

FIITJEE
ALL INDIA TEST SERIES
JEE (Advanced)-2023
FULL TEST – VIII
PAPER –1
TEST DATE: 07-05-2023

Time Allotted: 3 Hours

Maximum Marks: 180

General Instructions:

- The test consists of total 54 questions.
- Each subject (PCM) has 18 questions.
- This question paper contains **Three Parts**.
- **Part-I** is Physics, **Part-II** is Chemistry and **Part-III** is Mathematics.
- Each **Part** is further divided into **Two Sections: Section-A & Section-B**.

Section – A (01 –06, 19 – 24, 37 – 42): This section contains **EIGHTEEN (18)** questions. Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).

Section – A (07 – 10, 25 – 28, 43 – 46): This section contains **TWELVE (12)** Matching List Type Questions. Each question has **FOUR** statements in **List-I** entries (I), (II), (III) and (IV) and **FIVE** statements in **List-II** entries (P), (Q), (R), (S) and (T). The codes for lists have choices (A), (B), (C), (D) out of which, **ONLY ONE** of these four options is correct answer.

Section – B (11 – 18, 29 – 36, 47 – 54): This section contains **TWENTY FOUR (24)** numerical based questions. The answer to each question is a **NUMERICAL VALUE**. If the numerical value has more than two decimal places, truncate/round-off the value to **TWO** decimal places.

MARKING SCHEME

Section – A (One or More than One Correct): Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+4	If only (all) the correct option(s) is (are) chosen;
Partial Marks	:	+3	If all the four options are correct but ONLY three options are chosen;
Partial marks	:	+2	if three or more options are correct but ONLY two options are chosen and both of which are correct;
Partial Marks	:	+1	If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	-2	In all other cases.

Section – A (Single Correct): Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+3	If ONLY the correct option is chosen.
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	-1	In all other cases.

Section – B: Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+3	If ONLY the correct numerical value is entered at the designated place;
Zero Marks	:	0	In all other cases.

Physics

PART – I

Section – A (Maximum Marks: 24)

This section contains **SIX (06)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).

- A ball is thrown from ground under gravity with speed u at an angle θ with horizontal. It strikes the vertical wall and returns to ground. If x is distance of wall from point of projection and R is range of ball on ground if there was no wall. x_1 is distance of point of landing on ground from wall. Take wall to be smooth.

(A) If collision of ball with wall is inelastic time taken by ball to return to ground is $\frac{2u\sin\theta}{g}$ for $x = \frac{R}{2}$

(B) If collision of ball with wall is elastic time taken by ball to return to ground will be less than $\frac{2u\sin\theta}{g}$ for $x < \frac{R}{2}$

(C) If $e = \frac{1}{2}$ for collision between ball and wall then time of taken by ball to return ground will be equal to $\frac{2u\sin\theta}{g}$ for any value of x .

(D) If $e = 1$ for collision between ball and wall then $x + x_1 = R$
- Pressure variation inside a organ pipe is given as $\Delta P = P_0 \cos \frac{3\pi x}{2} \sin(200t + \phi)$, where x is in meter and t is in second. Then (take $x = 0$ at one of the ends of pipe)

(A) Orgain pipe is closed at one end and open at other end if length of organ pipe is one meter

(B) Orgain pipe is open at both ends if length of organ pipe $2m$ for some value of ϕ .

(C) Orgain pipe is closed at both ends if length of organ pipe $2m$ for any value of ϕ

(D) Orgain pipe is closed at one end and open at other end for some value of ϕ if length of orgain pipe $0.5 m$
- An artificial satellite is orbiting around earth in a circular orbit. Following is the gravitational force between satellite and earth given as

$$F = \frac{G\alpha m_1 m_2}{r^2}$$

Where α is dimensionless quantity other symbols have their usual meaning. Then

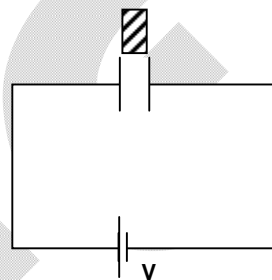
(A) If α decreases at constant rate then satellite will spiral away from earth

(B) If α decreases at constant rate then satellite will spiral towards earth

(C) If α changes with time, angular momentum of satellite will not be conserved about centre of earth

(D) Angular momentum of satellite will be conserved about centre of earth whether α increases, decreases or remain constant.

