FIITJEE
ALL INDIA TEST SERIES
JEE (Advanced)-2023
FULL TEST – VIII
PAPER –2
TEST DATE: 07-05-2023

ANSWERS, HINTS & SOLUTIONS

Physics

PART – I

Section – A

1. B

Sol. Flux enclosed is

$$\phi = \frac{q_{in}}{\varepsilon_0} = \frac{\rho\left(\frac{1}{2}\cdot 8R \cdot 3R\right)R}{\varepsilon_0}$$

$$\phi = \frac{12\rho R^3}{\varepsilon_0}$$

2. C

Sol. Force weight of liquid above the sphere

$$F = P_0\pi r^2 + (\pi r^2 \cdot 3r + \pi r^3 - \frac{2}{3}\pi r^3)\rho g$$

$$= P_0\pi r^2 + \frac{10}{3}\pi r^3\rho g = \frac{\pi r^2}{3}(3P_0 + 10\rho g r)$$

3. B

Sol. Since current is

$$I = \vec{J}\cdot\vec{S} = JS = \sigma ES = KE^2 \cdot 4\pi r^2$$

$$\therefore\ E = \frac{1}{r}\sqrt{\frac{I}{4\pi K}}$$

Also, $V = -\int_b^a \vec{E}\cdot d\vec{r} = \sqrt{\frac{I}{4\pi K}}\,\ell n\left(\frac{b}{a}\right)$

$$\therefore\ I = \frac{4\pi K V^2}{\left\{\ell n\left(\frac{b}{a}\right)\right\}^2}$$

4. C

Sol. Let density be ρ, where $\rho = \dfrac{2M}{\frac{4}{3}\pi\left[(2R)^3 - (R)^3\right]}$

$$\rho = \frac{2M}{\frac{4}{3}\pi(7R^3)}$$

Moment of inertia of remaining part (about point of contact)

$I = I_T - I_{cavity}$

$$I = \frac{7}{5}M_T(2R)^2 - \frac{7}{5}M_C R^2$$

$$M_T = \frac{16M}{7} \text{ and } M_C = \frac{2M}{7}$$

$$I = \frac{62}{5}MR^2$$

$$\text{K.E.} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{62}{5}\right)MR^2\left(\frac{V}{2R}\right)^2$$

$$\text{K.E.} = \frac{31}{20}Mv^2$$

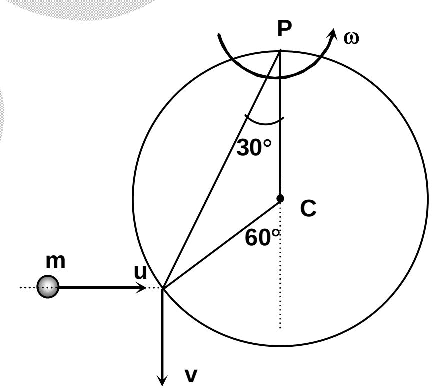
5. B, D

Sol. usin 30° = v sin60°

$$\Rightarrow v = \frac{u}{\sqrt{3}} \quad \ldots(i)$$

$$mu \times \frac{3R}{2} = mv \times \frac{\sqrt{3}R}{2} + \frac{3}{2}mR^2\omega$$

$$\Rightarrow \omega = \frac{2u}{3R} \quad \ldots(ii)$$



Coefficient of restitution,

$$e = \frac{\frac{v}{2} + \frac{\sqrt{3}}{2}\omega R}{\frac{\sqrt{3}}{2}u} = \frac{v + \sqrt{3}\omega R}{\sqrt{3}u}$$

Or e = 1

6. A, C

Sol. zero error = − 1.25 mm

Reading = 18 + 0.34 + 1.25 = 19.59 mm

7. C, D

Sol. At P, path difference

$$\Delta x = \left(\frac{4}{3} - 1\right)9\lambda - \left(\frac{3}{2} - 1\right)2\lambda = 2\lambda$$

$I_P = 4I_0$

8. B, C, D

Sol. $\ell n\left(\frac{\nu_n}{\nu_1}\right) = -3\ell nn$

$\ell n\left(\frac{r_n}{r_1}\right) = 2\ell nn$

$\ell_h\left(\frac{A_n}{A_1}\right) = 4\ell nn$

$r_h = r_0\left(\frac{n^2}{z}\right)$

9. A, D

Sol. $\frac{qE\ell}{2} = \frac{1}{2}mv^2$

$\therefore\ v = \sqrt{\frac{qE\ell}{m}}$

$a = \frac{v^2}{\ell} = \frac{qE}{m}$

$T - qE = \frac{m}{\ell}\left(\frac{qE\ell}{m}\right)$

$\therefore\ T = 2qE$

10. B, C

Sol. For A,

$J - J_1 = mu$ …(i)

For C,

$J_2 = mv$ …(ii)

Component of velocity of B in direction BA must be u.

