FIITJEE

ALL INDIA TEST SERIES

PART TEST – I

JEE (Main)-2022

TEST DATE: 20-11-2021

ANSWERS, HINTS & SOLUTIONS

Physics PART – A

SECTION – A

1. D

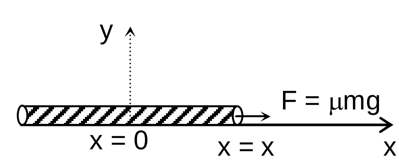
Sol. After every collision the velocity becomes half. When it comes down $\sqrt{2g\ell}$ is gained, again it goes up so that it never becomes zero.

2. D

Sol. $W_F + W_f = \Delta K$

$$\mu mg\ell - \int_0^{\ell} \frac{\mu mgx}{\ell} dx = \frac{mv^2}{2}$$

$$\Rightarrow \quad v = \sqrt{\mu g\ell}$$



3. A

Sol. Here net acceleration is

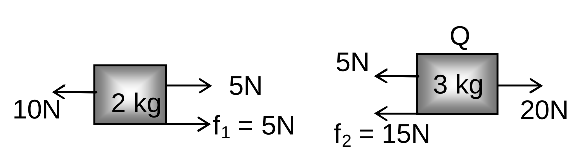
$$\frac{d^2\vec{r}}{dt^2} = -\omega^2(a\cos\omega t\hat{i} + b\sin\omega t\hat{j})$$

$$\Rightarrow \frac{d^2r}{dt^2} = -\omega^2\vec{r}$$

Hence angular acceleration is zero.

4. D

Sol. $\vec{f}_1 = 5N\hat{i}, \quad \vec{f}_2 = -15N\hat{i}, \quad T = 5N$



5. B

Sol. $w_g + w_{sp} + w_{fr} = 0$

$\Rightarrow mgx_m - \frac{1}{2}kx_m^2 - \mu mgx_m = 0$

$\Rightarrow \frac{3mg}{4} = \frac{kx_m}{2}$

Hence, $x_m = \frac{3mg}{2k}$

6. A

Sol. $v_B = \sqrt{2g\ell \sin\theta}$

$v_C = \sqrt{2g\ell}$

$\because\ 2\sqrt{2g\ell \sin\theta} = \sqrt{2g\ell}$

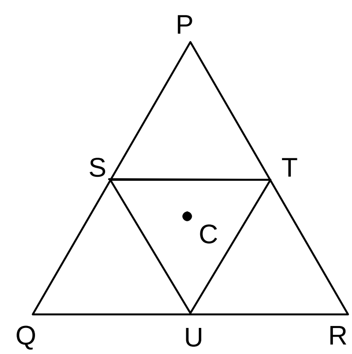
Hence, $\sin\theta = \frac{1}{4}$

7. A

Sol. $\vec{Q}$ cannot be $\hat{i} + \hat{j}$

8. B

Sol. Moment of inertia of STU about an axis passing through point C and perpendicular to its plane is $I_1 = \frac{I_0}{16}$



$I_{PST} = I_{QSU} = I_{TUR} = I_2$

Hence, $3I_2 + I_1 = I_0$

$\Rightarrow I_2 = \frac{5I_0}{16}$

Hence, Required moment of inertia = $I_0 - \frac{5I_0}{16} = \frac{11I_0}{16}$

9. C

Sol. From conservation of energy of rod,

$Mg\frac{R}{2} = \frac{1}{2}\frac{MR^2}{3}\omega^2$

$\omega = \sqrt{\frac{3g}{R}}$

From conservation of angular momentum about O;

$\frac{MR^2\omega}{3} = mvR \Rightarrow v = \frac{M}{3m}\sqrt{3gR}$

For complete the circle

$\frac{M}{3m}\sqrt{3gR} = \sqrt{5gR} \Rightarrow \frac{M}{m} = \sqrt{15}$

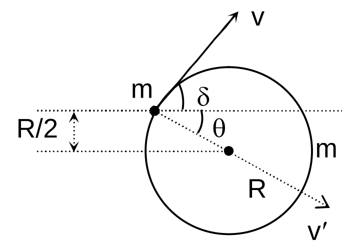
10. B

Sol. $\sin\theta = \frac{1}{2}$

$\theta = 30°$

Deviation;

