# FIITJEE
# ALL INDIA TEST SERIES
## JEE (Advanced)-2023
## FULL TEST – VIII
## PAPER –2
## TEST DATE: 07-05-2023

**Time Allotted: 3 Hours** **Maximum Marks: 180**

**General Instructions:**

- The test consists of total 54 questions.
- Each subject (PCM) has 18 questions.
- This question paper contains **Three Parts.**
- **Part-I** is Physics, **Part-II** is Chemistry and **Part-III** is Mathematics.
- Each **Part** is further divided into **Two Sections**: **Section-A & Section-B**.

**Section – A *(01 – 04, 19 – 22, 37 – 40)*:** This section contains **TWELVE (12)** questions. Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.

**Section – A *(05 –10, 23 – 28, 41 – 46)*:** This section contains **EIGHTEEN (18)** questions. Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).

**Section – B *(11 – 18, 29 – 36, 47 – 54)*:** This section contains **TWENTY FOUR (24)** numerical based questions. The answer to each question is a **Single Digit Integer, ranging from 0 to 9 both inclusive.**

**MARKING SCHEME**

**Section – A (Single Correct):** Answer to each question will be evaluated according to the following marking scheme:

| | | | |
|---|---|---|---|
| *Full Marks* | : | +3 | If ONLY the correct option is chosen. |
| Zero Marks | : | 0 | If none of the options is chosen (i.e. the question is unanswered); |
| *Negative Marks* | : | –1 | In all other cases. |

**Section – A (One or More than One Correct):** Answer to each question will be evaluated according to the following marking scheme:

| | | | |
|---|---|---|---|
| Full Marks | : | +4 | If only (all) the correct option(s) is (are) chosen; |
| Partial Marks | : | +3 | If all the four options are correct but ONLY three options are chosen; |
| Partial marks | : | +2 | if three or more options are correct but ONLY two options are chosen and both of which are correct; |
| Partial Marks | : | +1 | If two or more options are correct but ONLY one option is chosen and it is a correct option; |
| Zero Marks | : | 0 | If none of the options is chosen (i.e. the question is unanswered); |
| Negative Marks | : | –2 | In all other cases. |

**Section – B:** Answer to each question will be evaluated according to the following marking scheme:

| | | | |
|---|---|---|---|
| Full Marks | : | +3 | If ONLY the correct integer is entered; |
| Zero Marks | : | 0 | Question is unanswered; |
| Negative Marks | : | –1 | In all other cases. |

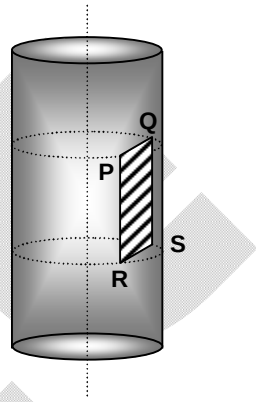
# Physics

## PART – I

### Section – A (Maximum Marks: 12)

*This section contains **FOUR (04)** questions. Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.*

1. A non conducting long cylinder having uniform charge density $\rho$ and radius of cylinder is 5R. If the flux passing through the surface PQRS as shown in the figure is (Here PQ = 8R, QS = R)



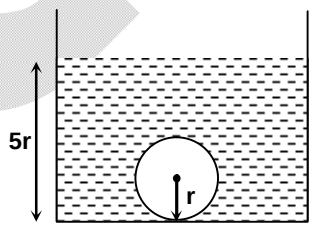

(A) $\frac{4\rho R^3}{\varepsilon_0}$

(B) $\frac{12\rho R^3}{\varepsilon_0}$

(C) $\frac{4\rho R^3}{3\varepsilon_0}$

(D) $\frac{24\rho R^3}{\varepsilon_0}$

2. A spherical ball of radius 'R' is lying at the bottom of stationary container containing liquid of density '$\rho$' as shown in the figure. The force exerted on the upper hemispherical portion of the sphere due to liquid is ($P_0$ = atmospheric pressure)




(A) $\pi r^2(3P_0 + 10\rho gr)$

(B) $\pi r^2(6P_0 + 10\rho gr)$

(C) $\frac{\pi r^2}{3}(3P_0 + 10\rho gr)$

(D) $\frac{\pi r^2}{3}(6P_0 + 10\rho gr)$

3. A metal sphere of radius 'a' is surrounded by a concentric metal sphere of inner radius 'b', where b > a. The space between the spheres is filled with a material whose electrical conducting $\sigma$ varies with electric field as $\sigma = KE$, where K is a constant and E is electric field. A potential difference V is maintained between the two spheres. What is the current between the two spheres?