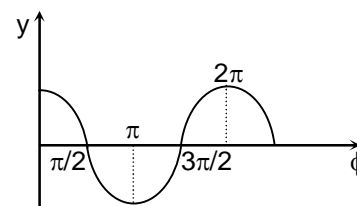
4. Which of the following is/are incorrect?
- (A) In Coolidge tube if accelerating potential is increased then frequency of K_{α} radiation will also increase
- (B) An hydrogen atom in first excited state emits radiation of frequency f_0 . If same hydrogen atom in first excited state is free to move and radiation of frequency emitted is f . The $f < f_0$
- (C) If surface of metallic sheet is exposed to radiation of frequency ν_0 then after some time photo electron will be ejected by sheet when electron gets sufficient energy.
- (D) A particle has zero rest mass. If it strikes the surface and loses 50% energy then speed of particle will become $\sqrt{2}$ times less than its initial velocity after the collision.
5. A parallel plate capacitor having capacitance C is connected in series with a battery of emf V as shown. A dielectric sheet of dielectric constant K is released in the gap (there is no external force on sheet after sheet starts going into gap between plates of capacitor. Dimensions of sheet are such that it just fills the gap between the plates when it is completely inside capacitor). Then which of the following is/are incorrect. (Assuming there is no drag force on sheet exerted by environment)
- (A) Part of energy supplied by the battery will be lost in form of heat.
- (B) Battery charges and discharges both during the process.
- (C) Rate of loss of energy by battery will be maximum when sheet fills the gap between plates completely
- (D) Battery discharges initially and then steady state of capacitor is achieved.
6. A long solenoid is made by wrapping a conducting wire of radius r , on a non magnetic core of radius R . Windings are insulated from each other. Which of the following is/are INCORRECT?
- (A) Magnetic field at midpoint of core on its axis is zero for any value of r .
- (B) Magnetic field at mid point of core on its axis is non zero for any value of r .
- (C) Magnetic field at mid point of core on its axis is zero
- (D) Any of the above depending upon current in solenoid



Section – A (Maximum Marks: 12)

This section contains **FOUR (04) Matching List Type Questions**. Each question has **FOUR** statements in **List-I** entries (I), (II), (III) and (IV) and **FIVE** statements in **List-II** entries (P), (Q), (R), (S) and (T). The codes for lists have choices (A), (B), (C), (D) out of which **ONLY ONE** of these four options is correct answer.

7. A particle is undergoing SHM. Its displacement from mean position versus phase angle graph is shown in the figure.



List –I describes the statement regarding particle motion and List-II gives graph of parameter of List-I versus Phase angle.

List –I		List –II	
(I)	Acceleration of particle Vs phase angle graph	(P)	

(II)	Velocity of particle Vs phase angle graph	(Q)	
(III)	Total energy of particle Vs phase angle graph	(R)	
(IV)	Kinetic energy of particle Vs phase angle graph	(S)	
		(T)	

Which one of the following options is correct ?

- (A) I → T, II → R, III → S, IV → R
- (B) I → P, II → Q, III → R, IV → S
- (C) I → S, II → T, III → P, IV → Q
- (D) I → T, II → P, III → Q, IV → S

8. An adiabatic chamber is divided into two equal parts by non-conducting piston which is free to move. Initially ideal diatomic gas in left part and ideal monoatomic gas in right part of container, are at same temperature T_i and at equal pressure P_i . Total volume of container is $2V_i$. Gas in left chamber is slowly heated so that gas in right chamber compresses to $\frac{V_i}{8}$. If P_f is final pressure of gas in right part, n_0 is number moles of diatomic gas, Q is heated supplied to left part and T_f is final temperature of left part.

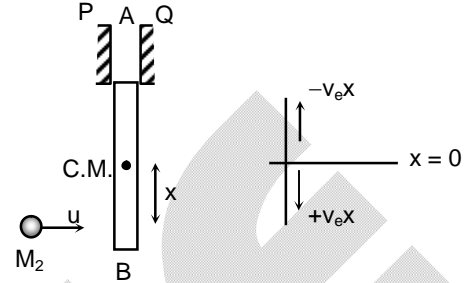
List -I describes the statement regarding the gases and List-II gives their respective values.

List -I		List -II	
(I)	$\frac{Q}{n_0RT_f}$	(P)	32
(II)	$\frac{T_f}{T_i}$	(Q)	2
(III)	$\frac{P_f}{P_i}$	(R)	60
(IV)	$\frac{Q}{P_iV_i}$	(S)	2.5
		(T)	152

Which one of the following options is correct ?

- (A) I → S, II → R, III → P, IV → T
- (B) I → P, II → Q, III → R, IV → S
- (C) I → S, II → T, III → P, IV → Q
- (D) I → T, II → S, III → P, IV → R

9. A rod of mass M_1 and length L is lying on smooth horizontal surface. Its end A is just fit in smooth gap as shown. A small ball of mass M_2 moving with speed u normal to length of rod strikes it at distance x from centre of mass of rod and stops after the collision. Assume mass distribution of rod to be uniform. Then during collision.



List –I gives value of x and List-II gives impact of collision on support during the collision.

List –I		List –II	
(I)	$x = \frac{L}{3}$	(P)	Reaction at P in non zero while reaction at Q is zero
(II)	$x = \frac{L}{6}$	(Q)	Reaction at P and Q is independent of x
(III)	$x = \frac{L}{8}$	(R)	Reaction at P is zero while at Q is non zero
(IV)	$x = -\frac{L}{6}$	(S)	Reaction at both P and Q is non zero
		(T)	No force is exerted by P or Q parts of gap

Which one of the following options is correct ?

