FIITJEE
ALL INDIA TEST SERIES
JEE (Advanced)-2023
FULL TEST – VIII
PAPER –1
TEST DATE: 07-05-2023

ANSWERS, HINTS & SOLUTIONS

Physics PART – I

Section – A

1. A, D
Sol. In case of smooth wall there is no change in vertical component of velocity during collision and therefore time of flight of ball is unchanged

2. A, C
Sol. At open end pressure amplitude is zero while at closed end pressure amplitude is maximum.

3. A, D
Sol. If α decreases, force between earth and the satellite will also decrease and therefore satellite will drift away from earth and vice versa.

4. A, C, D
Sol. In case hydrogen atom is free to move then to conserve linear momentum of the system it will gain some energy and therefore energy of the photon emitted will be less then the case when hydrogen atom was at rest.
A particle having zero rest mass can not travel with speed less then speed of light in free space

5. A, C, D
Sol. System will not loose any energy as there are no drag forces. Energy lost by battery will be partly stored in capacitor and partly will go into kinetic energy of sheet when sheet is entering into gap. Sheet will undergo oscillatory motion.

6. A, C, D
Sol. Current in solenoid can be resolved along length of solenoid (i_{ii}) and along curvature of solenoid $(i_{\perp})$. (i_{ii}) will produce field at mid point even if solenoid is long.

7. D
Sol. Basic concept of SHM

8. A
Sol. Final volume of right part = $\frac{V_i}{8}$

Final volume of left part = $2V_i - \frac{V_i}{8} = \frac{15V_i}{8}$

For right part using $P_iV_i^{\gamma} = P_fV_f^{\gamma}$

$$P_iV_i^{\gamma} = P_f\left(\frac{V_i}{8}\right)^{\gamma}$$

$$\Rightarrow \frac{P_f}{P_i} = 32 \qquad \ldots(i)$$

For left part using $\frac{PV}{RT} = \text{constant}$

$$\frac{P_iV_i}{RT_i} = \frac{P_f\left(\frac{15V_i}{8}\right)}{RT_f} \Rightarrow \frac{T_f}{T_i} = 60 \qquad \ldots(ii)$$

For left part

Q = W + ΔU

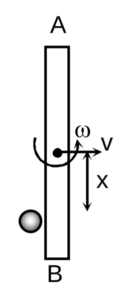
$$= \frac{P_fV_i = P_iV_i}{\gamma - 1} + n_0\frac{5}{2}R(T_f - T_i) = \frac{9}{2}P_iV_i + \frac{P_iV_i}{RT_i} + \frac{5}{2}R(60T_i - T_i)$$

Q = $152P_iV_i$ $\qquad \ldots(iii)$

$$\frac{Q}{n_0RT_f} = \frac{152P_iV_i}{\frac{P_iV_i}{RT_i}RT_f} = \frac{152T_i}{T_f} = \frac{152}{60} = 2.5$$

9. C

Sol. Conserving angular momentum about point just below centre of mass of rod.



Just after collision

$$3Mux = \frac{4ML^2}{12}\omega$$

$$\omega = \frac{9xu}{L^2} \qquad \ldots(i)$$

Conserving linear momentum

$3Mu = 4Mv \Rightarrow v = \frac{3u}{4}$

Velocity of end A

$$v_A = v - \frac{L}{2}\omega = \left(\frac{3u}{4} - \frac{9u}{2}\frac{x}{L}\right)$$

For $v_A = 0$

$$\frac{3u}{4} = \frac{9xu}{2L}$$

x = L/6

10. C

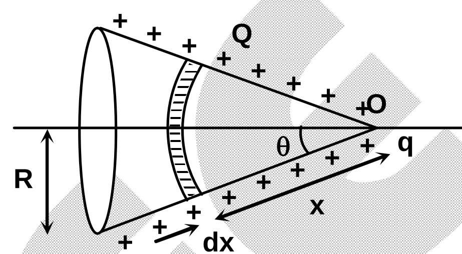
Sol. Information Based

Section – B

11. 0.00
Sol. No force is required to hold the barometric tube at equilibrium

12. 2.00
Sol. Potential at O due to circular ring of width dx

$$dV = \frac{kdq}{x} = \frac{1}{4\pi\varepsilon_0}\frac{\sigma \cdot 2\pi r dx}{x}$$ ($\sigma \rightarrow$ surface change density)