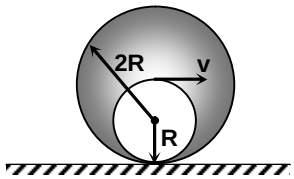
(A) $\frac{2\pi KV^2}{\left\{\ell n\left(\frac{b}{a}\right)\right\}^2}$

(B) $\frac{4\pi KV^2}{\left\{\ell n\left(\frac{b}{a}\right)\right\}^2}$

(C) $\frac{8\pi KV^2}{\left\{\ell n\left(\frac{b}{a}\right)\right\}^2}$

(D) $\frac{\pi KV^2}{\left\{\ell n\left(\frac{b}{a}\right)\right\}^2}$

4. A uniform sphere of radius 2R has a spherical cavity of radius R. Mass of the sphere with cavity is 2M. The sphere is rolling without slipping on a rough horizontal surface. When centre of cavity is at lowest point, centre of sphere has horizontal velocity v as shown in the figure. At this moment kinetic energy of sphere is



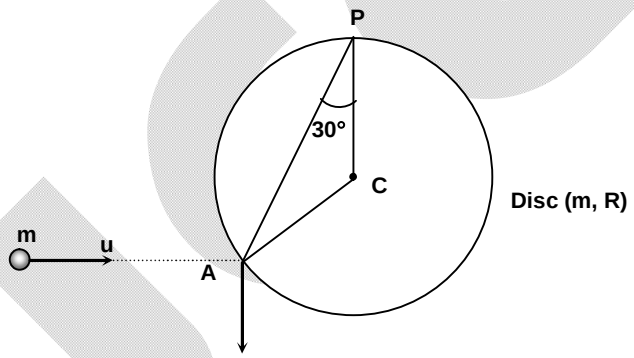

(A) $\frac{31}{40}Mv^2$ (B) $\frac{62}{20}Mv^2$

(C) $\frac{31}{20}Mv^2$ (D) $\frac{31}{10}Mv^2$

**Section – A (Maximum Marks: 24)**

*This section contains **SIX (06)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).*

5. A disc of mass m, radius R is hinged at its periphery and suspended in a vertical plane. A particle of same mass m strikes the disc with an initial velocity u in the horizontal direction. It is given that the particle rebounds in a vertical direction, with velocity v. The angular velocity of disc just after collision is $\omega$. Pick correct option(s):

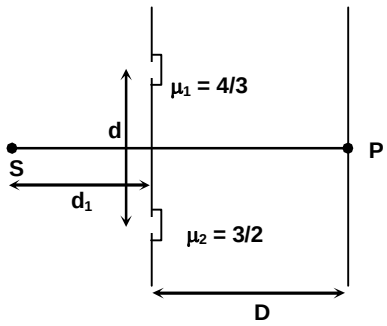



(A) $\omega = \frac{u}{3R}$ (B) $\omega = \frac{2u}{3R}$

(C) $e = \frac{1}{2}$ (D) e = 1

6. The pitch of screw gauge is 1 mm and its circular scale is divided into 100 divisions. When nothing is put between the studs the zero of main scale is not seen but when circular scale is rotated by 450° the zero of the main scale is just visible and the zero of main scale coincides with the zero of the circular scale. When a glass plate is placed between the studs the circular scale lies between $18^{th}$ and $19^{th}$ division of main scale and circular scale reads 34 divisions. Then
(A) There is negative zero error and its magnitude is 1.25 mm
(B) There is positive zero error end its magnitude is 1.25 mm
(C) Thickness of the glass plate is 19.59 mm
(D) Thickness of the glass plate is 17.09 mm

7. In YDSE, the upper slit $S_1$ is covered by a thin glass plate of refractive index $\frac{4}{3}$ and of thickness $9\lambda$, where $\lambda$ is wavelength of light used. The lower slits $S_2$ is also covered by another glass plate of thickness $2\lambda$ and refractive index $\frac{3}{2}$ as shown in the figure. If $I_0$ is intensity at P due to slits $S_1$ and $S_2$ each, then



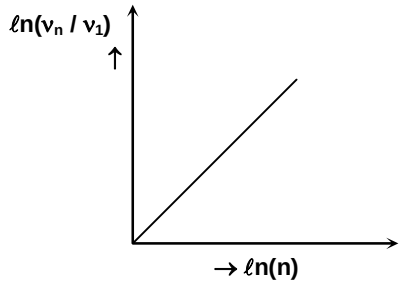

(A) Optical path difference between the waves from $S_1$ and $S_2$ at point P is $\lambda$
(B) Intensity at P is $2I_0$

(C) Intensity at P is $4I_0$

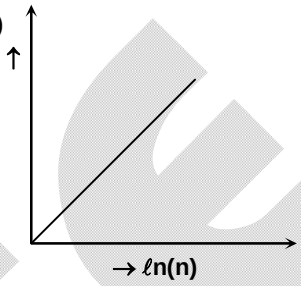
(D) If the source S is shifted upwards by a small distance $d_2$ then the fringe originally at P after inserting the plates, shifts downwards by $D\left(\frac{d_2}{d_1}\right)$

8. If in a hydrogen atom, radius, of nth Bohr orbit is $r_n$, frequency of revolution of electron in nth orbit is $\nu_n$ and area enclosed by the nth orbit is $A_n$, then which of the following graph are correct.