- (A) I → T, II → R, III → S, IV → R
 - (B) I → P, II → Q, III → R, IV → S
 - (C) I → P, II → T, III → R, IV → R
 - (D) I → T, II → S, III → P, IV → R
10. List –I gives different physical phenomena while List-II gives physical process involved in List-I.

List –I		List –II	
(I)	Blue colour of earth as seen from space	(P)	Refraction
(II)	Blue iris in eye	(Q)	Reflection
(III)	Twinkling of star	(R)	Dispersion
(IV)	Sun set 2 minutes later than actual setting time	(S)	Scattering of light
		(T)	Tyndall effect

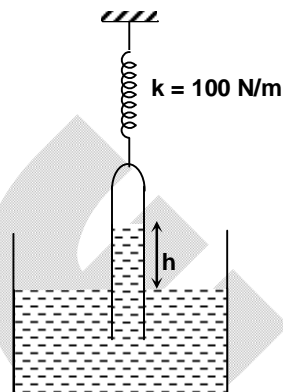
Which one of the following options is correct ?

- (A) I → T, II → R, III → S, IV → R
- (B) I → P, II → Q, III → R, IV → S
- (C) I → S, II → T, III → P, IV → P
- (D) I → T, II → S, III → P, IV → R

Section – B (Maximum Marks: 24)

This section contains **EIGHT (08)** numerical based questions. The answer to each question is a **NUMERICAL VALUE**. If the numerical value has more than two decimal places, truncate/round-off the value to **TWO** decimal places.

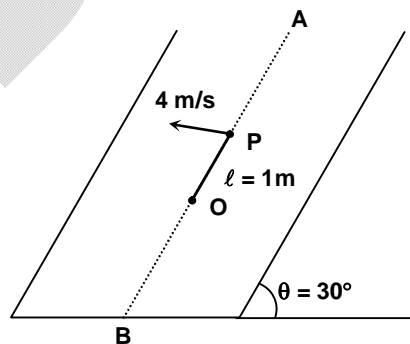
11. The tube of ideal barometer having negligible mass is suspended by spring as shown in the figure. Height of mercury in tube is h and area of cross-section of tube is A . The reading of spring balance is x newton. Find the value of x (Atmospheric pressure is P_0). Ignore surface tension and thickness of tube. (Given density of mercury is $13.6 \times 10^3 \text{ kg/m}^3$ and atmospheric pressure = 10^5 N/m^2)



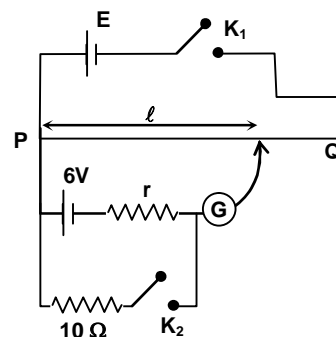
12. A charge Q is uniformly distributed over curved surface of a cone made of insulating material. Slant length of cone is L . Energy required to take test charge q from infinity to vertex of cone is $\frac{Qq}{k\pi\epsilon_0 L}$. Find the value of k .

13. A metallic stick has length 1 m and area of cross section 1 cm^2 . It is kept at 0°C . Now stick is kept between two reservoirs of temperature 100°C and 0°C . Given specific heat capacity of stick material is $10 \text{ J/kg-}^\circ\text{C}$ and mass of stick is 2 kg . Find heat absorbed by stick to reach at steady state in joule.

14. A particle of mass 1 kg is attached to one end of a string of length 1 m . Other end of string is fixed at O . When particle is at position P on line AB it is given velocity 4 m/s in direction perpendicular to line AB . Ratio of tension at position P and at position Q when velocity of particle is parallel to line AB (Line AB is along greatest slope of the inclined plane). Assume size of inclined plane is sufficiently large for circular motion of particle. (Given $OP = 1 \text{ m}$)

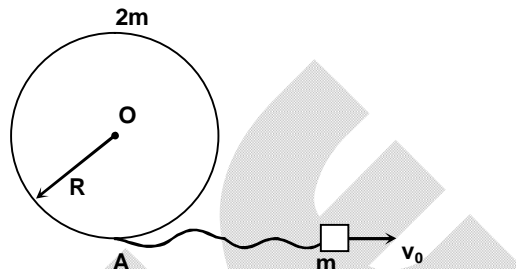


15. In given arrangement length of wire PQ is L_0 and resistance is R_0 . When key K_2 is open there is no deflection in galvanometer for $l = \frac{L_0}{2}$. If K_2 is closed galvanometer shows no deflection for $l = \frac{5L_0}{12}$. If the value of $E = x \text{ Volt}$ and $r = y \Omega$. Then find the value of $(x + y)$.