$\delta = 90° - 30° = 60°$



11. C

Sol. Range R = 12 +3 = 15 m

We know, $Y = x \tan\theta\left(1-\frac{x}{R}\right)$

$H = (12)\left(\frac{3}{4}\right)\left(1-\frac{12}{15}\right)$

$= 9\times\frac{3}{15} = 1.8$ m

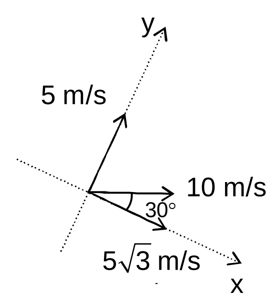
12. D

Sol. Acceleration of particle with respect to the inclined plane $= 10 + g = 20\ m/s^2$ (vertically downward)

$T = \frac{2v_y}{a_y}$

$T = \frac{2(5)}{10\sqrt{3}}$

$T = \frac{1}{\sqrt{3}}$ sec



13. B

Sol. $M = \int_0^R \sigma_0\left(1-\frac{r}{R}\right)2\pi r dr = \frac{\pi\sigma_0 R^2}{3}$

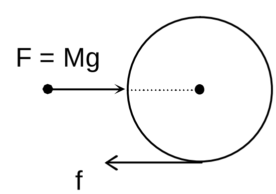
$I = \int_0^R \sigma_0\left(1-\frac{r}{R}\right)2\pi r^3 dr = \frac{3}{10}MR^2$

$F - f = Ma$ …(i)

$fR = \frac{3}{10}MR^2\alpha$ …(ii)

$a = R\alpha$ …(iii)

$\Rightarrow a = \frac{10g}{13}$



14. A

Sol. From conservation of angular momentum, velocity at maximum elongation is $\frac{4v_0}{5}$

Hence, $\frac{1}{2}mv_0^2 = \frac{1}{2}m\left(\frac{4v_0}{5}\right)^2 + \frac{1}{2}k\left(\frac{\ell_0}{4}\right)^2$

$\Rightarrow K = \frac{144}{25}\frac{mv_0^2}{\ell_0^2}$

15. C

Sol. Acceleration of centre of mass of the rod is vertical and due to constraint acceleration of A is horizontal.

16. A

Sol. For block B to move

$kx = 2\mu mg$ …(i)

If v = velocity of block A

$\frac{1}{2}mv^2 = \mu mgx + \frac{1}{2}kx^2$ …(ii)

From (i) and (ii)

$v = 2\mu g\sqrt{\frac{2m}{k}}$

17. B

Sol. $t_0 = \frac{2u\sin\alpha}{g}$

$-H = D\tan\alpha - \frac{gD^2}{2\left(\frac{g^2t_0^2}{4\sin^2\alpha}\right)\cos^2\alpha}$

$= \frac{gt_0^2}{4D}\left[1+\sqrt{1+\frac{8H}{gt_0^2}}\right]$

18. C

Sol. Along the rod,

$u \sin\theta = v\cos\theta$

$u = v\frac{\cos\theta}{\sin\theta}$ (u = velocity of B)

Acceleration of point B,

$a = \frac{du}{dt} \Rightarrow a = -v\,\text{cosec}^2\theta\left(\frac{d\theta}{dt}\right)$

Also, $-\frac{d\theta}{dt} = \frac{v}{\ell\sin\theta}$

Hence, $a = \frac{v^2}{\ell\sin^3\theta}$

19. B

Sol. $M = \rho\pi R^2\frac{R}{6}$

Also, $M = \rho\frac{4}{3}\pi r^3$ (r = radius of sphere)

