FIITJEE
ALL INDIA TEST SERIES
JEE (Advanced)-2023
FULL TEST – IX
PAPER –1
TEST DATE: 14-05-2023

ANSWERS, HINTS & SOLUTIONS

Physics PART – I

Section – A

1. A, B, D

Sol. Let V is emf of the battery

$$i = \frac{V}{r + R_1 + R_2} \quad \text{...(i)}$$

$$V_{AB} = \frac{VR_1}{r + R_1 + R_2} \quad \text{...(ii)}$$

$$V_{A'B'} = 4i\left(\frac{R_1 R}{R_1 + R}\right) = \frac{V_{AB}}{4} \quad \text{...(iii)}$$

From (i), (ii) and (iii)

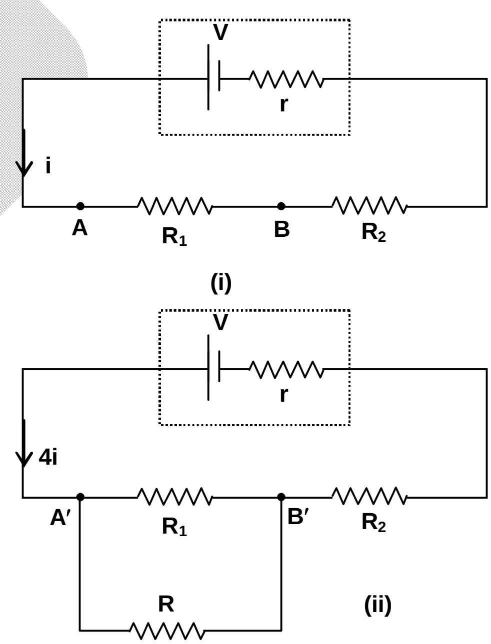
$R_1 = 180\ \Omega$

From (ii) circuit

$$4i = \frac{V}{\left(r + R_2 + \frac{R_1 R}{R_1 + R}\right)} \quad \text{...(iv)}$$

From (i) and (iv)

$R_2 = 40\Omega$



2. A, B, C, D

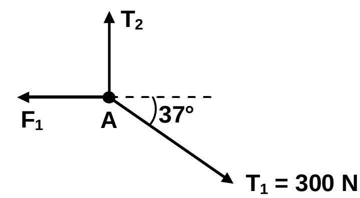
Sol. $T_2 = T_1 \sin 37°$ …(i)

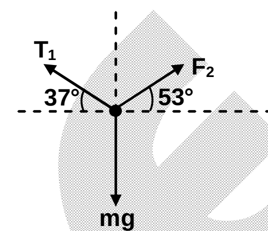
$F_1 = T_1 \cos 37°$ …(ii)

$F_2 = T_1 \cos 37° / \cos 53°$ …(iii)

$Mg = T_1 \sin 37° + F_2 \sin 53°$ …(iv)

From the above equation find F_1, F_2, T_2 and mass of the block





3. A, B, C, D

Sol. Let compression in the spring A, B and C are X_1, X_2 and X_3 respectively. Then in equilibrium

$$2KX_1 = 4KX_2 = 8KX_3$$

$$X_1 = 2X_2 = 4X_3 \quad \text{…(i)}$$

Total increase in length of rods

$$\Delta\ell = \Delta\ell_1 + \Delta\ell_2$$

$$= 2\ell\alpha\Delta T + 3\ell\alpha\Delta T == 5\ell\alpha\Delta T \quad \text{…(ii)}$$

$$X_1 + X_2 + X_3 = \Delta\ell = 5\ell\alpha\Delta T \quad \text{…(iii)}$$

From (i) and (iii)

$$X_1 = \frac{20\ell\alpha\Delta T}{7}, X_2 = \frac{10}{7}\ell\alpha\Delta T, X_3 = \frac{5}{7}\ell\alpha\Delta T$$

U (in the spring A) = $\frac{1}{2}(2K)\left(\frac{20\ell\alpha\Delta T}{7}\right)^2$

$$= \frac{400K(\ell\alpha\Delta T)^2}{49}$$

U (Total energy stored)

$$= \frac{1}{2}2KX_1^2 + \frac{1}{2}4KX_2^2 + \frac{1}{2}8KX_3^2 = \frac{100}{7}K(\ell\alpha\Delta T)^2$$

4. A, C

Sol. $dV = -\vec{E}.\vec{dr}$

$$dV = -\left[\left(24yz^3 - 8y\right)\hat{j} + 18y^2z^2\hat{k}\right].\left(dx\hat{i} + dy\hat{j} + dz\hat{k}\right)$$

$$\int dV = -\int\left[\left(24yz^3dy + 18y^2z^2dz\right) - 8y\,dy\right]$$

$$V = -12y^2z^3 + 4y^2 + c$$

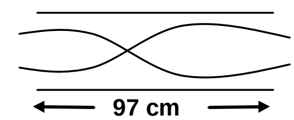
Hence electric field is conservative field

$$[V]_{400}^{V} = \left[-12y^2z^3 + 4y^2\right]_{(0,0,0)}^{(0,2,2)}$$

V = 32 Volts.

