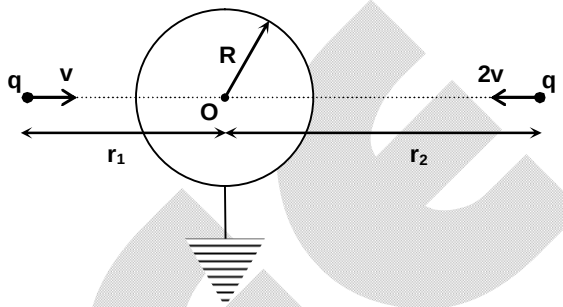
(A) The net acceleration of the particle after having covered a distance S = 4m is $a = 4\sqrt{5}\ m/s^2$

(B) The net acceleration of the particle after having covered a distance S = 4m is $a = 8\sqrt{5}\ m/s^2$

(C) The radius of curvature of the trajectory at S = 4 m is r = 4m.

(D) The radius of curvature of the trajectory at S = 8 m is r = 2m.

5. Two point charges each of charge 'q' are moving towards the centre 'O' of an earthed spherical conducting shell of radius R with the constant velocities v and 2v as shown in the figure. Then choose the correct option(s).



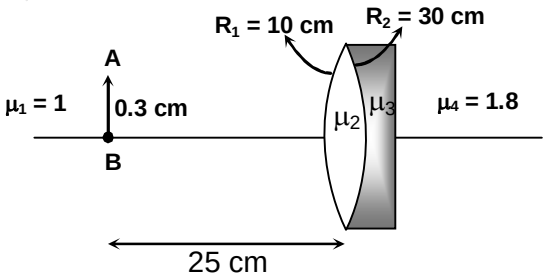
(A) When $r_1 = 2R$ and $r_2 = 4R$, the charge on the spherical conducting shell is $\left(\frac{-3q}{4}\right)$

(B) When $r_1 = 2R$ and $r_2 = 4R$, the charge on the spherical conducting shell is $\left(\frac{-3q}{2}\right)$

(C) When $r_1 = 2R$ and $r_2 = 4R$, the current flowing from the spherical conducting shell to the earth is $\left(\frac{3qv}{8R}\right)$

(D) When $r_1 = 2R$ and $r_2 = 4R$, the current flowing from the spherical conducting shell to the earth is $\left(\frac{3qv}{4R}\right)$

6. The refractive indices of the two thin lenses are $\mu_2 = 1.2$ and $\mu_3 = 1.5$ respectively. The medium on the two sides of the combined lens are having refractive indices $\mu_1 = 1$ and $\mu_4 = 1.8$ as shown in the figure. A linear object AB of size 0.30 cm is placed at a distance 25 cm from the combined lens as shown in the figure. Then choose the correct option(s).



(A) The image is formed at a distance 40 cm to the right of the combined lens.

(B) The image is formed at a distance 60 cm to the left of the combined lens.

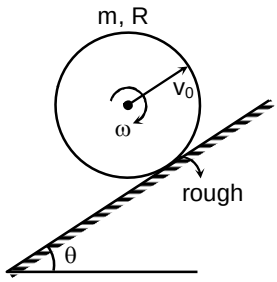
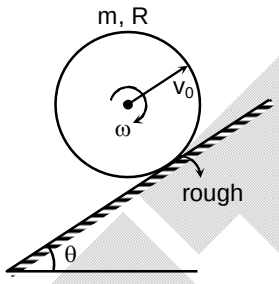
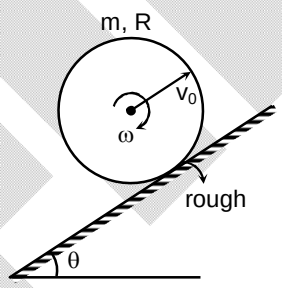
(C) The size of the image formed is 0.20 cm

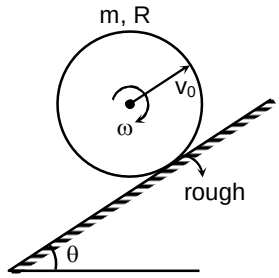
(D) The size of the image formed is 0.40 cm

Section – A (Maximum Marks: 12)

*This section contains **FOUR (04)** Matching List Type Questions. Each question has FOUR statements in **List-I** entries (I), (II), (III) and (IV) and FIVE statements in **List-II** entries (P), (Q), (R), (S) and (T). The codes for lists have choices (A), (B), (C), (D) out of which **ONLY ONE** of these four options is correct answer.*

7. List –I contains four different bodies placed on rough inclined planes which are having initial velocity v_0 and initial angular velocity ω_0 as shown. List-II contains the total time of rise of all different rigid bodies up the inclined planes.

List –I		List –II	
(I)	**Ring:-**  $\omega_0 = \frac{v_0}{2R},\ \mu = \tan\theta$	(P)	$\frac{4v_0}{3g\sin\theta}$
(II)	**Disc:-**  $\omega_0 = \frac{2v_0}{R},\ \mu = \tan\theta$	(Q)	$\frac{3v_0}{2g\sin\theta}$
(III)	**Hollow sphere**  $\omega_0 = \frac{v_0}{2R},\ \mu = \tan\theta$	(R)	$\frac{9v_0}{5g\sin\theta}$

(IV)	**Solid sphere**  $\omega_0 = \frac{2v_0}{R},\ \mu = \tan\theta$	(S)	$\frac{7v_0}{5g\sin\theta}$
		(T)	$\frac{2v_0}{g\sin\theta}$

Which one of the following options is correct ?