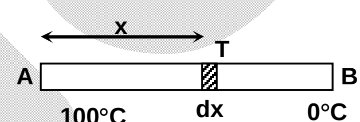
$$= \frac{\sigma}{2\varepsilon_0}\frac{x\sin\theta dx}{x}$$

$$\int dV = \frac{\sigma\sin\theta}{2\varepsilon_0}\int_0^L dx$$

$$V = \frac{\sigma L\sin\theta}{2\varepsilon_0} = \frac{\frac{Q}{\pi RL}L\frac{R}{L}}{2\varepsilon_0} = \frac{Q}{2\pi\varepsilon_0 L}$$

$\therefore$ work done = qV = $\frac{Qq}{2\pi\varepsilon_0 L}$

13. 1000.00
Sol. At steady state let temperature of stick at distance x from A is T.



$$\Rightarrow \frac{KA(100-0)}{L} = \frac{KA(100-T)}{x}$$

$$\Rightarrow \frac{100x}{L} = 100 - T$$

$$T = \left(100 - \frac{100x}{L}\right)$$

Heat absorbed by part dx

$$dQ = \left(\frac{2}{L}dx\right)\cdot 10\cdot(T-0)$$

$$= \frac{20}{L}dx\left(100 - \frac{100x}{L}\right)$$

$$dQ = \frac{2000}{L}\left(1 - \frac{x}{L}\right)dx$$

$$\Rightarrow Q = \frac{2000}{L}\int_0^L\left(1 - \frac{x}{L}\right)dx = \frac{2000}{L}\left[x - \frac{x^2}{2L}\right]_0^L$$

$$\frac{2000}{L}\left[L - \frac{L}{2}\right] = 1000\text{ J}$$

14. 0.42
Sol. $T_A + mg\sin\theta = \frac{mv^2}{R}$

$$T_A = \frac{1\times(4)^2}{1} - 1\times 10\times\sin 30°$$

T_A = 16 – 5 = 11 N …(i)

Using conservation of mechanical energy between points P and Q

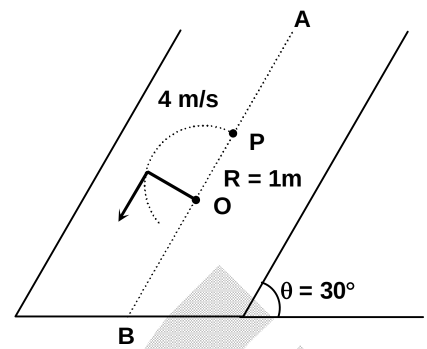
$$\frac{1}{2}m(4)^2 + mg \times R\sin\theta = \frac{1}{2}m(v_Q)^2$$

$$16 + 20 \times 1\sin 30° = v_Q^2$$

$$v_Q^2 = 26 \quad \text{...(ii)}$$

$$T_Q = \frac{m \times 26}{1} = 26\ \text{N}$$

$$\frac{T_P}{T_Q} = \frac{11}{26} = 0.42$$



15. 14.00

Sol. $r = R\left(\frac{\ell_1}{\ell_2} - 1\right)$

$$= 10\left(\frac{L_0/2}{\frac{5L_0}{12}} - 1\right) = 2\,\Omega$$

Also, $\frac{E}{2} = 6 \Rightarrow E = 12$ volt

16. 1.00

Sol. If source is at rest. Energy spread out in length c and area of cross section A is $\left(\frac{P}{4\pi a^2}A\right)$

$\Rightarrow$ Extra energy received by observer per second = $\left(\frac{P}{4\pi a^2}A \cdot \frac{1}{c} \cdot u\right)$

Total energy received by observer in one second per unit area = $\frac{P}{4\pi a^2} + \frac{P}{4\pi a^2}\frac{u}{c} = \frac{P}{4\pi a^2}\left(1 + \frac{u}{c}\right)$

Intensity received by observer = $\frac{\text{Power}}{\text{Area}} = \frac{P}{4\pi a^2} + \frac{P}{4\pi a^2}\frac{u}{c} = \frac{P}{4\pi a^2}\left(1 + \frac{u}{c}\right)$

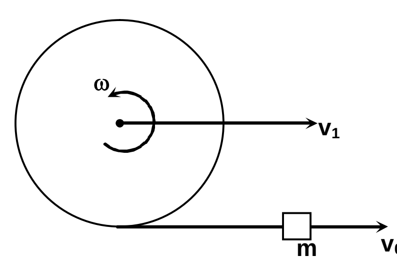
17. 0.60

Sol. Conservation of linear momentum

$mv_0 = mv + 2mv_1$

$v + 2v_1 + v_0$ ...(i)

