# FIITJEE
# ALL INDIA TEST SERIES
## JEE (Advanced)-2023
## FULL TEST – IX
## PAPER –2
## TEST DATE: 14-05-2023

**Time Allotted: 3 Hours** **Maximum Marks: 180**

**General Instructions:**

- The test consists of total 54 questions.
- Each subject (PCM) has 18 questions.
- This question paper contains **Three Parts.**
- **Part-I** is Physics, **Part-II** is Chemistry and **Part-III** is Mathematics.
- Each **Part** is further divided into **Two Sections**: **Section-A & Section-B**.

**Section – A *(01 – 04, 19 – 22, 37 – 40)*:** This section contains **TWELVE (12)** questions. Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.

**Section – A *(05 –10, 23 – 28, 41 – 46)*:** This section contains **EIGHTEEN (18)** questions. Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).

**Section – B *(11 – 18, 29 – 36, 47 – 54)*:** This section contains **TWENTY FOUR (24)** numerical based questions. The answer to each question is a **Single Digit Integer, ranging from 0 to 9 both inclusive.**

**MARKING SCHEME**

**Section – A (Single Correct):** Answer to each question will be evaluated according to the following marking scheme:

| | | | |
|---|---|---|---|
| *Full Marks* | : | +3 | If ONLY the correct option is chosen. |
| Zero Marks | : | 0 | If none of the options is chosen (i.e. the question is unanswered); |
| *Negative Marks* | : | –1 | In all other cases. |

**Section – A (One or More than One Correct):** Answer to each question will be evaluated according to the following marking scheme:

| | | | |
|---|---|---|---|
| Full Marks | : | +4 | If only (all) the correct option(s) is (are) chosen; |
| Partial Marks | : | +3 | If all the four options are correct but ONLY three options are chosen; |
| Partial marks | : | +2 | if three or more options are correct but ONLY two options are chosen and both of which are correct; |
| Partial Marks | : | +1 | If two or more options are correct but ONLY one option is chosen and it is a correct option; |
| Zero Marks | : | 0 | If none of the options is chosen (i.e. the question is unanswered); |
| Negative Marks | : | –2 | In all other cases. |

**Section – B:** Answer to each question will be evaluated according to the following marking scheme:

| | | | |
|---|---|---|---|
| Full Marks | : | +3 | If ONLY the correct integer is entered; |
| Zero Marks | : | 0 | Question is unanswered; |
| Negative Marks | : | –1 | In all other cases. |

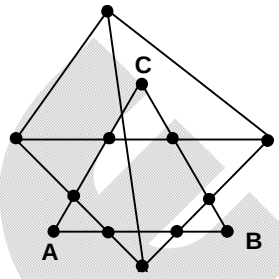
# Physics

## PART – I

### Section – A (Maximum Marks: 12)

*This section contains **FOUR (04)** questions. Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.*

1. Resistance of each wire connecting any two adjacent dots is R. The equivalent resistance between A and B is... [wires are connected only at the dot points]




(A) $\frac{8}{3}R$ (B) $\frac{8}{5}R$

(C) $\frac{8}{7}R$ (D) $\frac{8}{9}R$

2. Two thin lenses of different nature having radius of curvature $R_1 = R$ (convex) and $R_2 = R$ (concave) respectively. The lenses are put close together and immersed in water. The converging focal length of the system is $\frac{R}{2}$. Find the refractive index of the concave lens, if refractive index of convex lens is twice of concave lens. (Refractive index of water = 4/3)

(A) 4/3 (B) 8/3

(C) 3/2 (D) 5/3

3. A straight solenoid of length $\ell$ having a single layer winding of copper wire whose total mass is equal to m. The cross-sectional diameter of the solenoid is assumed to be very less than its length. Take resistivity of copper wire to be $\rho$ and density is equal to d. Then find the ratio of $\frac{L}{R}$. ($L \rightarrow$ inductance of solenoid and $R \rightarrow$ resistance of solenoid)