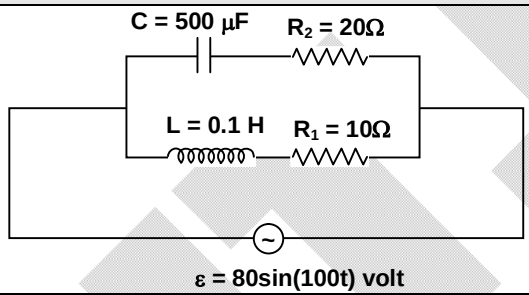
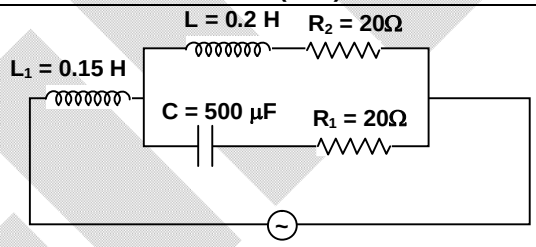
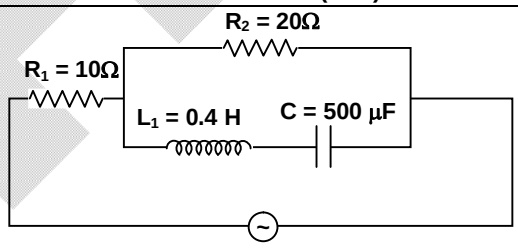
(A) I $\rightarrow$ T, II $\rightarrow$ R, III $\rightarrow$ P, IV $\rightarrow$ Q

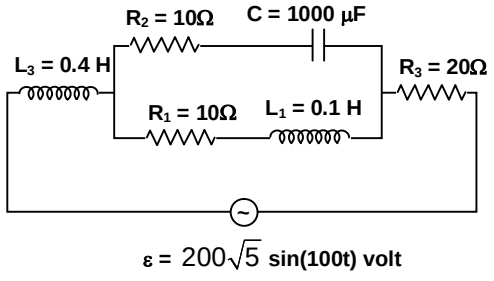
(B) I $\rightarrow$ Q, II $\rightarrow$ T, III $\rightarrow$ P, IV $\rightarrow$ R

(C) I $\rightarrow$ Q, II $\rightarrow$ S, III $\rightarrow$ R, IV $\rightarrow$ P

(D) I $\rightarrow$ P, II $\rightarrow$ T, III $\rightarrow$ R, IV $\rightarrow$ S

8. List –I contains four different AC circuits with AC sources as shown in the figure. List-II contains the rms currents through the different AC sources of the circuits.

List –I		List –II	
(I)	 ε = 80sin(100t) volt	(P)	$\sqrt{2}$ Amp
(II)	 ε = 50sin(100t) volt	(Q)	$4\sqrt{10}$ Amp
(III)	 ε = $40\sqrt{5}$ sin(100t) volt	(R)	$2\sqrt{10}$ Amp

(IV)		(S)	$2\sqrt{5}$ Amp
		(T)	$2\sqrt{2}$ Amp

Which one of the following options is correct ?

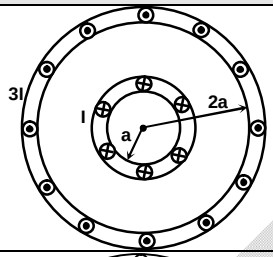
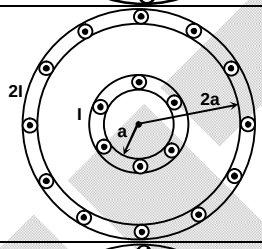
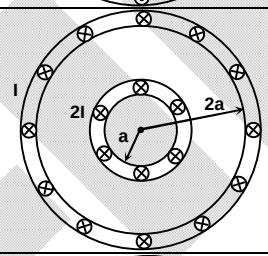
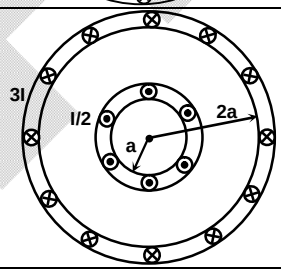
(A) I $\rightarrow$ S, II $\rightarrow$ P, III $\rightarrow$ T, IV $\rightarrow$ R

(B) I $\rightarrow$ P, II $\rightarrow$ S, III $\rightarrow$ R, IV $\rightarrow$ Q

(C) I $\rightarrow$ S, II $\rightarrow$ P, III $\rightarrow$ Q, IV $\rightarrow$ T

(D) I $\rightarrow$ T, II $\rightarrow$ R, III $\rightarrow$ Q, IV $\rightarrow$ P

9. List-I contains four different systems of two thin walled long current carrying coaxial cylinders. List-II contains the magnitude of magnetic pressure exerted on the wall of the outer cylinder.

List –I		List –II	
(I)		(P)	$\frac{5\mu_0 I^2}{32\pi^2 a^2}$
(II)		(Q)	$\frac{5\mu_0 I^2}{16\pi^2 a^2}$
(III)		(R)	$\frac{3\mu_0 I^2}{32\pi^2 a^2}$
(IV)		(S)	$\frac{3\mu_0 I^2}{16\pi^2 a^2}$
		(T)	$\frac{\mu_0 I^2}{4\pi^2 a^2}$

Which one of the following options is correct ?