Let its vertical component perpendicular to BA be v_1

Velocity component of B along CB is

$u \sin 30° - v_1 \cos 30° = v$ …(ii)

For B

$J_1 - J_2 \cos 60° = 2mu$

$2J_1 - J_2 = 4mu$ …(iv)

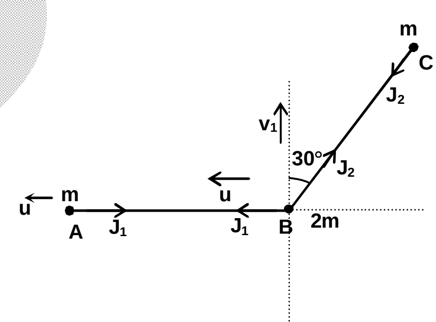
and $J_2 \cos 30° = 2mv_1$

$\frac{\sqrt{3}}{2}J_2 = 2mv_1$ …(v)

Solving (ii), (iii) and (iv) and (v)

$v = \frac{4u}{11}$, $v_1 = \frac{\sqrt{3}u}{11}$ and $v_B = \sqrt{u^2 + v_1^2}$

$v_B = \frac{4\sqrt{31}\,u}{11}$



Section – B

11. 5

Sol. $F_A = \frac{2m}{3}\omega^2 \times \frac{2\ell}{3}$, $F_C = \frac{m}{2}\omega^2 \times \frac{3\ell}{4}$

$\frac{F_A}{F_C} = \frac{32}{27} \Rightarrow n = 5$

12. 2

Sol. Elastic potential energy,

$U = \frac{1}{2}\left(\frac{YA}{\ell}\right)(\Delta\ell)^2$

Here y, ℓ, $\Delta\ell$ are same for both the wire

$\Rightarrow \frac{U_B}{U_A} = \frac{S_B}{S_A} = 2$

13. 2

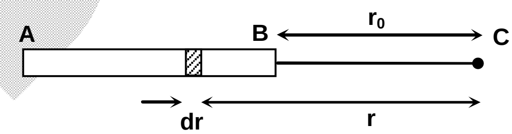
Sol. $I_0 = \frac{k_0 A}{\cos\left(\frac{\pi}{6L}x\right)}\frac{dT}{dx}$

$\Rightarrow T = T_0 \frac{6I_0L}{\pi k_0 A}\sin\left(\frac{\pi}{6L}x\right)$

$\Rightarrow T = 2T_0$ at $x = L$

14. 5

Sol. Potential due to rod at C



$V = -\frac{GM}{L}\int_{r_0}^{r_0+L}\frac{dx}{x}$

$v = -\frac{GM}{L}\ell n\left(1+\frac{L}{r_0}\right)$, where r_0 changes from L to $\frac{L}{2}$ then kinetic energy gained by m is

$\frac{1}{2}mu^2 = \frac{mGM}{L}\ell n\left(\frac{3}{2}\right)$

$v = \sqrt{\frac{2GM}{L}\ell n\left(\frac{3}{2}\right)}$

$\therefore$ x + y = 2 + 3 = 5

15. 4

Sol. $2T\sin\theta = BidR$

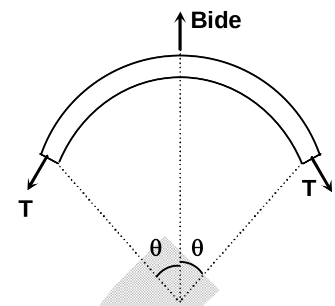
$2T\theta = Bi(R \times 2\theta)$

$T = BiR$

$v = \sqrt{\frac{T}{\rho S}} = \sqrt{\frac{BiR}{\rho S}}$

$B = \frac{V^2\rho S}{iR} = \frac{10\times10\times2\times10^3\times0.2\times10^{-4}}{1\times1}$

