FIITJEE

# ALL INDIA TEST SERIES

## JEE (Advanced)-2023

## OPEN TEST – II

## PAPER –1

## TEST DATE: 16-04-2023

# ANSWERS, HINTS & SOLUTIONS

## *Physics* PART – I

### Section – A

1. B, C

Sol. When x length of the sheet is pulled out of the table,

$$m\frac{dv}{dt} = F - \eta(\ell - x)\frac{v}{h}\ell$$

$$m\int_0^v dv = F\int_0^t dt - \eta\frac{\ell}{h}\int_0^{\ell/2}(\ell - x)dx$$

$$mv = Ft - \frac{\eta\ell}{h}\left[\ell x - \frac{x^2}{2}\right]_0^{\ell/2}$$

$$mv = Ft - \eta\frac{\ell}{h}\left(\frac{\ell^2}{2} - \frac{\ell^2}{8}\right)$$

$$0 = Ft - \frac{\eta\ell}{h}\frac{3\ell^2}{8} \text{ (since m = 0)}$$

$$t = \frac{3\eta\ell^3}{8Fh} = \frac{3\times 0.4\times 1}{8\times 25\times 1\times 10^{-3}} = 6 \text{ sec}$$

Since mass of the sheet, m = 0

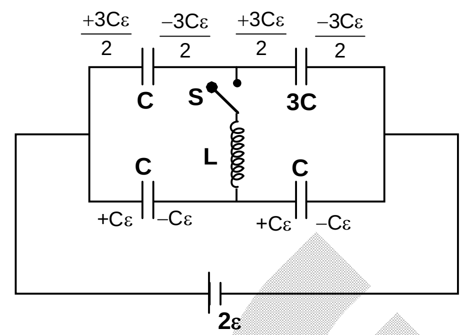
$$\eta\ell\frac{\ell}{2}\frac{v}{h} = F$$

$$v = \frac{2Fh}{\eta\ell^2} = \frac{2\times 25\times 1\times 10^{-3}}{0.4\times 1} = 12.5\times 10^{-2} \text{ m/s}$$

v = 12.5 cm/s

2. B, D

Sol. Before switch 'S' is closed.

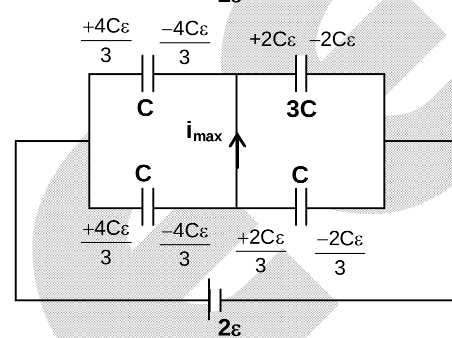



$$C_{eq} = \frac{3C}{4} + \frac{C}{2} = \frac{5C}{4}$$

$$Q = C_{eq} 2\varepsilon = \frac{5C}{4} \times 2\varepsilon$$

$$Q = \frac{5C\varepsilon}{2}$$

After switch 'S' is closed.




$$C'_{eq} = \frac{2C \times 4C}{(2C + 4C)} = \frac{4C}{3}$$

$$Q' = C'_{eq} 2\varepsilon = \frac{8C\varepsilon}{3}$$

The charge flown through the battery

$$\Delta Q = Q' - Q = \frac{8C\varepsilon}{3} - \frac{5C\varepsilon}{2} = \frac{C\varepsilon}{6}$$

Now, $\frac{C\varepsilon}{6} \times 2\varepsilon = \frac{1}{2} \times \frac{4C}{3}(2\varepsilon)^2 - \frac{1}{2} \times \frac{5C}{4} \times (2\varepsilon)^2 + \frac{1}{2} L i_{max}^2$

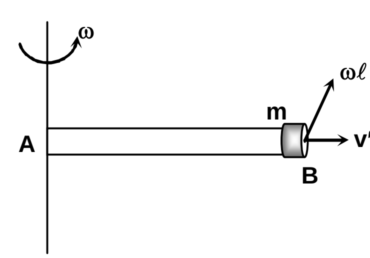
$$\frac{C\varepsilon^2}{3} = \frac{C\varepsilon^2}{6} + \frac{1}{2} L i_{max}^2$$

$$\Rightarrow \quad \frac{C\varepsilon^2}{6} = \frac{1}{2} L i_{max}^2$$

$$\Rightarrow \; i_{max} = \varepsilon\sqrt{\frac{C}{3L}}$$

3. A, B, D

Sol. Using conservation of angular momentum of the system,




$$\frac{M\ell^2}{3}\omega_0 = \left(\frac{M\ell^2}{3} + m\ell^2\right)\omega$$

$$m\ell^2\omega_0 = 2m\ell^2\omega \Rightarrow \; \omega = \frac{\omega_0}{2} = 5 \text{ rad/s}$$

Now using conservation of energy of the system,

$$\frac{1}{2}\frac{M\ell^2}{3}\omega_0^2 = \frac{1}{2}\left(\frac{M\ell^2}{3} + m\ell^2\right)\omega^2 + \frac{1}{2}mv'^2$$

$$\frac{1}{2}m\ell^2\omega_0^2 = \frac{1}{2} \times 2m\ell^2 \frac{\omega_0^2}{4} + \frac{1}{2}mv'^2$$

$$v'^2 = \frac{\omega_0^2\ell^2}{2}$$

$\Rightarrow v' = \frac{\omega_0 \ell}{\sqrt{2}} = 5\sqrt{2}$ m/s

$N_y = mg = 10N$ ...(i)

$N_x = 2mv'\omega - m\alpha\ell$ ...(ii)

$N_x \ell = \frac{M\ell^2}{3}\alpha \Rightarrow N_x = m\alpha\ell$ ...(iii)

Solving (ii) and (iii), we get

$2N_x = 2mv'\omega \Rightarrow N_x = mv'\omega = 1\times 5\sqrt{2}\times 5 = 25\sqrt{2}$ N

$N_x = 25\sqrt{2}$ N ...(iv)

Hence, $N = \sqrt{N_x^2 + N_y^2} = \sqrt{1250+100} = \sqrt{1350} = 15\sqrt{6}$ newton

The velocity of sleeve relative to the rod, $v' = 5\sqrt{2}$ m/s

The velocity of sleeve relative to the ground, $v = \sqrt{v'^2 + \omega^2\ell^2} = \sqrt{50+25}$

$v = 5\sqrt{3}$ m/s

4. A, C, D

Sol. $v^2 = u^2 + 2a_t S = 0 + 2\times 4S$

$\Rightarrow v = \sqrt{8S}$ ...(i)

$S = ut + \frac{1}{2}a_t t^2 \Rightarrow S = 0 + \frac{1}{2}\times 4t^2 = 2t^2$

$S = 2t^2$ ...(ii)

$a_n = 2t^4 = 2\times\frac{S^2}{4} = \frac{S^2}{2}$

$a_n = \frac{S^2}{2}$ ...(iii)