16. A source of natural frequency f_0 is kept in isotropic medium. Power of source is P . A person is standing at distance a from the source. If observer starts moving towards source with speed u . Then intensity of sound received by observer is $\frac{P}{4\pi a^2} \left(1 + \frac{xu}{c}\right)$. Find the value of x . (Take speed of light c)

17. A disc of radius R and mass $2m$ is placed on smooth horizontal surface. A string is loosely wound around disc and its other end is attached to a small block of mass m . Block is given velocity v_0 along tangent at point A at which string will become tight on disc. Just after string becomes tight velocity of point A is xv_0 . Find the value of x .



18. A box A contains capacitance of $1 \mu\text{F}$ and resistance of 32Ω connected in series. Box B contains inductor of self inductance 4.9 mH connected with resistance of 68Ω in series. Boxes A and B are connected in series with source 10V . Frequency of source is adjusted so that current in circuit becomes maximum. Then voltage across B is nearly x volt. Find the value of x .

Chemistry

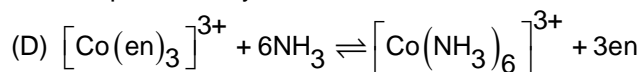
PART – II

Section – A (Maximum Marks: 24)

This section contains **SIX (06)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).

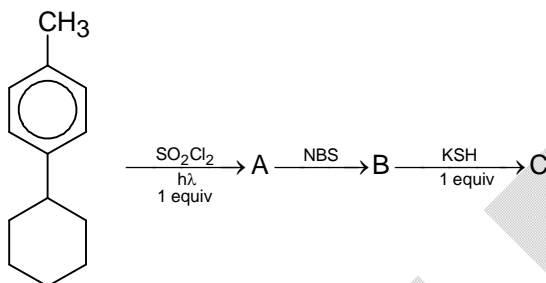
19. Select the correct statements:

- (A) Chelation effect in complex is maximum for five and six membered rings.
 (B) Complexes can not show geometrical isomers if it contains only chelating ligands.
 (C) An increase in chelate ring size beyond six membered produces a fairly uniform decrease in complex stability.



The above equilibrium is dominated by products.

20.



The correct statements about 'C'

- (A) Sodium fusion extract would give white ppt. with AgNO_3 solution.
 (B) Sodium fusion extract would give pale yellow ppt. with AgNO_3 solution.
 (C) C is more acidic than benzyl alcohol.
 (D) Sodium fusion extract gives a purple colour with sodium nitroprusside.

21. For reaction:

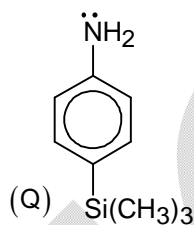
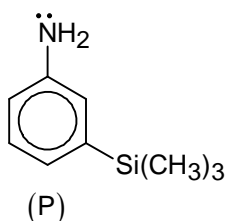
$\text{CH}_3\text{COCH}_3 + \text{Br}_2 \xrightarrow{[\text{H}^+]}$ $\text{CH}_3\text{COCH}_2\text{Br} + \text{H}^+ + \text{Br}^-$ following data was obtained. From the data, identify the correct statement(s)

[Acetone]	$[\text{Br}_2]$	$[\text{H}^+]$	[Rate of reaction]
0.15	0.25 M	0.025 M	$6 \times 10^{-4} \text{ M sec}^{-1}$
0.15	0.50 M	0.025 M	$6 \times 10^{-4} \text{ M sec}^{-1}$
0.15	0.25 M	0.050 M	$12 \times 10^{-4} \text{ M sec}^{-1}$
0.2	0.25 M	0.025 M	$8 \times 10^{-4} \text{ M sec}^{-1}$

- (A) The overall order of the reaction is 2nd order.
 (B) During the reaction, where $[\text{H}^+]$ does not change it will follow first order kinetics.
 (C) Data show that the halogen participates in the mechanism through a fast step which occurs after the rate determining step.
 (D) Order of reaction with respect to Br_2 will be 1.

22. Which statements are true?
 (A) The degree of dissociation of water is greater in 1 M HCN solution than in 1 M HCl solution.
 (B) The degree of dissociation of pure water at 25°C is greater than at 30°C.
 (C) The degree of dissociation of water in 0.1 M aqueous solution of HCl at a certain temperature (T) is 3.6×10^{-15} ; then T must be greater than 25°C.
 (D) The degree of dissociation of water always decreases on making it acidified or alkaline.
23. How many of the following on heating strongly produce two paramagnetic products?
 (A) LiNO_3 (B) KMnO_4
 (C) NH_4NO_3 (D) NaNO_3

24.



Identify the incorrect statement/s about (P) and (Q)

- (A) P is weaker base than Q.
 (B) C – Si bond is longer in P than that in Q.
 (C) In P, the trimethylsilyl group (Me_3Si) shows a small electron releasing effect.
 (D) Conjugate acid of Q is less stable than that of P.