$B = 4T$



16. 2

Sol. Amplitude of block = $\frac{mg}{k} = 1\,mm$

$\frac{1}{v} + \frac{1}{(-30)} = \frac{-1}{20} \Rightarrow v = -60$ cm

Magnification, $m = \frac{-v}{u} = -2$

Thus, amplitude of image = 2 mm

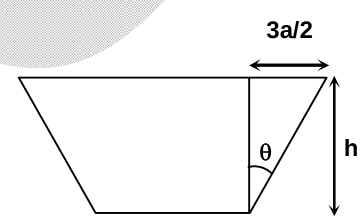
17. 3

Sol. $\tan\theta = \frac{3a}{2h}$

$2 \times 4aT \times \cos\theta = 4a\lambda g$

$\Rightarrow \cos\theta = \frac{\lambda g}{2T}$

$\Rightarrow h = \frac{3\lambda ga}{2\sqrt{4T^2 - \lambda^2 g^2}}$



18. 4

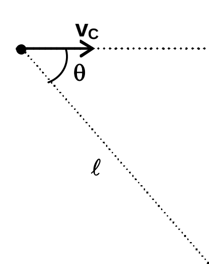
Sol. $\frac{v}{(v - v_C\cos\theta)}(1000) = 1500$

$\Rightarrow \theta = 30$

Also, $\frac{\ell}{v} + \frac{\ell\sin\theta}{v_{bullet}} = \frac{\ell\cos\theta}{v_C}$

$\Rightarrow \frac{1}{2v_{bullet}} = \frac{1}{v}\left(\frac{\sqrt{3}v}{2v_C} - 1\right) = \frac{5}{4v}$

$\Rightarrow v_{bullet} = \frac{2v}{5} = \frac{2}{5}(360) = 4(36)$ m/s



Chemistry

PART – II

Section – A

19. D

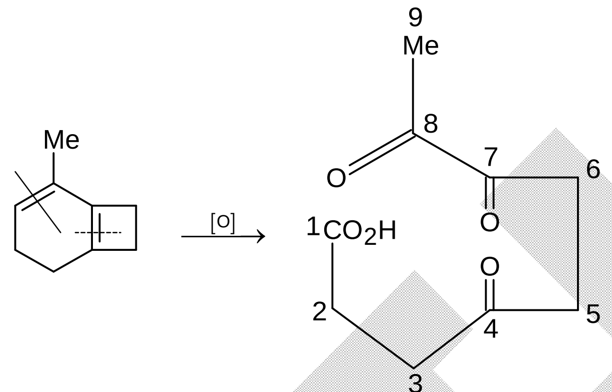
Sol. $K_P = \dfrac{n_{PCl_3} \times n_{Cl_2}}{n_{PCl_5}}\left(\dfrac{P}{\sum n}\right)^{\Delta n}$

$= \dfrac{1\times 1}{4}\times\left(\dfrac{1}{7}\right)^{1}$

$= \dfrac{1}{28}$

20. C

Sol.



4, 7, 8-trioxo nonanoic acid

21. B

Sol. In polar protic solvent down the group nucleophilicity increases and when the attacking atom is same generally stronger the base stronger the nucleophile provided there is no unfavourable steric hindrance.

22. B

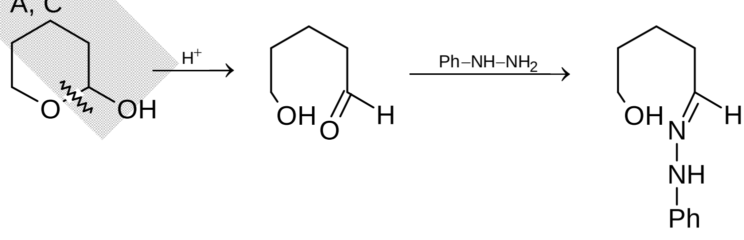
Sol. Alanine is a neutral amino acid. The isoelectric points of neutral amino acids are in the pH range 5.5 to 6.3