$I = \frac{MR^2}{2}$

For sphere,

$I' = \frac{2}{5}Mr^2 = \frac{2}{5}M\left(\frac{R}{2}\right)^2 = \frac{MR^2}{10} = \frac{I}{5}$

20. B

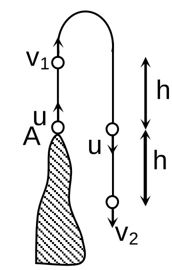
Sol. $v_1^2 = u^2 - 2gh$

$v_2^2 = u^2 + 2gh$

Also, $v_2^2 = 4v_1^2$

$\Rightarrow 3u^2 = 10gh$

Hence, $H_{max} = \frac{u^2}{2g} = \frac{5h}{3}$

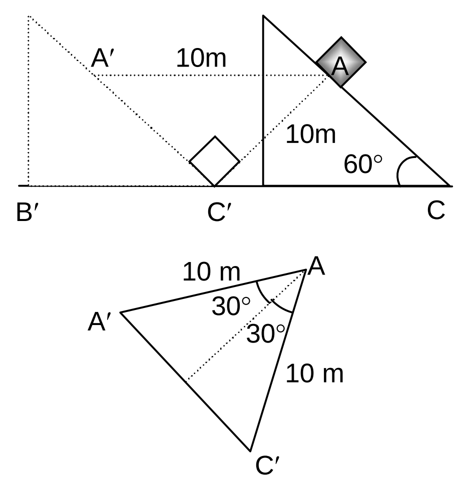


SECTION – B

21. 00010.00

Sol. AA′ = AC′ = 10 m

$\Rightarrow A'C' = 20\sin 30° = 10$ m

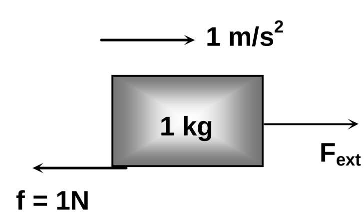


22. 00004.00

Sol. F_{ext} = 2 N

v = (1) (2) = 2 m/s

Hence, Power P = Fv = 4 watt



23. 00000.10

Sol. $a_C = \frac{v^2}{R} = 2\ \text{m/s}^2$

$a_A = \sqrt{(2a)^2 + \left(\frac{v^2}{R}\right)^2} = \sqrt{36+4} = \sqrt{40}\ \text{m/s}^2$

$\frac{a_C}{a_A} = \frac{2}{\sqrt{40}} = \sqrt{\frac{4}{40}} = \sqrt{0.1}$

24. 00034.00

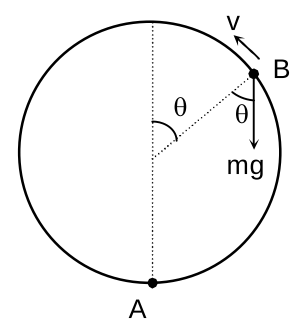
Sol. $a_{CM} = 10 = \frac{2m(-2) + m(a)}{2m + m}$

$\Rightarrow a - 4 = 30$

$\Rightarrow a = 34\ \text{m/s}^2$

25. 00053.00

Sol. When it leaves contact,



$$mg\cos\theta = \frac{mv^2}{R} \quad \ldots(i)$$

From conservation of mechanical energy at A and B,

$$\frac{1}{2}m\left(\sqrt{\frac{95}{25}Rg}\right)^2 = mgR(1+\cos\theta) + \frac{mv^2}{2} \quad \ldots(ii)$$

From (i) and (ii)

$$\cos\theta = \frac{3}{5}$$

$\Rightarrow \theta = 53°$

26. 00180.00

Sol. Smallest radius of curvature is at topmost point

$$\frac{(u\cos 53°)^2}{90} = 10$$

$\Rightarrow$ u = 50 m/s

After 1 sec from projection

v_x = 30 m/s, v_y = 30 m/s

Hence, $v = 30\sqrt{2}$ m/s

Normal acceleration $a_n = \frac{10}{\sqrt{2}}$ m/s^2

Hence, $\rho = \frac{v^2}{a_n} = \frac{(30\sqrt{2})^2}{(10/\sqrt{2})} = 180\sqrt{2}$