At S = 4m, $a_n = 8$ m/s$^2$

The net acceleration, $a = \sqrt{a_t^2 + a_n^2}$, at S = 4m, $a = \sqrt{16+64} = \sqrt{80} = 4\sqrt{5}$ m/s$^2$

The radius of curvature of the trajectory,

$r = \frac{v^2}{a_n} = \frac{8S}{S^2/2} = \frac{16}{S}$

$r = \frac{16}{S}$

At S = 4m, $r = \frac{16}{4} = 4$ m

At S = 8m, $r = \frac{16}{8} = 2$ m

5. A, C

Sol. $\frac{KQ}{R} + \frac{kq}{r_1} + \frac{kq}{r_2} = 0$

$Q = -qR\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$ ...(i)

When $r_1 = 2R$ and $r_2 = 4R$

$Q = -qR\left(\frac{1}{2R} + \frac{1}{4R}\right)$

$Q = -\frac{3q}{4}$

Now, $I = \frac{dQ}{dt} = qR\left(\frac{1}{r_1^2}\frac{dr_1}{dt} + \frac{1}{r_2^2}\frac{dr_2}{dt}\right)$

$I = -qR\left(\frac{v}{r_1^2} + \frac{2v}{r_2^2}\right)$

when $r_1 = 2R$ and $r_2 = 4R$

$I = -qRv\left(\frac{1}{4R^2} + \frac{2}{16R^2}\right)$

$I = -\left(\frac{3qv}{8R}\right)$

$|I| = \frac{3qv}{8R}$

6. B, D

Sol. $\frac{\mu_4}{v} - \frac{\mu_1}{u} = \left(\frac{\mu_2 - \mu_1}{R_1}\right) + \left(\frac{\mu_3 - \mu_2}{R_2}\right) + \left(\frac{\mu_4 - \mu_3}{R_3}\right)$

$\frac{1.8}{v} - \frac{1}{-25} = \left(\frac{1.2-1}{10}\right) + \left(\frac{1.5-1.2}{-30}\right) + \left(\frac{1.8-1.5}{\infty}\right)$

$\frac{1.8}{v} + \frac{1}{25} = \frac{0.2}{10} - \frac{0.3}{30} + 0$

$\frac{1.8}{v} + \frac{1}{25} = \frac{1}{50} - \frac{1}{100}$

$\frac{1.8}{v} = \frac{1}{100} - \frac{1}{25}$

$\Rightarrow v = -60$ cm

Lateral magnification, $m = \left(\frac{\mu_1}{\mu_4}\frac{v}{u}\right) = \frac{1}{1.8}\left(\frac{-60}{-25}\right)$

$m = +\frac{4}{3}$

The size of image formed $= \frac{4}{3} \times 0.3 = 0.40$ cm

7. B

Sol. Ring:

$f_k = \mu mg\cos\theta = mg\sin\theta$

$mg\sin\theta R = mR^2\alpha \Rightarrow \alpha_1 = \frac{g\sin\theta}{R}$

$a_1 = \frac{mg\sin\theta + f_k}{m} = 2g\sin\theta$

$v = v_0 - a_1t_1 = v_0 - 2g\sin\theta t_1$

$\omega = \omega_0 + \alpha_1 t_1 = \omega_0 + \frac{g\sin\theta}{R}t_1$

$v = \omega R$

$v_0 - 2g\sin\theta t_1 = \omega_0 R + g\sin\theta t_1$

$$v_0 - \frac{v_0}{2} = 3g\sin\theta t_1 \Rightarrow t_1 = \frac{v_0}{6g\sin\theta}$$

$$v = v_0 - 2g\sin\theta t_1 = v_0 - \frac{v_0}{3} = \frac{2v_0}{3}$$

$$t_2 = \frac{v}{a_2} = \frac{2v_0/3}{\frac{g\sin\theta}{2}} = \frac{4v_0}{3g\sin\theta}$$

$$t = t_1 + t_2 = \frac{v_0}{6g\sin\theta} + \frac{4v_0}{3g\sin\theta} = \frac{9v_0}{6g\sin\theta}$$

$$t = \frac{3v_0}{2g\sin\theta}$$

**Disc:**

$$a_1 = \frac{mg\sin\theta - mg\sin\theta}{m} = 0$$

$$mg\sin\theta R = \frac{mR^2\alpha_1}{2} \Rightarrow \alpha_1 = \frac{2g\sin\theta}{R}$$

$v = \omega R$

$v_0 = (\omega_0 - \alpha_1 t_1)R$

$v_0 = \omega_0 R - \alpha_1 R t_1$

$v_0 = 2v_0 - 2g\sin\theta t_1$

$$t_1 = \frac{v_0}{2g\sin\theta}$$

$$a_2 = \frac{2}{3}g\sin\theta$$

$$t_2 = \frac{v_0}{a_2} = \frac{3v_0}{2g\sin\theta}$$

$$t = t_1 + t_2 = \frac{v_0}{2g\sin\theta} + \frac{3v_0}{2g\sin\theta} = \frac{2v_0}{g\sin\theta}$$

**Hollow sphere:**

$$a_1 = \frac{mg\sin\theta + f_k}{m} = 2g\sin\theta$$

$$mg\sin\theta R = \frac{2mR^2\alpha_1}{3} \Rightarrow \alpha_1 = \frac{3g\sin\theta}{2R}$$

$v = \omega R$

$$v_0 - 2g\sin\theta t_1 = \left(\omega_0 + \frac{3g\sin\theta}{2R}t_1\right)R$$

$$v_0 - 2g\sin\theta t_1 = \frac{v_0}{2} + \frac{3g\sin\theta t_1}{2}$$

$$\frac{v_0}{2} = \frac{7g\sin\theta t_1}{2} \Rightarrow t_1 = \frac{v_0}{7g\sin\theta} \Rightarrow v = v_0 - \frac{2v_0}{7} = \frac{5v_0}{7}$$

$$a_2 = \frac{3g\sin\theta}{5}$$

$$t_2 = \frac{v}{a_2} = \frac{5v_0}{7} \times \frac{5}{3g\sin\theta} = \frac{25v_0}{21g\sin\theta}$$

$$t = t_1 + t_2 = \frac{v_0}{7g\sin\theta} + \frac{25v_0}{21g\sin\theta} = \frac{28v_0}{21g\sin\theta} = \frac{4v_0}{3g\sin\theta}$$