(A) $\frac{\mu_0 m}{4\pi\rho d\ell}$ (B) $\frac{\mu_0 m}{2\pi\rho d\ell}$

(C) $\frac{2\mu_0 m}{\pi\rho d\ell}$ (D) $\frac{\mu_0 m}{\pi\rho d\ell}$

4. An infinite long hollow metallic cylinder of radius R and surface charge density $\sigma$ is placed symmetrically with an imaginary surface of the shape of a prism. The length of prism is R and its three sides are all equal to 3R. The flux through the prism is

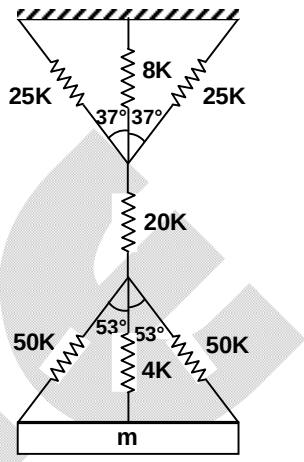
(A) $\frac{\pi R^2\sigma}{3\varepsilon_0}$ (B) $\frac{\pi R^2\sigma}{2\varepsilon_0}$

(C) $\frac{\pi R^2\sigma}{\varepsilon_0}$ (D) $\frac{2\pi R^2\sigma}{\varepsilon_0}$

**Section – A (Maximum Marks: 24)**

*This section contains **SIX (06)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).*

5. Initially system is in equilibrium, the plank is slightly displaced along vertical direction and then released. Then choose the correct option(s).




(A) Extension in the spring of spring constant 20k in equilibrium is $\frac{mg}{20k}$

(B) Extension in the spring of spring constant 20k in equilibrium is $\frac{mg}{10k}$

(C) The period of oscillation of the plank is $\frac{\pi}{5}\sqrt{\frac{m}{k}}$

(D) The period of oscillation of the plank is $2\pi\sqrt{\frac{m}{10k}}$

6. Light from a discharge tube containing hydrogen atoms falls on the surface of a piece of sodium. The kinetic energy of the fastest photo electron emitted from sodium is 5.23 eV. The work function for sodium is 7.52 eV. Then choose the correct option(s).

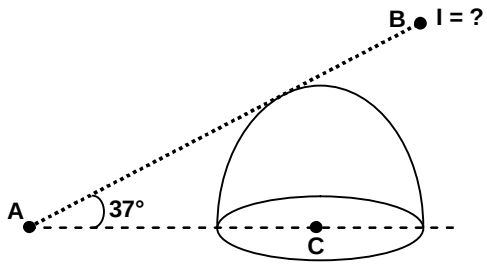
(A) The energy of photons causing the photo electric emission is 12.75 eV

(B) The quantum numbers of two levels involved in the emission of these photons are 1 and 4.

(C) The change in the angular momentum of the electron in the hydrogen atom in the above transition is $\frac{h}{\pi}$

(D) The change in the angular momentum of the electron in the hydrogen atom in the above transition is $\frac{3h}{2\pi}$

7. A hollow hemisphere of mass m and radius R lies as shown in the figure. The axis AC is passing through the centre C of the hollow hemisphere and line AB is tangent on the hollow hemisphere. Then choose the correct option(s).



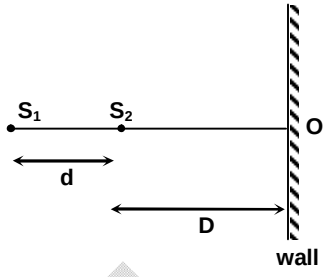

(A) Moment of inertia about an axis passing through the centre C and parallel to the line AB is $\frac{2}{3}mR^2$

(B) Moment of inertia about an axis passing through the centre of mass of hollow hemisphere and parallel to the line AB is $\frac{11}{12}mR^2$

