

**FIITJEE**  
**ALL INDIA TEST SERIES**  
**JEE (Advanced)-2023**  
**FULL TEST – IX**  
**PAPER –1**  
**TEST DATE: 14-05-2023**

**Time Allotted: 3 Hours**

**Maximum Marks: 180**

**General Instructions:**

- The test consists of total 54 questions.
- Each subject (PCM) has 18 questions.
- This question paper contains **Three Parts**.
- **Part-I** is Physics, **Part-II** is Chemistry and **Part-III** is Mathematics.
- Each **Part** is further divided into **Two Sections: Section-A & Section-B**.

**Section – A (01 –06, 19 – 24, 37 – 42):** This section contains **EIGHTEEN (18)** questions. Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).

**Section – A (07 – 10, 25 – 28, 43 – 46):** This section contains **TWELVE (12)** Matching List Type Questions. Each question has **FOUR** statements in **List-I** entries (I), (II), (III) and (IV) and **FIVE** statements in **List-II** entries (P), (Q), (R), (S) and (T). The codes for lists have choices (A), (B), (C), (D) out of which, **ONLY ONE** of these four options is correct answer.

**Section – B (11 – 18, 29 – 36, 47 – 54):** This section contains **TWENTY FOUR (24)** numerical based questions. The answer to each question is a **NUMERICAL VALUE**. If the numerical value has more than two decimal places, truncate/round-off the value to **TWO** decimal places.

**MARKING SCHEME**

**Section – A (One or More than One Correct):** Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+4	If only (all) the correct option(s) is (are) chosen;
Partial Marks	:	+3	If all the four options are correct but <b>ONLY</b> three options are chosen;
Partial marks	:	+2	if three or more options are correct but <b>ONLY</b> two options are chosen and both of which are correct;
Partial Marks	:	+1	If two or more options are correct but <b>ONLY</b> one option is chosen and it is a correct option;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	-2	In all other cases.

**Section – A (Single Correct):** Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+3	If <b>ONLY</b> the correct option is chosen.
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	-1	In all other cases.

**Section – B:** Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+3	If <b>ONLY</b> the correct numerical value is entered at the designated place;
Zero Marks	:	0	In all other cases.

# Physics

## PART – I

### Section – A (Maximum Marks: 24)

This section contains **SIX (06)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).

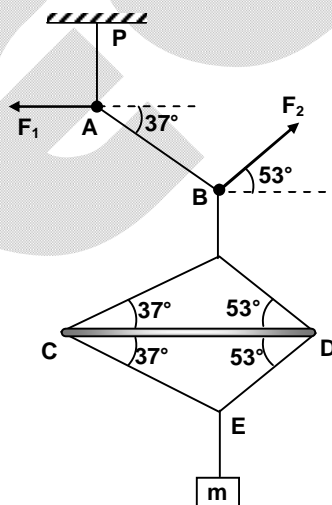
1. A voltmeter of resistance  $R_1$  and an ammeter of resistance  $R_2$  are connected in series across a battery of emf  $V = 450$  volts and having internal resistance  $5\Omega$ . When a resistance  $12\Omega$  is connected in parallel to voltmeter, reading of ammeter increases four times while that of voltmeter reduces to one fourth. Then choose the correct option(s).

- (A) The resistor  $R_1 = 180\Omega$   
 (B) The resistor  $R_2 = 40\Omega$   
 (C) Current read by the ammeter is 3 ampere when a  $12\Omega$  resistance is not connected  
 (D) Current read by the ammeter is 8 ampere when a  $12\Omega$  resistance is connected.

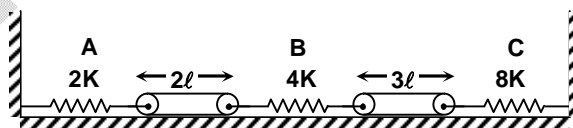
2. Block is in equilibrium and the tension in the string AB is 300 N. Then choose the correct option(s). (Take  $g = 10$

$\text{m/s}^2$ ,  $\tan 37^\circ = \frac{3}{4}$  and neglect the mass of the rod CD)

- (A) The tension in the string CE is 300 N  
 (B) The ratio of force  $F_2$  and  $F_1$ ,  $\left(\frac{F_2}{F_1}\right)$  is  $\frac{5}{3}$ .  
 (C) Force of compression in the rod CD is 480 N.  
 (D) Mass of the block is 50 Kg



3. Two rigid rods of same material of length  $2l$  and  $3l$  are placed in equilibrium on a smooth horizontal surface at temperature  $T$  as shown in figure. Springs are in its natural length. If the temperature is increased by  $\Delta T$ , then which of the following is/are correct(s) [coefficient of linear expansion of material of the rods is  $\alpha$  and neglect the change in springs constant due to increase in temperature]