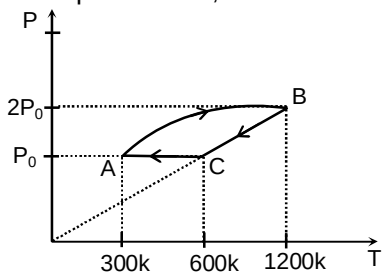
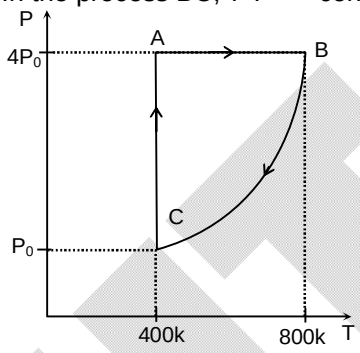
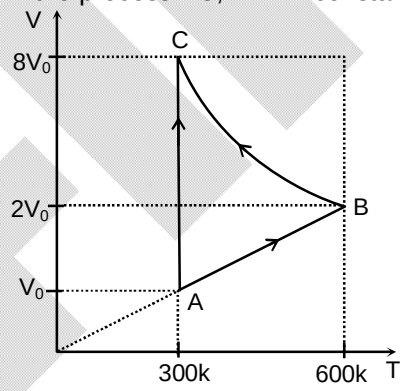
(A) $I \rightarrow R,\ II \rightarrow T,\ III \rightarrow Q,\ IV \rightarrow S$

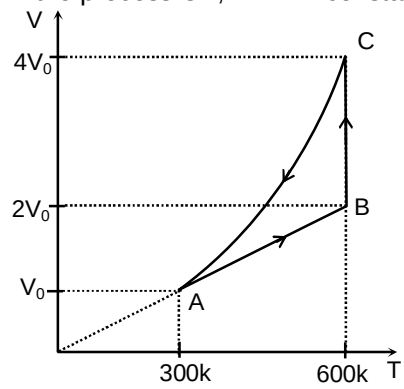
(B) $I \rightarrow P,\ II \rightarrow T,\ III \rightarrow R,\ IV \rightarrow Q$

(C) $I \rightarrow Q,\ II \rightarrow S,\ III \rightarrow T,\ IV \rightarrow P$

(D) $I \rightarrow R,\ II \rightarrow T,\ III \rightarrow P,\ IV \rightarrow S$

10. Two moles of an ideal monoatomic gas is taken through four different cyclic processes as depicted in the List-I and List-II gives the total work done by the gas during each cyclic process. (Take $\ell n2 = 0.7$)

List –I		List –II	
(I)	In the process AB, $PT^{-\frac{1}{2}} = \text{constant}$ 	(P)	480 R
(II)	In the process BC, $PT^{-2} = \text{constant}$ 	(Q)	240 R
(III)	In the process BC, $VT^2 = \text{constant}$ 	(R)	600 R

(IV)	In the process CA, VT^{-2} = constant 	(S)	540 R
		(T)	300 R

Which one of the following options is correct ?

(A) I $\rightarrow$ T, II $\rightarrow$ P, III $\rightarrow$ R, IV $\rightarrow$ S

(B) I $\rightarrow$ T, II $\rightarrow$ P, III $\rightarrow$ S, IV $\rightarrow$ Q

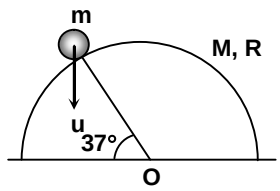
(C) I $\rightarrow$ P, II $\rightarrow$ R, III $\rightarrow$ S, IV $\rightarrow$ Q

(D) I $\rightarrow$ Q, II $\rightarrow$ S, III $\rightarrow$ T, IV $\rightarrow$ P

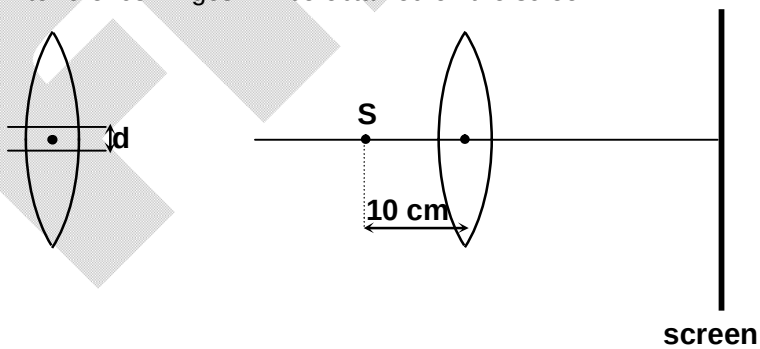
Section – B (Maximum Marks: 24)

*This section contains **EIGHT (08)** numerical based questions. The answer to each question is a **NUMERICAL VALUE.** If the numerical value has more than two decimal places, truncate/round-off the value to **TWO** decimal places.*

11. A particle of mass m = 0.1 kg moving vertically downward with a velocity u = 20 m/s collides with a rough hemispherical body of mass M = 0.3 kg with coefficient of restitution e = 0.5 as shown. The hemispherical body is placed on a rough horizontal surface with coefficient of friction μ_0 = 0.2. The coefficient of friction between the particle and the hemispherical body is μ = 0.5. Find the velocity (in m/s) of the hemispherical body just after collision.