Conserving angular momentum about point on the ground just below centre of the disc



$$mv_0R = mvR + \frac{2mR^2}{2}\omega$$

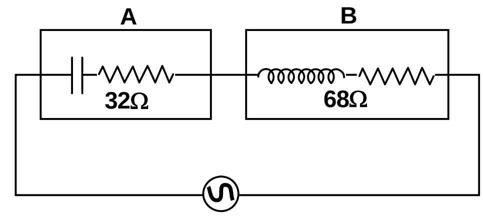
Or $v_0 = v + R\omega$ ...(ii)

$v = R\omega + v_1$ ...(iii)

Solving, $\omega = \frac{2v_0}{5R}$, $v_1 = \frac{v_0}{5}$

$$v_A = R\omega + v_1 = \frac{2}{5}v_0 + \frac{v_0}{5} = \frac{3v_0}{5}$$

18. 9.76



Sol. $i = \frac{V}{Z}$

At resonance, $i = \frac{10}{100} = 0.1A$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4.9 \times 10^{-3} \times 10^{-6}}} = \left(\frac{10^5}{7}\right) \text{ rad/s}$$

$$X_C = \frac{1}{\omega_0 C} = \frac{7}{10^5 \times 10^{-6}} = 70\Omega$$

$$Z_A = \sqrt{(32)^2 + (70)^2} = 76.97$$

$$V_A = iz_A = 0.1 \times 76.97 = 7.7 \text{ Volt}$$

$$X_L = \omega L = \frac{10^5}{7} \times 4.9 \times 10^{-3} = \frac{490}{7} = 70\Omega$$

$$Z_B = \sqrt{(68)^2 + (70)^2} = 97.6\Omega$$

$V_B = 0.1 \times 97.6 = 9.76$ volt

Chemistry

PART – II

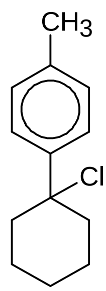
Section – A

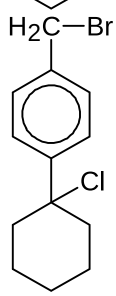
19. A, C

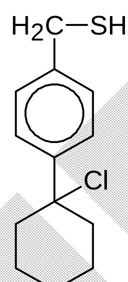
Sol. An increase in chelate ring size beyond six membered ring produces a fairly uniform decrease in complex stability. Which does not show any strong dependence on the size of the metal ion.

20. A, C, D

Sol.

A → 

B → 

C → 

21. A, B, C

Sol. $\text{rate} = k\left[\text{Acetone}\right]^1\left[H^+\right]^1\left[Br_2\right]^0$.

22. A, C, D

Sol.

$H_2O \rightleftharpoons H^+ + OH^-$

C

$C(1-\alpha) \quad 0.1 \quad C\alpha$

$K_W = \left[H^+\right]\left[OH^-\right]$

$= 0.1 \times 55.5 \times 3.6 \times 10^{-15}$

$= 2 \times 10^{-14}$

Hence, T must be > 25^{o}C.

23. A, B

Sol. $4LiNO_3 \longrightarrow 2Li_2O + \underline{4NO_2} + \underline{O_2}$

$2KMnO_4 \longrightarrow \underline{K_2MnO_4} + MnO_2 + \underline{O_2}$

$NH_4NO_3 \longrightarrow N_2O + H_2O$

$NaNO_3 \longrightarrow NaNO_2 + \frac{1}{2}\underline{O_2}$

24. A

Sol. Trimethylsilyl group (Me_3Si) shows a strong electron withdrawing effect from the ortho and para positions due to the resonance effects involving the 3d orbital of the silicon atom.

25. A

Sol. Oxidation always takes place at anode.
In electrolytic cell $\Delta G > 0$.
Fuel cell based breath alcohol sensor oxidizes the alcohol in a breath sample and produces an electric current.

26. A

Sol. All Monosaccharides are reducing sugars.
(I) is Mannose and is a C-2 epimer of glucose.
The formation of osazone involves C-1 and C-2.
Glucose, Mannose and Fructose have identical configuration at C-3, C-4 and C-5. Hence, they form the same osazone.