**Solid sphere**

$$a_1 = \frac{mg\sin\theta - f_k}{m} = \frac{mg\sin\theta - mg\sin\theta}{m} = 0$$

$$f_k R = \frac{2}{5} mR^2 \alpha_1$$

$$mg\sin\theta R = \frac{2}{5} mR^2 \alpha_1$$

$$\alpha_1 = \frac{5g\sin\theta}{2R}$$

v = ωR

$$v_0 = \left(\omega_0 - \frac{5g\sin\theta t_1}{2R}\right)R$$

$$v_0 = \omega_0 R - \frac{5g\sin\theta t_1}{2}$$

$$v_0 = \frac{5g\sin\theta t_1}{2} \Rightarrow t_1 = \frac{2v_0}{5g\sin\theta}$$

$$a_2 = \frac{5g\sin\theta}{7}$$

$$t_2 = \frac{v_0}{a_2} = \frac{7v_0}{5g\sin\theta}$$

$$t = t_1 + t_2 = \frac{9v_0}{5g\sin\theta}$$

8. A

Sol. (I) $z_1 = 10 + 10i = 10(1 + i)$

$Z_2 = 20 - 20i = 20(1 - i)$

$$\frac{1}{z} = \frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{10(1+i)} + \frac{1}{20(1-i)}$$

$$\frac{1}{z} = \left(\frac{1-i}{20}\right) + \left(\frac{1+i}{40}\right) = \left(\frac{3-i}{40}\right)$$

$$z = \frac{40}{(3-i)} \times \frac{(3+i)}{(3+i)} = 4(3+i)$$

$$I_{rms} = \frac{\varepsilon_{rms}}{|z|} = \frac{40\sqrt{2}}{4\sqrt{10}} = 2\sqrt{5} \text{ amp}$$

(II) $z_1 = 20(1 - i)$

$Z_2 = 20(1 + i)$

$$\frac{1}{z'} = \frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{20(1-i)} + \frac{1}{20(1+i)}$$

$$\frac{1}{z'} = \frac{(1+i)}{40} + \frac{(1-i)}{40} = \frac{1}{20}$$

$z' = 20\ \Omega$

$z = 20 + 15i$

$\Rightarrow I_{rms} = \frac{\varepsilon_{rms}}{|z|} = \frac{25\sqrt{2}}{25} = \sqrt{2}$ amp

(III) $x = (x_L - x_C) = 40 - 20 = 20\,\Omega$

$\frac{1}{z'} = \frac{1}{20} + \frac{1}{20i} = \frac{1-i}{20}$

$z' = \frac{20}{1-i} = 10(1+i)$

z = 10 + 10(1+i) = 10(2+i)

$\Rightarrow I_{rms} = \frac{\varepsilon_{rms}}{|z|} = \frac{20\sqrt{10}}{10\sqrt{5}} = 2\sqrt{2}$ amp

(IV) $\frac{1}{z'} = \frac{1}{10(1+i)} + \frac{1}{10(1-i)} = \frac{(1-i)}{20} + \frac{(1+i)}{20} = \frac{1}{10}$

$z' = 10\Omega$

$\Rightarrow z = 10 + (20 + 40i) = 30 + 40i$

$I_{rms} = \frac{\varepsilon_{rms}}{|z|} = \frac{100\sqrt{10}}{50} = 2\sqrt{10}$ amp

9. D

Sol. (I) Magnetic pressure exerted on the wall of the outer cylinder,

$P_0 = \frac{3I}{4\pi a}\left(\frac{3\mu_0 I}{8\pi a} - \frac{\mu_0 I}{4\pi a}\right)$

$P_0 = \frac{3I}{4\pi a} \times \frac{\mu_0 I}{8\pi a} = \frac{3\mu_0 I^2}{32\pi^2 a^2}$

(II) Magnetic pressure exerted on the wall of the outer cylinder,

$P_0 = \frac{2I}{4\pi a}\left(\frac{\mu_0 I}{4\pi a} + \frac{\mu_0 I}{4\pi a}\right)$

$P_0 = \frac{I}{2\pi a} \times \frac{\mu_0 I}{2\pi a} = \frac{\mu_0 I^2}{4\pi^2 a^2}$

(III) Magnetic pressure exerted on the wall of the outer cylinder,

$P_0 = \frac{I}{4\pi a}\left(\frac{\mu_0 I}{2\pi a} + \frac{\mu_0 I}{8\pi a}\right)$

$P_0 = \frac{5\mu_0 I^2}{32\pi^2 a^2}$

(IV) Magnetic pressure exerted on the wall of the outer cylinder,

$P_0 = \frac{3I}{4\pi a}\left(\frac{3\mu_0 I}{8\pi a} - \frac{\mu_0 I}{8\pi a}\right)$

$P_0 = \frac{3I}{4\pi a} \times \frac{\mu_0 I}{4\pi a} = \frac{3\mu_0 I^2}{16\pi^2 a^2}$

10. B

Sol. (I) In the process AB,

$PT^{-1/2}$ = constant $\Rightarrow PV^{-1}$ = constant

$\Delta W_{AB} = \frac{nR\Delta T}{(1-x)} = \frac{2R \times 900}{2} = 900$ R

$\Delta W_{BC} = 0$

$\Delta W_{CA} = nR\Delta T = 2R(-300) = -600\ R$
$\Delta W_{cycle} = 900\ R + 0 - 600\ R = 300\ R$

(II) $\Delta Q_{AB}$ **= $nC_P\Delta T$ =** $2 \times \frac{5R}{2} \times 400 = 2000R$

In the process BC, $PT^{-2} = \text{constant} \Rightarrow PV^2 = \text{constant}$

$C = C_V + \frac{R}{(1-x)} = \frac{3R}{2} + \frac{R}{(1-2)} = \frac{R}{2}$

$\Delta Q_{BC} = nC\Delta T = 2 \times \frac{R}{2} \times (400 - 800) = -400R$

$\Delta Q_{CA} = \Delta W_{CA} = nRT_0 \ell n\left(\frac{P_0}{4P_0}\right) = 2R \times 400(-2\ell n2) = -1120R$

$\Delta W_{cycle} = \Delta Q_{cycle} = 2000R - 400R - 1120R = 480R$

(III) $\Delta Q_{AB} = nC_P\Delta T = 2 \times \frac{5R}{2} \times 300 = 1500R$

In the process BC, $VT^2 = \text{constant} \Rightarrow PV^{3/2} = \text{constant}$

$C = C_V + \frac{R}{(1-x)} = \frac{3R}{2} + \frac{R}{\left(1-\frac{3}{2}\right)} = \frac{3R}{2} - 2R = -\frac{R}{2}$

$\Delta Q_{BC} = nC\Delta T = 2 \times \left(-\frac{R}{2}\right) \times (-300) = 300R$

$\Delta Q_{CA} = \Delta W_{CA} = nRT_0 \ell n\left(\frac{V_0}{8V_0}\right) = 2R \times 300(-3\ell n2) = -1260R$

$\Delta W_{cycle} = \Delta Q_{cycle} = 1500R + 300R - 1260R = 540R$

(IV) $\Delta Q_{AB} = nC_P\Delta T = 2 \times \frac{5R}{2} \times 300 = 1500R$

$\Delta Q_{BC} = \Delta W_{BC} = nRT\ell n2 = 2R \times 600 \times 0.7 = 840R$

In the process CA, $VT^{-2} = \text{constant} \Rightarrow PV^{1/2} = \text{constant}$