27. 00005.00

Sol. Acceleration of each block = a

$$a = \frac{30\cos 60°}{5+10} = 1 \text{ m/s}^2$$

Hence, Tension T = $m_A a = 5N$

28. 00002.00

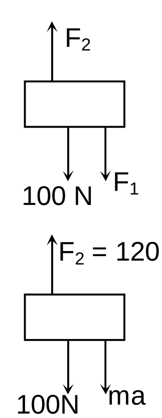
Sol. When the lift is at rest, tension in the upper spring is

F_2 = 10 + 100 = 110 N

Since both springs are identical, so extension of upper spring is 11 times that of lower spring. When tension in lower spring is zero, then upper spring stretches 12 times.

ma = 20

a = 2 m/s^2



29. 00020.00

Sol. W = μmgx = 20 J

30. 00037.50

Sol. Centre of mass of the system stops moving after both the blocks stop.
Left block stops at t = 10 sec and the right block stops at t = 5 sec

$$v_{cm} = \frac{(20)(2)-(10)(2)}{2+2} = 5 \text{ m/s}$$

Displacement of centre of mass in 5 sec S_1 = 25 m
Retardation of centre of mass between t = 5 to t = 10 sec is $a_{CM} = -1$ m/s^2
Further displacement of centre of mass before it stops

$$S_2 = (5)(5) - \frac{1}{2}(1)(5)^2 = 12.5 \text{ m}$$

Hence, total displacement of centre of mass of the system = 37.50 m

Chemistry

PART – B

SECTION – A

31. D

Sol. Since, $\sqrt{\nu} = a(Z-b)$

$\Rightarrow \sqrt{\nu} = aZ - ab$

Slope $a = \tan 45^o = 1$

Intercept $ab = OX = 1$

$\Rightarrow \sqrt{\nu} = 52 - 1 = 51$

$\Rightarrow \nu = 51^2$

$= 2601\,Hz$

32. A

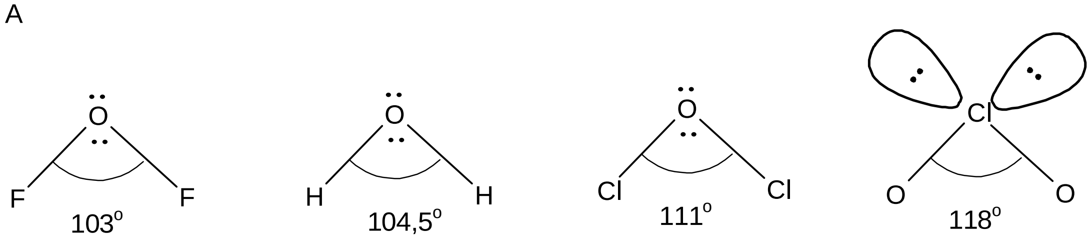
Sol. $K_2Cr_2O_7 + 14HCl \longrightarrow 2KCl + 2CrCl_3 + 3Cl_2 + 7H_2O$

$x = 2, y = 2, z = 3, w = 7$

x + y + z = 7

33. A

Sol.



34. A

Sol. At $25^oC(298K), [H^+] = 10^{-7}$

$\Rightarrow K_w = 10^{-14}$

At $35^oC(308\,K), [H^+] = 10^{-6}$

$\Rightarrow K_w = 10^{-12}$

Since, $\log\frac{K_{w_2}}{K_{w_1}} = \frac{\Delta H^o}{2.303R}\left(\frac{T_2 - T_1}{T_1T_2}\right)$

$\Rightarrow \log\frac{10^{-12}}{10^{-14}} = \frac{\Delta H^o}{2.303\times 2}\left(\frac{308-298}{308\times 298}\right)$