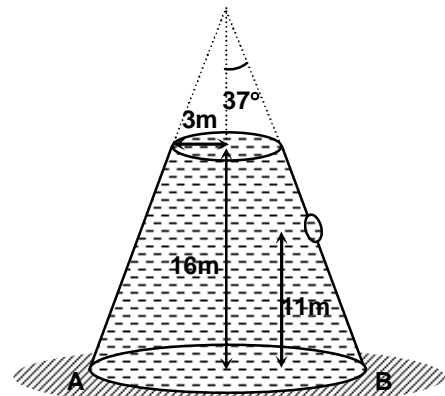
- (A) Compression in the spring B is  $\frac{10}{7}l\alpha\Delta T$   
 (B) Compression in the spring C is  $\frac{5l\alpha\Delta T}{7}$   
 (C) Energy stored in the spring A is  $\frac{400K(l\alpha\Delta T)^2}{49}$   
 (D) Total energy stored in all the springs is  $\frac{100}{7}K(l\alpha\Delta T)^2$

4. Electric field intensity at a point  $(0, y, z)$  is given by  $\vec{E} = (24yz^3 - 8y)\hat{j} + 36y^2z^2\hat{k}$  where all parameters are in S.I. units. Then choose the correct statements out of the following :
- (A) The given electric field is a conservative electric field.  
 (B) The given electric field is a non-conservative electric field.  
 (C) If field is conservative then electric potential at point  $(0, 2m, 2m)$  is 32 volts, if electric potential at origin is 400 volts.  
 (D) Electric potential at point  $(0, 2m, 2m)$  can not be defined.
5. A tube of certain diameter of length 97 cm is open at both ends. Its fundamental frequency of resonance is found to be 160 Hz. The velocity of sound in air is 320 m/s. Then choose the correct statement (s)
- (A) Diameter of the tube is 5 cm  
 (B) When one end of the tube is closed, the lowest frequency of resonance for the tube is 81.22 Hz (approximately upto two decimal)  
 (C) When one end of the tube is closed the frequency of 3<sup>rd</sup> harmonic is 243.65 Hz (approximately upto two decimal)  
 (D) When both end are open the frequency of 3<sup>rd</sup> harmonic is 480 Hz
6. An earth satellite is revolving in a circular orbit of radius  $\ell_0$  with velocity  $v_0$ . A gun is in the satellite and is aimed such that the direction of net velocity of bullet is at angle of  $\theta = \tan^{-1}\left(\frac{1}{2}\right)$  towards earth with the satellite velocity. (Neglect the air resistance), then choose the correct obtain(s)
- (A) Bullet is fired from the gun with a velocity  $\frac{v_0}{2}$  with respect to satellite towards the centre of the earth.  
 (B) Maximum distance of bullet from the centre of earth during its subsequent motion is  $2\ell_0$   
 (C) Minimum distance of bullet from the centre of earth during its subsequent motion is  $\frac{2}{3}\ell_0$   
 (D) Bullet is fired from the gun with velocity  $\frac{\sqrt{5}}{4}v_0$  with respect to satellite

### Section – A (Maximum Marks: 12)

This section contains **FOUR (04) Matching List Type Questions**. Each question has **FOUR** statements in **List-I** entries (I), (II), (III) and (IV) and **FIVE** statements in **List-II** entries (P), (Q), (R), (S) and (T). The codes for lists have choices (A), (B), (C), (D) out of which **ONLY ONE** of these four options is correct answer.

7. Curved surface of a vessel shape of a truncated cone having semi-vertex angle  $37^\circ$ . Vessel is filled with water upto height of 16 m and is placed on a horizontal plane. Upper surface is opened to atmosphere. A very small hole is made on curved wall at a height of 11 m from bottom as shown in the figure. Area of water surface in the vessel is large as compared to the area of the hole. (take  $g = 10 \text{ m/s}^2$ , density of waver =  $1000 \text{ kg/m}^3$ , area of hole =  $0.5 \text{ cm}^2$  and  $\sin 37^\circ = 3/5$ )

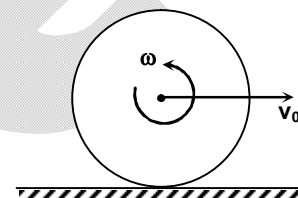


List –I		List –II (S.I. unit)	
(I)	Initial velocity of efflux (in m/s)	(P)	18.70
(II)	Distance (in meter) of the water (coming out from the hole just after opening the hole) where it strikes the horizontal surface from the point B, is	(Q)	3
(III)	The horizontal force (in newton) exerted by the water coming out from the hole (just after opening the hole) on the container, is	(R)	4
(IV)	The vertical force (in newton) exerted by the water coming out from the hole (just after opening the hole) on the container, is	(S)	9.35
		(T)	10

Which one of the following options is correct ?

- (A) I → T, II → S, III → R, IV → Q
- (B) I → P, II → Q, III → R, IV → S
- (C) I → S, II → T, III → P, IV → Q
- (D) I → R, II → T, III → P, IV → Q

8. A solid cylinder of radius R is projected with velocity  $v_0$  on a rough horizontal surface of coefficient of friction  $\mu$  at  $t = 0$  and at the same instant it has an angular velocity  $\omega_0 = \frac{4v_0}{R}$  as shown in the figure.