Section – A (Maximum Marks: 12)

This section contains **FOUR (04) Matching List Type Questions**. Each question has **FOUR** statements in **List-I** entries (I), (II), (III) and (IV) and **FIVE** statements in **List-II** entries (P), (Q), (R), (S) and (T). The codes for lists have choices (A), (B), (C), (D) out of which **ONLY ONE** of these four options is correct answer.

25. Match the following:

List – I		List – II	
(I)	Daniel cell	(P)	Oxidation takes place at anode
(II)	Concentration cell	(Q)	$\Delta G = -nFE$
(III)	Fuel cell	(R)	Chemical energy is converted to electrical energy
(IV)	Electrolytic cell	(S)	$\Delta G \leq 0$
		(T)	Used for detecting breath alcohol.

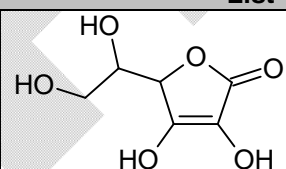
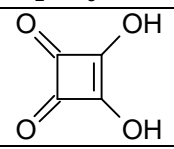
- (A) I→P, Q, R, S; II→P, Q, S; III→P, Q, R, S, T; IV→P, Q
 (B) I→P, R, S; II→Q, R, S, T; III→P, R, S, T; IV→Q, S
 (C) I→Q, S, T; II→P, Q, R, S, T; III→P, S, T; IV→S, T
 (D) I→P, Q, S, T; II→P, Q, R, S; III→Q, R, T; IV→P, Q

26. Match the following

List – I		List – II	
(I)	$ \begin{array}{c} \text{CHO} \\ \\ \text{HO} - \text{H} \\ \\ \text{HO} - \text{H} \\ \\ \text{H} - \text{OH} \\ \\ \text{H} - \text{OH} \\ \\ \text{CH}_2\text{OH} \end{array} $	(P)	Functional isomer of D-glucose
(II)	$ \begin{array}{c} \text{CHO} \\ \\ \text{H} - \text{OH} \\ \\ \text{HO} - \text{H} \\ \\ \text{HO} - \text{H} \\ \\ \text{H} - \text{OH} \\ \\ \text{CH}_2\text{OH} \end{array} $	(Q)	Epimer of D-glucose
(III)	$ \begin{array}{c} \text{CHO} \\ \\ \text{HO} - \\ \\ \text{HO} - \text{OH} \\ \\ \text{HO} - \\ \\ \text{HO} - \\ \\ \text{CH}_2\text{OH} \end{array} $	(R)	Reducing sugar
(IV)	$ \begin{array}{c} \text{CH}_2\text{OH} \\ \\ \text{C} = \text{O} \\ \\ \text{HO} - \\ \\ \text{OH} - \\ \\ \text{OH} - \\ \\ \text{CH}_2\text{OH} \end{array} $	(S)	Forms same osazone on treatment $\text{Ph} - \text{NH} - \text{NH}_2$ as D-glucose forms
		(T)	Enantiomer of D-glucose

- (A) I → Q, R, S; II → Q, R; III → R, T; IV → P, R, S
 (B) I → P, Q; II → R, S, T; III → R, S; IV → Q, S, T
 (C) I → Q, R, S; II → P, S, T; III → P, Q; IV → P, R, S
 (D) I → P, S; II → Q, T; III → Q, R, S; IV → P, Q, T

27. Match the following:

List – I		List – II	
(I)		(P)	All the C – O bonds are equal in its dianion form.
(II)	H_2CO_3	(Q)	Diprotic acid
(III)		(R)	Decolourises purple colour of the MnO_4^- ion

(IV)	$\begin{array}{c} \text{CO}_2\text{H} \\ \\ \text{CO}_2\text{H} \end{array}$	(S)	Produces effervescence on addition of NaHCO_3
		(T)	Stronger acid than phenol

- (A) I→P, Q, S, T; II→P, R, T; III→P, Q, R, T; IV→P, Q, R, S, T
 (B) I→Q, R, S, T; II→P, Q, T; III→P, Q, S, T; IV→P, Q, R, S, T
 (C) I→P, R, S, T; II→Q, R, S, T; III→P, Q, R, T; IV→P, Q, R, T
 (D) I→R, Q, S, T; II→P, S, T; III→P, Q, R, T; IV→Q, R, S, T

28. Match the following:

List – I		List – II	
(I)	CrO_4^{2-}	(P)	Contains peroxo linkage
(II)	$\text{Cr}_2\text{O}_7^{2-}$	(Q)	Unstable compound
(III)	CrO_5	(R)	Cr in +VI state
(IV)	CrO_8^{3-}	(S)	Oxidizes 1° alcohol to carboxylic acid
		(T)	All Cr – O bonds are equivalent