$C = C_V + \frac{R}{(1-x)} = \frac{3R}{2} + 2R = \frac{7R}{2}$

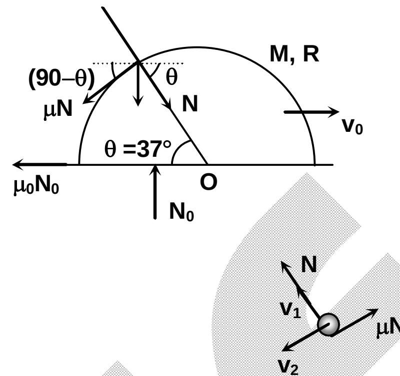
$\Delta Q_{CA} = nC\Delta T = 2 \times \frac{7R}{2} \times (-300) = -2100R$

$\Delta W_{cycle} = \Delta Q_{cycle} = 1500R + 840R - 2100R = 240R$

## Section – B

11. 1.67

Sol. $\int N dt = m(v_1 + u\sin\theta)$ …(i)




$\int \mu N dt = m(u\cos\theta - v_2)$ …(ii)

$\int N_0 dt = \int N(\sin\theta + \mu\cos\theta)dt$

$\Rightarrow \int N_0 dt = \int N dt$

Now, $(\cos\theta - \mu\sin\theta)\int N dt - \mu_0 \int N_0 dt = Mv_0$

$\left(\frac{4}{5} - 0.5\times\frac{3}{5}\right)\int N dt - 0.2\int N dt = Mv_0$

$0.3\int N dt = Mv_0$

$\Rightarrow 0.3m(v_1 + u\sin\theta) = Mv_0$

$\Rightarrow v_1 + 12 = 10v_0$

$10v_0 - v_1 = 12$ …(iii)

Now, $v_0\cos\theta + v_1 = eu\sin\theta$

$0.8v_0 + v_1 = 6$

Solving (iii) and (iv), we get

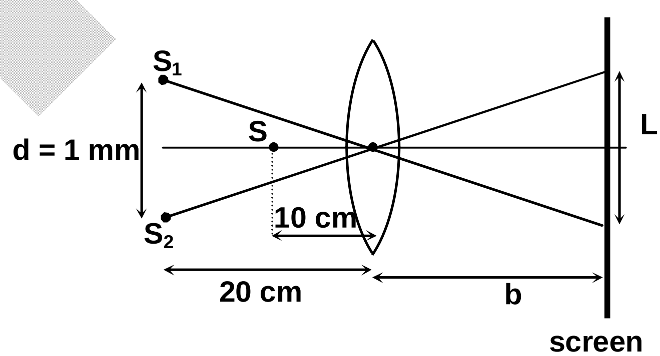
$10.8v_0 = 18$

$\Rightarrow v_0 = \frac{18}{10.8} = \frac{5}{3} = 1.67$ m/s

$v_0 = 1.67$ m/s

12. 30.00

Sol. $\frac{1}{v} - \frac{1}{-10} = \frac{1}{20}$




$\frac{1}{v} = \frac{1}{20} - \frac{1}{10}$

v = –20 cm

$m = \frac{v}{u} = \frac{-20}{-10} = +2$

$\frac{L}{d} = \frac{b}{20} \Rightarrow L = \frac{bd}{20}$

D = (b + 20) cm

d = 1 mm

Fringe width, $\omega = \frac{\lambda D}{d} = \frac{\lambda(b+20)}{d}$

No. of fringes obtained on the screen,

$n = \frac{L}{\omega} = \frac{bd^2}{20\lambda(b+20)}$

Putting n = 6, we get

$6 = \frac{b\times 1\times 10^{-6}}{20\times 5\times 10^{-7}\times(b+20)\times 10^{-2}}$

$6 = \frac{10b}{(b+20)}$

$\Rightarrow$ b = 30 cm

13. 2.88

Sol. $P + \frac{\rho C_V T}{M} = \frac{1}{2}\rho v^2$

$\frac{\rho RT}{M} + \frac{\rho C_V T}{M} = \frac{1}{2}\rho v^2$

$\frac{\rho T}{M}(R + C_V) = \frac{1}{2}\rho v^2$

$\frac{\rho T C_P}{M} = \frac{1}{2}\rho v^2$

$\frac{\rho T}{M}\frac{5R}{2} = \frac{1}{2}\rho v^2$

$v = \sqrt{\frac{5RT}{M}} = \sqrt{\frac{5\times 8.3\times 800}{4\times 10^{-3}}} = 2.88\times 10^3$ m/s

v = 2.88 km/s

14. 33.00

Sol. $U_1 = \frac{1}{2}C_1V_1^2 = \frac{1}{2}\times 2\times 16 = 16\ \mu J$

$U_2 = \frac{1}{2}C_2V_2^2 = \frac{1}{2}\times 4\times 36 = 72\ \mu J$

Total heat dissipated in the two resistors,

H = 16 + 72 = 88 $\mu$J

$v_1 = 4e^{-t/\tau}$ and $v_2 = 6e^{-t/\tau}$

Now, the total heat dissipated in the resistor $R_1$,

$H_1 = \int_0^\infty \frac{(10e^{-t/\tau})^2}{5\times 10^3}dt = \frac{100}{5\times 10^3}\int_0^\infty e^{-2t/\tau}dt = 10\times 10^{-3}\ \tau$

Total heat dissipated in the resistor $R_2$,

$H_2 = \int_0^\infty \frac{(6\times e^{-t/\tau})^2}{3\times 10^3}dt = 6\times 10^{-3}\tau$

Now, $H_1 + H_2 = 88\ \mu J$ and $\frac{H_1}{H_2} = \frac{5}{3}$

$H_1 = \frac{5}{8}\times 88 = 55\ \mu J$

$H_2 = \frac{3}{8}\times 88 = 33\ \mu J$

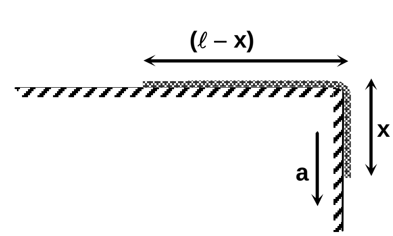
15. 5.60

Sol. Let mass per unit length of the rope is $\lambda$

The breaking tension of the rope, $T_{max} = \lambda \ell_0 g$

The acceleration of the rope,

$a = \frac{\lambda x g}{\lambda \ell}$



$a = \frac{gx}{\ell}$ …(i)

$T = \lambda(\ell - x)a$

$T = \lambda(\ell - x)\frac{gx}{\ell}$

$T = \frac{\lambda g}{\ell}(\ell - x)x$ …(ii)

For T to be maximum, $\frac{dT}{dx} = 0$

$\frac{\lambda g}{\ell}(\ell - 2x) = 0$

$x = \frac{\ell}{2}$

Hence, $T_{max} = \frac{\lambda g}{\ell}\left(\ell - \frac{\ell}{2}\right)\frac{\ell}{2}$