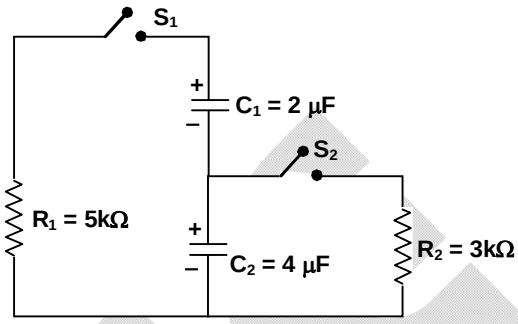


12. A central portion of width 1 mm is cut out of a convex lens of focal length 20 cm and both halves are tightly fitted against each other. A point source 'S' emitting light of wavelength λ = 5000Å is placed at a distance 10 cm from the lens as shown in the figure. Find the distance (in cm) from the lens at which a screen should be placed on the opposite side of the lens so that six interference fringes will be obtained on the screen.

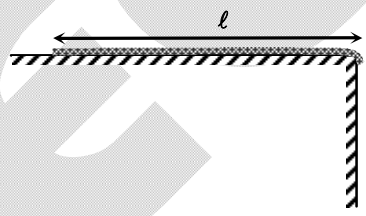


13. Find the velocity (in km/s) with which helium gas flows out of a thermally insulated vessel into vacuum through a small hole made in the vessel. The flow rate of the gas inside the vessel is assumed to be negligible under these conditions. The temperature of helium in the vessel is T = 800K. (Take R = 8.3 J/m-k)

14. In the circuit shown, initially the capacitors C_1 = 2μF and C_2 = 4μF are charged to potential drops V_1 = 4V and V_2 = 6V respectively according to the polarities shown. Initially both the switches S_1 and S_2 are open. If both the switches are closed simultaneously at t = 0. Find the total heat (in μJ) dissipated in the resistor R_2 = 3kΩ.



15. If you hang vertically some length of a long uniform inextensible rope, the longest hanging length is ℓ_0 = 1.40 m that does not break due to its own weight. Now you cut a length of the same rope and place it straight on a smooth horizontal table with a little length hanging over the edge so that it begins to slide down when released. What maximum length (in meter) can this piece have so that it does not break during sliding.



16. A string of mass 2.5 gm and length 1 meter is fixed at both ends. A pipe closed at one end has a length of 80 cm. When the string vibrates in its fourth overtone and the air column in the pipe in its first overtone, they produce a beats frequency of 5 Hz. It is observed that decreasing the tension in the string increases the beats frequency. Neglect the end correction in the pipe. The velocity of sound in air is 320 m/s. Find the tension (in newton) in the string.

17. Photoelectrons are emitted when light of wavelength 400 nm is incident on a surface of metal. When the emitted electrons enter a region having constant magnetic field of strength B = 2.5 mT at an angle of 60° with the field direction, the maximum pitch of the helix described by the electron is found to be 2.7 mm. The specific charge of electron is 1.76×10^{11} C/kg. Find the work function (in eV) of the metal.

18. The pitch of a screw gauge is 1 mm and its cap is divided into 100 divisions. When nothing is placed between its studs, the zero marking of the circular scale is getting 4 divisions above the reference line. When a wire is placed between its studs, the main scale reading is 2 mm and 56^{th} division of circular scale coincides with the reference line. If the length of the wire is 10.5 cm, then find the curved surface area (in cm^2) of the wire in true significant figures.

Chemistry

PART – II

Section – A (Maximum Marks: 24)

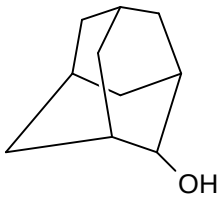
*This section contains **SIX (06)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).*

19. In which of the following molecule plane of symmetry is/are present

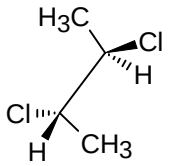
(A) $(Br)(Cl)C = C = C(Cl)(Br)$

(B) $\left[Cu(en)_3\right]^{2+}$

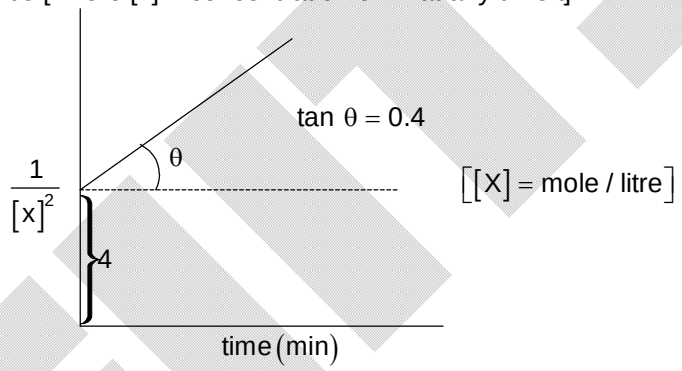
(C)



(D)

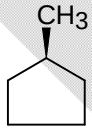


20. For a reaction $X \rightarrow Z$, a graph plotted between $\frac{1}{[x]^2}$ vs. time is found to be linear with slope = 0.4 and y intercept equal to 4 as shown. The rate of disappearance of X at the initial stages will be [where [x] = concentration of 'x' at any time t].