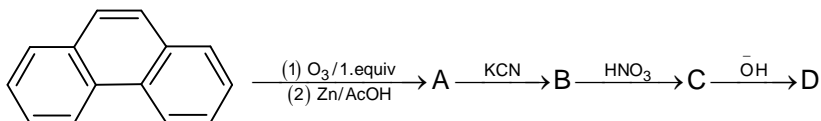
- (A) I→P, R, S; II→S, T; III→P, Q, R, S; IV→Q, R, S
 (B) I→Q, S, T; II→P, S; III→Q, R, S, T; IV→Q, R, S
 (C) I→R, S, T; II→R, S; III→P, Q, R, S; IV→P, Q, S
 (D) I→P, S, T; II→Q, S; III→P, R, S, T; IV→P, Q, S

Section – B (Maximum Marks: 24)

This section contains **EIGHT (08)** numerical based questions. The answer to each question is a **NUMERICAL VALUE**. If the numerical value has more than two decimal places, truncate/round-off the value to **TWO** decimal places.

29. A compound (A) on heating with an excess of caustic soda solution liberates a gas B which gives white fumes on exposure to HCl. Heating is continued to expel the gas completely. The resultant alkaline solution again liberates the same gas B, when heated with zinc powder. However, the compound A, when heated alone, does not give nitrogen. If molar mass of A is 'x' then find the value of $\frac{x}{3}$?
30. 100 ml 0.1 M HCl is added to 200 ml 0.1 M Na_2CO_3 solution. What would be the pH of the final solution.
 [For H_2CO_3 , $K_{a1} = 10^{-7}$, $K_{a2} = 10^{-11}$]
31. If the spin only magnetic moments of $\text{K}_3[\text{Fe}(\text{oxalate})_3]$ and $\text{K}_3[\text{Ru}(\text{oxalate})_3]$ are \sqrt{x} and \sqrt{y} BM. (Bohr magnetron) respectively. Then the value of $\frac{x-y}{5}$ would be
32. The emf of a standard cadmium cell is 1.02 V at 300 K. The temperature coefficient of the cell is $-5.0 \times 10^{-5} \text{ VK}^{-1}$. If the value of ΔH° for the cell is x kJ mol^{-1} , then value of |x| would be

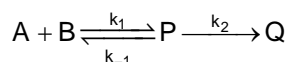
33.



If number of C atoms, number of O atoms and number of N atoms in D are x, y and z respectively then determine the value of $\frac{x+y+z}{3}$?

34. If the number of five membered rings in fullerene (C – 60), cis-1,2 diol-boric acid complex and in Ca(II) – EDTA chelate are x, y and z respectively then value of $\frac{x \times y \times z}{10}$ is

35. For the reaction



$k_1 = 1.5 \times 10^5 \text{ L mol}^{-1} \text{ s}^{-1}$, $k_{-1} = 10^4 \text{ s}^{-1}$ and $k_2 = 10 \text{ s}^{-1}$. Under steady state approximation, the rate constant for the overall reaction in $\text{L mol}^{-1} \text{ s}^{-1}$ would be

36. Optically pure R-2-methylbutan-1-ol has a specific rotation of $+13.5^\circ$. The specific rotation of 2-methylbutan-1-ol containing 35% of the S-enantiomer is.....

Mathematics

PART – III

Section – A (Maximum Marks: 24)

This section contains **SIX (06)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).

37. Let two circles c_1 and c_2 be given the equation of c_1 is $|z^2| + \bar{a}z + a\bar{z} + c = 0$ and equation of c_2 is $|z^2| + \bar{b}z + b\bar{z} + d = 0$ where a and b are complex numbers and c and d are real number, then
- (A) radius of c_1 is $\sqrt{|a|^2 - c}$
- (B) c_1 and c_2 will touch externally iff $|a - b| = \sqrt{|a|^2 - c} + \sqrt{|b|^2 - d}$
- (C) c_1 and c_2 will intersect orthogonally iff $a\bar{b} + \bar{a}b = c + d$
- (D) radical axis of c_1 and c_2 is $(\bar{a} - \bar{b})z + (a - b)\bar{z} + c - d = 0$
38. If $(x^{2006} + x^{2008} + 2)^{2010} = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$, then the value of $a_0 - \frac{1}{2}a_1 - \frac{1}{2}a_2 + a_3 - \frac{1}{2}a_4 - \frac{1}{2}a_5 + a_6 + \dots$ is
- (A) less than 2
- (B) greater than 0
- (C) less than 4
- (D) greater than 2
39. If $\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = -\frac{3}{2}$, then
- (A) $\sum \cos \alpha = 0$
- (B) $\sum \sin \alpha = 0$
- (C) $\sum \cos \alpha \sin \alpha = 0$
- (D) $\sum (\cos \alpha + \sin \alpha) = 0$
40. If $f(x) = \begin{vmatrix} \cos(x + \alpha) & \cos(x + \beta) & \cos(x + \gamma) \\ \sin(x + \alpha) & \sin(x + \beta) & \sin(x + \gamma) \\ \sin(\beta - \gamma) & \sin(\gamma - \alpha) & \sin(\alpha - \beta) \end{vmatrix}$, then $f(\alpha)$, $f(\beta)$ and $f(\gamma)$ are in
- (A) Arithmetic Progression
- (B) Geometric Progression
- (C) Harmonic Progression
- (D) none of these
41. If a, b, c are three positive real number and n is some real number p and q are defined as $p = \frac{a^n + b^n + c^n}{3}$; $q = \left(\frac{a+b+c}{3}\right)^n$, then $p > q$ for
- (A) all $n > 0$
- (B) all $n < 0$
- (C) for all $n \in (0, 1)$
- (D) for all $n > 1$
42. For $n, k \in \mathbb{N}$ let $T(n, k) = \frac{1}{(n+1)!} + \frac{1}{(n+2)!} + \frac{1}{(n+3)!} + \dots + \frac{1}{(n+k)!}$, then
- (A) $T(n, k) \leq \frac{1}{n} \left(\frac{1}{n!} - \frac{1}{(n+k)!} \right)$
- (B) $\lim_{n \rightarrow \infty} T(n, k) \leq \frac{1}{n!}$
- (C) $T(n, k) > \frac{1}{(n-1)!}$
- (D) $T(n, k) > \frac{1}{2} \forall n, k \in \mathbb{N}$