$\lambda \ell_0 g = \frac{\lambda g \ell}{4}$

$\ell = 4\ell_0$

$\ell = 4 \times 1.40 = 5.60$ m

16. 34.81

Sol. The frequency of vibration of air column in the pipe,

$f_o = \frac{3v_0}{4\ell_0} = \frac{3 \times 320}{4 \times 0.8} = 300$ Hz

Now, $f_o - f_s = f_b$

$f_S = f_o - f_b$

$f_S = 300 - 5 = 295$ Hz

Now, $f_S = \frac{5}{2\ell}\sqrt{\frac{F}{\mu}}$

$295 = \frac{5}{2 \times 1}\sqrt{\frac{F}{2.5 \times 10^{-3}}}$

F = 34.81 newton

17. 2.69

(Range 2.65 to 2.70)

Sol. Energy of each photon, $E = \frac{hC}{\lambda} = \frac{1240}{400} = 3.10$ eV

Now, $v_{max}\cos 60\left(\frac{2\pi m}{eB}\right) = 2.7 \times 10^{-3}$

$v_{max} = \frac{7 \times 2.5 \times 10^{-3} \times 2.7 \times 10^{-3} \times 1.76 \times 10^{11}}{22}$

$v_{max} = 3.78 \times 10^{5}$ m/s

$k_{max} = \frac{1}{2}mv_{max}^2$

$$k_{max} = \frac{1}{2} \times 9.1 \times 10^{-31} \times \frac{(3.78)^2 \times 10^{10}}{1.6 \times 10^{-19}}$$

$k_{max}$ = 0.4063 eV
$k_{max}$ = 0.41 eV
Now, $k_{max} = E - \phi$
$\phi = E - k_{max}$
$\phi = 3.10 - 0.41$
Hence, work function $\phi$ = 2.69 eV

18. 8.58

Sol. Pitch, P = 1 mm

Least count = $\frac{P}{N} = \frac{1\,mm}{100} = 0.01\,mm$

Diameter of the wire, d = 2 mm + 56 × L.C. + 4 × L.C.
d = 2mm + 56 × 0.01 mm + 4 × 0.01 mm
d = 2 mm + 60 × 0.01 mm
d = 2.60 mm
d = 0.260 cm
length of the wire, $\ell$ = 10.5 cm
The curved surface area of the wire,
$S = \pi d\ell$

$$S = \frac{22}{7} \times 0.260 \times 10.5 = 8.58\ cm^2$$

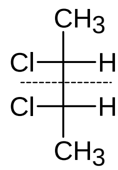
# Chemistry

## PART – II

### Section – A

19. C, D

Sol. In C and D P. O. S. is present




20. A

Sol. For $n^{th}$ order reaction,

$$\Rightarrow t = \frac{1}{k(n-1)} \times \left[X_t^{1-n} - X_o^{1-n}\right]$$

As we know,

n = 3

$$t = \frac{1}{k(3-1)}\left[x_t^{1-3} - x_o^{1-3}\right] = \frac{1}{2k}\left[\frac{1}{x_{t^2}} - \frac{1}{x_{o^2}}\right]$$

$$\because 2k = 0.4$$

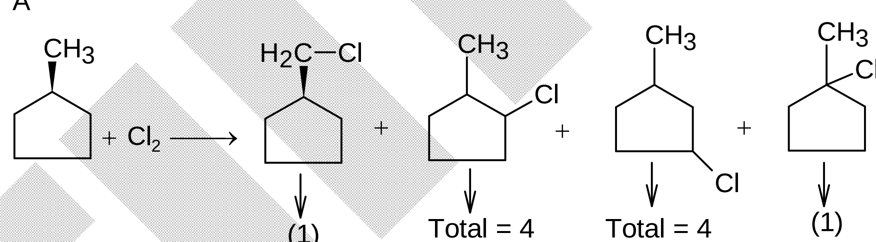
$$k = 0.2 \text{ unit and } \frac{1}{x_{o^2}} = 4 \Rightarrow x_o = \sqrt{\frac{1}{4}} = 0.5$$

$$\therefore \frac{-d[x]}{dt} = k[x_o]^3 = 0.2 \times 0.5 \times 0.5 \times 0.5$$

= 0.025 M/min

21. A

Sol.

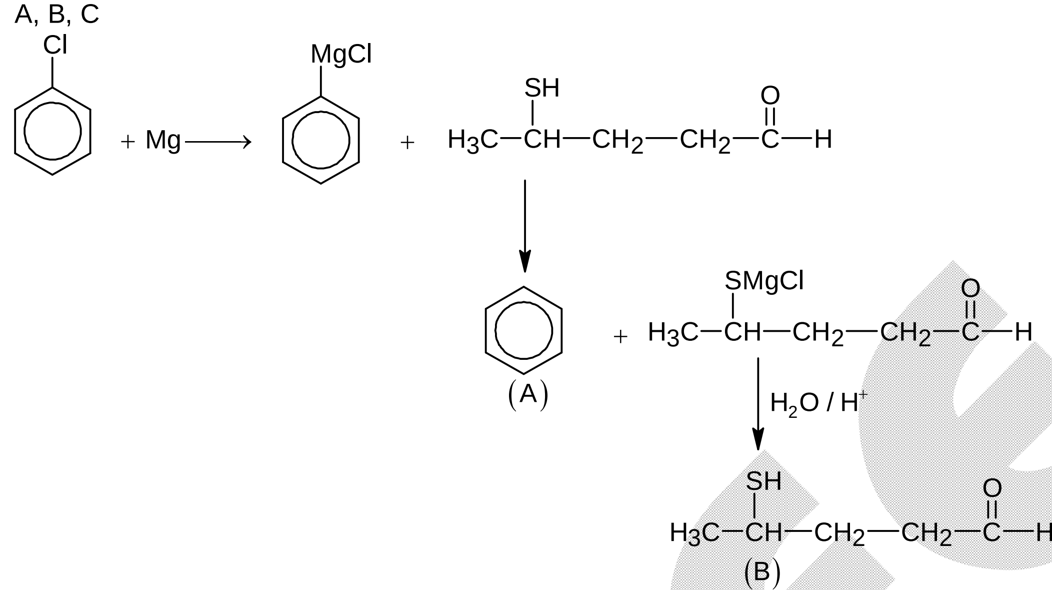



22. A, B, C, D

Sol. All are aromatic.