5. A, B, C, D

Sol. (A) $\frac{\lambda}{2} = 97 + 0.6D$ …(i)



$v = f\lambda$

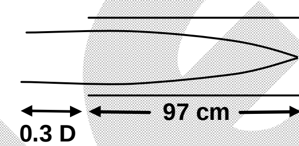
$\lambda = \frac{320}{160} = 2m = 200\,cm$ …(ii)

From (i) and (ii)

$\frac{200}{2} = 97 + 0.6D$

D = 5 cm

(B) $\frac{\lambda}{4} = 97 + 0.3D$

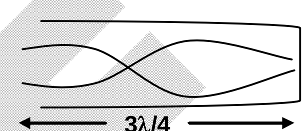


$\lambda = (97 \times 4 + 1.2D)$

$\lambda = (97 \times 4 + 1.2 \times 5) = 394\,cm$

$f = \frac{v}{\lambda} = \frac{32000}{394} = 81.22\,Hz$

(C) $\frac{3\lambda}{4} = 97 + 0.3D$



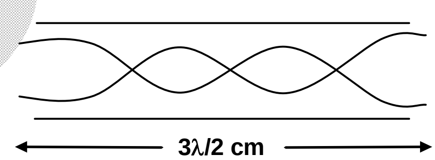
$= 97 + (0.3)(5)$

= 98.5

$\lambda = (98.5)\left(\frac{14}{3}\right) = \frac{394}{3}$

$f = \frac{v}{\lambda} = \frac{32000 \times 3}{394} = 243.65\,Hz$

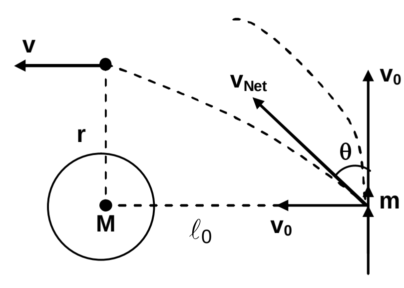
(D) $\frac{3\lambda}{2} = 97 + 0.6D$



$f = \frac{v}{\lambda} = \frac{320 \times 3 \times 100}{200} = 480\text{ Hz}$

6. A, B, C

Sol. $v_{Net}\cos\theta = v_0$



$v_{Net} = \frac{\sqrt{5}v_0}{2}$

$v_0 = \sqrt{\frac{Gm}{\ell_0}}$

$mv_0\ell_0 = mvr$

By the conservation of energy

$\frac{1}{2}m\left(\frac{\sqrt{5}v_0}{2}\right)^2 - \frac{GMm}{\ell_0} = \frac{1}{2}mv^2 - \frac{GMm}{r}$

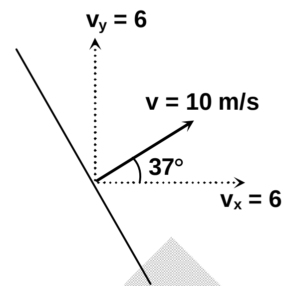
$\frac{5}{8}mv_0^2 - \frac{GMm}{\ell_0} = \frac{1}{2}mv^2 - \frac{GMm}{r}$

From (i) (ii) and (iii)

$r = 2\ell_0$ and $r = \frac{2\ell_0}{3}$

7. A

Sol. efflux velocity = $\sqrt{2gH} = \sqrt{2 \times 10 \times 5}$



Time of flight

$-11 = 6t - \frac{1}{2}(10)t^2$

t = 2.2 sec

Horizontal range from B

$R = 8 \times 2.2 - \left(\frac{33}{4}\right)$

R = 9.35 m

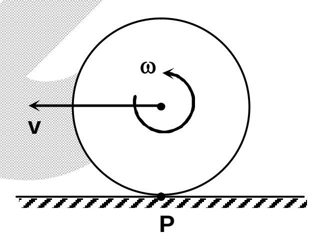
$F_x = \rho a v^2 \cos 37°$

$= (10^3)(0.15 \times 10^{-4})(100)\left(\frac{4}{5}\right) = 4$

$F_y = \rho a v^2 \sin 37° = 3$

8. D

Sol. Conservation of angular momentum about P



$\frac{3}{2} mR^2\omega = -mv_0R + \frac{mR^2}{2} \cdot \frac{4v_0}{R}$

$\omega = \frac{2}{3}\frac{v_0}{R}$

$v = R\omega = \frac{2}{3}v_0$

$\vec{v} = \vec{u} + \vec{a}t$

$-\frac{2}{3}v_0\hat{i} = v_0\hat{i} - \mu g\hat{i}t$

$t = \frac{5v_0}{3\mu g}$

Displacement when pure rolling starts

$\Delta\vec{r} = v_0\hat{i}\left(\frac{5v_0}{3\mu g}\right) - \frac{1}{2}(\mu g\hat{i})\left(\frac{5v_0}{3\mu g}\right)^2$