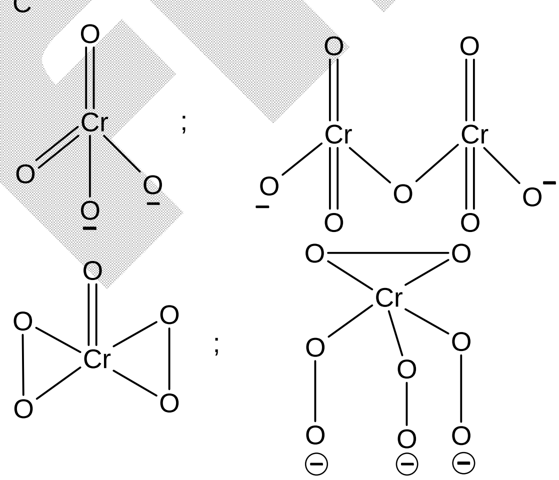
27. B

Sol. (I) is Ascorbic acid (Vitamin C) is a diprotic acid $\left(K_{a_1} = 9 \times 10^{-3}\right)$.

(III) is a Squaric acid and also a diprotic acid. Both are stronger acid than H_2CO_3 and produce effervescence on addition of $NaHCO_3$.

28. C

Sol.



Section – B

29. 26.67

Sol. A is NH_4NO_3

$$NH_4NO_3 + NaOH \xrightarrow{\Delta} NaNO_3 + NH_3 \uparrow + H_2O \uparrow$$

$$NH_3 + HCl \longrightarrow NH_4Cl$$

$$NaNO_3 + 4Zn + 7NaOH \longrightarrow NH_3 + 4Na_2ZnO_2 + 2H_2O$$

$$NH_4NO_3 \xrightarrow{\Delta} N_2O \uparrow + 2H_2O \uparrow$$

30. 11.00

Sol.

Na_2CO_3	$+ HCl$	$\longrightarrow$	$NaHCO_3$	$+ NaCl$
20	10			
10	–		10	

$pH = pK_{a_2} = 11$

31. 6.40

Sol. $K_3\left[Fe(oxalate)_3\right] \to t_{2g}^3 eg^2$ HS complex

$K_3\left[Ru(oxalate)_3\right] \to t_{2g}^5 eg^o$ LS complex

μ_s of $K_3\left[Fe(oxalate)_3\right] = \sqrt{35}$ BM

μ_s of $K_3\left[Ru(oxalate)_3\right] = \sqrt{3}$ BM

$$\therefore \frac{x-y}{5} = \frac{35-3}{5} = 6.40$$

32. 19.75

Sol.

$$\Delta S^{\ominus} = nF\frac{dE}{dT}$$

$$= 2 \times 96500 \times -5 \times 10^{-5}$$

$$= -2 \times 5 \times 965 \times \frac{1}{1000}$$

$$= -965 \times 10^{-2}$$

$$\Delta G^{\ominus} = -nFE^{\ominus}$$

$$= -2 \times 96500 \times 1.02$$

$$\Delta H^{\ominus} = \Delta G^{\ominus} + T\Delta S^{\ominus}$$

$$\Delta G^{\ominus} = \Delta H^{\ominus} - T\Delta S^{\ominus}$$

$$= -2 \times 965 \times 102 - 300 \times 965 \times 10^{-2}$$

$$= -965(204 + 3)$$

$$= -965 \times 207$$

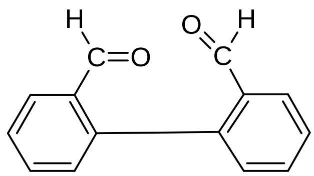
$$= -199755$$

$$= -19.75 \text{ kJ mol}^{-1}$$

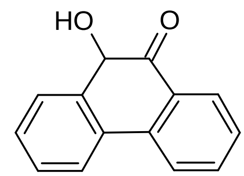
33. 5.67

Sol.

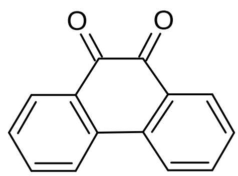
A →



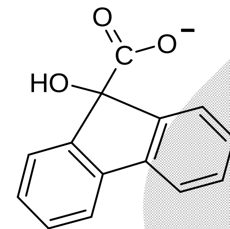
B →



C →



D →



Nuber of C atoms – 14 (x)

Nuber of O atoms – 3 (y)

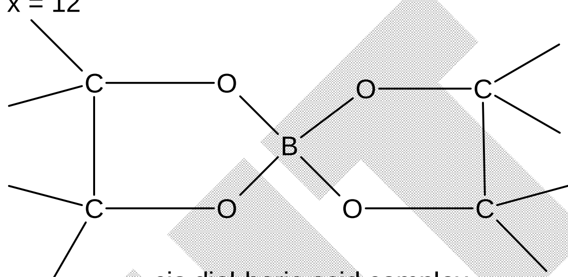
Nuber of N atoms – 0 (z)