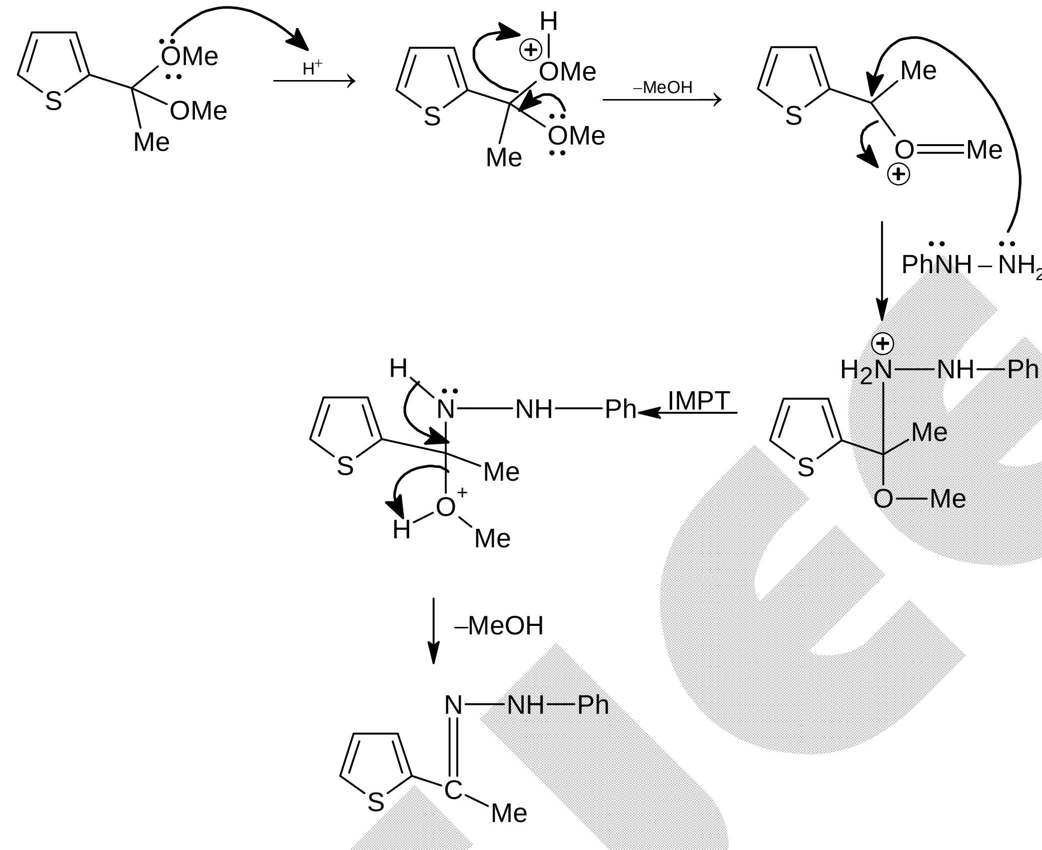
23. B, D

Sol. If the diol/polyol involves the OH group oriented in such a way that they accurately match the structural parameters required by a tetrahedrally coordinated boron, a strong complex is formed.

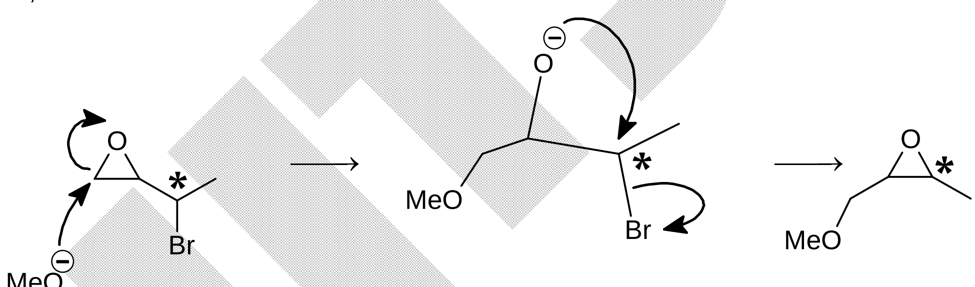
24. A, C

Sol.





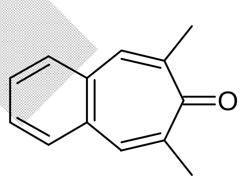
25. C, D
Sol.

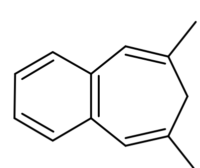


26. B, D
Sol. $b_{N_2} > b_{O_2}$

Since volume of N_2 molecule is greater than O_2 molecule. Both are non-polar molecule, in that case a is proportional to molecular mass.