(C) Moment of inertia about the line AB is $\frac{13}{15}mR^2$

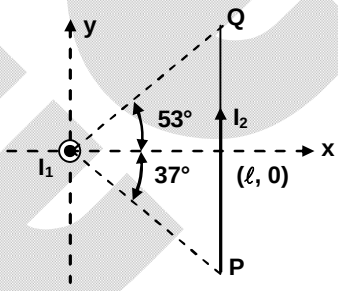
(D) Moment of inertia about the line AB is $\frac{2}{3}mR^2$

8. Two point sources are placed on a straight line separated by distance d = 10λ where λ is wavelength of the light produced by sources. Both sources having same frequency and are placed at a distance D(>>d) = 2m from a wall which is perpendicular to straight line as shown in the figure. Both the sources are sending waves of equal intensity. Then which of the following is/are correct(s).




(A) The locus of point on the wall having equal intensity are circle.
(B) No. of dark rings will be observed on the wall are 10.
(C) Radius of 4th bright ring form O is $\frac{8}{3}$ m (exclude the position of O)
(D) Radius of 4th bright ring from of is $\sqrt{21}$ m (exclude the position of O)

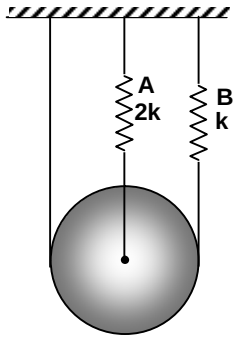
9. An infinite wire, placed along z-axis has current $I_1$ in positive z-direction. A conducting rod PQ placed in xy plane parallel to y-axis has current $I_2$ in positive y-direction. The ends of the rod subtend angles 37° and 53° at the origin with positive x-axis as shown in figure. The rod is at a distance $\ell$ from the origin.




Then choose the correct option(s). $\left[\text{Given } \tan 37° = \frac{3}{4}\right]$

(A) The direction of net force acting on the wire PQ is along the negative z-axis.
(B) The direction of net force acting on the wire PQ is along the positive z-axis.
(C) The magnitude of net force acting on the rod PQ is $\frac{\mu_0 I_1 I_2}{4\pi}\ln\left(\frac{16}{9}\right)$
(D) The magnitude of net force acting on the rod PQ is $\frac{\mu_0 I_1 I_2}{2\pi}\ln\left(\frac{16}{9}\right)$

10. Initially (disc) is hold such that both spring is in its natural length and then released (assume that there is no slipping between the string and the pulley). Mass of the pulley is m and radius is R. Then choose the correct option(s).

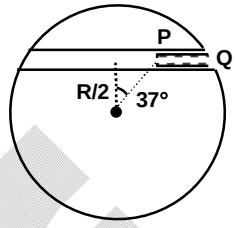



(A) Maximum displacement of centre of mass of the pulley is $\frac{2mg}{3k}$
(B) Maximum extension in the spring B is $\frac{2mg}{3k}$
(C) The magnitude of velocity centre of mass of the pulley when extension in the spring B is half of its maximum value, is $v = \frac{g}{3}\sqrt{\frac{m}{k}}$
(D) Angular velocity of the pulley is not zero when the extension in the spring A is maximum

**Section – B (Maximum Marks: 24)**

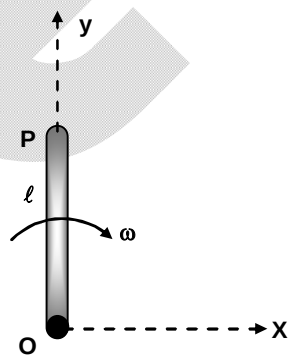
*This section contains **EIGHT (08)** numerical based questions. The answer to each question is a **Single Digit Integer, ranging from 0 to 9 both inclusive.***

11. A thin tunnel is dug along the chord of earth at a distance R/2 from the centre of the earth. A tube PQ which is fully filled with liquid fixed along the tunnel. One end of the tube is at the point P of the tunnel have cross-sectional area $A_1$ whereas other end Q of the tube is at the surface of the earth of cross section area $A_2$ ($A_1 \ll A_2$) as shown in the figure. The initial efflux velocity of the liquid at point P is $\frac{1}{k}\sqrt{\frac{39GM}{R}}$. Find the value of k.