$= \left(\frac{5}{3} - \frac{25}{18}\right)\frac{v_0^2}{\mu g}\hat{i} = -\frac{5}{18}\frac{v_0^2}{\mu g}\hat{i}$

Distance at $t = \frac{4v_0}{3\mu g}$

Distance traveled by centre of mass when it stops

$S_1 = \frac{v_0^2}{2\mu g}$

Distance traveled at

$t = \frac{4v_0}{3\mu g} - \frac{v_0}{\mu g} = \frac{v_0}{3\mu g}$

$S_2 = \frac{1}{2}(\mu g)\left(\frac{v_0}{3\mu g}\right)^2 = \frac{v_0^2}{18\mu g}$

So, total distance $S = S_1 + S_2 = \frac{5}{9}\frac{v_0^2}{\mu g}$

Angular velocity at $t = \frac{4}{3}\frac{v_0}{\mu g}$

$$\vec{\omega} = \vec{\omega}_0 + \vec{\alpha}t$$

$$\vec{\omega} = \frac{4v_0}{R}\hat{k} - \frac{2\mu g}{R}\hat{k}\left(\frac{4}{3}\frac{v_0}{\mu g}\right) = \frac{4}{3}\frac{v_0}{R}\hat{k}$$

9. B

Sol.
$$Q = \int_R^r 4\pi R^2 \rho\left(\frac{r}{R}\right)dr$$

$$Q = \frac{4\pi\rho}{4R}\left(r^4 - R^4\right)$$

$Q(r = 2R) = 15\pi\rho R^3$

For electric field

$$E \cdot 4\pi r^2 = \frac{4\pi\rho}{\varepsilon_0 R}\left(\frac{r^4}{4} - \frac{R^4}{4}\right)$$

$$E\left(r = \frac{3}{2}R\right) = \frac{65\rho R}{144\varepsilon_0}$$

$$V_{2R} = \frac{KQ}{2R} = \frac{15\rho R^2}{8\varepsilon_0}$$

$$\int_{V_{2R}}^{V_R} dV = -\int_{2R}^{R} E dr$$

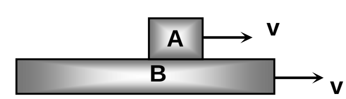
$$V_R - V_{2R} = \frac{11}{24}\frac{\rho R^2}{\varepsilon_0}$$

$$V_R = \left(\frac{15}{8} + \frac{11}{24}\right)\frac{\rho R^2}{\varepsilon_0}$$

$$W = (V_R - V_{2R})q$$

10. A

Sol. using conservation of momentum



$10 \times 8 + 12 \times 20 = 20\,v$

$v = 16$ m/s

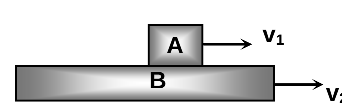
work done by friction on A

$$w_f = \frac{1}{2}(8)\left[(16)^2 - (10)^2\right] = 624 \text{ J}$$

work done by friction on B

$$w_f = \frac{1}{2}(12)\left[(16)^2 - (20)^2\right] = -864 \text{ J}$$

Velocity of block A when the loss of energy becomes half of its maximum value



$$\frac{1}{2}(8)(10)^2 + \frac{1}{2}(12)(20)^2 = \frac{1}{2}(8)v_1^2 + \frac{1}{2}(12)v_2^2 + 120 \quad \ldots\text{(i)}$$

By using of conservation of linear momentum

$$10 \times 8 + 12 \times 20 = 8v_1 + 12\,v_2 \quad \ldots\text{(ii)}$$

From (i) and (ii)
$v_1 = (16 - 3\sqrt{2})$ m/s

Section – B

11. 480.00

Sol. Power obtained = 1200 mega watt
$= 1200 \times 10^6 \times 3600 = 432 \times 10^{10}$ J
The output energy from the power house
$$E = \frac{(432 \times 10^{10}) \times 100}{20} = 216 \times 10^{11}\,\text{J}$$
Let this energy is obtained from Δm kg
$\Delta mc^2 = 216 \times 10^{11=}$
$$\Delta m = \frac{216 \times 10^{11}}{9 \times 10^{16}} = 24 \times 10^{-5}\,\text{kg}$$
Hence the uranium required
$$m = \frac{3 \times 24 \times 10^{-5}}{1.5 \times 10^{-3}} = 48 \times 10^{-2}\,\text{kg}$$
m = 480 g

12. 25.83

Sol. Error $= \left(\frac{270^\circ}{360^\circ} \times 100\right) \times \frac{1\,\text{mm}}{100} = 0.75\,\text{mm}$

Measured value = 28 mm + 0.20 mm – 3 mm – 0.12 mm + 0.75 mm = 25.83 mm

13. 33.75

Sol. $y = 36\left(\frac{x}{3}\right)^2 = 4x^2$
$$R = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$
$$x\left[t = \left(\frac{1}{72}\right)^{1/6}\right] = 3\left(\frac{1}{72}\right)^{1/2} = \frac{1}{2\sqrt{2}}$$
$y = 4x^2$
$$\frac{dy}{dx} = 8x,\ \left.\frac{dy}{dx}\right|_{x=\frac{1}{2\sqrt{2}}} = \frac{8}{2\sqrt{2}} = 2\sqrt{2}$$
$$\frac{d^2y}{dx^2} = 8$$
$$R = \frac{\left[1 + (2\sqrt{2})^2\right]^{3/2}}{8} = \frac{27}{8}\,\text{m}$$