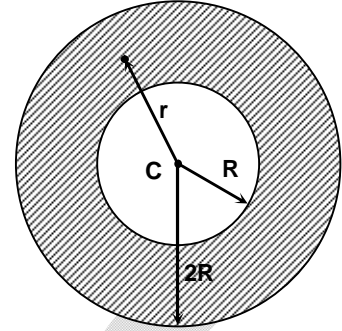


List –I		List –II	
(I)	The time when pure rolling starts is $\frac{Wv_0}{3\mu g}$ . The value of W is	(P)	18
(II)	Distance traveled by centre of mass of disc at time $t = \frac{4v_0}{3\mu g}$ is $S = \frac{5v_0^2}{X\mu g}$ . The value of X is	(Q)	9
(III)	Angular velocity of the cylinder at time $t = \frac{4v_0}{3\mu g}$ is $\omega = \frac{4v_0}{YR}$ . The value of Y is	(R)	6
(IV)	Displacement of centre of mass of the cylinder when pure rolling starts is $\frac{5v_0^2}{Z\mu g}$ . The value of Z is	(S)	5
		(T)	3

Which one of the following options is correct ?

- (A) I → T, II → R, III → S, IV → R
- (B) I → P, II → Q, III → R, IV → S
- (C) I → S, II → T, III → P, IV → Q
- (D) I → S, II → Q, III → T, IV → P

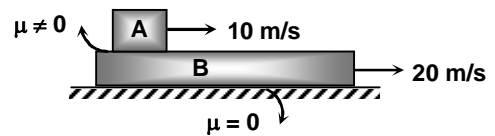
9. Consider a spherical volume has charge density  $\rho(r) = \frac{\rho r}{R}$  from  $r = R$  to  $r = 2R$ , where  $r$  is distance from centre. There is no charge from  $r = 0$  to  $r = R$  and from  $r = 2R$  to infinity.



List -I		List -II	
(I)	Total charge enclosed by the given spherical volume is $W\pi\rho R^3$ . The value of $W$ is	(P)	11
(II)	Electric field due to charge at a distance $r = \frac{3R}{2}$ is $\frac{65\rho R}{X\epsilon_0}$ . The value of $X$ is	(Q)	144
(III)	Electric potential due to charge at a distance $r = R$ is $\frac{28\rho R^2}{Y\epsilon_0}$ . The value of $Y$ is	(R)	65
(IV)	Work done to bring a charge particle slowly from $r = 2R$ to $r = R$ , is $\frac{Z\rho R^2 q}{24\epsilon_0}$ . Then the value of $Z$ is	(S)	15
		(T)	12

Which one of the following options is correct ?

- (A) I  $\rightarrow$  T, II  $\rightarrow$  R, III  $\rightarrow$  S, IV  $\rightarrow$  R  
 (B) I  $\rightarrow$  S, II  $\rightarrow$  Q, III  $\rightarrow$  T, IV  $\rightarrow$  P  
 (C) I  $\rightarrow$  S, II  $\rightarrow$  T, III  $\rightarrow$  P, IV  $\rightarrow$  Q  
 (D) I  $\rightarrow$  R, II  $\rightarrow$  T, III  $\rightarrow$  P, IV  $\rightarrow$  Q
10. Block A of mass 8 kg is placed on the rough surface of a long block B of mass 12 kg. Block B is placed on smooth horizontal surface. Block A and B are given velocities 10 m/s and 20 m/s respectively as shown in the figure.



List -I		List -II	
(I)	Common velocity (in m/s) of blocks when sliding between block A and B stops	(P)	624
(II)	Magnitude of work done (in joule) by the friction on the block A, when both block A and B starts moving with same velocity	(Q)	$(16 - 6\sqrt{2})$
(III)	Magnitude of work done (in joule) by the friction on the block B, when both block A and B starts moving with same velocity	(R)	16
(IV)	Velocity of block A when the loss of energy becomes half of its maximum value	(S)	864
		(T)	$(16 - 3\sqrt{2})$

Which one of the following options is correct ?

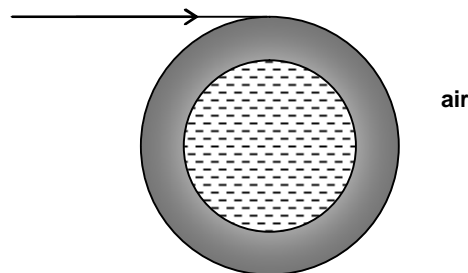
- (A) I → R, II → P, III → S, IV → T
- (B) I → P, II → Q, III → R, IV → S
- (C) I → S, II → T, III → P, IV → Q
- (D) I → R, II → T, III → P, IV → Q

**Section – B (Maximum Marks: 24)**

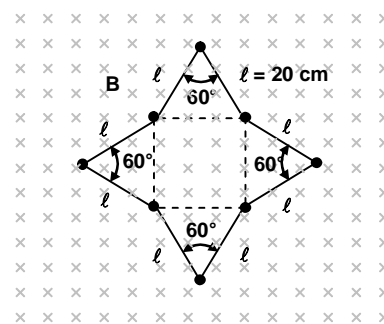
This section contains **EIGHT (08)** numerical based questions. The answer to each question is a **NUMERICAL VALUE**. If the numerical value has more than two decimal places, truncate/round-off the value to **TWO** decimal places.