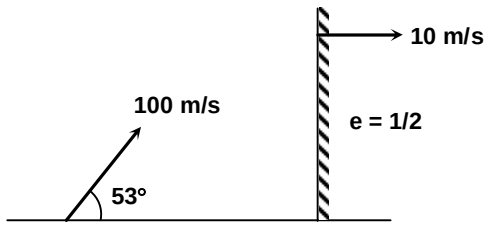

12. A string of 0.2 kg/m and length 1.2 m is fixed at both ends are stretched such that tension generated in the string is 320 N. The string vibrates in six segments with amplitude of 0.25 cm. Then maximum transverse velocity of a particle which is at a distance of $\frac{1}{30}$m from the fixed end is $5k\pi$ cm/s. Find the value of k.

13. A conductor OP of length $\ell$ placed in x-y plane with one end hinged at origin. In this region a non-uniform magnetic field exist along positive z-direction of which magnitude depends on its y-coordinate which is given as $B = B_0\left(1+\frac{y^2}{\ell^2}\right)$ tesla. If conductor OP starts rotating with angular velocity $-\omega_0\hat{k}$. Then the potential difference across OP is $\frac{n}{4}B_0\omega\ell^2$ when the conductor lies along y-axis. Find the value of n.

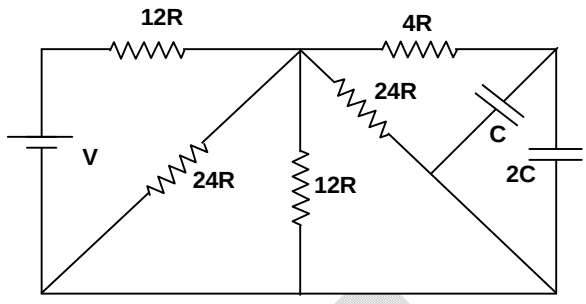



14. Two identical gases whose adiabatic exponent is $\gamma$ are filled in two identical containers at equal pressure. In both the containers the volume of gas is doubled. In first container it is done by isothermal process and in the second container it is done by adiabatic process. The condition for which the work done by the gas in two expansion is same, is $x-(2)^y = z(\ell n2)$. Find the value of (x + y + z).

15. A ball is projected with velocity 100 m/s at an angle of 53° with the horizontal towards a vertical wall (massive mass) and wall is moving away from the projection point with velocity 10 m/s as shown in the figure. The coefficient of restitution between ball and wall is $e = \frac{1}{2}$. Ball collide with the wall and comes back at the point of projection. If initial separation between ball and wall along horizontal direction is (32k) meter. Then find the value of k. (Take g = 10 m/s$^2$ and tan 37° = 3/4)



16. In the circuit shown, the battery is an ideal one with emf V volts. The time constant of the circuit is 4nRC. Find the value of n.




17. Two vernier calipers A and B both have M.S.D. of 1 mm. 8 vernier scale division have same length as 2 main scale division of vernier A and 5 vernier scale division have same length as 3 main scale division for vernier B. The magnitude of difference in least count of vernier A and vernier B is $\frac{k}{100}$ mm. Find the value of k.

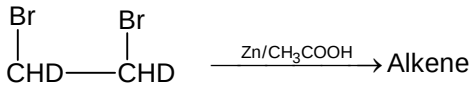
18. When a soap bubble of radius r is charged to a uniform density $\sigma$, its radius increases to triple value. Assume temperature remains constant during expansion and atmospheric pressure as $P_0$. The pressure inside the initially uncharged bubble of radius r is $P = \left(\frac{\sigma^2}{X\varepsilon_0} - \frac{Y}{4}P_0\right)$. Find the value of $(X - 5Y)$. [Charge resides only on the outer surface of the bubble]