14. 23.15

Sol. $\tan 53^\circ = \frac{\omega L}{R} = \frac{4}{3}$

$\tan 37^\circ = \frac{1}{\omega CR} = \frac{3}{4}$

$\omega L - \frac{1}{\omega C} = \frac{7}{12}R$

$Z = \sqrt{R^2 + \left(\frac{7}{12}R\right)^2} = 23.15$

15. 60.00

Sol. 1 sin 90° = 2 sin r

r = 30°

For inner surface

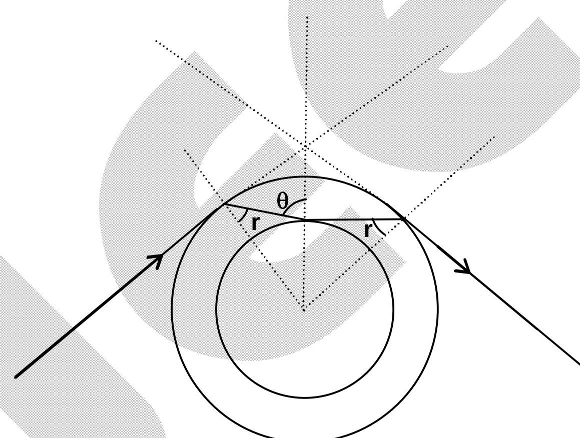
2 sinC = 1.5 sin 90°

sin C = 3/4 = 0.75

using sine rule

$\frac{\sqrt{3}R}{\sin(\pi-\theta)} = \frac{R}{\sin r}$

$\sin\theta = \frac{\sqrt{3}}{2} = 0.865$



As θ > C hence total internal reflection occurs.

From the geometry angle of deviation is 60°.

16. 1092.80

Sol. $W = |I\Delta\phi|$

$\phi_i = BAN,\ \phi_f = 0$

$W = I\left(4\frac{\sqrt{3}}{4}\ell^2 + \ell^2\right)NB$

$= I\left(\sqrt{3}+1\right)\ell^2 NB$

$= I(1.732+1)\left(20\times10^{-2}\right)^2(50)(100)$

$= (2.0\times50\times100)(2.732)\left(4\times10^{-2}\right)$

= 1092.80

17. 29.16

Sol. $\mu = \frac{m}{L} = \frac{5}{50} = 0.01\ \text{kg/m}$

For string $f_1 = \frac{1}{\lambda}\sqrt{\frac{T}{\mu}} = \frac{1}{\ell}\sqrt{\frac{T}{\mu}} = \left(\frac{1}{0.5}\right)\sqrt{\frac{T}{0.01}} = 20\sqrt{T}$

For origin pipe

$f_2 = \frac{v}{4\ell} = \frac{320}{4\times0.80} = 100$

$100 - 20\sqrt{T} = 8$ …(i)

or $20\sqrt{T} - 100 = 8$ …(ii)

Now, as decreasing tension decreases the beat frequency

Hence $20\sqrt{T} = 108$

$$T = \left(\frac{108}{20}\right)^2 = 29.16\text{ N}.$$

18. 1200.00

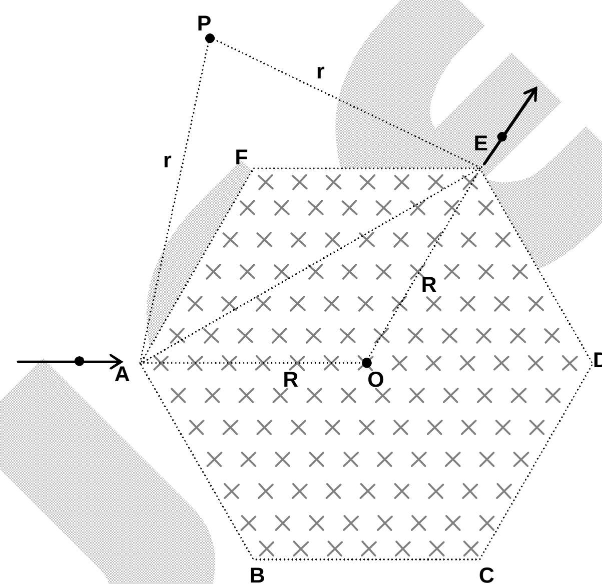
Sol. AE = 2r cos 60° = 2Rcos30°

$r = \sqrt{3}R$

$$\frac{mv}{qB} = \sqrt{3}R$$

$$v = \frac{\sqrt{3}qBR}{m}$$

$$v = \frac{(\sqrt{3})(4)(10)(\sqrt{3})}{100 \times 10^{-3}} = 1200\text{ m/s}$$



Chemistry

PART – II

Section – A

19. A, C, D

Sol. Be and Al forms BeO and Al_2O_3 on their surface, on reaction with conc. HNO_3.
$BeCl_2$ forms polymer while $AlCl_3$ forms dimer.
BeO and Al_2O_3 reacts with acid as well as with base.
$BeCl_2$ and $AlCl_3$ both are electron deficient and have vacant orbital in valency shell.