27. A, D
Sol.

P → 

Q → 

28. A, C, D

Sol. (A) $\Delta T_f = K_f \times m$

$$= 1.86 \times \frac{0.04}{200} \times 1000$$

$= 0.372$

(B) $\Delta G^- = -RT \ln K_{eq}$

$\Delta G^- = \Delta H^- - T\Delta S^-$

$$\ln K_{eq} = \frac{\Delta H^-}{RT} + \frac{\Delta S^o}{R}$$

(C)

$$A(g) \longrightarrow 2B(g) + C(g)$$

$t = 0$	a		
$t = t_{eq}$	$a - x$	$2x$	x

$P_o \propto a$

$P_t \propto a + 2x$

$$x \propto \frac{P_t - P_o}{2}$$

$$a - x \propto P_o - \left(\frac{\{P_t - P_o}{2}\right)$$

$$\propto \frac{3P_o - P_t}{2}$$

$$k = \frac{1}{t}\ell n \frac{a}{a - x}$$

$$k = \frac{1}{t}\ell n \frac{P_o}{\frac{3P_o - P_t}{2}}$$

$$k = \frac{1}{t}\ell n \frac{2P_o}{3P_o - P_t}$$

(D) $2\pi r = n\lambda$

$2\pi \times 0.5n^2 = n \times 12.56\ \text{Å}$

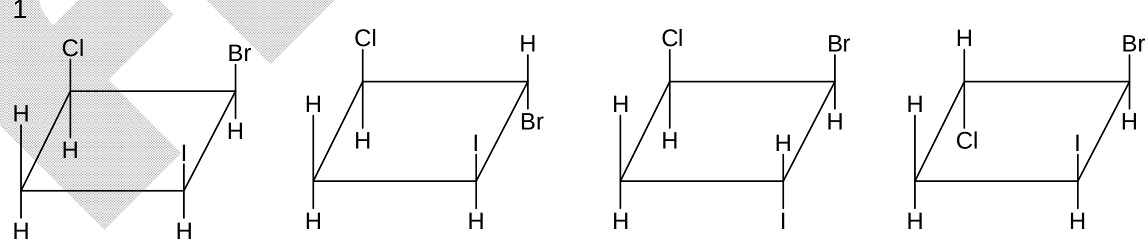
$3.14 \times n = 12.56$

$n = 4$

Section – B

29. 1

Sol.



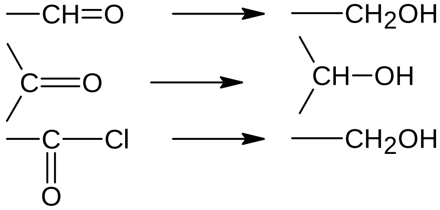
P has four diastereomers and each of the diastereomers exhibits enantiomerism.

30. 5

Sol. CO_2, CH_3CN, NO_2^+, N_2O & C_3O_2

31. 7

Sol. $NaBH_4$ can reduce only following group.



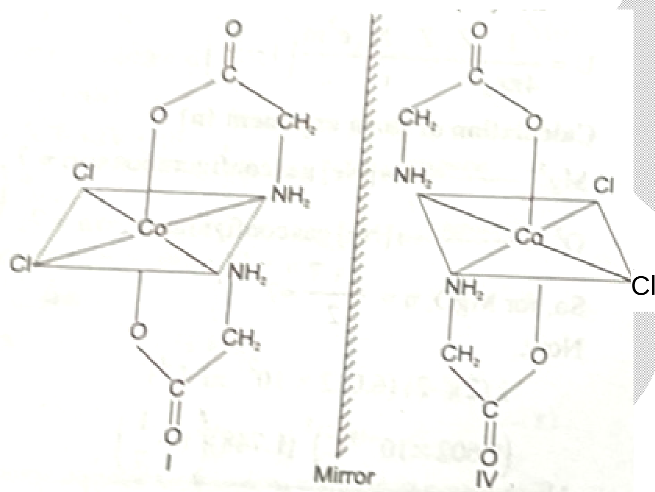
R - X $\longrightarrow$ R - H

($2^o/3^o$ halide)