# Chemistry

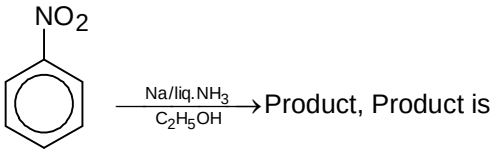
## PART – II

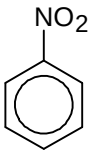
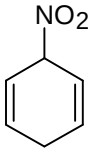
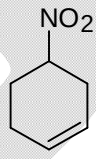
### Section – A (Maximum Marks: 12)

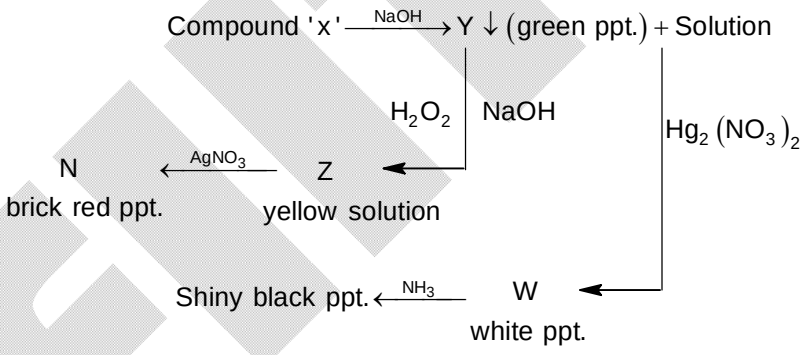
*This section contains **FOUR (04)** questions. Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.*

19. 

Which of the following option is incorrect?
(A) It is an example of anti elimination
(B) Reaction follows $E_2$ mechanism
(C) If substrate is meso then product will be trans alkene
(D) If substrate is 'd' or '$\ell$' then product will be trans alkene

20. 

(A) 
(B) 
(C) 
(D) 

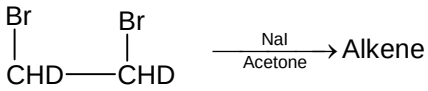
21. 

What is compound 'x'?
(A) $NiCl_3$
(B) NiS
(C) $CrCl_3$
(D) $Cr_2(SO_4)_3$

22. The radial probability distribution curve of an orbital of 'H' atom has 3 local maxima. If orbital has 2 angular node then orbital will be
(A) 5d
(B) 5f
(C) 6p
(D) 7f

## Section – A (Maximum Marks: 24)

*This section contains **SIX (06)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).*

23. 

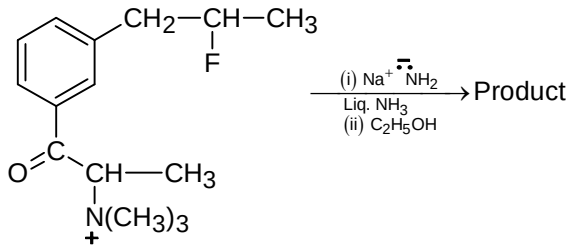

Which option(s) is/are correct?

(A) Product is an example of syn elimination.

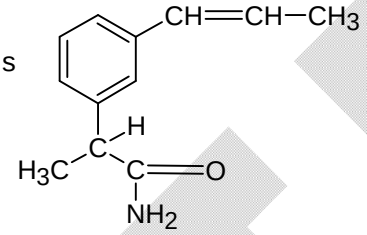
(B) Reaction takes place via $S_N2$ followed by $E_2$ mechanism.

(C) If substrate is meso then product will be cis alkene.

(D) If substrate is d or $\ell$ isomer then product will be trans alkene.

24. 


Which of the following statement(s) is/are correct?

(A) Product is 


(B) One of the intermediate is carbanion.

(C) Rearrangement takes place via carbene formation.

(D) One of the product is $3^o$ Amine.