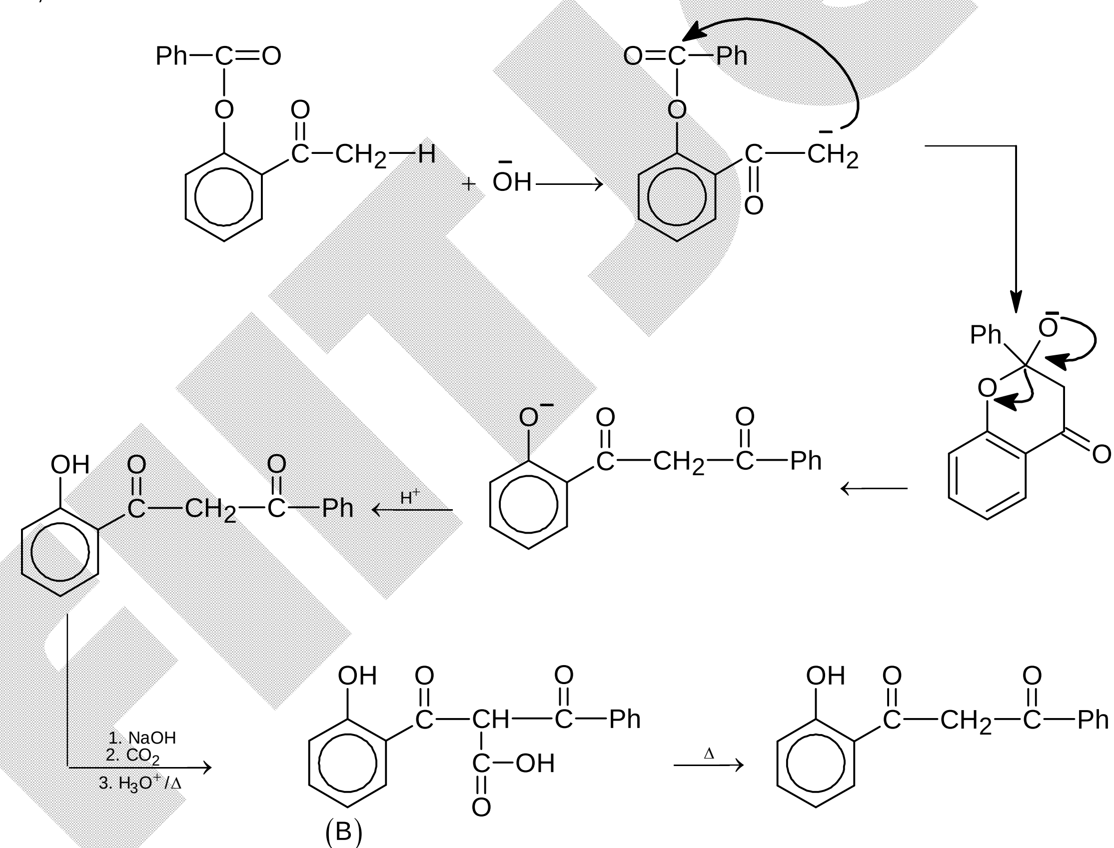
23. A, B, C

Sol.




24. C, D

Sol.



25. B

Sol. $\left[Fe(CN)_6\right]^{4-} \rightarrow$ Low spin, $t_{2g}$ contain 6 electron.

$\left[Cu(NH_3)_4\right]^{2+} \rightarrow$ Square planar complex.

26. B

Sol. $XeO_4 + XeF_6 \longrightarrow XeOF_4 + XeO_3F_2$

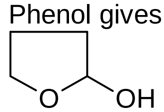
$XeF_6 + H_2O \longrightarrow XeOF_4 + 2HX$

$2XeF_6 + SiO_2 \longrightarrow 2XeOF_4 + SiF_4$

$XeF_6 + POF_3 \longrightarrow XeOF_4 + PF_5$

$XeF_6 + OH^- \longrightarrow XeO_6^{4-} + Xe + O_2 + 2H_2O$

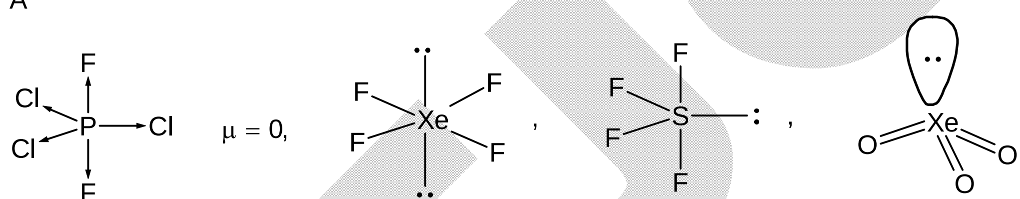
27. A

Sol.
1. Anilines gives carbylamines test and $Br_2/H_2O$ test.
2. Phenol gives Bromine water test and neutral $FeCl_3$ test.
3. 

$\longrightarrow$ Respond Tollen's test and Fehling's test

28. A

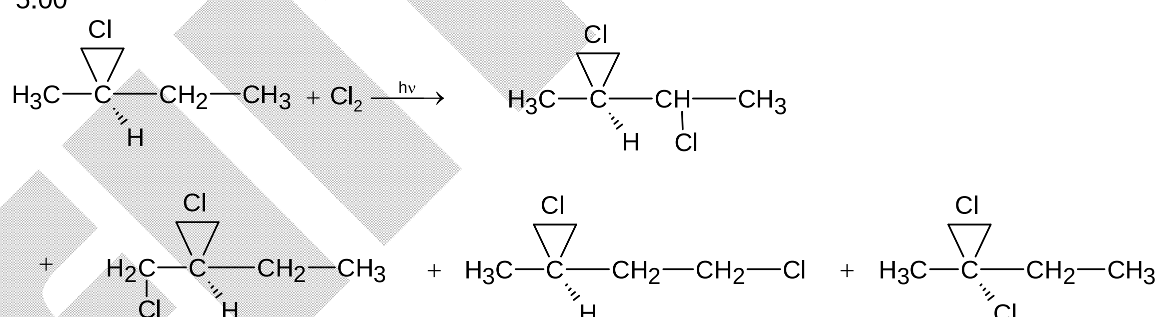
Sol.




**Section – B**

29. 5.00

Sol.




$\therefore$ Total number of dihalogen derivative including stereo isomers = 5.

$\therefore$ After fractional distillation total numbers = 5

$\therefore \dfrac{x+y}{2} = \dfrac{10}{2} = 5$

30. 1.00

Sol. $PO_4^{3-} + H_2O \rightleftharpoons HPO_4^{2-} + OH^-$

$0.4 - x \qquad x \qquad x$

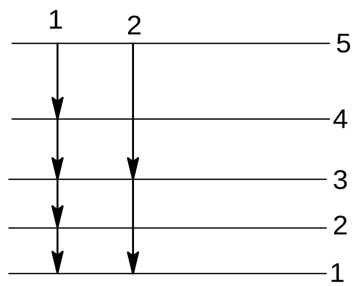
$\left[OH^-\right] = \sqrt{\dfrac{K_W}{K_{a_3}} \times C} = \sqrt{\dfrac{10^{-14}}{4 \times 10^{-13}} \times 0.4}$