(A) 0.025 M/Min
(B) 4.16×10^{-4} M / Min
(C) 1.8 M/sec
(D) 0.65 M/Min

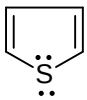
21. Number of monohalogen derivative including stereo isomers for the given molecule is

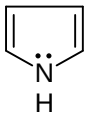


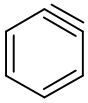
(A) 10
(B) 8
(C) 6
(D) 4

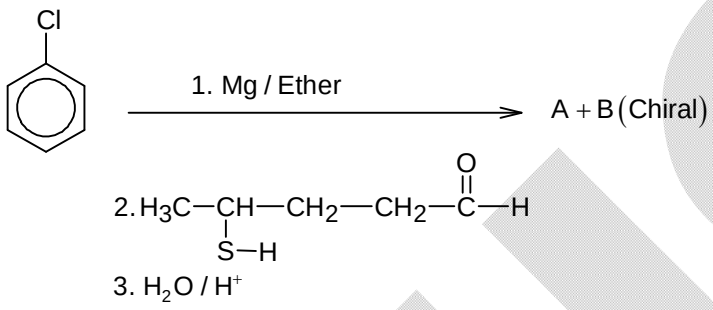
22. Which of the following compound is/are aromatic?

(A) 

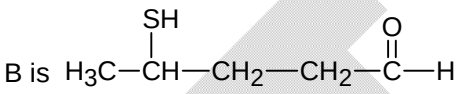
(B) 

(C) 

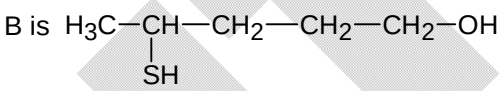
(D) 

23. 

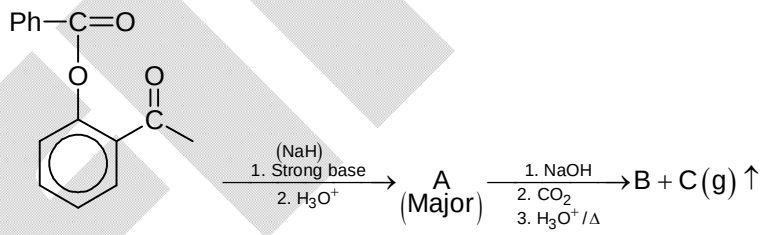
(A) A is benzene

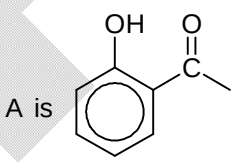
(B) 

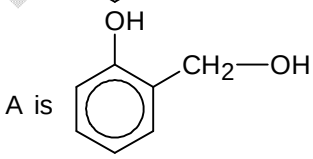
(C) Number of stereoisomer in B is 2

(D) 

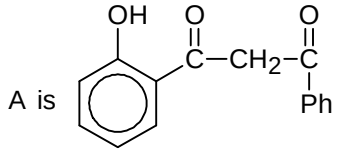
24. Considering the reaction sequence given below, the correct statement (s) is/are



(A) 

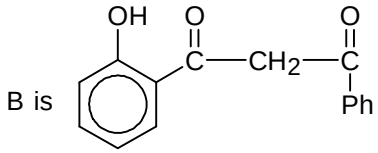
(B) 

(C)



A is

(D)



B is

Section – A (Maximum Marks: 12)

*This section contains **FOUR (04)** Matching List Type Questions. Each question has FOUR statements in **List-I** entries (I), (II), (III) and (IV) and FIVE statements in **List-II** entries (P), (Q), (R), (S) and (T). The codes for lists have choices (A), (B), (C), (D) out of which **ONLY ONE** of these four options is correct answer.*

25. List – I contain complex species and List – II contain their properties

List – I		List – II	
(I)	$\left[Fe(CN)_6\right]^{4-}$	(P)	High spin
(II)	$\left[Fe(H_2O)_6\right]^{2+}$	(Q)	Low spin
(III)	$\left[CO(H_2O)_6\right]^{2+}$	(R)	Paramagnetic
(IV)	$\left[Cu(NH_3)_4\right]^{2+}$	(S)	Square planar
		(T)	t_{2g} contain 6 electron

[Given atomic number of Fe = 26, Cu = 29, CO = 27]

Match each complex species in List-I with their properties in List-II, and choose the correct option

(A) I→R,T; II→P, S; III→Q,T; IV→P, Q

(B) I→Q, T; II→P, R; III→P, R; IV→S, R

(C) I→P,R; II→R,S; III→R, T; IV→P, T

(D) I→Q,T; II→S, T; III→P,T; IV→Q, R

26. List – I contains compounds and List – II contains reactions

List – I		List – II	
(I)	SiF_4	(P)	$XeO_4 + XeF_6 \longrightarrow$
(II)	$XeOF_4$	(Q)	$XeF_6 + H_2O \longrightarrow$
(III)	XeO_3F_2	(R)	$XeF_6 + SiO_2 \longrightarrow$
(IV)	Xe	(S)	$XeF_6 + POF_3 \longrightarrow$
		(T)	$XeO_3 + OH^- \longrightarrow$

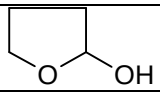
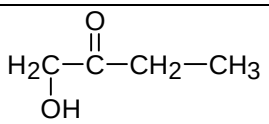
(A) I→R; II→P; III→S; IV→R

(B) I→R; II→P, Q, R, S; III→P; IV→T

(C) I→R; II→S; III→P, S; IV→S, T

(D) I→R; II→Q, T; III→S, T; IV→T

27. Match the compounds in List-I with the observations in List – II, and choose the correct option

List – I		List – II	
(I)	Aniline	(P)	Compounds gives carbylamines test
(II)	Phenol	(Q)	The compound reacts with Br_2/H_2O to give a white precipitate
(III)		(R)	Treating the compound with neutral $FeCl_3$ solution produces violet colour
(IV)		(S)	Compound gives Tollen's test
		(T)	Compound gives Fehling's test