$\Rightarrow \Delta H^o = 84551\,\text{cal mol}^{-1}$

$\Rightarrow \Delta H^o = 84.55\ \text{Kcal mol}^{-1}$

$\downarrow$

$\left(\text{for } H_2O \longrightarrow H^+ + OH^-\right)$

35. C

Sol. $2SO_3(g) \rightleftharpoons 2SO_2(g) + O_2(g)$

At equili. 400 CC 200 CC 100 CC

$$P_{SO_3} = \frac{400}{700} \times 10 = \frac{40}{7} \text{ atm}$$

$$P_{SO_2} = \frac{200}{700} \times 10 = \frac{20}{7} \text{ atm}$$

$$P_{O_2} = \frac{100}{700} \times 10 = \frac{10}{7} \text{ atm}$$

$(\because \text{mol\%} \propto \text{vol. \%})$

$\Rightarrow (\text{mol fraction} \propto \text{vol. fraction})$

$$K_P = \frac{{P_{SO_2}}^2 \times P_{O_2}}{{P_{SO_3}}^2}$$

$$K_P = \frac{\frac{20}{7} \times \frac{20}{7} \times \frac{10}{7}}{\frac{40}{7} \times \frac{40}{7}}$$

$$\Rightarrow K_P = \frac{5}{14}$$

36. A

Sol. Saline/Ionic hydrides are
1. Formed by s-Block elements.
2. Good reducing agents.
3. Readily decomposed by water and alcohols to liberate $H_2(g)$.
4. Saline hydirides have high melting point

37. C

Sol. $MnO_4^- + H_2O_2 + H^+ \longrightarrow Mn^{++} + O_2$ (H_2O_2 acting as reductant)

$PbS \xrightarrow{H_2O_2} PbSO_4$ (H_2O_2 acting as oxidant)

$SO_2 \xrightarrow{H_2O_2} H_2SO_4$ (H_2O_2 acting as oxidant)

$Na_2CO_3 + H_2O_2 \longrightarrow Na_2O_2 + H_2O + CO_2$

(Acid – Base reaction)

38. B

Sol. $B \equiv 801\ KJ\ mol^{-1}$

$Al \equiv 577\ KJ\ mol^{-1}$

$Ga \equiv 579\ KJ\ mol^{-1}$

$In \equiv 558\ KJ\ mol^{-1}$

$Tl \equiv 589\ KJ\ mol^{-1}$

The irregular variation is due to poor shielding offered by d- and f-electrons which effects Z_{eff}.

39. C

Sol. $Pb_3O_4 + 4HNO_3 \longrightarrow 2Pb(NO_3)_2 + PbO_2 + H_2O$

40. D

Sol. (A), (B) and (C) are correct (fact based).

Beryl is $Be_3Al_2[Si_6O_{18}]$ is a cyclic silicate.

41. C
Sol. Amphoteric oxides $\rightarrow Cr_2O_3, ZnO, Al_2O_3, BeO, SnO, SnO_2, PbO, PbO_2$
Acidic oxides $\rightarrow CrO_3, B_2O_3$
Basic oxide $\rightarrow CrO$

42. C
Sol. For isostructural species, the hybridization and shape must be same
$\left.\begin{matrix} CO_3^{--} \\ NO_3^- \end{matrix}\right\} sp^2$ planar $\left.\begin{matrix} PCl_4^+ \\ SiCl_4 \end{matrix}\right\} sp^3$ tetrahedral $\left.\begin{matrix} AlF_6^{3-} \\ SF_6 \end{matrix}\right\} sp^3d^2$ octahedral

$PF_5 \Rightarrow$ Trigonal bipyramidal (sp^3d) $BrF_5 \Rightarrow$ square pyramidal (sp^3d^2)

43. D
Sol. $H_2S \rightleftharpoons H^+ + HS^-$
$HCl \rightarrow H^+ + Cl^-$
Due to common ion effect the conc. of H^+ will increase, so the equilibrium will shift in backward direction.
$\therefore [H^+]\uparrow; [HS^-]\downarrow; [H_2S]\uparrow$ and K_{eq} remains same since temperature do not charge.