$$\left[OH^-\right] = \sqrt{10^{-2}}$$
pOH = 1

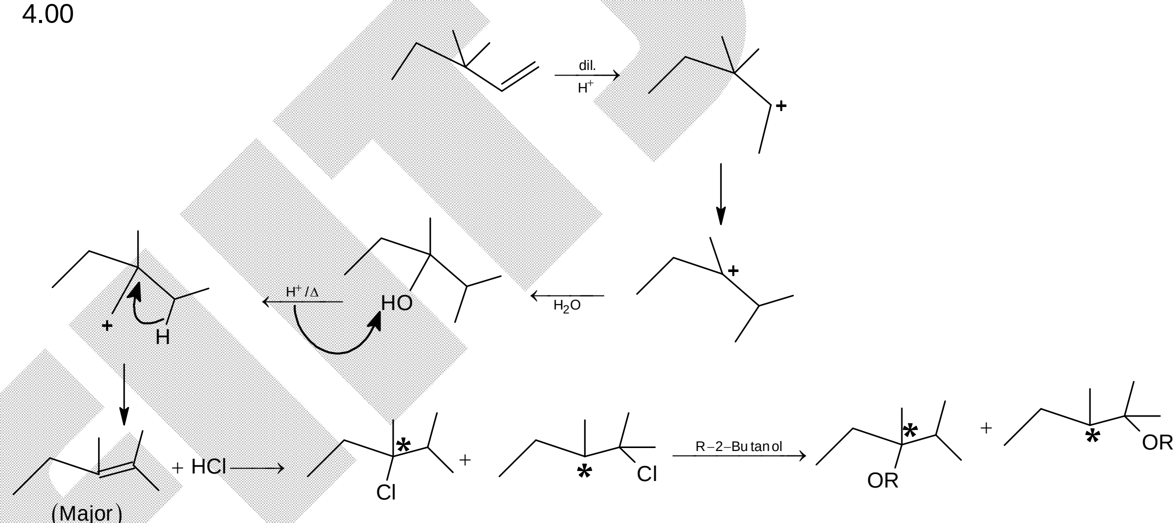
31. 8.00

Sol. $$H_3PO_4 + NaOH \longrightarrow NaH_2PO_4 + H_2O$$
$$\begin{matrix} 10-10 & 15-10 & 10 \end{matrix}$$
$$NaH_2PO_4 + NaOH \longrightarrow Na_2HPO_4 + H_2O$$
$$\begin{matrix} 10-5 & 5-5 & 5 \end{matrix}$$
∴ Resulting solution is buffer,
$$pH = pK_a + \log\frac{S}{A} = 8$$

32. 7.00

Sol.



33. 4.00

Sol.



Total products = 4.

34. 1.00

Sol. $$\left[OH^-\right] = \sqrt{\frac{K_W}{K_{a_2}} \times C} = \sqrt{\frac{10^{-14}}{10^{-7}} \times 0.1}$$
$$= 10^{-4}$$
$$\therefore \left[H^+\right]_{cathode} = 10^{-10}$$

$\therefore E_{cell} = E_{H_2/H^+} + E_{H^+/H_2} = 0.54 \times 2$

$2x = 1.08 \approx 1$

35. 48.00

Sol. $N_2O_5(g) + O_2(g) \xrightarrow{40\%} 2NO_2(g) + O_3(g)$

$NO_2(g) + O_2(g) \xrightarrow{60\%} NO(g) + O_3(g)$

$(n)_{N_2O_5} = 40$ then moles of $O_2$ consumed in 1st reaction = 40

$\therefore (n)_{NO_2} = 40 \times 2 \times \frac{40}{100} = 32$ mole

$(n)_{O_3} = 40 \times \frac{40}{100} = 16$ mole

Now, for 2nd reaction, $NO_2$ is L.R.

$\therefore [n_{O_2}]_{consumed} = 32$

$[(n)_{O_2}]_{left} = 85 - \{40 + 32\} = 13$ mole

$\therefore [(n)_{O_3}]_{formed} = 60 \times \frac{32}{100} = 3.2 \times 6 = 19.2$

$\therefore$ Sum of mole of $O_3$ and $O_2$ after reaction
= 13 + 19.2 + 16 = 48.2

36. 13.00

Sol. $H_3C-\underset{\underset{Br}{|}}{CH}-CH=CH-\underset{\underset{Br}{|}}{CH}-CH_3$

Possible stereo isomers are

| | |
|---|---|
| RCS | RTS |
| SCS | STS |
| RCR | RTR |

$\therefore$ Number of diastereo pair $= {}^6C_2 - 2 = \frac{5 \times 6}{2} - 2 = 13$

# Mathematics

## PART – III

### Section – A

37. B, D

Sol. Let $f(x) = g(x) + \frac{4}{3}x \Rightarrow g(x-3) = g(x)$, g(x) is periodic with period 3.

$\int_0^6 \left(g(x) + \frac{4}{3}x\right)dx = 0$ ; $\int_0^3 g(x)dx = -12$

38. A, B, C

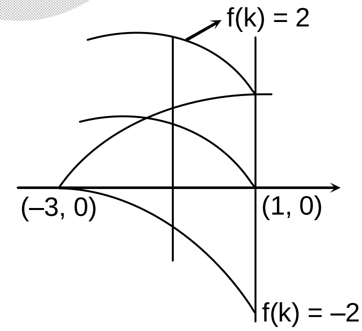
Sol. $\alpha = \frac{1}{2\cos\frac{2\pi}{7}}$ ; $\beta = \frac{1}{2\cos\frac{4\pi}{7}}$ ; $\gamma = \frac{1}{2\cos\frac{6\pi}{7}}$ ;

So, $\cos\frac{2r\pi}{7}$ for r = 1, 2, 3 are roots of $8\cos^3\theta + 4\cos^2\theta - 4\cos\theta - 1 = 0$

$\Rightarrow \alpha, \beta, \gamma$ are roots of $8\left(\frac{1}{2t}\right)^3 + 4\left(\frac{1}{2t}\right)^2 - 4\frac{1}{2t} - 1 = 0 \Rightarrow t^3 + 2t^2 - t - 1 = 0$

39. A, B, C

Sol. Shown in the graph




40. A, B

Sol. $(\hat{r}\times\vec{a})\cdot(\hat{r}\times\vec{b}) = \vec{a}\cdot\vec{b} - (\vec{a}\cdot\hat{r})(\vec{b}\cdot\hat{r}) = \vec{a}\cdot\vec{b}$

$\Rightarrow (\vec{a}\cdot\hat{r})(\vec{b}\cdot\hat{r}) = 0 \Rightarrow \hat{r} = \lambda\left(\hat{a}\times\hat{b}\right) \Rightarrow \hat{r} = 2\left(\hat{a}\times\hat{b}\right)$

41. A, C

Sol. Let $a_n = a + (n-1)d$ and $b_n = br^{n-1}$ subtracting (ii) from (i) $b(r + r^3) = -4 \times 5$