- 11. In the process of nuclear fission of 3g uranium, the mass lost is 1.5 mg. The efficiency of power house run by the fission reactor is 20%. To obtain 1200 megawatt power from the power house, how much uranium (in gram) will be required per hour?
- 12. The pitch of screw gauge is 1 mm and its circular scale is divided into 100 divisions. When nothing is put between the studs the circular scale lies between 2<sup>nd</sup> and 3<sup>rd</sup> division of main scale, but when circular scale is rotated by 270° the 3<sup>rd</sup> division of main scale is seen and the 3<sup>rd</sup> division of main scale coincide with the 12<sup>th</sup> division of circular scale. When a glass plate is placed between the studs, the circular scale lies between 28<sup>th</sup> to 29<sup>th</sup> division of main scale and circular scale reads 20 divisions. Find the thickness of the glass plate in millimeter. (Assume value of one main scale division is same as pitch)
- 13. A particle is moving in a plane according to the law  $x = 3t^3$  and  $y = 36t^6$ . The radius of curvature of particle at  $t = \left(\frac{1}{72}\right)^{1/6}$  sec is  $\frac{\lambda}{10}$  meter. Find the value of  $\lambda$ .
- 14. In series RLC circuit with 20 Ω resistance is connected to an AC source. When only the capacitance is removed the current lags behind the voltage by 53° when only the inductance is removed, the current leads the voltage by 37°. The impedance of the circuit is NΩ. Find the value of N. [take  $\tan 37^\circ = 3/4$  and  $\sqrt{193} = 13.89$ ]

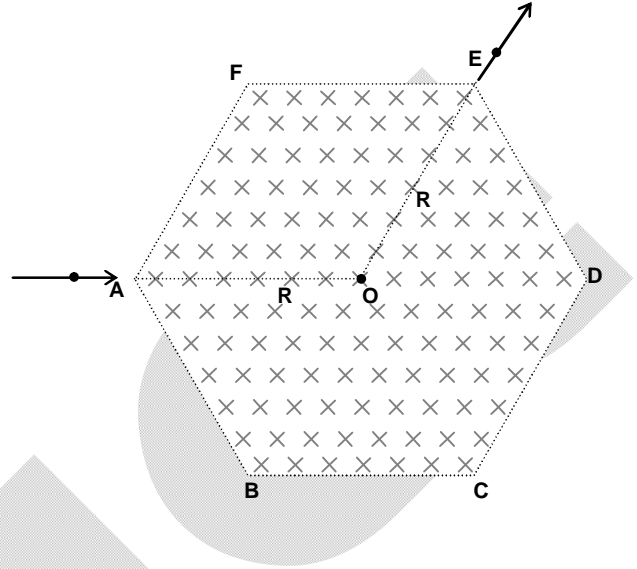
- 15. A capillary tube made of a material of refractive index 2. The outer radius of the tube is  $\sqrt{3}R$  and inner radius is R. The tube is filled with a liquid of refractive index 1.5. A light ray is incident on the tube at grazing incidence (angle of incidence approximately  $\approx 90^\circ$ ) on the outer surface of capillary tube as shown in the figure. The net angle of deviation is  $k^\circ$ . Find the value of k.



- 16. A close loop having  $N = 50$  turns with a current  $I = 2$  amp is placed in uniform magnetic induction  $B = 100$  Tesla. The work required to pull this coil out from the magnetic field is K joule. Find the value K. [take  $\sqrt{3} = 1.732$ ]



17. A string 50 cm long and having mass 5 g is under tension. A pipe closed at one end is 80 cm long when the string is set vibrating in first overtone and air in the pipe in its fundamental frequency, 8 beats per second are heard. It is observed that decreasing the tension in the string decreases the beat frequency. If the speed of sound in air is 320 m/s, then find the tension in the string in Newton.
18. A charged particle of mass  $m = 100 \text{ gm}$  and charge  $q = 4 \text{ coulomb}$  enters along AO at point A in a uniform magnetic field of magnitude  $B = 10 \text{ Tesla}$  existing in a hexagonal region of side  $R = \sqrt{3} \text{ m}$  and into the plane of paper as shown in figure. The particle leaves the region at 'E' along OE. The speed of the charge particle (in m/s) as it leaves the region is



# Chemistry

## PART – II

### Section – A (Maximum Marks: 24)

This section contains **SIX (06)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).