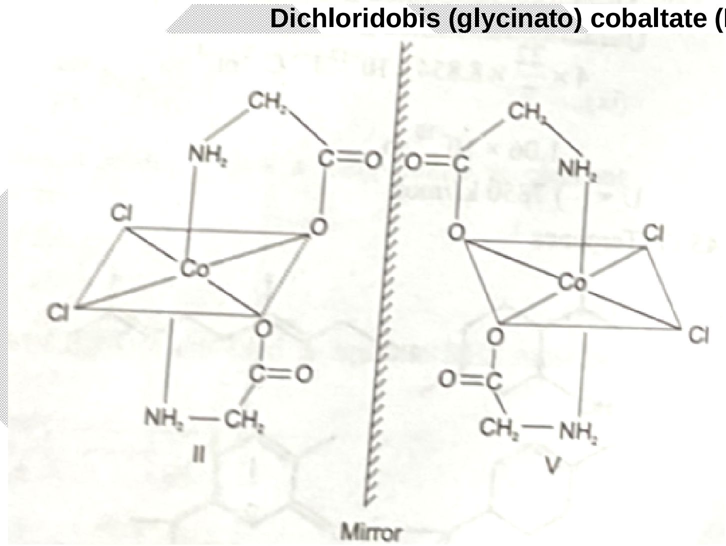
32. 6

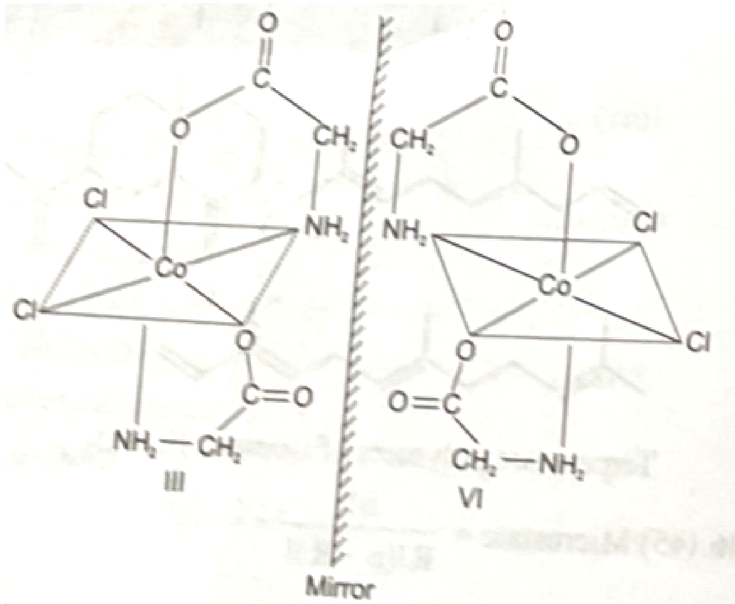
Sol.

Dichloridobis (glycinato) cobaltate (III) ion $\left[CoCl_2\left(gly\right)_2\right]^{\ominus}$



Dichloridobis (glycinato) cobaltate (III) ion





Total number of optical active isomers = 6(six)

33. 5

Sol.

$$7A \longrightarrow B$$

$t = 0 \quad a_0$

$t = t \quad a_0 - 7x \quad x$

Now, $a_0 = 8x$

$8x \xrightarrow{t_{1/2}} 4x \xrightarrow{t_{1/2}} 2x \xrightarrow{t_{1/2}} x$

Given $3t_{1/2} = 900$ hr

$t_{1/2} = 300$ hr

$= \frac{300}{24}$ days

$\therefore 0.4t_{1/2} = \frac{300}{24} \times \frac{4}{10}$ days

= 5 days

34. 4

Sol. In the given molecule there are 4 acidic hydrogen atoms are present.

35. 9

Sol. $FeSO_4.(NH_4)_2 SO_4.6H_2O \longrightarrow Fe^{2+} + 2NH_4^+ + 2SO_4^{2-}$

x = 5

$2CuSO_4 + 4KCN \longrightarrow 2CuCN + K_2SO_4 + (CN)_2 \uparrow$

$3KCN + CuCN \longrightarrow K_3[Cu(CN)_4]$

$K_3[Cu(CN)_4] \longrightarrow +3K^+ + [Cu(CN)_4]^{3-}$

y = 4

36. 4

Sol. Lattice energy $\propto$ charge of cation × charge of anion.

Mathematics

PART – III

Section – A

37. C

Sol. $T_n = 2 + \frac{1}{n} - \frac{1}{n+2}$

38. D

Sol. Use the function $f(x) = (\cot^{-1}(x))^2 + \frac{2}{\sqrt{x^2+1}}$

39. A

Sol. Sum of elements = $\sum_{r=0}^{3}\left(\frac{4\pi}{3} + 2r\pi\right) = \frac{52\pi}{3}$

40. C

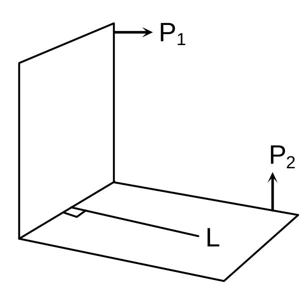
Sol. Put $e^{\frac{y^2}{x}} = t \Rightarrow y(y - x) = x \log_e(ce^y - 1)$