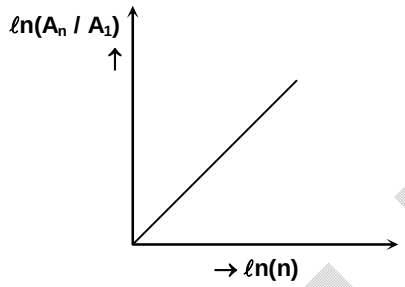
(A)



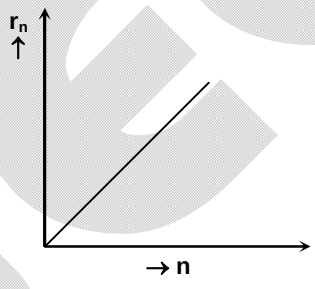

(B)



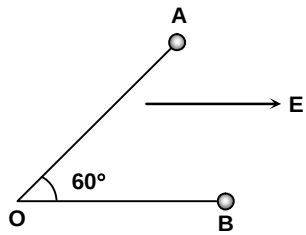

(C)




(D)




9. A particle of mass 'm' and charge 'q' is fastened to other end of a string fixed at point O. The whole system lie on a frictionless horizontal plane. Initially, the mass was at rest at point A. A uniform electric field E in the direction shown is then switched on



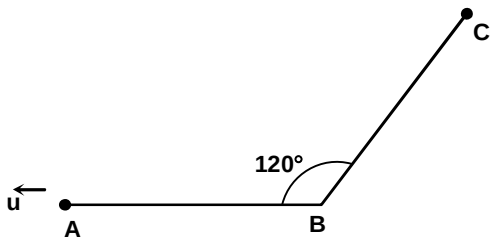

(A) The speed of the particle when it reaches at point B is $\sqrt{\frac{qE\ell}{m}}$

(B) Acceleration of particle at point B is $\frac{q^2E^2\ell}{2m}$

(C) Angular momentum of particle about point O is conserved

(D) Tension in string when particle reaches at point B is 2qE

10. Three particles A, B and C have mass m, 2m and m respectively. They lie on a smooth horizontal table connected by light inextensible string AB and BC. The strings are just taut and $\angle ABC = 120°$. An impulse is applied to particle A along BA so that it acquires a velocity u.




(A) Initial speed of B will be $\frac{\sqrt{3}u}{11}$

(B) Initial speed of C will be $\frac{4u}{11}$

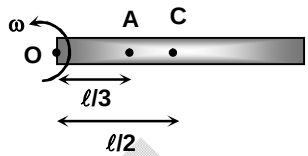
(C) Initial speed of B will be $\frac{2\sqrt{31}u}{11}$

(D) Initial speed of C will be $\frac{\sqrt{3}u}{4}$

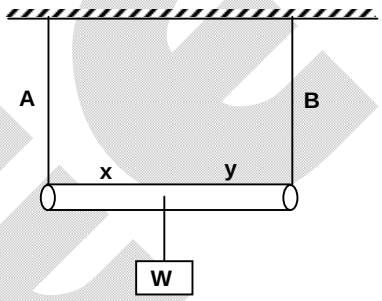
**Section – B (Maximum Marks: 24)**

*This section contains **EIGHT (08)** numerical based questions. The answer to each question is a **Single Digit Integer, ranging from 0 to 9 both inclusive.***

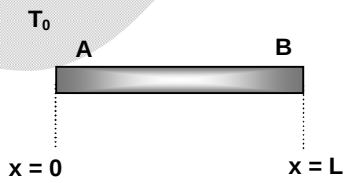
11. A uniform rod of mass m, length $\ell$ rotates about its end point O in horizontal plane. If the rod is rotating with a constant angular speed on a frictionless surface and the ratio of restoring force developed in the rod at points A and C is $\frac{F_A}{F_C}=\left(1+\frac{n}{27}\right)$, where 'n' is an integer value, find n.



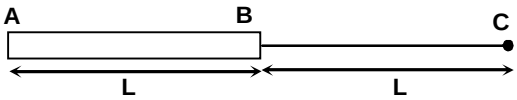

12. Two wires of same material A and B, support a massless rod and a weight w as shown in the figure. The cross-section area of A is $S_A$ and that of B is $S_B$ with $2S_A = S_B$. They have same length $\ell$ and are extended by same length $\Delta\ell$. If elastic potential energy stored in each wire are $U_A$ and $U_B$ then find $\frac{U_B}{U_A}$.



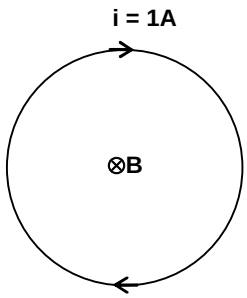

13. Figure shows a metal rod of uniform cross section area A, with variable thermal conductivity given by $k(x)=k_0\sec\left(\frac{\pi}{6L}x\right)$. If the end A is maintained at temperature $T_0$, the rod carries a thermal current $I_0$ (from B to A) in steady state and $\frac{I_0L}{k_0AT_0}=\frac{\pi}{3}$; find the temperature of the end B of the rod. Let's say the temperature is $PT_0$, find integer value P.