44. B
Sol. $X \rightleftharpoons 2Y,\ K_C = 4$
At point (A), $Q_C = \frac{[Y]^2}{[X]} = 0$
Since, [Y] = 0.
So, (I) is incorrect.
When [X] = [Y] = 0.1 M
$Q_C = \frac{0.01}{0.1} = 0.1\,M$
$\Rightarrow Q_C < K_C$
Reaction is in forward direction. So, (II) is correct.
At point (D) or (E), the conc. of [X] and [Y] is constant w.r.t. time. So, equilibrium is reached, i.e. $Q_C = K_C$
$\therefore$ (III) is correct.

45. A
Sol. Since, $K = \frac{[X]^2}{[X_2]}$
$[X] = K^{1/2}[X_2]^{1/2}$ … (1)
$\frac{d[Y]}{dt} = 0 = K_1[X][Y_2] - K_2[Y][X_2]$
or
$[Y] = \frac{K_1[X][Y_2]}{K_2[X_2]} = \frac{K_1K^{1/2}[X_2]^{1/2}[Y_2]}{K_2[X_2]}$

$$\left[Y\right]=\frac{K_1K^{1/2}}{K_2}\left[X_2\right]^{-1/2}\left[Y_2\right]$$

Now, $\frac{d\left[Z\right]}{dt}=K_1\left[X\right]\left[Y_2\right]+K_2\left[Y\right]\left[X_2\right]$

$$K_1.K^{1/2}\left[X_2\right]^{1/2}\left[Y_2\right]+K_2.\frac{K_1K^{1/2}}{K_2}\left[X_2\right]^{-1/2}\left[Y_2\right]\left[X_2\right]$$

$$\frac{d\left[Z\right]}{dt}=2K_1K^{1/2}\left[X_2\right]^{1/2}\left[Y_2\right]$$

46. C

Sol.

$$A_2\left(g\right)\longrightarrow B\left(g\right)+\frac{1}{2}C\left(g\right)$$

$t=0,$	100	–	–
$t=5$	$100-x$	x	$x/2$

Since, $100-x+x+x/2=120$

$x=40$

$\therefore$ Rate of disappearance of $A_2\left(g\right)=\frac{-d\left[A_2\right]}{dt}=\frac{40}{5}=8\text{ mm min}^{-1}$

47. C

Sol.

$$CaSO_4.2H_2O\xrightarrow{120^oC}CaSO_4.\frac{1}{2}H_2O+\frac{3}{2}H_2O$$

$X\left(POP\right)$

$CaSO_4.2H_2O\xrightarrow[\Delta]{200^oC}CaSO_4+2H_2O$

$Y\left(DBP\right)$

$$\xrightarrow[\text{heating}]{\text{Strong}}CaO+\frac{1}{2}O_2+SO_2$$

$\left(Z\right)$

$SO_2\left(\text{Acidic gas}\right)\Rightarrow$ Irritating burning tyre smell gas.

48. C

Sol. Thermal stability of Gr. (2) carbonates increases down the group due to increase in the size of M^{++} ion. While K_2CO_3 is thermall stable and decomposes at a very high temperature.

49. D

Sol. $PCl_5\left(\text{vapour phase}\right)\Rightarrow P\left(sp^3d\right)$

$PCl_5\left(\text{Solid phase}\right)\Rightarrow\left[PCl_4\right]^+\quad\left[PCl_6\right]^-$

$sp^3\quad sp^3d^2$

$SiF_4+2HF\longrightarrow H_2\left[SiF_6\right]$

$sp^3\quad sp^3d^2$

$BeCl_3(s) \longrightarrow$ Polymeric

sp^3 ($\because$ Be is bonded to four Cl atoms)

$[SiCl_6]^{2-}$ do not exist because six large Cl^- ions cannot be octahedrally arranged around small Si^{4+} ion.