20. A, B, D

Sol.
$$XeF_6 + 3H_2O \longrightarrow XeO_3 + 6HF$$
$$XeF_6 + H_2O \longrightarrow XeOF_4 + 2HF$$
$$XeF_6 + 2H_2O \longrightarrow XeO_2F_2 + 4HF$$

21. A, B, C, D

Sol. All are facts.

22. B, C, D

Sol. Work done is zero in adiabatic expansion of gas in vacuum.
$KE_{average} = \frac{3}{2}KT,$ where K is Boltzmann's constant.
$$NH_4NO_2 \xrightarrow{\Delta} N_2 \uparrow + 2H_2O$$
$$(NH_4)_2 Cr_2O_7 \xrightarrow{\Delta} N_2 \uparrow + 4H_2O + Cr_2O_3$$
$$V_C = 3b$$

23. A, B, C

Sol. Ag_2CrO_4 is red in colour.

24. A, C, D

Sol. Aldehydes which don't have 'α' 'H' atom can give Cannizzaro reaction.

25. B

Sol. **Solution (I)**
$$E_{cell} = E^o - \frac{0.059}{2}\log\frac{[Cu^{+2}]}{[Ag^+]^2}$$
$$= E^o - \frac{0.059}{2}\log\frac{10^{-4}}{(10^{-2})^2}$$
$$E_{cell} = E^o - \frac{0.059}{2}\log 1 = E^o$$
Solution (II)
$$E_{cell} = E^o - \frac{0.059}{2}\log\frac{[Ni^{+2}]}{[Cu^{+2}]}$$
$$E_{cell} = 0.59 - \frac{0.059}{2}\log\frac{1}{(0.1)} = 0.59 - \frac{0.059}{2}$$

$E_{cell} = 0.5605$

Solution (III)

E^o of SHE = 0

Solution (IV)

E^o of concentration cell = 0

$$E = 0 - \frac{0.059}{2}\log\frac{C_2}{C_1} = \frac{0.059}{2}\log\frac{1}{0.1}$$

$$= \frac{0.059}{2} = 0.0295\ V$$

26. C

Sol. At equilibrium $\Delta G = 0,\ \Delta H = T\Delta S$

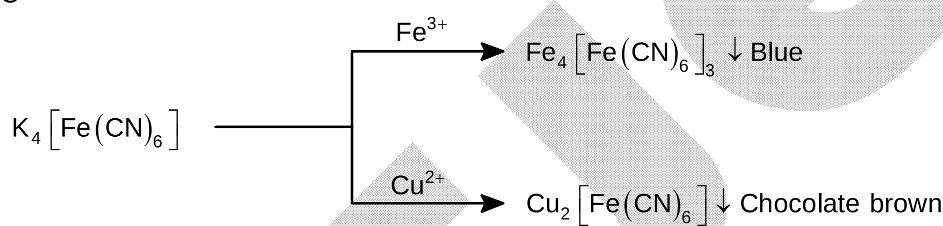
In reversible isothermal expansion $|\Delta S_{system}| = |\Delta S_{surrounding}|$

$|W_{rev}| < |W_{irrev}|$ in isothermal compression.

27. C

28. C

Sol.



$Ni^{2+} + DMG \xrightarrow{OH^-} \text{Red ppt.}$

$Pb^{2+} + KI \longrightarrow PbI_2 \downarrow$ Yellow

$PbI_2 + KI(excess) \longrightarrow K_2PbI_4$

$Cu^{2+} + KCN \longrightarrow Cu(CN)_2 \downarrow$ Yellow

$Cu(CN)_2 \longrightarrow \underset{(White)}{CuCN\downarrow} + (CN)_2\uparrow$

$CuCN + KCN \longrightarrow \underset{(Colourless)}{K_3[Cu(CN)_4]}$

Section – B

29. 2.98

[Range : 2.90 – 3.00]

Sol. $2Pb(NO_3)_2 \xrightarrow{\Delta} 2PbO + 4NO_2\uparrow + O_2\uparrow$

2×332 gm $Pb(NO_3)_2$ gives $\longrightarrow 2\times 224$ gm Pb

$\therefore$ x gm $Pb(NO_3)_2$ gives $\longrightarrow \frac{224}{332}\times x$ gm