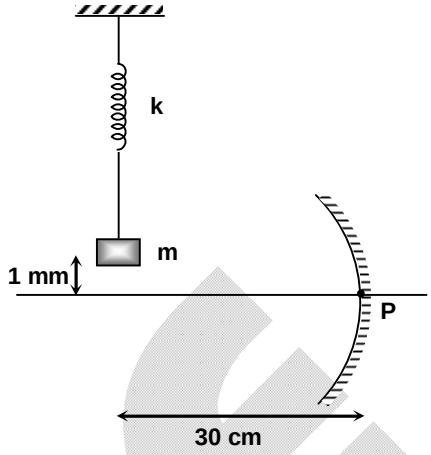

14. A rod AB of mass M and length L is fixed. A point mass is released from point C as shown in the figure. Point mass starts moving towards rod when set free. The speed of the point mass was found to be $v=\sqrt{\frac{xGM}{L}\ell\left(\frac{y}{2}\right)}$, when point mass is at a distance of $\frac{L}{2}$ from end B. Find x + y.




15. A current carrying string is in the shape of ring of radius 1 m. It is placed in a uniform magnetic field perpendicular to plane of the ring. The current in the ring is 1A and area of cross-section of ring is 0.2 $cm^2$. The density of string is $2\times10^3$ $kg/m^3$. The speed of wave in the string (ring) is 10 m/s. Find magnetic field (in T)? (Neglect any other interaction)

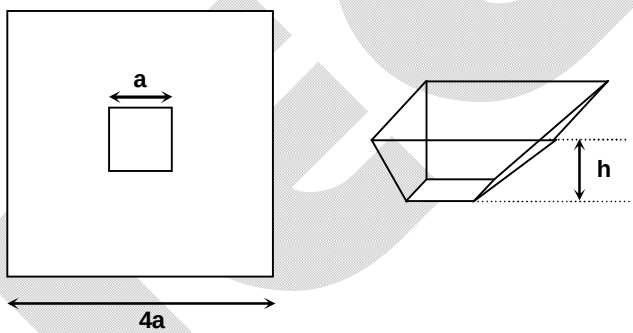


16. Figure shows a small block suspended vertically using an ideal spring. Initially spring is in natural length and the mass is 1 mm above the principal axis of a concave mirror whose focal length is 20 cm. given that mass m = 10 gm, k = 100 N/m and g = 10 m/s$^2$, the amplitude of the image of the block formed by concave mirror is A millimeters; find the integer value 'A'.

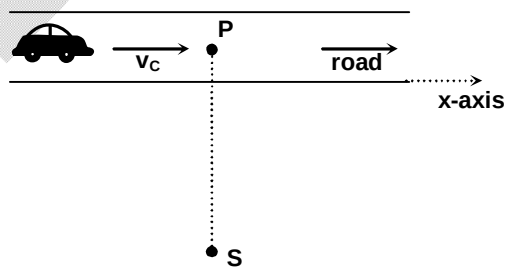



17. Figure shows a soap film formed between two square figures made of a uniform wire. The bigger square is held keeping it in a horizontal plane and the smaller square is slowly allowed to drop vertically. It reaches an equilibrium state after dropping a height h. Let surface tension of soap = T, Mass per unit length of wire = $\lambda$ and acceleration due to gravity = g

Given that $h = \dfrac{n\lambda ga}{2\sqrt{4T^2 - \lambda^2 g^2}}$; find the integer value n.




18. A car is moving with velocity $v_C$ along positive x-axis on the road as shown in the figure. A shooter S is at some distance from the road. He has a detector which can detect signal only of frequency 1500 Hz. The car blows horn of frequency 1000 Hz. When detector detects a signal, shooter immediately shoots towards the road along PS and bullet hits the car. If velocity of sound in medium is 360 m/s and velocity of car $v_C = 80\sqrt{3}$ m/s, then velocity of bullet (in m/s) is 36 N. Find N.



# Chemistry

## PART – II

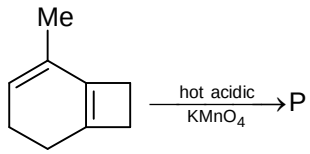
### Section – A (Maximum Marks: 12)

*This section contains **FOUR (04)** questions. Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.*

19. 5 mol $PCl_5$ (g) and 1 mol $N_2$ gas is placed in a closed vessel. At equilibrium 20% of $PCl_5$(g) decomposes and total pressure in the container is found to be 1 atm. The $K_P$ for the reaction $PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$ is

(A) $\frac{1}{24}$ atm (B) $\frac{1}{4}$ atm

(C) $\frac{1}{16}$ atm (D) $\frac{1}{28}$ atm

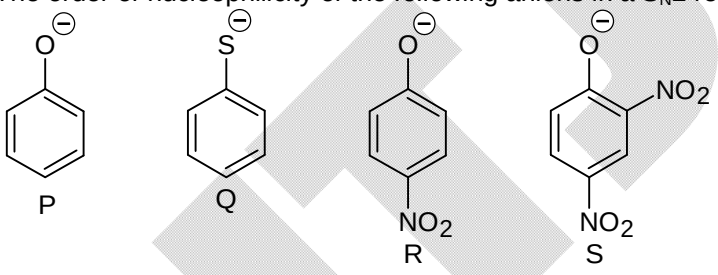
20. 