$$\therefore \frac{x+y+z}{3} = \frac{17}{3} = 5.67$$

34. 12.00

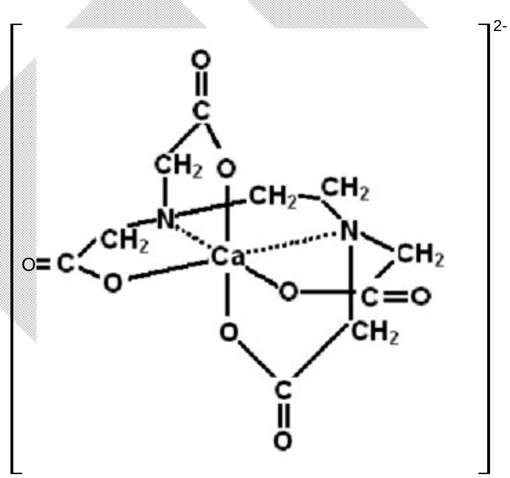
Sol. Fullerene has 12 five membered rings

x = 12



y = 2

cis diol-boric acid complex



z = 5

$$\therefore \frac{x \times y \times z}{10} = \frac{12 \times 2 \times 5}{10} = 12$$

35. 149.85

Sol. $A + B \xrightarrow{k_1} P$

$P \xrightarrow{k_{-1}} A + B$

$P \xrightarrow{k_2} Q$

$\text{Rate} = k_2[P]$ … (1)

Applying steady state assumption on [P]

$\frac{d[P]}{dt} = k_1[A][B] - k_{-1}[P] - K_2[P] = 0$

$\therefore k_1[A][B] = k_{-1}[P] + k_2[P]$

$[P] = \frac{k_1[A][B]}{k_{-1} + k_2}$ … (2)

From (1) and (2)

$\text{Rate} = \frac{k_2 k_1 [A][B]}{k_{-1} + k_2}$

$k_{\text{overall}} = \frac{k_2 k_1}{k_{-1} + k_2}$

$= \frac{10 \times 1.5 \times 10^5}{10^4 + 10}$

$= \frac{1.5 \times 10^6}{10010}$

= 149.85

36. 4.05

Sol. Mixture is containing 70%. Racemic mixture and 30% R-2-methylbutan-1-ol.

Mathematics

PART – III

Section – A

37. A, B, C, D

Sol. The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ can be reduced to $|z|^2 + (g - if)z + (g + if)\overline{z} + c = 0$ by putting $z = x + iy \Rightarrow \overline{z} = x - iy$

(i) Radius of $c_1 = \sqrt{|a|^2 - c}$

(ii) If c_1 and c_2 touch extremely, then distance between centres = $r_1 + r_2$

$\Rightarrow |\overline{a} - \overline{b}| = \sqrt{|a|^2 - c} + \sqrt{|b|^2 - c}$

(iii) Let $a = g_1 + if_1$; $b = g_2 + if_2$

c_1c_2 will cut orthogonally if $2g_1g_2 + 2f_1f_2 = c + d$

Now, $a\overline{b} + b\overline{a} = (g_1 + if_1)(g_2 - if_2) + (g_2 + if_2)(g_1 + if_1) = 2g_1g_2 + 2f_1f_2$

(iv) Radical axis can be obtained by subtracting the equations

38. A, C, D

Sol. Put $x = \omega, \omega^2$

(i) $1 = a_0 + a_1\omega + a_2\omega^2 + a_3\omega^3 + ...$ (1)

(ii) $1 = a_0 + a_1\omega^2 + a_2\omega^4 + a_3\omega^6 + ...$ (2)

By adding equation (1) and (2), we get $2 = 2a_0 - a_1 - a_2 + 2a_3 - a_4 - a_5$

$\Rightarrow 1 = a_0 - \frac{a_1}{2} - \frac{a_2}{2} + a_3 - \frac{a_4}{2} - \frac{a_5}{2} + ...$

39. A, B, C, D

Sol. The given equation can be written as $3 + 2\cos(\beta - \gamma)\ \cos(\gamma - \alpha) + \cos(\alpha - \beta)$

Also, $3 = 1 + 1 + 1 = \cos^2\alpha + \sin^2\alpha + \cos^2\beta + \sin^2\beta + \cos^2\gamma + \sin^2\gamma$

Hence, the equation may be

$$\left(2\cos^2\alpha + \cos^2\beta + \cos^2\gamma + 2\cos\alpha\cos\beta + 2\cos\beta\cos\gamma + 2\cos\gamma\cos\alpha\right) +$$
$$\left(\sin^2\alpha + \sin^2\beta + \sin^2\gamma + 2\sin\alpha\sin\beta + \sin\beta\sin\gamma + \sin\gamma\sin\alpha\right) = 0$$