50. B

Sol. $\dot{N}\ddot{O}:$ $\Rightarrow$ Odd e^- $\Rightarrow$ paramagnetic

$:C \equiv O:$ $\Rightarrow$ Diamagnetic (isoelectronic with N_2)

MnO_4^{2-}, i.e. Mn^{6+} $\Rightarrow$ one unpaired electron $\Rightarrow$ paramagnetic

B_2, i.e. $\sigma(1s)^2 \sigma^*(2s)^2 \sigma(2s)^2 \sigma^*(2s)^2$

$\left\{\begin{matrix}\pi(2p_x)^1 \\ \pi(2p_y)^1\end{matrix}\right. \Rightarrow$ paramagnetic

SECTION – B

51. 00005.00

Sol. $K-\text{shell} \Rightarrow 1s \Rightarrow 1s^2 = (n-5)s^2$

$\Rightarrow n = 6$

$1s^2........\underset{\substack{\downarrow \\ \text{Pre-penul} \\ (23)}}{\underline{4s^2p^6d^{10}f^5}}\ \underset{\substack{\downarrow \\ \text{Penultimate} \\ (8)}}{\underline{5s^2 5p^6}}\ \underset{\substack{\downarrow \\ \text{Outermost} \\ (2)}}{\underline{6s^2}}$

Number of unpaired electrons = f^5
i.e. 5.

52. 00001.25

Sol.

$$\left.\begin{matrix} N_2^+, BO = \frac{1}{2}(9-4) = 2.5 \\ N_2^-, BO = \frac{1}{2}(10-5) = 2.5 \\ O_2^+, BO = \frac{1}{2}(10-5) = 2.5 \end{matrix}\right\}$$

$O_2^-, BO = \frac{1}{2}(10-7) = 1.5$

NO and N_2^- are isoelectronic.
So, BO (NO) = 2.5
CN and N_2^+ are isoelectronic
So, BO (CN) = 2.5
$\therefore$ Total species 'x' = 5

$\therefore \frac{x}{4} = 1.25$

53. 00008.00

Sol. Borax is $Na_2[B_4O_5(OH)_4].8H_2O$

54. 00003.20

Sol. $C_{60} \Rightarrow$ 5 membered rings $= 12$

6 membered rings $= \frac{60}{2} - 10 = 20$

T = 12 + 20 = 32

$\therefore \frac{T}{10} = \frac{32}{10} = 3.20$

55. 00600.00

Sol. $\frac{-d[RX]}{dt} = K_2[RX][OH^-]\;(S_N2)$

$\frac{-d[RX]}{dt} = K_1[RX] \qquad (S_N1)$

$\Rightarrow \frac{-d[RX]}{dt} = K_2[RX][OH^-] + K_1[RX]$

or

$$-\underset{\downarrow\atop Y}{\frac{1}{RX}.\frac{d[RX]}{dt}} = \underset{\downarrow\atop \text{Slope}}{K_2}\,\underset{\downarrow\atop X}{[OH^-]} + \underset{\downarrow\atop \text{Intercept}}{K_1}$$

$K_2 = 4\times10^3\ M^{-1}s^{-1}$

$K_1 = 2\times10^2\ s^{-1}$

$[RX] = 1\,M \qquad [OH^-] = 0.1\,M$

$\Rightarrow \frac{-d[RX]}{dt} = 4\times10^3\times0.1 + 2\times10^2\times1$

$\Rightarrow \frac{-d[RX]}{dt} = 600\ mol\ L^{-1}S^{-1}$

56. 00002.00

Sol. Given : First order reaction

	$C_8H_{18}O_2(g) \longrightarrow$	$2CH_3COCH_3(g)$	$+\,C_2H_6(g)$
$t = 0$	1000	0	0
$t = 2$	$100 - P$	2P	P

After 2 minutes

$P_{total} = 1000 - P + 2P + P$

2000 = 1000 + 2P

2P = 1000

P = 500 mm Hg

$\therefore$ Pressure of acetone $= 2P = 2\times500 = 1000$ mm Hg .