IUPAC name of 'P' would be

(A) 3, 6, 7-trioxo octanoic acid (B) 3, 6, 7-trioxo octanal

(C) 4, 7, 8-trioxo nonanoic acid (D) 2, 3, 6-trioxo nonanoic acid

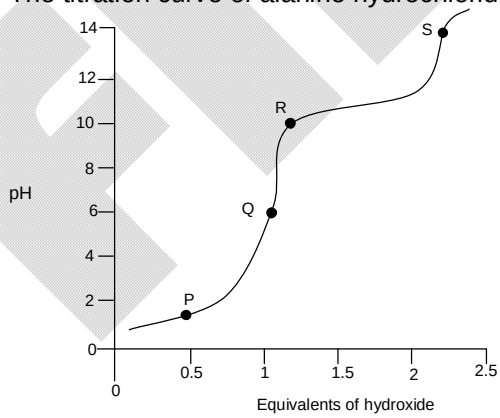
21. The order of nucleophilicity of the following anions in a $S_N2$ reaction in a protic solvent is




(A) Q > R > S > P (B) Q > P > R > S

(C) Q > R > P > S (D) P > S > R > Q

22. The titration curve of alanine hydrochloride is given below




The position in the graph that corresponds to the isoelectric point of alanine is

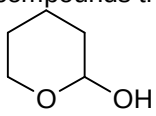
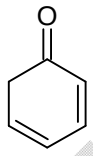
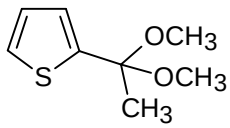
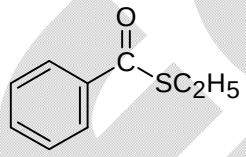
(A) P (B) Q

(C) R (D) S

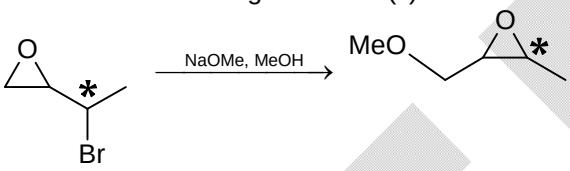
## Section – A (Maximum Marks: 24)

*This section contains **SIX (06)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).*

23. Which of the following substance/s can not enhance acidic strength of 'Boric acid' significantly on addition to boric acid?
(A) Catechol
(B) trans-1, 2-cyclo pentane diol
(C) D-Mannitol
(D) 1, 3-propanediol

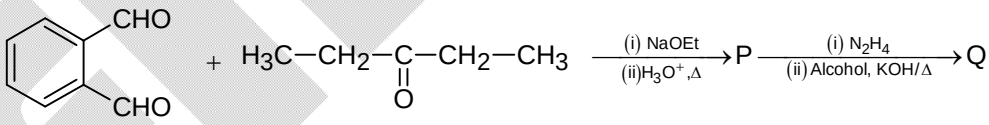
24. The compounds that form phenyl hydrazone derivatives under acidic condition is/are
(A) 
(B) 
(C) 
(D) 

25. Which of the following statement(s) is/are not true about the reaction given below?




(A) It involves a carbocation intermediate
(B) Rearrangement is due to $S_N1$ reaction mechanism
(C) It proceeds via a concerted $S_N2$ pathway
(D) It involves neighbouring group participation

26. Which of the following relationships is/are correct between the van der Waals constant of $N_2$ and $O_2$
(A) $b_{(N_2)} = b_{(O_2)} \neq 0$
(B) $b_{N_2} > b_{O_2}$
(C) $a_{N_2} > a_{O_2}$
(D) $a_{O_2} > a_{N_2}$

27. 


Which of the following is/are correct for Q?
(A) It gives positive test with $Br_2$ / $H_2O$ .
(B) It gives positive test with 2-4-DNPH.
(C) Degree of unsaturation of the compound is 6.
(D) On oxidative ozonolysis it produces a dicarbonyl compound as one the product.

28. Number of correct statement/s is/are

$\left(K_{f\ H_2O} = 1.86,\ \text{radius of first Bohr orbit} = 0.5\,\overset{o}{A}\right)$

(A) It 0.01 mole of HCl and 0.02 mole of sucrose are dissolved in 200 gm water, then depression in freezing point of water in $^oC$ will be $0.372^o$.

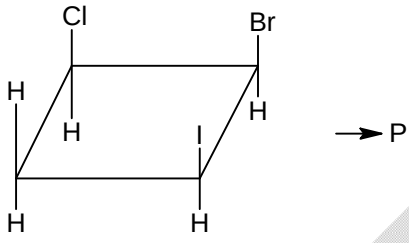
(B) A straight line having a slope of $\frac{-\Delta H^o}{R}$ is obtained in a plot between $\ell nK_p$ versus T.