19. Be and Al resemble in following aspects  
 (A) both becomes passive on reaction with conc.  $\text{HNO}_3$  due to formation of oxide layer  
 (B) their chlorides exist in polymeric form  
 (C) both forms amphoteric oxides  
 (D)  $\text{BeCl}_2$  and  $\text{AlCl}_3$  both are Lewis acid
20.  $\text{XeF}_6 \xrightarrow{\text{hydrolysis}}$  Product(s) is/are  
 (A)  $\text{XeO}_3$  (B)  $\text{XeO}_2\text{F}_2$   
 (C)  $\text{XeO}_4$  (D)  $\text{XeOF}_4$
21. In an NaCl structure which of the following statement(s) is/are correct?  
 (A) All octahedral sites are occupied.  
 (B) Coordination number of  $\text{Na}^+$  is six.  
 (C) Number of ion pairs of  $\text{Na}^+\text{Cl}^-$  present effectively in an unit cell is four.  
 (D) Edge length of unit cell is equal to  $(r_{\text{Na}^+} + r_{\text{Cl}^-}) \times 2$
22. Which of the following statements is/are correct?  
 (A) The temperature of a real gas changes when it expands adiabatically in vacuum.  
 (B) Average kinetic energy of the gas molecules at any given temperature is independent of the average speed.  
 (C)  $\text{NH}_4\text{NO}_2$  and  $(\text{NH}_4)_2\text{Cr}_2\text{O}_7$  both gives same gas on heating.  
 (D) van der Waal's constant 'b' is one-third of critical volume.
23. How many of the following compound(s) is/are yellow in colour?  
 (A)  $\text{PbI}_2$  (B)  $\text{CdS}$   
 (C)  $\text{As}_2\text{S}_3$  (D)  $\text{Ag}_2\text{CrO}_4$
24. Cannizzaro reaction is given by  
 (A)  $\text{HCHO}$  (B)  $\text{CH}_3\text{CHO}$   
 (C)  $\text{PhCHO}$  (D)  $(\text{CH}_3)_3\text{CCHO} + \text{HCHO} \xrightarrow{\text{OH}^-}$



## Section – A (Maximum Marks: 12)

This section contains **FOUR (04)** Matching List Type Questions. Each question has **FOUR** statements in **List-I** entries (I), (II), (III) and (IV) and **FIVE** statements in **List-II** entries (P), (Q), (R), (S) and (T). The codes for lists have choices (A), (B), (C), (D) out of which **ONLY ONE** of these four options is correct answer.

25. Match the following:

List – I		List – II	
(I)	$2\text{Ag}^+ + \text{Cu} \longrightarrow 2\text{Ag} + \text{Cu}^{2+}$ $E_{\text{Cu}^{2+}/\text{Cu}}^{\circ} = 0.34$ $E_{\text{Ag}^+/\text{Ag}}^{\circ} = 0.80$ $[\text{Ag}^+] = 10^{-2} \text{ M}, [\text{Cu}^{2+}] = 10^{-4}$	(P)	$E_{\text{Cell}} = 0.5605$
(II)	$\text{Ni}   \text{Ni}^{2+} (1 \text{ M})    \text{Cu}^{2+} (0.1 \text{ M})   \text{Cu}$ $E_{\text{Ni}^{2+}/\text{Ni}}^{\circ} = -0.25 \text{ V}, E_{\text{Cu}^{2+}/\text{Cu}}^{\circ} = 0.34 \text{ V}$	(Q)	$E_{\text{Cell}} = 0.0295$
(III)	$\text{Pt}   \text{H}_2 (P_1 \text{ atm})   \text{H}^+ (1 \text{ M})    \text{H}^+ (1 \text{ M})   \text{H}_2 (P_2 \text{ atm})$	(R)	$E_{\text{Cell}} = E_{\text{cell}}^{\circ}$
(IV)	$\text{Zn} - \text{Hg} (C_1 \text{ M})   \text{Zn}^{2+} (\text{aq})   \text{Hg} - \text{Zn} (C_2 \text{ M})$ $C_1 = 1 \text{ gm}   100 \text{ gm of Hg}, C_2 = 10 \text{ gm} / 100 \text{ gm of Hg}$	(S)	$E_{\text{Cell}} = 0.59 \text{ V}$
		(T)	$E_{\text{cell}}^{\circ} = 0$

(A) I→S; II→T; III→P; IV→Q

(B) I→R; II→P; III→T; IV→Q

(C) I→S; II→Q; III→R; IV→T

(D) I→R; II→P; III→Q; IV→T

26. Match the following:

List – I		List – II	
(I)	$(\Delta G_{\text{system}})_{T,P} = 0$	(P)	Irreversible isothermal expansion
(II)	$ \Delta S_{\text{system}}  =  \Delta S_{\text{surrounding}} $	(Q)	Isothermal compression
(III)	$ W_{\text{irrev}}  >  W_{\text{rev}} $	(R)	$\Delta S = \frac{\Delta H}{T}$
(IV)	$ \Delta S_{\text{system}}  >  \Delta S_{\text{surrounding}} $	(S)	Isothermal expansion
		(T)	Reversible isothermal expansion

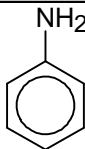
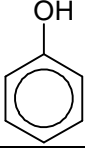
(A) I→S; II→Q; III→T; IV→P

(B) I→Q; II→R; III→P; IV→S

(C) I→R; II→T; III→Q; IV→P

(D) I→T; II→P; III→S; IV→Q

27. Match the following:

List – I		List – II	
(I)		(P)	Gives 2,4 DNP test
(II)		(Q)	Gives isocyanide test
(III)	$\text{Ph}-\overset{\text{O}}{\parallel}{\text{C}}-\text{NH}_2$	(R)	Tri substitution product with $\text{Cl}_2/\text{H}_2\text{O}$
(IV)	$\text{Ph}-\overset{\text{O}}{\parallel}{\text{C}}-\text{CH}_3$	(S)	Gives Hoffmann bromamide test
		(T)	Gives Azo-coupling reaction with diazonium salt in basic medium