$NaNO_3 \longrightarrow NaNO_2 + \frac{1}{2}O_2\uparrow$

$85 \text{ gm } NaNO_3 \text{ gives} \longrightarrow 69 \text{ gm } NaNO_2$

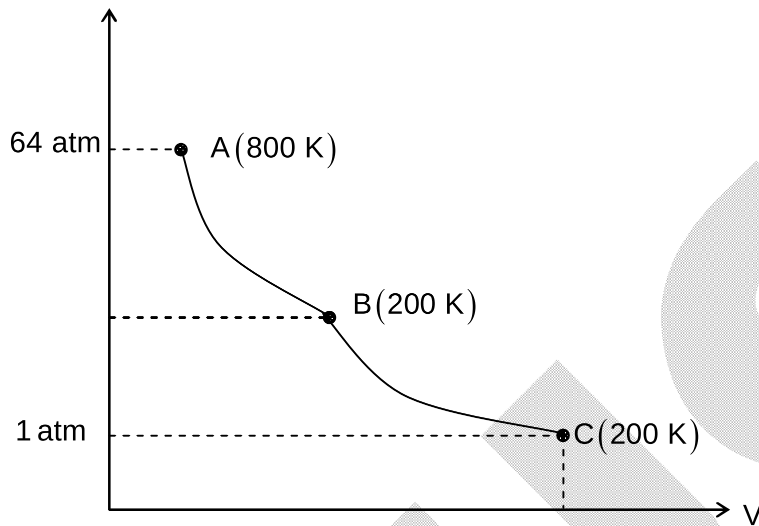
$\therefore (5.0 - x) \text{gm } NaNO_3 \text{ will give} \Rightarrow \frac{69}{85} \times (5.0 - x) \text{gm}$

$\therefore \frac{224}{332} x + \frac{69}{85}(5.0 - x) = (5.0 - 1.35) \text{gm}$

$x = 2.98$

30. 2080.00

Sol.



For AB path :

$$P_2 = P_1\left(\frac{T_1}{T_2}\right)^{\frac{1}{\frac{1}{7}-1}} = 64\left(\frac{800}{200}\right)^{\left(\frac{1}{\frac{3}{5}-1}\right)} = 2 \text{ atm}$$

Total $W_{done} = W_{AB} + W_{BC}$

$= nC_V \Delta T + \left(-nRT \ell n \frac{P_B}{P_C}\right)$

$= 1 \times \frac{3}{2} R \times (200 - 800) + \left[-1 \times R \times 200 \ell n \frac{2}{1}\right]$

$= (-900R) + (-140R)$

$= 2080.00$ cal

31. 4.51

Sol. Mili moles of $H_2SO_3 = 15 \times 0.30 = 4.50$

Mili moles of $NaOH = 30 \times 0.15 = 4.50$

	H_2SO_3	$+ NaOH$	$\longrightarrow NaHSO_3$
Initially	4.5	4.5	–
Finally	–	–	4.5

$$\therefore pH = \frac{pK_{a_1} + pK_{a_2}}{2} = \frac{1.82 + 7.2}{2} = 4.51$$

32. 100.00

Sol. $\left(\frac{\ell n2}{15}\right) \times t = \ell n\frac{100}{1}$

$t = \frac{2 \times 15}{0.30} = 100$

33. 327.00

Sol. Meq. of silver salt = Meq. of silver

$\frac{0.800}{E} \times 1000 = \frac{0.40}{108} \times 1000$

$\therefore$ Equivalent mass of silver salt = 216

Valency of acid = 3

$\therefore$ Molar mass of silver salt $= 216 \times 3 = 648$

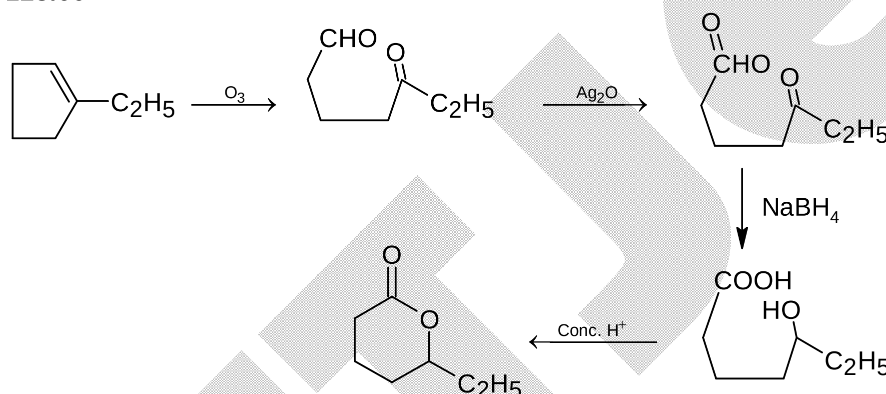
$\therefore$ Molar mass of acid = 648 $+3 - 3 \times$ Atomic mass of Ag

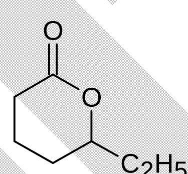
$= 648 + 3 - (3 \times 108)$

= 327

34. 128.00

Sol.