25. Consider the following equilibria

1. $P(s) \rightleftharpoons Q(g) + NH_3(g)$, $P_T$ at equilibrium $= 8$ cm Hg
2. $Y(s) \rightleftharpoons Z(g) + NH_3(g)$, $P_T$ at equilibrium $= 26$ cm Hg

Thus, choose the correct option(s)

(A) $K_{eq}(I) = 16$

(B) $K_{eq}(II) = 169$

(C) $P_{Total} = 27.20$ when P and Y decomposed together

(D) $P_{Total} = 34.0$ when P and Y decomposed together

26. In a constant pressure calorimeter with heat capacity of $450\ JK^{-1}$, 200 ml of 0.9 M HCl and 200 ml of 0.45 M $Ba(OH)_2$ are mixed. (density $= 1$ gm $ml^{-1}$). Initially, both solutions are at $21^oC$. Select correct statement $\left[\text{Given}: \Delta H^o_{Neutralization} = -56.2 \text{ kJ mol}^{-1}\right]$

Specific heat of $H_2O = 4.184$ Joule $gm^{-1}$.

(A) $\Delta H^o$ of reaction is $10.116 \times 10^3$ J

(B) Some HCl is left unreacted

(C) Final temperature is $25.76^oC$

(D) Some $Ba(OH)_2$ is left unreacted

27. Which of the following method(s) can be used for the preparation of anhydrous aluminium chloride?
(A) Heating $AlCl_3.6H_2O$ with $SOCl_2$
(B) Heating a mixture of alumina and coke in a current of dry chlorine
(C) Passing dry chlorine over heated aluminium
(D) Passing dry HCl gas over heated aluminium powder

28. Basic radical(s) which can not be identified by Borax bead test:
(A) $Fe^{3+}$ (B) $Ag^{+}$
(C) $Co^{3+}$ (D) $Mg^{2+}$

**Section – B (Maximum Marks: 24)**

*This section contains **EIGHT (08)** numerical based questions. The answer to each question is a **Single Digit Integer, ranging from 0 to 9 both inclusive.***

29. What is the density in gm $ml^{-1}$ of urea solution. If its molarity is 4.0 mol $lit^{-1}$ and molality is 2.273 mol $kg^{-1}$.

30. A metal 'x' exists in the bcc structure having empty space between the atoms along the edge equal to 1.35 $\overset{o}{A}$. If molar mass of the metal is 600 gm/mole, then calculate density of the crystal in gm/ml.
$\left[\begin{array}{l}\text{Given } \sqrt{3}=1.73 \\ \qquad N_{av}=6.0\times10^{23}\end{array}\right]$

31. A radioactive element decays by following three different parallel paths:
$A \xrightarrow{\lambda_1} B \quad \lambda_1 = 6\times10^{-2}\ sec^{-1}$
$2A \xrightarrow{\lambda_2} C \quad \lambda_2 = 2.5\times10^{-2}\ sec^{-1}$
$3A \xrightarrow{\lambda_3} D \quad \lambda_3 = 5\times10^{-3}\ sec^{-1}$
Average life of element is $=\dfrac{1}{\lambda}$
Where $\lambda$ is net rate constant of the net decay reaction of A. $\lambda_1,\lambda_2,\lambda_3$ are decay constant for respective reaction. Calculate the average life of decay in sec.

32. In a particular case of physisorption, magnitude of enthalpy change and entropy change were observed to be 27.6 kJ/mole and 100 J/mole K. Calculate the minimum temperature (in $^oC$) above which physisorption will become non spontaneous.

33. The EMF of the cell
$M\,|\,M^{n+}(0.01\,M)\,||\,H^{+}(1\,M)\,|\,H_2(g)(1\,atm)\,|\,Pt$
At $25^o$ C is 0.58 V. Calculate the valency of the metal if the standard oxidation potential of the metal is 0.521 V

34. For a fixed amount of a real gas when a graph of Z vs P was plotted then at very high pressure slope was observed to be 0.02 $atm^{-1}$ at the same temperature, if a graph is plotted between PV vs. P then for 2 moles of the gas the y-intercept is found to be 50 atm litre. Calculate excluded value in litres for 10 moles of the real gas.