Section – A (Maximum Marks: 12)

This section contains **FOUR (04) Matching List Type Questions**. Each question has **FOUR** statements in **List-I** entries (I), (II), (III) and (IV) and **FIVE** statements in **List-II** entries (P), (Q), (R), (S) and (T). The codes for lists have choices (A), (B), (C), (D) out of which **ONLY ONE** of these four options is correct answer.

43. Match the following List-I with List-II

List-I		List-II	
(I)	a, b, c are in harmonic progression, then $\ln(a + c), \ln(a - c), \ln(a - 2b + c)$ are in	(P)	A.P.
(II)	If $a_1, a_2, a_3, \dots, a_n$ are n non-zero real numbers such that $(a_1^2 + a_2^2 + a_3^2 + \dots + a_{n-1}^2)(a_2^2 + a_3^2 + \dots + a_n^2) \leq (a_1a_2 + a_2a_3 + a_3a_4 + \dots + a_{n-1}a_n)^2$, then $a_1, a_2, a_3, a_4, \dots, a_n$ are in	(Q)	G.P.
(III)	If a, b, c, d, e are five numbers in which first three are in A.P. and last three are in H.P. If the three numbers in the middle are in G.P., then the numbers in odd places are in	(R)	H.P.
(IV)	If $x^{18} = y^{21} = z^{28}$, then $3 + 3\log_y x, 3\log_z y, 7\log_x z$ are in	(S)	A.G.P.
		(T)	none of these

The correct option is:

- (A) (I) → (P); (II) → (Q); (III) → (R); (IV) → (S)
- (B) (I) → (P); (II) → (Q); (III) → (S); (IV) → (T)
- (C) (I) → (P); (II) → (Q); (III) → (Q); (IV) → (R)
- (D) (I) → (P); (II) → (Q); (III) → (Q); (IV) → (T)

44. Match the following List-I with List-II

List-I		List-II	
(I)	If $\alpha, \beta, \gamma, \delta$ satisfy the equation $\tan\left(x + \frac{\pi}{4}\right) = 3 \tan 3x$, then $\tan \alpha + \tan \beta + \tan \gamma + \tan \delta$ is equal to	(P)	$\frac{3}{4}$
(II)	In ΔABC the least value of $\sin^2\left(\frac{A}{2}\right) + \sin^2\left(\frac{B}{2}\right) + \sin^2\left(\frac{C}{2}\right)$ is equal to	(Q)	0
(III)	Maximum value of $\sin x \sin(60 - x) \sin(60 + x)$ is equal to	(R)	$\frac{1}{4}$
(IV)	If $\sin x + \sin^2 x + \sin^3 x = 1$, then $\log_{\sec^2 \frac{\pi}{3}}(\cos^6 x - 4 \cos^4 x + 8 \cos^2 x)$ is equal to	(S)	1
		(T)	none of these

The correct option is:

- (A) (I) → (Q); (II) → (P); (III) → (S); (IV) → (T)
- (B) (I) → (Q); (II) → (P); (III) → (R); (IV) → (T)
- (C) (I) → (Q); (II) → (P); (III) → (R); (IV) → (S)
- (D) (I) → (Q); (II) → (P); (III) → (R); (IV) → (S)

45. Match the following List-I with List-II

List-I		List-II	
(I)	If e_1 and e_2 are the eccentricities of the hyperbolas $xy = c^2$ and $x^2 - y^2 = c^2$, then $e_1^2 + e_2^2$ is equal to	(P)	2
(II)	If the angle between the asymptotes of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $\frac{\pi}{3}$, then the eccentricity of the conjugate hyperbola is	(Q)	4
(III)	P and Q be two points on the hyperbola $10x^2 - 3y^2 = 1$ be such that OP and OQ are perpendicular, O is origin then $\frac{1}{OP^2} + \frac{1}{OQ^2}$ is equal to	(R)	8
(IV)	If the sum of the slopes of the normal from point P to hyperbola $xy = 4$ is equal to 2, the locus of point p is $x^2 = \lambda$ where λ is equal to	(S)	7
		(T)	none of these