(C) For a first order reaction $A(g) \longrightarrow 2B(g) + C(g)$; the rate constant in terms of initial pressure $P_o$ and pressure at time t (Pt) is given by $\frac{1}{t}\ell n\frac{2P_o}{3P_o - P_t}$.

(D) The electron in a H atom is in its $4^{th}$ orbit having de-Broglie wavelength of $12.56\,\overset{o}{A}$.

**Section – B (Maximum Marks: 24)**

*This section contains **EIGHT (08)** numerical based questions. The answer to each question is a **Single Digit Integer, ranging from 0 to 9 both inclusive.***

29. 

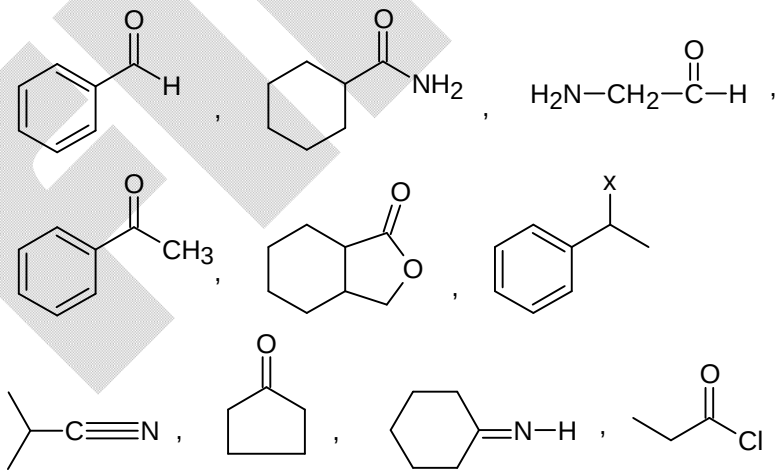
$\longrightarrow$ P

If 'P' has x pairs of diastereomers and y pairs of enantiomers then value of $\frac{x}{y}$ would be

30. Total number of species among the following in which at least one atom has same hybridisation as the N-atom in azide $\left(N_3^-\right)$ ion.

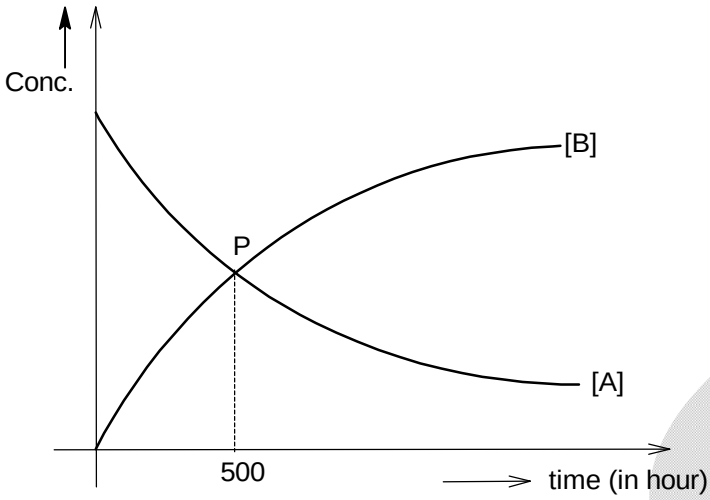
$I_3^-$, $CO_2$, $XeF_2$, $CH_3CN$, $NO_2^+$, $N_2O$, $SO_2$, $C_3O_2$

31. How many of the following compounds can be reduced by $NaBH_4$ ?

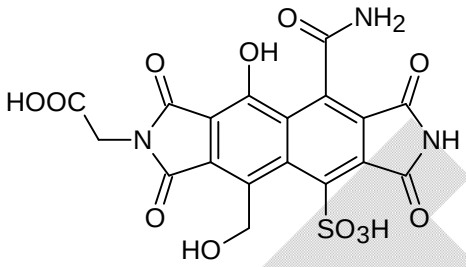


32. The total number of optically active isomers of dichloridobis(glycinato) cobaltate (III) ion is

33. Concentration vs. time graph is given below for first order reaction $7A \longrightarrow B$




If half-life for A in days is 'x' then 0.4x will be

34. 


If the amount in gms of NaOH required for complete neutralization of 1 mol of the above compound is x, then $\frac{x}{40}$ would be

35. If number of moles of ions produced by complete dissociation of one mole of Mohr's salt in water is x and the number of moles of ions produced by the complex (perfect) formed by the addition of $CuSO_4$ in excess KCN in water is y; then value of x + y is

36. If the lattice energy of LiF and MgO calculated from Born-Lande equation is – x kJ/mol and – y kJ/mol respectively then what would be the value of $\frac{y}{x}$? [Assume that for both LiF and MgO, the Modelung constants, interionic distances and Born exponents have the same value]

# Mathematics

## PART – III

### Section – A (Maximum Marks: 12)

*This section contains **FOUR (04)** questions. Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.*