Hence, $(\cos\alpha + \cos\beta + \cos\gamma)^2 + (\sin\alpha + \sin\beta + \sin\gamma)^2 = 0$

$\Rightarrow \sum\cos\alpha = 0 \Rightarrow \sum\sin\alpha = 0$ and $\sum(\cos\alpha + \sin\alpha) = 0$

40. A, B, C

Sol. $$f'(x) = \begin{vmatrix} -\sin(x+\alpha) & -\sin(x+\beta) & -\sin(x+\gamma) \\ \sin(x+\alpha) & \sin(x+\beta) & \sin(x+\gamma) \\ \sin(\beta-\gamma) & \sin(\gamma-\alpha) & \sin(\alpha-\beta) \end{vmatrix} + \begin{vmatrix} \cos(x+\alpha) & \cos(x+\beta) & \cos(x+\gamma) \\ \cos(x+\alpha) & \cos(x+\beta) & \cos(x+\gamma) \\ \sin(\beta-\gamma) & \sin(\gamma-\alpha) & \sin(\alpha-\beta) \end{vmatrix}$$

$\Rightarrow f'(x) = 0 \Rightarrow f(x)$ is a constant function $\Rightarrow f(\alpha) = f(\beta) = f(\gamma)$

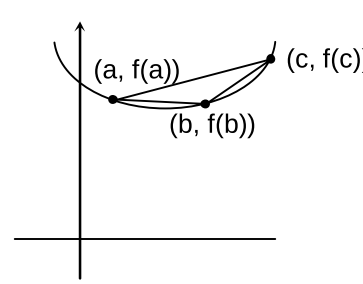
41. B, D

Sol. Let $f(x) = x^n \Rightarrow f'(x) = nx^{n-1} \Rightarrow f''(x) = n(n - 1)x^{n-2}$

For ($n > 1$ or $n < 0$ and $x > 0$) $f''(x) > 0$

$\Rightarrow$ Convexity downward

$\Rightarrow f\left(\frac{a+b+c}{3}\right) < \frac{f(a)+f(b)+f(c)}{3}$



42. A, B

Sol. For $1 \le j \le k$

$$\frac{n}{(n+j)!} \le \frac{n+j-1}{(n+j)!} = \frac{1}{(n+j-1)!} - \frac{1}{(n+j)!}$$

$$n\sum_{j=1}^{k}\frac{1}{(n+j)!} \le \frac{1}{n!} - \frac{1}{(n+k)!} \Rightarrow \lim_{k\to\infty} T(n,k) \le \frac{1}{(n!)n} \le \frac{1}{n!}$$

Now, $T(n,n) = \frac{1}{(n+1)!} + \frac{1}{(n+2)!} + ... + \frac{1}{(2n)!}$

$\Rightarrow T(n,h) < \frac{1}{n!} + \frac{1}{n!} + ... + \frac{1}{n!} = \frac{1}{(n-1)!} \Rightarrow T(n, k)$

43. D

Sol. (I) $(a + c)(a + c - 2b) = (a + c)^2 - 2b(a + c) = (a + c)^2 - 4ac = (a - c)^2$

$\Rightarrow 2\log_e(a-c) = \log_e(a+c) + \log_e(a+c-2b) \Rightarrow$ A.P.

(II) $(a_1 x - a_2)^2 + (a_2 x - a_3)^2 + ... + (a_{n-1} x - a_n)^2 \ge 0$

$\Rightarrow \left(a_1^2 + a_2^2 + a_3^2 ..... a_n^2\right)\left(a_2^2 + a_3^2 ..... a_{n-1}^2\right) \ge \left(a_1a_2 + a_2a_3 + a_3a_4 ..... a_{n-1}a_n\right)$

$\Rightarrow$ Only equality holds when $\frac{a_1}{a_2} = \frac{a_2}{a_3} = \frac{a_3}{a_3}.....$ $\Rightarrow$ G.P.

(III) G.P.

(IV) $x^{18} = y^{21} = z^{28} \Rightarrow 18 \ln x = 21 \ln y = 28 \ln z$

$\Rightarrow \log_y^x = \frac{7}{6}$, $\log_z^y = \frac{4}{3}$, $\log_x^z = \frac{9}{14} \Rightarrow$ A.P.