35. Mond's process take place in following steps

$$Ni + CO(\text{excess}) \xrightarrow{Ti} \left[Ni(CO)_x\right]$$

$$\left[Ni(CO)_x\right] \xrightarrow{T_2} Ni + CO$$

When $T_2 > T_1$, what is the value of x in complex.

36. The ratio of cis and trans-isomer of the complex $\left[Ma_2bcde\right]^{n\pm}$ is 'x', then value of 'x' is

# Mathematics

## PART – III

### Section – A (Maximum Marks: 12)

*This section contains **FOUR (04)** questions. Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.*

37. Number of $3 \times 3$ matrices that can be formed using first 9 natural numbers atmost once such that sum of elements in atleast one row or column is greater than 21 is
(A) $12 \times 6! \times 4!$ (B) $6 \times 6! \times 4!$
(C) $177 \times 4! \times 4!$ (D) $119 \times 3! \times 3! \times 4!$

38. Let tangents be drawn from any point P on directrix of parabola S : $y^2 = 4x$ to S intersecting it at points A and B. Locus of circumcentre of $\Delta PAB$ is
(A) $y^2 = 2x - 2$ (B) $y^2 = x - 1$
(C) $y^2 = 4x - 4$ (D) none of these

39. Let $S = \{(\alpha, \beta, \gamma) : \alpha, \beta, \gamma \in \{1, 2, 3, ....., 9\}\}$ . Number of elements in S for which there are 2 distinct solutions of the set of equations $x + y + z = 0$, $\alpha x + \beta y + \gamma z = 0$, $\alpha^2 x + \beta^2 y + \gamma^2 z = 0$ is
(A) 9 (B) 45
(C) 108 (D) 225

40. A student gave 20 JEE Advanced mock tests over a month. Given below are his marks in some of those tests.

| Test Numbers | 1 | 5 | 10 | 15 | 20 |
|---|---|---|---|---|---|
| Marks | 10 | 30 | 60 | 100 | 180 |

Given that the student kept on improving his score in every subsequent mock test, his average marks over the given month lies in the interval
(A) (60, 76) (B) (60, 67.5)
(C) (67.5, 92.5) (D) (58.5, 91.5)

### Section – A (Maximum Marks: 24)

*This section contains **SIX (06)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).*

41. Let 'f' be a continuous function on R satisfying the relation $f(f(f(x))) = x \ \forall \ x \in R$, then
(A) there exists 'f' which is non-decreasing function
(B) there exists 'f' which is non-increasing function
(C) 'f' has to be differentiable $\forall \ x \in R$
(D) there exists only one function that satisfies the given conditions

42. Area bounded by $y = \sin^{-1}\left[\cos^{-1} x\right] + \cos^{-1}\left[\sin^{-1} x\right]$ with x-axis is given by
$a \sin 1 + b \cos 1 + c \sin 2 + d \cos 2 + e$, then (where [.] represents greatest integer function)
(A) a = b (B) c = d
(C) $a + b = \frac{3}{2} d$ (D) a + c = b + e

43. Let a relation R be defined on real numbers such that $R = \left\{(a, b) \left| \frac{a}{b} \in I; b \in \left\{\sqrt{6}, \sqrt{10}, \sqrt{15}\right\}\right.\right\}$, then
(A) domain of R consists of irrational numbers only
(B) R is not a reflexive relation but is a transitive relation
(C) R is equivalence relation
(D) $R^{-1}$ is not a reflexive relation but is a transitive relation

44. Let f(r) be area bounded by the graphs $y = \sin^{-1}(\sin x)$ and $y = \frac{\pi}{2} - \sqrt{r^2 - \left(x - \frac{\pi}{2}\right)^2}$ $(r \geq 0)$, then