41. A, D

Sol. $A^4 - 7A^3 + 15A^2 - 9A = 0$ (Null matrix)
$\Rightarrow B + C = 4A$

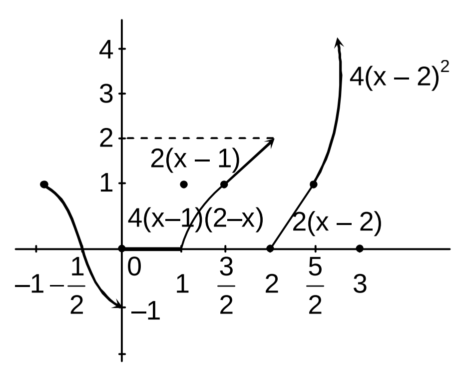
42. A, B, C, D

Sol. $P_1 : 8x - 9y + 4z = -40$
$P_2 : 3x + 4y + 3z = -10$
Line L must lie in P_2 and perpendicular to P_1
$\therefore\ A\left(x_1, y_1, \frac{10}{3}\right)$ must lie in both the plane
$\therefore\ x_1 = -\frac{20}{3},\ y_1 = 0,\ z_1 = \frac{10}{3}$



43. A, B, C

Sol. Shown in the figure



44. B, C

Sol. $f(\alpha) = \frac{\alpha}{\sqrt{1-\alpha^2}}$

45. A, B, C, D

Sol. $$\lim_{h\to 0}\frac{(1+h)^a-5(1+h)+4}{h\left\{(1+h)^a-4(1+h)+3\right\}} = \lim_{h\to 0}\frac{\left[\left\{1+ah+a\left(\frac{a-1}{2}\right)h^2+.....\right\}-5(1+h)+4\right]}{h\left[\left\{(1+ah)+a\left(\frac{a-1}{2}\right)h^2+.....\right\}-4(1+h)+3\right]}$$

$$= \lim_{h\to 0}\frac{(a-5)h+a\left(\frac{a-1}{2}\right)h^2+.....}{h\left\{(a-4)h+a\left(\frac{a-1}{2}\right)h^2+.....\right\}} \Rightarrow (a = 5) \text{ and } b=\frac{a\left(\frac{a-1}{2}\right)}{a-4}=10$$

46. A, D

Sol. Let $z_1=2e^{ia},\ z_2=\frac{14\sqrt{2}}{5}e^{ic},\ z_3=2e^{ib}$

$$\therefore\ \arg(z_2-z_1)=\alpha\ \Rightarrow\ \frac{\frac{14\sqrt{2}}{5}\sin(a+\theta)-2\sin a}{\frac{14\sqrt{2}}{5}\cos(a+\theta)-2\cos a}=\frac{4}{3}\ ;\ \cos\theta=\frac{7\sqrt{2}}{10}$$

$$\Rightarrow\ \tan a=\frac{3}{4}\ \therefore\ \tan(a+2\theta)=\frac{4}{3}\ \therefore\ \arg(z_1)+\arg(z_3)=\frac{\pi}{2}$$

Section – B

47. 4

Sol. Number of favourable cases = 7788

48. 5

Sol. $$I=\int_0^1\frac{x^{15}-x^{11}+x^7}{\left(3x^{16}-4x^{12}+6x^8\right)^{\frac{3}{4}}}dx\ \Rightarrow I=\frac{1}{48}\int_0^5\frac{dt}{t^{\frac{3}{4}}}=\frac{5^{\frac{1}{4}}}{12}$$

49. 2

Sol. Let $\cot^{-1}(2x-1)=\theta\ ;\ 0<\theta<\pi$

$$\Rightarrow \sin 2\theta=\frac{1+\cot\theta}{2}\ \Rightarrow\ \frac{2t}{1+t^2}=\frac{t+1}{2t}\Rightarrow t^3-3t^2+t+1=0$$