44. C

Sol. (I) $\frac{1-\tan\alpha}{1+\tan\alpha} = \frac{3\left(3\tan x - \tan^3 x\right)}{1-3\tan^2 x} \Rightarrow 3\tan^4 x - 6\tan^2 x + 8\tan x - 1 = 0$

(II) Least value exist for equilateral triangle

(IV) $\sin x(\sin^2 60 - \sin^2 x) = \frac{1}{4}\sin 3x$

$\sin x + \sin^2 x + \sin^3 x = 1 \Rightarrow \sin x(1 + \sin^2 x) = 1 - \sin^2 x = \cos^2 x$

$(1 - \cos^2 x)(4 - 4\cos^2 x + \cos^4 x) = \cos^4 x \Rightarrow \cos^6 x - 4\cos^4 x + 8\cos^2 x = 4$

45. B

Sol. (I) Both are rectangular hyperbola $\Rightarrow e_1^2 + e_2^2 = 2 + 2 = 4$

(II) $\tan^{-1}\left(\frac{\frac{b}{a}+\frac{b}{a}}{1-\frac{b^2}{a^2}}\right) = \frac{\pi}{3} \Rightarrow \frac{b^2}{a^2} = \frac{1}{3} \Rightarrow e^2 = \frac{4}{3}$

$\frac{1}{e^2} + \frac{1}{e'^2} = 1 \Rightarrow e' = 2$

(III) $OP = r_1$, $OQ = r_2$, $\angle POX = \theta$

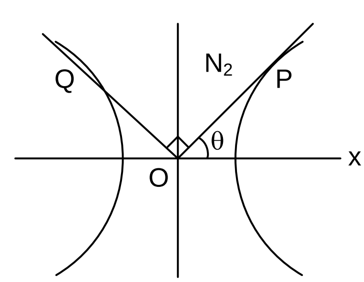
$P \equiv (r_1 \cos\theta, r_1 \sin\theta)$

$Q \equiv (-r_2 \sin\theta, r_2 \cos\theta)$

P and Q lies on $10x^2 - 3y^2 = 1$

$\Rightarrow \frac{1}{r_1^2} = 10\cos^2\theta - 3\sin^2\theta$

$\Rightarrow \frac{1}{r_2^2} = 10\sin^2\theta - 3\cos^2\theta \Rightarrow \frac{1}{r_1^2} + \frac{1}{r_2^2} = 7$



(IV) Normal at $\left(2t, \frac{2}{t}\right)$ is $2t^4 - xt^3 + yt - 2 = 0$

Let slope of normal is $m = t^2$

Let P be (h, k) $\Rightarrow 2t^4 - ht^3 + kt - 2 = 0$

$\Rightarrow 2m^2 - t(hm - k) - 2 = 0 \Rightarrow 4m^4 - h^2m^3 + (2hk - 8)m^2 - k^2m + 4 = 0$

Sum of slopes = 2 $\Rightarrow \frac{h^2}{4} = 2 \Rightarrow h^2 = 8$

46. B

Sol. (I) 54 is multiple of 3 and 2

So, common divisor 2 can be if the other number is multiple of 2 but not multiple of 6

$\Rightarrow$ Total 3000 numbers exist

(II) $3^{2025} = 3(3)^{2024} = 3(81)^{506} = 3(1+80)^{506} = 3(1+506\times80+100k)$

$= 3(1 + 80 + 505 \times 80 + 1001k) = 3(81) + 100k = 243 + 100k$

10^{th} place = 4

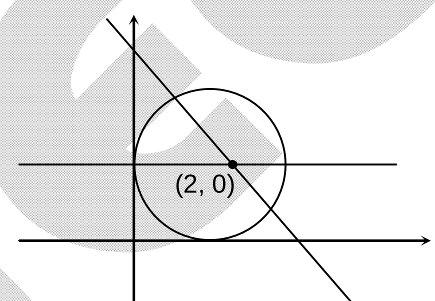
(III) $|z - 2| = 2$ is circle with centre (2, 0) and r = 2