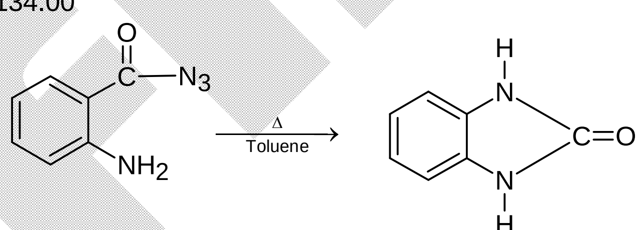


Molar mass of 

is 128

35. 134.00

Sol.



(A)

Molar mass of ‘A’ is 134.

36. 323.00

Sol. 'C' is $PbCrO_4$

$\therefore$ Molar mass $= 207.0 + 52 + 64$ **= 323**

Mathematics

PART – III

Section – A

37. A, C

Sol. $\frac{a}{b}-1=\frac{1}{2} \Rightarrow \frac{a}{b}=\frac{3}{2}$, therefore locus is an ellipse with eccentricity $\frac{\sqrt{5}}{3}$

38. A, C

Sol. tan x = 1 $\Rightarrow$ $x = n\pi + \frac{\pi}{4}$; $n \in I$

$\Rightarrow$ a_1, a_2, a_3, ... form an A.P. and b_1, b_2, b_3, ... form a G.P.

39. A, B

Sol. Angle between asymptotes = 2 $\sec^{-1}$ e

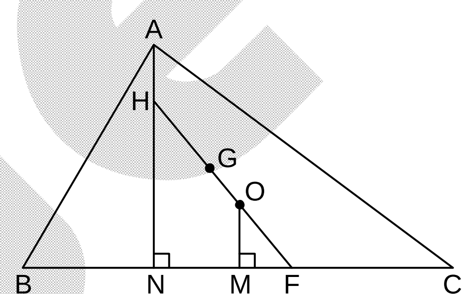
40. B, C

Sol. H is orthocentre

$\frac{OF}{HF}=\frac{OM}{HN}=\frac{MF}{NF}$

$\Rightarrow \frac{1}{4}=\frac{\cos A}{\cos B\cos C}=\frac{MF}{BM-BN+MF}$

$\Rightarrow MF=\frac{a}{6}-\frac{c}{3}\cos B$ and $\tan B\tan C=\frac{5}{4}$



41. A, C, D

Sol. $AA^T = A^TA = I$

$B^2 = I$

AB = –BA

AB is idempotent $\Rightarrow$ (AB)(AB) = AB

$\Rightarrow$ – BAAB = AB

$-B^2$ = AB

B = –A $\Rightarrow$ A + B = O

$B^2 = I \Rightarrow B = B^{-1}$. Hence, $A + B^{-1} = O$

and AB = –BA $\Rightarrow$ |A| |B| = $(-1)^n$ |B| |A|

$\Rightarrow$ n must be even because $|A| \neq 0$ and $|B| \neq 0$

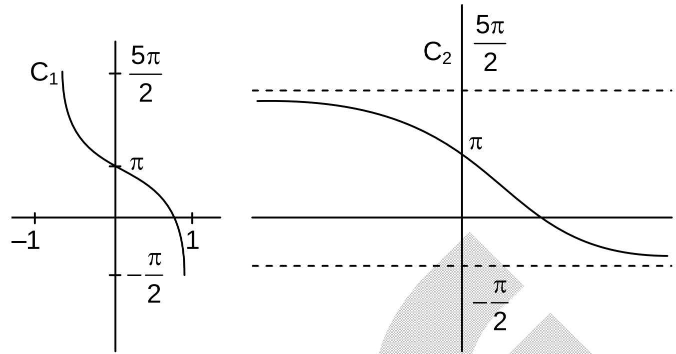
42. B, D

Sol. a = 2b – c ; c^2 = bd $\Rightarrow$ $d=\frac{c^2}{b}$

$\frac{1}{e}=\frac{2}{d}-\frac{1}{c}=\frac{2b}{c^2}-\frac{1}{c} = \frac{2b-c}{c^2}=\frac{a}{c^2}$ $\Rightarrow c^2$ = ae also c^2 = bd

43. C

Sol. $C_1 + C_2$ is decreasing function and its domain will be [–1, 1]



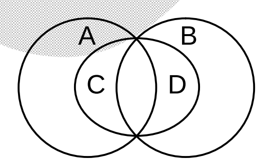
44. B

Sol. (I) $\int \frac{2\tan^2 \frac{x}{2}}{\left(1-\tan\frac{x}{2}\right)^2} dx = \int \left(\sec^2 x - \sec x + \sec x \tan x - \tan x\right) dx$

$= \left(\sec x + \tan x\right) - \ln\left(\sec x + \tan x\right) + \ln\cos x + C$

$\therefore\ g\left(\frac{\pi}{3}\right) = \frac{1}{2}$

(II) $P\left(\frac{A \cap B}{C \cap D}\right) = 1$



(III) $\left(\sqrt{5+2\sqrt{6}}\right)^{4n} + \left(\sqrt{5-2\sqrt{6}}\right)^{4n} + 2 = \left(\sqrt{3}+\sqrt{2}\right)^{4n} + \left(\sqrt{3}-\sqrt{2}\right)^{4n} + 2$