37. The sum of the series

$$\sqrt{4+\frac{1}{1^2}+\frac{1}{3^2}+2}+\sqrt{4+\frac{1}{2^2}+\frac{1}{4^2}+\frac{3}{4}}+\sqrt{4+\frac{1}{3^2}+\frac{1}{5^2}+\frac{2}{5}}+\ldots..\ \sqrt{4+\frac{1}{(2021)^2}+\frac{1}{(2023)^2}+\frac{6}{2021\times 2023}}$$

(A) $8084-\frac{1}{2022}-\frac{1}{2023}$ (B) $\frac{8087}{2}-\frac{1}{2021}-\frac{1}{2023}$

(C) $\frac{8087}{2}-\frac{1}{2022}-\frac{1}{2023}$ (D) $\frac{8085}{2}-\frac{1}{2022}-\frac{1}{2023}$

38. Let $I_1=\frac{\pi^2}{16}+\sqrt{2}$, $I_2=\left(\cot^{-1}\left(\frac{1}{\pi}\right)\right)^2+\frac{2\pi}{\sqrt{\pi^2+1}}$, $I_3=\left(\cot^{-1}\pi\right)^2+\frac{2}{\sqrt{\pi^2+1}}$, then which of the following is TRUE?

(A) $I_1 < I_2 < I_3$ (B) $I_1 > I_2 > I_3$

(C) $I_2 > I_3 > I_1$ (D) $I_2 > I_1 > I_3$

39. Consider three sets

$$A\equiv\left\{x: 2\cos 2x+2\left(\sqrt{3}+1\right)\cos x+2+\sqrt{3}=0\right\}$$

$$B\equiv\left\{x: 2\left(\sin 3x+\sin x\right)+\sqrt{3}=0\right\}$$

$$C\equiv\left\{x: \sqrt{2}-1<\tan x<\sqrt{2}+1\right\}$$

If the sum of elements in $A\cap B\cap C$ is equal to $5k\pi$; $x\in\left[0,\frac{15\pi}{2}\right]$, then the value of k is

(A) $\frac{52}{15}$ (B) $\frac{52}{25}$

(C) $\frac{52}{5}$ (D) 3

40. Let y(x) be a solution of the differential equation $y^2\frac{dx}{dy}+x^2e^{\frac{y(x-y)}{x}}=2x\left(y-x\right)$. If y(1) = 0 and $y(x_0)=1$, then the value of $\frac{1}{x_0}-\log_e\left(2e-1\right)$ is equal to

(A) 0 (B) –1

(C) 1 (D) $\frac{1}{2}$

**Section – A (Maximum Marks: 24)**

*This section contains **SIX (06)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).*

41. Let $A=\begin{bmatrix}1 & 3 & 0\\ 0 & 3 & 0\\ 0 & -2 & 3\end{bmatrix}$ be a given matrix. If $B = A^4 - 4A$ and $C = 15A^2 - 7A^3 - A$, then (Tr (M) and Det (M) represents trace and determinant of matrix M respectively)

(A) Tr(B + C) = 28

(B) Tr(B + C) = 14

(C) Det(Adj(B + C)) = 576

(D) $\sqrt{\text{Det}\left(\text{Adj}\left(\text{Adj}(B+C)\right)\right)}=(576)^2$

42. The two adjacent faces of a cuboid are represented by planes $P_1 : 2x - 3\sin^2\theta.y + z = -10$ and $P_2 : 3x + 4y + 12\cos^2\theta.z = -10$. If there is a line L which is perpendicular to the line of intersection of planes $P_1$ and $P_2$ and whose one point $A\left(x_1, y_1, \frac{10}{3}\right)$ lies on $P_1$ and two points $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$ lies on plane $P_2$. If equation of L is $\frac{x-x_1}{l}=\frac{y-y_1}{-m}=\frac{z-\frac{10}{3}}{n}$; $(l, m, n \in N)$, then

(A) $(l + m + n)_{min} = 21$

(B) $3x_1 + y_1 = -20$

(C) $\left|\frac{x_3-x_2}{z_3-z_2}\right|=2$

(D) line of intersection of $P_1$ and $P_2$ is $\frac{x+\frac{250}{59}}{43}=\frac{y-\frac{40}{59}}{12}=\frac{z-0}{-59}$

43. Consider a function defined as

$$f(x)=\sin\left(\frac{\pi}{2}\left(|x|-\{x\}\right)\right);-1\le x<1,$$

$$\{x\}\sqrt{8(x-2)^2+2+2|4x^2-16x+15|};1\le x\le 3$$

(where {x} represents fractional part of function)

(A) number of points where f(x) is either discontinuous or non differentiable in (–1, 3) are 5

(B) range of f(x) is (–1, 4)

(C) $\int_{\frac{3}{2}}^{\frac{5}{2}} f(x)\,dx = 1$

(D) f(x) has no local extremum in (–1, 3)