$z(1-i)+\bar{z}(1+i)=4$

$\Rightarrow (z+\bar{z})-i(z-\bar{z})=4 \Rightarrow 2x - i(2iy) = 4$

$\Rightarrow 2x + 2y = r \Rightarrow x + y = 2$

i.e., 2 common points



(IV) Counting of terms are $1025 \ge 2^\circ + 2^1 + 2^2 + 2^3 + ... + 2^k$

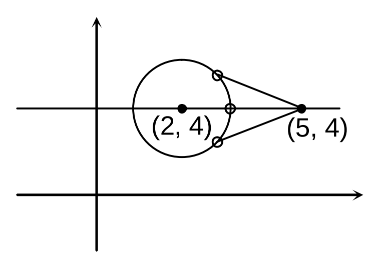
$1025 \ge \frac{2^k - 1}{2-1}$

$1026 \ge 2^k \Rightarrow k = 11$

Section – B

47. 3.00

Sol. Given in the figure



48. 10.50

Sol. $n^3 = 5x_3$, $m^2 = 2x_3$

$\frac{n^3 + m^2}{2} = \frac{7x_3}{2} = \frac{7\times3}{2} = 10.5$

Since, minimum value of $x_3 = 3$

49. 9.00

Sol. $g'(x) = f'(x) - 2f'(2x)$

$g'(1) = f'(1) - 2f'(2) = 5$

$g'(2) = f'(2) - 2f'(4) = 7$

$g'(1) + 2g'(2) = f'(1) - 4f'(4) = 19$

$h'(x) = f'(x) - 4f'(4x) - 10$
$h'(1) = f'(1) - 4f'(4) - 10 = 19 - 10 = 9$

50. 0.00

Sol. $g(f(x)) = x \Rightarrow g'(f(x))\, f'(x) = 1 \Rightarrow g''(f(x)) = \dfrac{-f''(x)}{(f'(x))^3}$

$\Rightarrow g'(3) = \dfrac{1}{23}$ and $g''(3) = \dfrac{-43}{5(23)^3}$

51. 0.00

Sol. $\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \dfrac{f(x)\cdot e^{\frac{x^2}{2}}}{e^{\frac{x^2}{2}}} = \lim\limits_{x\to\infty} \dfrac{f''(x) + 2xf'(x) + (x^2+1)f(x)}{(1+x^2)} \Rightarrow \lim\limits_{x\to\infty} f(x) = 0$

52. 3.00

Sol. $f(x) = 1 - (a \cos x + b \sin x) - (A \cos 2x + B \sin 2x)$

$\Rightarrow f(x) = 1 - \sqrt{a^2+b^2}\cos(x-\alpha) - \sqrt{A^2+B^2}\sin(2x-\beta)$

$\Rightarrow f\left(\dfrac{\pi}{4}+\alpha\right) = 1 - \dfrac{\sqrt{a^2+b^2}}{\sqrt{2}} - \sqrt{A^2+B^2}\cos(2\alpha-\beta)$

$\Rightarrow f\left(\alpha-\dfrac{\pi}{4}\right) = 1 - \dfrac{\sqrt{a^2+b^2}}{\sqrt{2}} + \sqrt{A^2+B^2}\cos(2\alpha-\beta)$

$\Rightarrow f\left(\alpha-\dfrac{\pi}{4}\right) + f\left(\alpha+\dfrac{\pi}{4}\right) = 2 - \dfrac{2\sqrt{a^2+b^2}}{2} \ge 0 \Rightarrow \sqrt{a^2+b^2} \le \sqrt{2}$

Similarly, $f(\beta) + f(\pi+\beta) = 2 - 2\sqrt{A^2+B^2} \ge 0 \Rightarrow A^2 + B^2 \le 1$

53. 4.00

Sol. Applying $R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$

$\Delta = \begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ [x] & [y] & [z]+1 \end{vmatrix} \Rightarrow [z] + [y] + [x] + 1 = \Delta$

Maximum value of $\Delta = 2 + 0 + 1 + 1 = 4$

54. 0.00

Sol. $2\left[\cos((\alpha+\theta)-(\beta+\theta)) + \cos((\beta+\theta)-(\gamma+\theta)) + \cos(\gamma+\theta)-(\alpha-\theta)\right] + 3 = 0$

$\Rightarrow 2\sum\cos(\alpha+\theta)\cos(\beta+\theta) + \sum\cos^2(\alpha+\theta) + 2\sum\sin(\alpha+\theta)\sin(\beta+\theta) + \sum\sin^2(\beta+\theta)$

$\Rightarrow \left(\sum\sin(\alpha+\theta)\right)^2 + \left(\sum\sin(\alpha+\theta)\right)^2 = 0$

$\Rightarrow \sin(\alpha + \theta) + \sin(\beta + \theta) + \sin(\gamma + \theta) = 0$

$\Rightarrow d\sin(\alpha + \theta)\ d\sin(\beta + \theta) + d\sin(\gamma + \theta) = 0$

$\Rightarrow \cos(\alpha + \theta)\, d\alpha + \cos(\beta + \theta)\, d\beta + \cos(\gamma + \theta)\, d\gamma = 0$

and $\sin(\alpha + \theta)\, d\alpha + \sin(\beta + \theta)\, d\beta + \sin(\gamma + \theta)\, d\gamma = 0$