$= \left(\left(\sqrt{3}+\sqrt{2}\right)^{2n} + \left(\sqrt{3}-\sqrt{2}\right)^{2n}\right)^2 = 4\left\{{}^{2n}C_0\left(\sqrt{3}\right)^{2n} + {}^{2n}C_2\left(\sqrt{3}\right)^{2n-2}\left(\sqrt{2}\right)^2 + ...\right\}$

(IV) Let roots are a, b, c, then $\frac{ab+bc+ca}{3} = 4$ and abc = 8

$\Rightarrow \left(\frac{ab+bc+ca}{3}\right)^3 = a^2b^2c^2$

$\Rightarrow$ a = b = c = 2

$\therefore\ \alpha$ = a + b + c = 6

45. C

Sol. $n(S) = \underline{|6} = 720$

(I) $n(E) = \underline{|3} \times 2^3 + 4 \times 2 \times 2 \times 2 \times 2 = 48 + 64 = 112$

p(E) = 0.1555

(II) Suppose three couples are $(H_1, W_1), (H_2, W_2)$ and (H_3, W_3) for our event E

Suppose: **Case-I:** All gents are on first row number of ways = $\underline{|3}\,\underline{|3}\left(1 - \frac{1}{\underline{|1}} + \frac{1}{\underline{|2}} - \frac{1}{\underline{|3}}\right)$

$= 36 \times \frac{1}{3} = 12$ ways

Case-II: Two gents and one lady (no wife of selected gents for first row)

Number of ways = $12 \times {}^3C_2 \times 1 = 36$

Case-III: Two gents and one lady (wife of one of the selected gents for first row)

Number of ways = 0

n(E) = 2 × (12 + 36 + 0) = 96 ⇒ p(E) = 0.1333

(III) The couple which sit infront of each other or adjacently can be selected in 3 ways and this couple when sit

Case-I: Infront of each other. Number of ways = 3 × 2 × 2 × 2 = 24

Case-II: Adjacently number of ways = 4 × 4 × 2 × 1 = 32

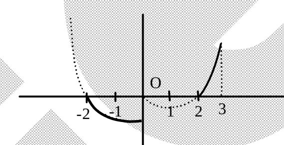
Total number of ways so that exactly one couple sit infront of each other or adjacently

n(E) = 3 × (24 + 32) = 168. p(E) = 0.2333

(IV) Number of way $n(E) = 2 \times \underline{|3} \times \underline{|3} = 72$ required probability = 0.1000

46. D

Sol. g(x) can be defined as



$$g(x) = \begin{cases} f(x), & -2 \le x < -1 \\ -1, & -1 \le x < 0 \\ 0, & 0 \le x < 2 \\ f(x), & 2 \le x \le 3 \end{cases}$$

$$\text{or } g(x) = \begin{cases} x^2 + 2x, & -2 \le x < -1 \\ -1, & -1 \le x < 0 \\ 0, & 0 \le x < 2 \\ x^2 - 2x, & 2 \le x \le 3 \end{cases}$$

Clearly g(x) is discontinuous at x = 0, g(x) is not differentiable at x = 0, 2

Section – B

47. 0.83

Sol. $f(x) = \dfrac{\pi}{2\cos^{-1}x} + \dfrac{\pi}{2\cot^{-1}x} - 2$

f(x) is increasing function and its domain is [–1, 1). Hence range = [f(–1), f(1))

$f(-1) = -\dfrac{5}{6}$

48. 4.25

Sol. Required area = $\dfrac{4\sqrt{3}}{2}\int_1^2 \sqrt{4 - x^2} = 4.25$

49. 0.33

Sol. If an equilateral triangle is inscribed in the parabola $y^2 = 4ax$, then its side length is equal to $8\sqrt{3}a$ unit.

50. 5.00

Sol. $b_n^2 = 5a_n$

51. 15.00

Sol. $\sin^{-1}\dfrac{r_1}{r + r_1} = \dfrac{\pi}{16} \Rightarrow \alpha + \beta = 15$

52. 1.00

Sol. Ellipses cut each other at line, y = x and y = –x due to symmetry, and radius of circle will be 1

53. 24.00

Sol. $$\underline{|4}\left(1-\frac{1}{\underline{|1}}+\frac{1}{\underline{|2}}-\frac{1}{\underline{|3}}+\frac{1}{\underline{|4}}\right)+{}^4C_1\underline{|3}\left(1-\frac{1}{\underline{|1}}+\frac{1}{\underline{|2}}-\frac{1}{\underline{|3}}\right)+{}^4C_2\underline{|2}\left(1-\frac{1}{\underline{|1}}+\frac{1}{\underline{|2}}\right)+1$$

$= 9 + 8 + 6 + 1 = 24$

54. 5.00

Sol. $2\alpha - \beta + 2\gamma = 0$ ..... (1)

$\alpha + 1 = 0$ ..... (2)

$3\alpha + 6\beta - 2\gamma = 0$ ..... (3)

By solving these equation, we get $\alpha = -1$, $\beta = 1$ and $\gamma = \frac{3}{2}$