44. Let $\alpha$ and f($\alpha$) be the eccentricity of the ellipse $\frac{x^2}{3b^2-4a^2}+\frac{y^2}{2\left(b^2-a^2\right)}=1$ and $\frac{x^2}{2\left(b^2-a^2\right)}+\frac{y^2}{b^2}=1;\ \left(|b|>\sqrt{2}|a|\right)$ respectively, then

(A) $f(\alpha)=\frac{\alpha}{2\sqrt{1-\alpha^2}}$ (B) $f(\alpha)=\frac{\alpha}{\sqrt{1-\alpha^2}}$

(C) $\int_0^{\frac{1}{\sqrt{2023}}} \underbrace{fff.....f(\alpha)}_{2023 \text{ times}} d\alpha=\frac{1}{2023}$ (D) $\int_0^{\frac{1}{\sqrt{2}}} e^{\alpha}\left[f(\alpha)+f'''(\alpha)\right]d\alpha=9e^{\frac{1}{\sqrt{2}}}+1$

45. Let a be a natural number such that $\lim_{x\to1}\left(\frac{1}{x-1}-\frac{1}{x^a-4x+3}\right)=b$ $(b\neq0)$, then

(A) $\int_a^b \frac{\left[x^2\right]}{\left[x^2\right]+\left[x^2-30x+225\right]}=\frac{5}{2}$ (where [.] represents greatest integer function)

(B) least value of n for which $(n-2)x^2-8x+n+4>k\ \forall\ x\in R$ where $k=\tan^{-1}\tan(a+b)+\sin^{-1}\sin(a+b)$ and $n\in N$ is 5

(C) the number of points of discontinuity of f(x) = [a sin x], $x\in[\pi, 2\pi]$ is equal to b (where [.] represents greatest integer function)

(D) the value of c of the chord ax + by = c, which subtends right angle at the centre of the conic $2x^2+3y^2=1$ is $\pm5$

46. Consider a rhombus OABC; O is origin having its vertices $A(z_1)$ and $C(z_3)$ lying on the circle |z| = 2 and vertex $B(z_2)$ lies on the circle $|z|=\frac{14\sqrt{2}}{5}$. Also, $\arg(z_2-z_1)=\alpha$ ; $\alpha=\tan^{-1}\frac{4}{3}$ ($z_1$, $z_2$, $z_3$ represent complex numbers lying in the 1st quadrant), then

(A) $\arg z_1=\tan^{-1}\frac{3}{4}$ (B) $\arg z_1=\tan^{-1}\frac{4}{3}$

(C) $\arg z_2=\tan^{-1}\frac{4}{3}$ (D) $\arg z_1+\arg z_3=\frac{\pi}{2}$

## Section – B (Maximum Marks: 24)

*This section contains **EIGHT (08)** numerical based questions. The answer to each question is a **Single Digit Integer, ranging from 0 to 9 both inclusive.***

47. Five cards are drawn from a pack of 52 cards. If the probability of getting exactly 3 Diamonds and 2 Jack is P, then the value of $\frac{866320}{649}P$ is

48. Let $I=\int_0^1 \frac{\left(x^9-x^5+x\right)}{\left(3x^8-4x^4+6\right)^{\frac{3}{4}}}dx$, then the value of $(12\ I)^4$ is

49. The square of the product of reciprocal of roots of the equation $2\cot^{-1}(2x-1)+\sin^{-1}x=\pi$ is

50. If $\sum_{r=1}^{2023} \frac{(-1)^{r-1} r}{{}^{2024}C_r}$ is equal to k, then the value of $\frac{3039}{506}k$ is

51. Let PQ be the normal chord of maximum length of the curve $y^2 = 4ax$, $a > 0$ where $P \equiv \left(at^2, 2at\right)$; $1 \le t \le 3$. If the angle between tangents drawn at P and Q is $\tan\theta$, then $|3\tan\theta|$ is

52. Let $\vec{a}$ and $\vec{b}$ be two non collinear vectors such that $|\vec{a}| = |\vec{b}| = 2$ and $|\vec{a} \times \vec{b}| = |\vec{a} - 2\vec{b}|$. If $\vec{r}$ is any vector such that $|\vec{r}| = \frac{4}{\sqrt{3}}$ and $\vec{r} \times (\vec{a} \times \vec{b}) = 3\vec{a} - 2\vec{b}$, then the value of $\left|\left[\vec{r}\ \ \vec{a}\ \ \vec{b}\right]\right|$ is

53. Let A be the set of $2 \times 2$ matrices given by $A = \left[a_{ij}\right]$; where $a_{ij} \in \{0, 1, 2, 3, 4, 5, 6\}$ such that $a_{11} + a_{12} + a_{21} + a_{22} = 6$. If the number of matrices in A which are invertible is equal to k, then $\left[\sqrt{k}\right]$ is
(where [.] represents greatest integer function)

54. Let S be the region in argand plane that consists of all points z such that real and imaginary parts of $\frac{z}{k}$ and $\frac{k}{\bar{z}}$ lie between 0 and 1, $k > 0$. If area (A) of region S is $250(6 - \pi)$, then $\left[\sqrt{k}\right]$ is
(where [.] represents greatest integer function)