- (A) I→P; II→Q; III→R; IV→S  
 (B) I→Q; II→R; III→P; IV→T  
 (C) I→R; II→T; III→S; IV→P  
 (D) I→T; II→S; III→Q; IV→R

28. Match the following

List – I		List – II	
(I)	Gives coloured ppt. with $\text{K}_4[\text{Fe}(\text{CN})_6]$	(P)	$\text{Ni}^{2+}$
(II)	Gives red ppt. with DMG in alkaline medium	(Q)	$\text{Cu}^{2+}$
(III)	Gives coloured ppt. with KI which dissolves in excess of KI to give colourless complex	(R)	$\text{Pb}^{2+}$
(IV)	Gives coloured ppt. with KCN and ppt. changes its colour immediately which further gives complex with excess of KCN	(S)	$\text{Co}^{2+}$
		(T)	$\text{Fe}^{3+}$

- (A) I→Q; II→S; III→R; IV→P  
 (B) I→P; II→R; III→T; IV→Q  
 (C) I→T; II→P; III→R; IV→Q  
 (D) I→T; II→Q; III→S; IV→P

### Section – B (Maximum Marks: 24)

This section contains **EIGHT (08)** numerical based questions. The answer to each question is a **NUMERICAL VALUE**. If the numerical value has more than two decimal places, truncate/round-off the value to **TWO** decimal places.

29. A solid mixture (5.0 mg) containing lead nitrate and sodium nitrate was heated below  $600^\circ\text{C}$  until the weight of residue becomes constant. If the loss in weight is 1.35 gm, then the amount of lead nitrate (in mg) in the mixture is (Molar mass of Pb = 208)

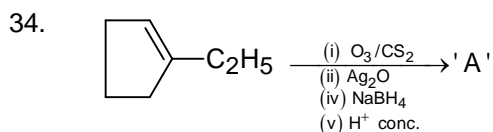
30. One mole of an ideal monoatomic gas initially at 800 K and 64 atm is expanded to a final state at 200 K and 1 atm. To achieve the above change a reversible path is contracted that involve an adiabatic expansion in the beginning followed by an isothermal expansion to the final state. The magnitude of net work done by the gas (in cal) is

$$(\ln 2 = 0.7), (R = 2 \text{ cal mol}^{-1} \text{ K}^{-1})$$

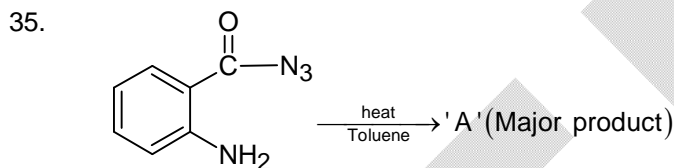
31.  $\text{pK}_{a_1}$  and  $\text{pK}_{a_2}$  of  $\text{H}_2\text{SO}_3$  are 1.82 and 7.2 respectively. The pH after 30 ml of 0.15 N NaOH is added to 15 ml of 0.30 M  $\text{H}_2\text{SO}_3$  is?

32. The half-life period of a first order chemical reaction is 15 minutes. Time required for the completion of 99% of the chemical reaction will be ( $\log 2 = 0.30$ )

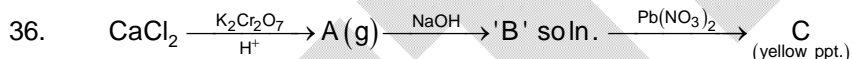
33. 0.800 gm of silver salt of a tribasic acid on combustion gave 0.40 gm pure Ag. Calculate the molar mass of acid (Atomic mass of Ag = 108)



Molar mass of A is M, then  $M = ?$



Molar mass of A is M, then  $M = ?$



Molar atomic masses

$\text{K} = 39.09, \text{Cl} = 35.5, \text{Cr} = 52, \text{Pb} = 207.0, \text{N} = 14, \text{O} = 16$

What is the molar mass of 'C'?

# Mathematics

## PART – III

### Section – A (Maximum Marks: 24)

This section contains **SIX (06)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).