(A) f(r) is increasing function $\forall\ r \geq 0$
(B) f(r) is continuous and differentiable function $\forall\ r \geq 0$
(C) number of solutions of the equation $f(r) = \frac{\pi r^2}{2}$ form an arithmetic progression
(D) if $r = r_1, r_2, r_3, .....$ be critical points of y = f(r), then $r_1, r_2, r_3$ ..... form a geometric progression

45. Let '÷' be an operator defined for vectors such that $\vec{a} \div \vec{b} = \vec{a} \times \vec{b} + \vec{a} + \vec{b}$. Let $\vec{x}, \vec{y}, \vec{z}$ be three coplanar vectors, then

(A) $\vec{x} \div (\vec{y} \div \vec{z}) = (\vec{x} \div \vec{y}) \div \vec{z}$
(B) $\vec{x}, \vec{y}, \vec{x} \div \vec{y}$ are always coplanar
(C) $\vec{x} \div \vec{y}, \vec{y} \div \vec{z}, \vec{z} \div \vec{x}$ are always coplanar
(D) none of these

46. Let the planes passing through line $L : \vec{r} = -\frac{2}{3}\hat{j} + \frac{2}{3}\hat{k} + \lambda(\hat{i} - \hat{k})$ and at farthest possible distance from origin be $P_1$. Let $P_2$ be the plane passing through L and at nearest possible distance from origin. Let r, R be minimum and maximum possible radius of spheres passing through $(1, \sqrt{2} - 2, -1)$ and touching both the planes $P_1$ and $P_2$ respectively, then

(A) $r = 2\sqrt{3} - \frac{4}{\sqrt{3}}$
(B) $r = 2\sqrt{3} - 4\sqrt{\frac{2}{3}}$
(C) $R = \frac{2}{\sqrt{3}}$
(D) $R = 2\sqrt{3}$

**Section – B (Maximum Marks: 24)**

*This section contains **EIGHT (08)** numerical based questions. The answer to each question is a **Single Digit Integer, ranging from 0 to 9 both inclusive.***

47. Number of solutions of the equation $xyz\omega = 2^5 3^7 5^4$; x, y, z, $\omega \in N$ such that HCF of x, y, z, $\omega$ equals 1 is k, then $\frac{k}{22100}$ equals

48. Cards from a deck of 52 playing cards are distributed among 4 players equally. Probability that each of them receives a Ace, King, Queen and Jack of different suit is P, then $P \times \frac{52! \times (9!)^4}{4! \times 2 \times 36! \times (13!)^4}$ equals

49. If $\sum\sum\sum_{x+y+z=n} {}^nC_x\ {}^nC_y\ {}^nC_z = {}^pC_q$, then highest possible value of $\frac{p+q}{n}$ is

50. Let $P_1\left(2, \frac{1}{2}\right)$, $P_2\left(3, \frac{1}{3}\right)$, $P_3\left(4, \frac{1}{4}\right)$. Let $P_n$ be orthocentre of triangle formed by point $P_{n-1}$, $P_{n-2}$ and $P_{n-3}$ $\forall\ n \geq 4$. Abscissa of $P_{2023}$ is

51. Let $S : x^2 + y^2 = 1$. Tangents are drawn to the circle at A, B, C which form a triangle DEF such that A lies on DE, B lies on DF and C lies on EF respectively. Let G, H, I be mirror images of A in OF, B in OE and C in OD respectively. Radius of circumcircle of $\Delta GHI$ is

52. Let the range of eccentricity for which a square having sides parallel to coordinate axes can be inscribed in ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (a, b $\in$ R) be $(e_1, e_2)$, then $9(e_2 - e_1)$ equals

53. Let there be a cuboid having volume V and surface area A. If $V \le \left(\frac{A}{\lambda}\right)^{\frac{3}{2}}$, then $\lambda$ equals

54. Number of roots of the equation $z^{10} - 2z^7 - z^6 + 2z^4 + 2z^3 - 2 = 0$ for which imaginary part is non-negative is