(A) I→P, Q; II→R, Q; III→S, T; IV→S, T
(B) I→P; II→R,S; III→R; IV→Q,S
(C) I→Q,S; II→P,T; III→P; IV→S
(D) I→P,S; II→T; III→Q,R; IV→P

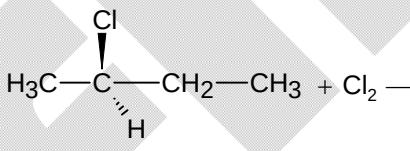
28. Match the following List – I with List - II

List – I		List – II	
(I)	PF_2Cl_3	(P)	All 'P – Cl' bonds are identical
(II)	XeF_4	(Q)	$\mu = 0$
(III)	SF_4	(R)	Only one lone pair on central atom
(IV)	XeO_3	(S)	sp^3d^2 hybridisation with 2 lone pair on central atom
		(T)	d-orbital involved in hybridisation is d_{z^2}

(A) I→P, Q, T; II→Q, S; III→T, R; IV→R
(B) I→S; II→Q, P; III→R, P; IV→Q
(C) I→Q,S; II→P,T; III→P, T; IV→T
(D) I→P, S; II→R, S; III→T; IV→Q

Section – B (Maximum Marks: 24)

*This section contains **EIGHT (08)** numerical based questions. The answer to each question is a **NUMERICAL VALUE.** If the numerical value has more than two decimal places, truncate/round-off the value to **TWO** decimal places.*

29.  if number of dihalogen derivative including stereo isomers is x and after fraction distillation total number is y then the value of $\frac{x+y}{2}$ is

30. The pOH of the solution which is formed by mixing 0.4 mole of Na_3PO_4 in sufficient water to make 1 L of solution
$\left[K_{a_1} = 10^{-4}, K_{a_2} = 10^{-8}, K_{a_3} = 4 \times 10^{-13}\right]$

31. Solution is prepared by mixing 0.1 M 100 ml H_3PO_4 and 0.1 M 150 ml NaOH. pH of the resulting solution is
[Given $K_{a_1} = 10^{-4}, K_{a_2} = 10^{-8}, K_{a_3} = 10^{-12}$]

32. If in the sample of hydrogen atom 2 atoms are only present, total number of maximum different spectral line for the transition n = 5 to n = 1 is x and corresponding lines in Balmer series is y, value of x + y is

33. 
$\xrightarrow[H^+/H_2O]{\text{dil.}} A\,(\text{Major}) \xrightarrow[H_2SO_4]{\text{conc.}} B\,(\text{Major}) \xrightarrow{HCl} \text{Mixture of products} \xrightarrow{R-2-\text{Butanol}}$

Number of products in final solution is x, value of 'x' is

34. The emf of the cell

$Pt \mid H_2\,(1\,atm) \mid Na_2CO_3\,(0.1\,M) \parallel \underset{(0.1\,M)}{H^+} \mid \underset{(1atm)}{H_2} \mid Pt$

[Given $\frac{2.303RT}{F} = 0.06$] $\left[K_{a_1}\right.$ and $\left.K_{a_2}\right.$ of H_2CO_3 is 10^{-3} and $\left.10^{-7}\right]$ is x volt. Then 2x is?

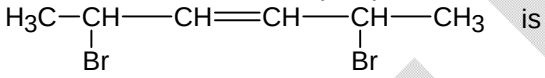
35. For the following reaction

$N_2O_5\,(g) + O_2\,(g) \xrightarrow{40\%} 2NO_2\,(g) + O_3\,(g)$

$NO_2\,(g) + O_2\,(g) \xrightarrow{60\%} NO\,(g) + O_3\,(g)$

If initially 40 mole of N_2O_5 and 85 mole of O_2 are taken then sum of moles of O_2 and O_3 after the reaction is. [Efficiency of reaction mentioned above the arrow].

36. Number of diastereomeric pair possible for



is

Mathematics

PART – III

Section – A (Maximum Marks: 24)

*This section contains **SIX (06)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).*

37. f(x) is a continuous and increasing function satisfying f(x) = f(x – 3) + 4 and $\int_0^6 f(x)\,dx = 0$. Choose the **CORRECT** option(s)

(A) Area bounded by y = f(x), x-axis and lines x = 6 and x = 9 is 9
(B) Area bounded by y = f(x), x-axis and lines x = 6 and x = 9 is 18
(C) y = f(x) is a periodic function
(D) if y = f(x) + h(x) is periodic function (where h(x) is continuous function), then

$$\lim_{x\to\infty} \frac{(h(x)-x)(h(x)+x)}{x^2} = \frac{7}{9}$$

38. If $z = \cos\frac{2\pi}{7} + i\sin\frac{2\pi}{7}$ and $\alpha = \frac{z}{z^2+1}, \beta = \frac{z^2}{z^4+1}, \gamma = \frac{z^3}{z^6+1}$. Choose the **CORRECT** option(s)

(A) $\alpha + \beta + \gamma = -2$
(B) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = -1$
(C) β divides the line joining α and γ internally
(D) $|\beta| < |\alpha| < |\gamma|$