37. P is a moving point on the circle  $x^2 + y^2 = 4$ . If a point Q divide the perpendiculars dropped from point P on the line  $x + y = 0$ , such that Q divide PN (N is foot of perpendicular) in the ratio 1 : 2, then locus of point Q is
- (A) An ellipse (B) A hyperbola  
 (C) A conic with eccentricity  $\frac{\sqrt{5}}{3}$  (D) A conic with eccentricity  $\frac{3}{\sqrt{5}}$
38. Consider two functions  $f(x) = e^x \sin x$  and  $g(x) = e^x \cos x$ . If  $f(x)$  and  $g(x)$  intersect at the points  $(a_n, b_n)$  where  $n \in I$ , then
- (A)  $a_1, a_2, a_3, \dots$  form an A.P. (B)  $a_1, a_2, a_3, \dots$  form an G.P.  
 (C)  $b_1, b_2, b_3, \dots$  form an A.P. (D)  $b_1, b_2, b_3, \dots$  form an G.P.
39. If the equation of asymptotes of a hyperbola is  $3x^2 + 10xy + 3y^2 + 5x + 6y + \mu = 0$ , then eccentricity of the hyperbola is/are
- (A)  $\sqrt{5}$  (B)  $\frac{\sqrt{5}}{2}$   
 (C)  $\sqrt{3}$  (D)  $\frac{\sqrt{3}}{2}$
40. In a triangle  $\Delta ABC$ , line joining circumcenter and centroid cut the side BC at point F, such that circumcenter is midpoint of centroid and point F. If M is midpoint of side BC, then which of the following(s) is/are TRUE? (where a, b, c are lengths of sides BC, CA and AB respectively)
- (A)  $MF = \frac{a}{3} - \frac{c}{6} \cos B$  (B)  $MF = \frac{a}{6} - \frac{c}{3} \cos B$   
 (C)  $\tan B \tan C = \frac{5}{4}$  (D)  $\tan B \tan C = \frac{3}{2}$
41. If A is an orthogonal and symmetric matrix, B is involutory matrix also A and B anti commute and AB is idempotent matrix, then which of the following(s) is/are TRUE?
- (A)  $\det(A + B)$  is 0  
 (B)  $A + B$  is not null matrix  
 (C)  $\det(A + B^{-1}) = 0$   
 (D) A and B must be square matrix of even order
42. If a, b, c are in A.P., b, c, d are in G.P. and c, d, e are in H.P., then which of the following is/are FALSE? Given that no two of a, b, c, d, e are equal
- (A) a, c, e are in G.P. (B) a, c, e are in A.P.  
 (C)  $ae = bd$  (D) a, c, e are in H.P.

**Section – A (Maximum Marks: 12)**

This section contains **FOUR (04)** Matching List Type Questions. Each question has **FOUR** statements in **List-I** entries (I), (II), (III) and (IV) and **FIVE** statements in **List-II** entries (P), (Q), (R), (S) and (T). The codes for lists have choices (A), (B), (C), (D) out of which **ONLY ONE** of these four options is correct answer.

43.  $C_1$  is a curve which is obtained by taking image of  $\sin^{-1} x$  w.r.t the curve  $\cos^{-1} x$  along y-axis and  $C_2$  is a curve which is obtained by taking image of  $\tan^{-1} x$  w.r.t  $\cot^{-1} x$  along y-axis. Then based on the following grid, give your answer (Where  $[\cdot]$  represents greatest integer function)

List-I		List-II	
(I)	If maximum value of $C_2$ is $\alpha$ , then $[\alpha]$ is equal to	(P)	0
(II)	If minimum value of $C_1$ is $\beta$ , then $[\beta]$ is equal to	(Q)	7
(III)	If $C_1$ and $C_2$ intersect at $x = \gamma$ , then $[\gamma]$ is equal to	(R)	4
(IV)	If curve $C_1 + C_2$ takes $\delta$ integral values, then $\delta$ is equal to	(S)	-2
		(T)	14

The correct option is:

- (A) (I)  $\rightarrow$  (S); (II)  $\rightarrow$  (Q); (III)  $\rightarrow$  (P); (IV)  $\rightarrow$  (T)  
 (B) (I)  $\rightarrow$  (R); (II)  $\rightarrow$  (Q); (III)  $\rightarrow$  (S); (IV)  $\rightarrow$  (P)  
 (C) (I)  $\rightarrow$  (Q); (II)  $\rightarrow$  (S); (III)  $\rightarrow$  (P); (IV)  $\rightarrow$  (T)  
 (D) (I)  $\rightarrow$  (P); (II)  $\rightarrow$  (Q); (III)  $\rightarrow$  (S); (IV)  $\rightarrow$  (T)

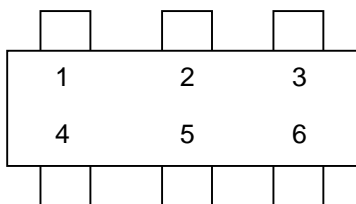
44. Based on the following grid chose the correct option

List-I		List-II	
(I)	$\int \frac{2 \tan^2 \frac{x}{2}}{\left(1 - \tan \frac{x}{2}\right)^2} dx = f(x) - \ln f(x) + \ln g(x) + C$ , where C is integration constant, then $g\left(\frac{\pi}{3}\right)$ is equal to	(P)	0
(II)	If A, B, C, D are four sets such that $A \cap B \subset C$ , $A \cap B \subset D$ and $A \cap B = C \cap D$ , then $P\left(\frac{A \cap B}{C \cap D}\right)$ is equal to	(Q)	0.5
(III)	What would be remainder if $\left(\sqrt{5+2\sqrt{6}}\right)^{4n} + \left(\sqrt{5-2\sqrt{6}}\right)^{4n} + 2$ ; $n \in \mathbb{N}$ is divided by 4	(R)	6
(IV)	If the equation $x^3 - \alpha x^2 + 12x - 8 = 0$ have three positive real roots, then $\alpha$ is equal to	(S)	1
		(T)	none of these

The correct option is:

- (A) (I)  $\rightarrow$  (R); (II)  $\rightarrow$  (S); (III)  $\rightarrow$  (Q); (IV)  $\rightarrow$  (P)  
 (B) (I)  $\rightarrow$  (Q); (II)  $\rightarrow$  (S); (III)  $\rightarrow$  (P); (IV)  $\rightarrow$  (R)  
 (C) (I)  $\rightarrow$  (Q); (II)  $\rightarrow$  (P); (III)  $\rightarrow$  (R); (IV)  $\rightarrow$  (S)  
 (D) (I)  $\rightarrow$  (T); (II)  $\rightarrow$  (T); (III)  $\rightarrow$  (Q); (IV)  $\rightarrow$  (R)