The correct option is:

- (A) (I) → (Q); (II) → (P); (III) → (S); (IV) → (T)
- (B) (I) → (Q); (II) → (P); (III) → (S); (IV) → (R)
- (C) (I) → (P); (II) → (Q); (III) → (R); (IV) → (S)
- (D) (I) → (P); (II) → (Q); (III) → (R); (IV) → (T)

46. Match the following List-I with List-II

List-I		List-II	
(I)	The total number of 4 digit numbers whose greatest common divisor with 54 is 2 is 1000k, then	(P)	10
(II)	The 10 th place digit of 3^{2025} is	(Q)	4
(III)	The number of points in the argand plane which satisfy the conditions $z(1-i) + \bar{z}(1+i) = 4$ and $ z - 2 = 4$ are	(R)	3
(IV)	The 1025 th term of sequence 1, 2, 2, 4, 4, 4, 4, 8, 8, 8, 8, 8, 8, 8, 8, is 2^k , then k is equal to	(S)	2
		(T)	none of these

The correct option is:

- (A) (I) → (R); (II) → (S); (III) → (Q); (IV) → (P)
- (B) (I) → (R); (II) → (Q); (III) → (S); (IV) → (P)
- (C) (I) → (R); (II) → (Q); (III) → (P); (IV) → (S)
- (D) (I) → (R); (II) → (Q); (III) → (S); (IV) → (T)

Section – B (Maximum Marks: 24)

This section contains **EIGHT (08)** numerical based questions. The answer to each question is a **NUMERICAL VALUE**. If the numerical value has more than two decimal places, truncate/round-off the value to **TWO** decimal places.

47. Let $\alpha_k = 2 + 4i + e^{i\frac{2k\pi}{17}}$ $k \in \mathbb{N}$, $k \leq 50$, then the number of values of k such that $|\alpha_k - 5 - 4i|$ is minimum

48. If x_1, x_2, x_3, x_4, x_5 are five consecutive natural numbers and $x_1 + x_2 + x_3 + x_4 + x_5 = n^3$ and $x_2 + x_3 + x_4 = m^2$, then the least possible value $\frac{n^3 + m^2}{2}$
49. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ is a function $f(x)$ now $g(x)$ is defined as $g(x) = f(x) - f(2x)$ and $g'(1) = 5$ and $g'(2) = 7$. Let $h(x)$ is defined as $h(x) = f(x) - f(4x) - 10x$, then $h'(1)$ is equal to
50. Let f is a real valued function on $\mathbb{R} \rightarrow \mathbb{R}$ such that $e^{-x}f(x) = \frac{3}{e^2} + 4e^{-x} \int_2^x \sqrt{2t^2 + 6t + 5} dt \quad \forall x \in \mathbb{R}$ and let $g(x) = f^{-1}(x)$, then $[g'(3)] + [[g''(3)]]$ is equal to
51. Let $f : (0, \infty) \rightarrow \mathbb{R}$ the a twice differentiable function such that $|f''(x) + 2xf'(x) + (x^2 + 1)f(x)| \leq 2 \quad \forall x \in \mathbb{R}$ and $\lim_{x \rightarrow \infty} (xf(x) + f'(x)) = 5$, then $\lim_{x \rightarrow \infty} f(x)$ is equal to
52. If a, b, A, B are four real numbers and $f(x) = 1 - a \cos x - b \sin x - A \cos 2x - B \sin 2x$. If $f(x) \geq 0 \quad \forall x \in \mathbb{R}$ and P & Q are the maximum possible values of $a^2 + b^2$ and $A^2 + B^2$ respectively then $P + Q$ is equal to
53. If $0 \leq [x] < 2, -1 \leq [y] < 1$ and $1 \leq [z] < 3$ (where $[.]$ represents greatest integer function), then maximum value of the determinant $\Delta = \begin{vmatrix} [x]+1 & [y] & [z] \\ [x] & [y]+1 & [z] \\ [x] & [y] & [z]+1 \end{vmatrix}$ is
54. If $2(\cos(\alpha - \beta) + \cos(\beta - \gamma) + \cos(\gamma - \alpha)) + 3 = 0$, then $\frac{d\alpha}{\sin(\beta + \theta)\sin(\gamma + \theta)} + \frac{d\beta}{\sin(\alpha + \theta)\sin(\gamma + \theta)} + \frac{d\gamma}{\sin(\alpha + \theta)\sin(\beta + \theta)}$ is equal to
(Consider α, β, γ are variable and θ is constant)