$\Rightarrow t = 1,\ 1\pm\sqrt{2}\ ;\ t=1-\sqrt{2}$ does not satisfy

$\therefore\ x=1,\frac{1}{\sqrt{2}}$

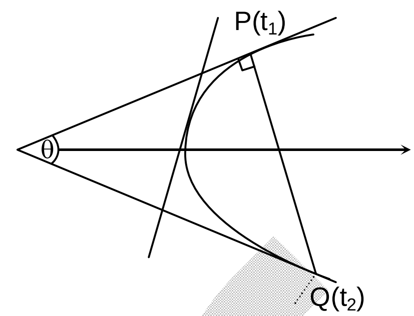
50. 6

Sol. $$\frac{1}{{}^{2024}C_r}=\frac{2025}{2026}\left\{\frac{1}{{}^{2025}C_r}+\frac{1}{{}^{2025}C_{r+1}}\right\}$$

$$\Rightarrow\ \sum_{r=1}^{2023}\frac{(-1)^{r-1}r}{{}^{2024}C_r}=\frac{2025}{2026}\left\{\left(\frac{1}{C_1}+\frac{1}{C_2}\right)-2\left(\frac{1}{C_2}+\frac{1}{C_3}\right)+3\left(\frac{1}{C_3}+\frac{1}{C_4}\right)+.....2023\left(\frac{1}{C_{2023}}+\frac{1}{C_{2024}}\right)\right\}$$

$$=\frac{2025}{2026}\left\{\frac{1}{C_1}-\frac{1}{C_1}+\frac{1}{C_3}-\frac{1}{C_4}+....-\frac{1}{C_{2024}}+\frac{2024}{C_{2024}}\right\}\ =\ \frac{2024}{2026}=\frac{1012}{1013}$$

51. 2



Sol. $PQ = \frac{4a\left(t_1^2+1\right)^{\frac{3}{2}}}{t_1^2}$

PQ is max at $t_1 = 3$ and $t_2 = -3-\frac{2}{3} = -\frac{11}{3}$

$$\tan\theta = \left|\frac{\frac{1}{3}+\frac{3}{11}}{1-\frac{1}{3}\cdot\frac{3}{11}}\right| = \frac{20}{33}\times\frac{11}{10} = \frac{2}{3}$$

52. 6

Sol. $\vec{r} = x\vec{a} + y\vec{b} + z\left(\vec{a}\times\vec{b}\right)$ ..... (1)

$\vec{r}\times\left(\vec{a}\times\vec{b}\right) = \left(\vec{r}\cdot\vec{b}\right)\vec{a} - \left(\vec{r}-\vec{a}\right)\vec{b} \Rightarrow \vec{r}\cdot\vec{b} = 3,\ \vec{r}\cdot\vec{a} = 2$

From equation (1), we get $2 = x|a|^2 + y\vec{a}\cdot\vec{b} \Rightarrow 2x + y = 1$

$3 = x\vec{a}\cdot\vec{b} + y|b|^2 \Rightarrow 2x + 4y = 3$

$\therefore\ x = \frac{1}{6}; y = \frac{2}{3}$. Also, $|\vec{r}| = \frac{4}{\sqrt{3}} \Rightarrow z = \pm\frac{1}{2}$

$\therefore\ \left|\left[\vec{r}\ \ \vec{a}\ \ \vec{b}\right]\right| = |z||a\times b|^2$

53. 7

Sol. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $ad - bc \neq 0 \Rightarrow a + b + c + d = 6$

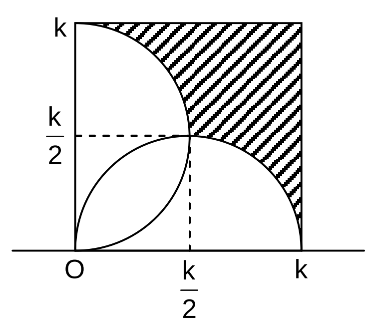
(A) Exactly one zero. Number of matrices = $\frac{4!}{2!} + 4! + \frac{4!}{3!}$

(B) Exactly two zeros. Number of matrices $2\cdot {}^{6-1}C_{2-1} = 10$

(C) No zeroes number of matrices = 6

54. 6

Sol. Let z be x + iy then $0 < x < k$, $0 < y < k$
And $x^2 + y^2 > kx$, $x^2 + y^2 > ky$



$$\text{Area} = k^2 - \left[\frac{\pi k^2}{4} - 2\left\{\frac{\pi}{4}\left(\frac{k}{2}\right)^2 - \frac{1}{2}\cdot\frac{k^2}{4}\right\}\right]$$

$= \frac{3}{4}k^2 - \frac{\pi}{2}\cdot\frac{k^2}{4} = \frac{k^2}{4}\left(3-\frac{\pi}{2}\right) = 250(6-\pi)$

$\Rightarrow k^2 = 2000 \Rightarrow k = 20\sqrt{5}$