$\Rightarrow$ Either b = 10, r = –1 or b = 2 and r = –2

$\Rightarrow \sum_{n=1}^{5}|b_n| = 50$ or 62 so $\sum_{n=1}^{5}a_n = 17$ and $\sum_{n=1}^{5}|a_n| = 31$ or $\sum_{n=1}^{5}a_n = 5$ and $\sum_{n=1}^{5}|a_n| = 19$

On solving, we get a = 7, d = –3

42. A, B, C

Sol. Let $a_n = f(a_{n-1})$, $a_0 = \sin^2\theta$, $0 \le \theta \le \frac{\pi}{2}$

$a_n = \sin^2 2^n\theta$ so $f_n(x) = f_0(x)$

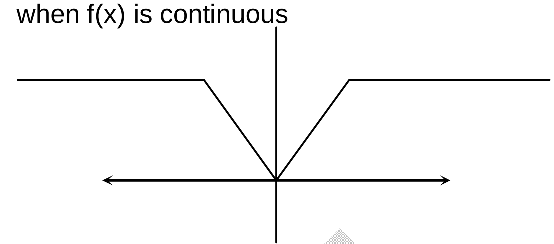
$\Rightarrow \sin^2 2^n\theta = \sin^2\theta$, $\theta = \frac{m\pi}{2^n \pm 1}$

43. B

Sol. (f(x) – 1)(f(x) – x)(f(x) + x) = 0

Similarly, solve graphically

when f(x) is continuous



44. D

Sol. For maximum f(i) = 15 and can be 14 so minimum is –12

If $\sum_{i=1}^{10} f(i) = 55 \Rightarrow \sum_{i=1}^{10} f(i) - i = 0 \Rightarrow$ five modulus opens with (+) sign and remaining with (–) sign

45. B

Sol. Let $\overrightarrow{AB'} = \alpha\hat{i}$, $\overrightarrow{AC'} = \alpha\hat{i} + \alpha\hat{j}$, $\overrightarrow{AD'} = \alpha\lambda\hat{i} + \alpha\hat{j} + \alpha\hat{k}$ for some $\alpha \in (0, 1)$

Volume of tetrahedron AB′C′D′ = $\frac{\alpha^3}{6}$

Area of ΔB′C′D′ = $\frac{\alpha^2}{2}\sqrt{\lambda^2 - 2\lambda + 2}$

$\Rightarrow$ Height of tetrahedron from vertex A = $\frac{\alpha}{\sqrt{\lambda^2 - 2\lambda + 2}}$

46. A

Sol. f(x) = (x + 1)($x^2$ +ax +a) so f(0) = a ; f′(0) = 2a

$g(x) = \frac{x+2}{x^2 + ax + a}$ ; $g'(x) = \frac{-(x^2 + 4x + a)}{(x^2 + ax + a)^2}$

D = $a^2$ – 4a < 0 $\Rightarrow$ a $\in$ (0, 4)

**Section – B**

47. 200.00

Sol. $x = \frac{(4n+1)\pi}{6}$, $x = n\pi - \frac{\pi}{6}$, $x = n\pi \pm \frac{\pi}{6}$, so common solution $x = \frac{(4n+1)\pi}{6}$ and $x = m\pi - \frac{\pi}{6}$

48. 0.25

Sol. Let g(x) = $t^3$ ln (x – t) – $2e^{x-a}$

$g'(x) = \frac{t^3}{x-t} - 2e^{x-a}$ which has only are solution say x = β, then g(β) = g′(β) = 0

$\Rightarrow t^3 \ln(\beta - t) = 2e^{\beta - a}$ and $\frac{t^3}{\beta - t} = 2e^{\beta - a}$, $\ln(\beta - t) = \frac{1}{\beta - t}$

$\Rightarrow$ β – t = k (some constant)

$\frac{t^3}{k} = 2e^{k+t-f(t)}$ ; k + t – f(t) = 3 ln t – ln k – ln 2

f′(t) = $1 - \frac{3}{t}$ ; $f'(4) = \frac{1}{4}$

49. 6.33

Sol. $\int_0^2 f(x)dx + \int_2^3 f(x)dx = \int_0^2 \left(f(x) + \frac{1}{2}f\left(\frac{x}{2}+2\right)\right)dx = \frac{1}{2}\int_0^2 (x^2+2x+3)dx = \frac{19}{3} = 6.33$

50. 0.49

Sol. Probability that Vishi wins $i^{th}$ match is $\frac{1}{2}\left(1-\frac{i}{i+1}\right) = \frac{1}{2(i+1)}$

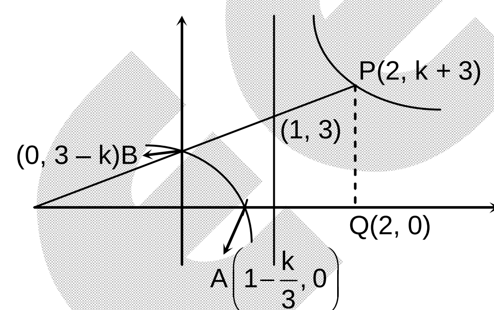
Required probability = $\frac{1}{2}\left(1-\prod_{i=1}^{99}\left(1-\frac{1}{i+1}\right)\right) = \frac{99}{200}$

51. 5.00

Sol. Area of quadrilateral ABPQ is

$\frac{27+6k-k^2}{6} = 6-\frac{(k-3)^2}{6}$

Which lies in (4.5, 6)



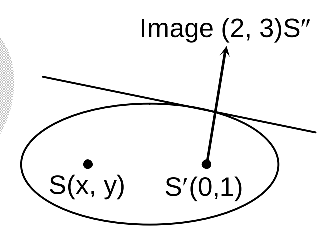

52. 0.22

Sol. Now, SS″ = 2a

SS′ = 2ae

$\frac{SS''}{SS'} = \frac{1}{e}$

$\Rightarrow 3x^2 + 3y^2 + 4x - 2y - 9 = 0$




53. 75.00

Sol. AB is symmetric so AB = $B^TA^T$ by taking $A^T$ = B using $A = \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \\ c_1 & c_2 \end{bmatrix}$ and $B = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{bmatrix}$ on solving we get, $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 3 & 1 \end{bmatrix}$ so $BA = \begin{bmatrix} 14 & 11 \\ 11 & 14 \end{bmatrix}$

54. 2508.00

Sol. In $\Delta A$ we have 45 elements where element r is repeated 10 – r times for r = 1, ..... 9.

So sum of elements of $\Delta^2 A$ is $1\sum_{r=1}^{8} r(r+1) + 2\sum_{r=1}^{7} r(r+2) + ..... 8\sum_{r=1}^{1} r(r+8)$

$= \sum_{k=1}^{8}\sum_{r=1}^{9-k} kr(r+k) = \sum_{k=1}^{8}\frac{(9-k)(10-k)(19+k)k}{6}$

$= \frac{1}{6}\sum_{k=1}^{8}\left(k(k+1)(k+2)(k+3) - 42k(k+1)(k+2) + 330k(k+1)\right) = 2508$