39. The graph of $y = \sqrt{x+3}$ and $y = \sqrt{1-x} + f(k)$ intersect, where k is real parameter

(A) if $f(k) = k^2 - 7k + 12$, then maximum value of k is 5
(B) if f(k) = k, then number of integral values of k is 5
(C) if $f(k) = k^2 - 7k + 12$, then minimum value of k is 2
(D) if f(k) = k, then number of integral values of k is 2

40. Let $\hat{r}$ be unit vector satisfying $(\hat{r}\times\vec{a})\cdot(\hat{r}\times\vec{b}) = \vec{a}\cdot\vec{b}$, $(\vec{a}+2\vec{b})\cdot\hat{r} = 0$ and angle between vectors $\vec{a}$ and $\vec{b}$ is 30°. ($\hat{a}, \hat{b}$ represents unit vectors)

(A) $[\hat{r}\ \ \hat{a}\ \ \hat{b}] = \frac{1}{2}$
(B) $[2\hat{a}\times\hat{b}\quad \hat{b}\times\hat{r}\quad \hat{r}\times\hat{a}] = \frac{1}{2}$
(C) $[\hat{a}\times\hat{b}\quad \hat{b}\times\hat{r}\quad \hat{r}\times\hat{a}] = 3$
(D) $\hat{r} = \vec{a} + 2\vec{b} + (\vec{a}\times\vec{b})$

41. Given an A.P. $\{a_n\}$ with a natural number as first term and a negative integer as common difference and a G.P. $\{b_n\}$ with a natural number as first term and a negative integer as common ratio. The following condition holds (i) $\sum_{n=1}^{5}(a_n + b_n) = 27$, (ii) $\sum_{n=1}^{5}(a_n + |b_n|) = 67$ and (iii) $\sum_{n=1}^{5}|a_n| + |b_n| = 81$

(A) $a_7 + b_7 = 117$
(B) $a_7 + b_7 = -5$
(C) if $a_n \geq -23$, then maximum value of n is 11
(D) if $a_n \geq -23$, then maximum value of n is 9

42. $f : [0, 1] \to R$, $f(x) = 4x(1 - x)$, $f_n(x) = f(f_{n-1}(x)) \ \forall\ n \geq 1$ and $f_0(x) = x$. Let $\{b_k(n)\}$ represents a sequence formed by solution of the equation $f_n(x) = f_0(x)$ and B(n) represents sum of all terms of sequence $\{b_k(n)\}$

(A) Number of terms in the sequence $\{b_k(4)\}$ is 16

(B) The value of $\frac{B(2)}{B(3)}$ is $\frac{147}{310}$

(C) Number of non-zero terms is $\{b_k(3)\}$ is 7

(D) The value of $\frac{B(2)}{B(3)} = \frac{121}{240}$

Section – A (Maximum Marks: 12)

*This section contains **FOUR (04)** Matching List Type Questions. Each question has FOUR statements in **List-I** entries (I), (II), (III) and (IV) and FIVE statements in **List-II** entries (P), (Q), (R), (S) and (T). The codes for lists have choices (A), (B), (C), (D) out of which **ONLY ONE** of these four options is correct answer.*

43. $f : R \to R$ satisfies the equation $(f(x))^3 - (f(x))^2 - x^2 f(x) + x^2 = 0$. Maximum value of f(x) is 1 and minimum value is 0.

List-I		List-II	
(I)	f(x) is a continuous function	(P)	f(x) is non-differentiable at maximum two points
(II)	$\frac{f(x)}{x}$, $(x \neq 0)$ is non-differentiable at only integral values of x	(Q)	f(x) is non-differentiable at exactly three points
(III)	f(x) is discontinuous at $x = \frac{n^2+1}{6n+1}$ $(n \in N)$	(R)	f(x) may be discontinuous at infinite many points
(IV)	f(x) is a discontinuous as well as even function	(S)	f(x) is non-differentiable at five points
		(T)	f(x) is non-differentiable at 7 points

The correct option is:

(A) (I) $\to$ (Q); (II) $\to$ (R); (III) $\to$ (S); (IV) $\to$ (P)

(B) (I) $\to$ (Q); (II) $\to$ (P); (III) $\to$ (S); (IV) $\to$ (R)

(C) (I) $\to$ (S); (II) $\to$ (R); (III) $\to$ (T); (IV) $\to$ (P)

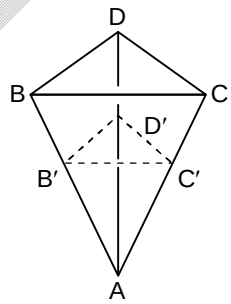
(D) (I) $\to$ (S); (II) $\to$ (P); (III) $\to$ (T); (IV) $\to$ (R)

44. Let f : {1, 2, 3, ..., 10} $\to$ I be a function such that $|f(i) - i| = p$, $\forall\, i \in \{1, 2, ..., 10\}$

List-I		List-II	
(I)	If the maximum value of f(x) is 15	(P)	then the number of possible values of p is 1
(II)	If the maximum value of f(x) is 10	(Q)	then the number of such function is 512
(III)	If the maximum value of f(x) is 12 and $\sum_{i=1}^{10} f(i) = 55$	(R)	then the minimum value of f(x) is –12
(IV)	If the range of f(x) is same as domain	(S)	then the number of such function is 252
		(T)	then number of possible values of p is 3

The correct option is:
(A) (I) $\to$ (T); (II) $\to$ (Q); (III) $\to$ (R); (IV) $\to$ (S)
(B) (I) $\to$ (S); (II) $\to$ (T); (III) $\to$ (R); (IV) $\to$ (Q)
(C) (I) $\to$ (S); (II) $\to$ (P); (III) $\to$ (T); (IV) $\to$ (Q)
(D) (I) $\to$ (R); (II) $\to$ (Q); (III) $\to$ (S); (IV) $\to$ (T)