45. There is a rectangular table as shown in the figure, which has three chairs on two sides each



There married couples sit on these chairs. Then answer the following grid

List-I		List-II	
(I)	Probability that couples sit either infront of each other or adjacently	(P)	0.2333
(II)	Probability that no couple sit either infront of each other or adjacently	(Q)	0.1000
(III)	Probability that exactly one couple sit infront of each other or adjacently	(R)	0.1555
(IV)	Probability that no two ladies sit infront of each other or adjacently	(S)	0.1322
		(T)	0.1333

The correct option is:

- (A) (I) → (P); (II) → (Q); (III) → (R); (IV) → (S)  
 (B) (I) → (T); (II) → (R); (III) → (P); (IV) → (Q)  
 (C) (I) → (R); (II) → (T); (III) → (P); (IV) → (Q)  
 (D) (I) → (R); (II) → (T); (III) → (Q); (IV) → (P)
46. Let  $f(x)$  be a real valued function defined by  $f(x) = x^2 - 2|x|$  and

$$g(x) = \begin{cases} \text{minimum } \{f(t) : -2 \leq t \leq x\}, & x \in [-2, 0) \\ \text{maximum } \{f(t) : 0 \leq t \leq x\}, & x \in [0, 3] \end{cases}$$

List-I		List-II	
(I)	$f(x)$ is not continuous at $x$ equal to	(P)	-2
(II)	$g(x)$ is not differentiable at $x$ equal to	(Q)	0
(III)	Number of points of local extremum of $g(x)$ is equal to	(R)	1
(IV)	Absolute maximum value of $g(x)$ is equal to	(S)	2
		(T)	3

The correct option is:

- (A) (I) → (P); (II) → (Q, R); (III) → (Q); (IV) → (T)  
 (B) (I) → (T); (II) → (P, T); (III) → (P); (IV) → (T)  
 (C) (I) → (Q); (II) → (Q, T); (III) → (P); (IV) → (R)  
 (D) (I) → (Q); (II) → (Q, S); (III) → (Q); (IV) → (T)

**Section – B (Maximum Marks: 24)**

This section contains **EIGHT (08)** numerical based questions. The answer to each question is a **NUMERICAL VALUE**. If the numerical value has more than two decimal places, truncate/round-off the value to **TWO** decimal places.

47. If range of the function  $f(x) = \frac{\sin^{-1} x}{\cos^{-1} x} + \frac{\tan^{-1} x}{\cot^{-1} x}$  is  $[a, \infty)$ . Then the value of  $|a|$  is equal to

48. A variable complex number  $z$  moves such that it satisfies both the inequalities  $|z + 1| + |z - 1| \leq 4$  and  $|z - 1| + |z - 3| \leq 4$  simultaneously, then area of the region in which  $z$  moves is equal to
49. Three circles of equal radius  $\frac{1}{\sqrt{3}}$  unit touch each other externally, if a parabola passes through the centres of these circles, then length of latus rectum of the parabola is equal to
50. For the family of parabola  $y = a_n x^2 + b_n x + 1$  and  $a_n > 0 \forall n \in \mathbb{I}^+$  and length of latus rectum of these parabolas are in H.P., also distance between the points where these parabolas cut the x-axis is always 1, then  $b_1^2, b_2^2, b_3^2, \dots$  are in A.P. with common difference equal to  $\alpha$  times the common difference of A.P. corresponding to latus rectum H.P., then  $\alpha$  is equal to
51.  $C$  is a circle with radius  $r$  and  $C_i$  ( $i = 1, 2, 3, 4$ ) are the circles such that  $\forall i = 1, 2, 3, C_i$  touches  $C_{i+1}$  and  $C$  externally, also  $C_1$  touches x-axis and  $C_4$  touches  $C_3$  and  $C$  and y-axis. Radius of  $C_i \forall i = 1, 2, 3, 4$  is  $r_i$ . If  $\frac{r}{r_i} = \alpha + \operatorname{cosec} \frac{\pi}{\beta}$ , then  $\alpha + \beta$  is equal to
52. Two ellipse  $3x^2 + y^2 = 2$  and  $x^2 + 3y^2 = 2$  cut each other at four points A, B, C, D, then radius of circle passes through these four points is equal to
53. There are four colour shifting balls of colours  $C_1, C_2, C_3$  and  $C_4$  initially, each ball can change into any other colour or can regain its colour at every second. The number of way of their colours change, if maximum possible number of arrangements of all four balls is possible after 1 second is equal to
54. There is a plane  $\alpha x + \beta y + \gamma z + 1 = 0$  which is perpendicular to the plane  $2x - y + 2z + 5 = 0$  and containing the line  $\vec{r} = (3\lambda - 1)\hat{i} + 6\lambda\hat{j} - 2\lambda\hat{k}$ . Then the value of  $3\alpha + 2\beta + 4\gamma$  is equal to