45. A tetrahedron vessel has vertices ABCD such that $\overrightarrow{AB} = \hat{i}$, $\overrightarrow{AC} = \hat{i} + \hat{j}$, $\overrightarrow{AD} = \lambda\hat{i} + \hat{j} + \hat{k}$ and it is completely filled with water



List-I		List-II	
(I)	If the water evaporates from the top surface at the rate proportional to height of water level from the bottom (vertex A)	(P)	$\frac{19}{27}$ of tetrahedron when $\lambda = 3$
(II)	If the water evaporates from the top surface at positive rate proportional to area of water exposed to air	(Q)	time to empty $\frac{7}{8}$ of total volume is $\frac{3}{4}$th of the time to empty whole of the vessel
(III)	If the tetrahedron is cut through a parallel plane such that area of $\Delta B'C'D'$ is $\frac{4}{9}$ of area of ΔBCD then volume of figure BDCB′D′C′ is	(R)	time to empty $\frac{7}{8}$ of the total volume is $\frac{1}{2}$ of the time to empty whole of the vessel
(IV)	For $\lambda = 2$ the volume of the tetrahedron is	(S)	$\frac{27}{8}$ of tetrahedron when $\lambda = 3$
		(T)	same as of the tetrahedron having $\lambda = 1$

The correct option is:
(A) (I) → (R); (II) → (S); (III) → (T); (IV) → (P)
(B) (I) → (Q); (II) → (R); (III) → (P); (IV) → (T)
(C) (I) → (R); (II) → (Q); (III) → (P); (IV) → (S)
(D) (I) → (Q); (II) → (R); (III) → (S); (IV) → (T)

46. f(x) is cubic polynomial with leading coefficient 1 and g(x) is continuous function on R such that $f(x)\cdot g(x) = (x^2 + 3x + 2)$ and $g(-1) = 1$

List-I		List-II	
(I)	If f(0) is integer, then	(P)	when f(0) also takes maximum integral value is $-\frac{1}{3}$
(II)	If f′(0) is integer, then	(Q)	the number of integral values of f(0) is 7
(III)	The maximum value of g(x)	(R)	when f(0) also takes maximum integral value is $\frac{1}{7}$
(IV)	The minimum value of g(x)	(S)	when f(0) also takes minimum integral value is $-\frac{1}{8}$
		(T)	number of integral value of f(0) is 3

The correct option is:
(A) (I) → (T); (II) → (Q); (III) → (R); (IV) → (P)
(B) (I) → (Q); (II) → (T); (III) → (R); (IV) → (P)
(C) (I) → (T); (II) → (Q); (III) → (P); (IV) → (S)
(D) (I) → (Q); (II) → (T); (III) → (P); (IV) → (R)

Section – B (Maximum Marks: 24)

*This section contains **EIGHT (08)** numerical based questions. The answer to each question is a **NUMERICAL VALUE.** If the numerical value has more than two decimal places, truncate/round-off the value to **TWO** decimal places.*

47. The number of solution of the equation $(\sin 3x - 1)\left(\left|\sqrt{3}\tan x + 1\right| + \left|2\cos 2x - 1\right|\right) = 0$ in the interval $[0, 100\pi]$ is

48. For $x > t > 0$ and $a > 0$, a is function of t such that the equation $t^3 \ln(x - t) = 2e^{x-a}$ (for $x > t$) has only one solution. If for this condition $a = f(t)$, then value of $4(f'(4))^2$ is

49. $f : R \to R$ be a continuous function satisfying $2f(x) + f\left(\frac{x}{2} + 2\right) = x^2 + 2x + 3$, then $\int_0^3 f(x)\,dx$ is

50. Two chess players Vishi and Ritti has same level of expertise, both play a series of n games. The probability that i^{th} game ends in a draw is $\frac{i}{i+1}$. P is the probability that Vishi wins odd number of games. The value of P when n = 99, is

51. The hyperbola $y = \frac{k}{x-1} + 3$ intersects (k > 0) positive x-axis and positive y-axis at points A and B respectively. P is image of B in the point (1, 3), Q is foot of perpendicular from P on the x-axis. The integral value of area of quadrilateral ABPQ is

52. An ellipse has $x + y - 3 = 0$ as its tangent and (0, 1) is one of the foci. If the eccentricity of the ellipse is $\frac{1}{2}$, then locus of other focus is $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$. The value of $\frac{a+b+h+g+f+c}{c}$ is

53. A and B are two 3×2 and 2×3 matrices such that $AB = \begin{bmatrix} 5 & 8 & 5 \\ 8 & 13 & 9 \\ 5 & 9 & 10 \end{bmatrix}$ find the value of determinant of BA

54. A set A = {1, 2, 3, ... n} contains first n natural numbers ΔA represent set of all possible difference of any two element of set A. Similarly $\Delta^2 A$ represent difference of any two elements of set ΔA and so on, {for example if A = {1, 2, 3}, ΔA = {1, 1, 2}, $\Delta^2 A$ = {0, 1, 1}, $\Delta^3 A$ = {0, 1, 1}.....}. So, sum of all elements of $\Delta^2 A$ when n = 10 is _____ (Note that repeated elements are also listed in ΔA, $\Delta^2 A$, ...)