57. 00011.00

Sol. n = 3

$\Rightarrow 3s, (3P_x, 3P_y, 3P_z), (3d_{xy}, 3d_{yz}, 3d_{zx}, 3d_{x^2-y^2}, 3d_{z^2})$

Total nodal planes = 3(1) + 4(2) = 11.

3s $\Rightarrow$ No nodal plane

$3d_{z^2} \Rightarrow$ No nodal plane rather have two nodal cones.

58. 00002.75

Sol. The balanced equation is

$$C_2H_5OH + 4I_2 + 6OH^- \longrightarrow CHI_3 + HCOO^- + 5H_2O + 5I^-$$

$$\Rightarrow x = 6, y = 4, z = 5$$

$$\Rightarrow \frac{x+z}{y} = \frac{6+5}{4} = \frac{11}{4} = 2.75$$

59. 00006.20

Sol. Hydrazine is a weak base

$$\therefore pOH = \frac{1}{2}\left(pK_b - \log C\right)$$

$$2\left(14 - 9.7\right) = pK_b - \log 4 \times 10^{-3}$$

$$8.6 = pK_b + 3 - 2\left(0.30\right)$$

$$\Rightarrow pK_b = 6.20$$

60. 00040.24

Sol. $M_{mixture} = \frac{dRT}{P} = \frac{0.925 \times 0.08 \times 900}{1}$

$$M_{mixture} = 66.6 \text{ g mol}^{-1}$$

Now, $D = \frac{80}{2} = 40$

$$d = \frac{66.6}{2} = 33.3$$

$$\therefore \alpha = \frac{D-d}{(n-1)d} = \frac{40 - 33.3}{\left(\frac{3}{2} - 1\right)33.3}$$

$$\alpha = \frac{6.7 \times 2}{33.3} = 0.4024$$

i.e. 40.24%

Mathematics PART – C

SECTION – A

61. D

Sol. $\lim_{x\to\infty} \frac{x^{2013}}{e^{3x}} = 0$; $\lim_{x\to\infty} \left(\cos\frac{2}{x}\right)^{x^2} = e^{-2}$

62. C

Sol. R is not reflexive as (2, 2) $\notin$ R (3, 3) $\notin$ R. Also R is not transitive as (2, 1) $\in$ R and (1, 2) $\in$ R. But (2, 2) $\notin$ R. However R is symmetric

63. C

Sol. $\int \frac{x^3 - x^2 + x - 1}{x^5 + 1} dx = \int \frac{x^4 - 1}{(x^5 + 1)(x + 1)} dx = \int \frac{x^4}{x^5 + 1} dx - \int \frac{1}{x + 1} dx$

$= \frac{1}{5}\ln|x^5 + 1| - \ln|x + 1| + c$

64. B

Sol. $a_2^2 > (a_1 + a_3)^2 \Rightarrow (a_1 - a_2 + a_3)(a_1 + a_2 + a_3) < 0 \Rightarrow f(-1)\cdot f(1) < 0$

65. A

Sol. Let (3, α) be the point on y = g(x) $\Rightarrow$ (α, 3) lies on y = f(x) $\Rightarrow$ α = 1

Also, x = f(g(x)) $\Rightarrow$ $g'(3) = \frac{1}{f'(g(3))} = \frac{1}{f'(1)} = \frac{1}{4}$

Hence, tangent is $y - 1 = \frac{1}{4}(x - 3) \Rightarrow x - 4y + 1 = 0$

66. B

Sol. Let $x = \frac{1-u}{1+u}$ $\therefore$ $I = \int_0^1 \frac{\ln 2 - \ln(1+u)}{1+u^2} dx \Rightarrow 2I = \ln 2(\tan^{-1} u)_0^1 \Rightarrow I = \frac{\pi \ln 2}{8}$

67. A

Sol. $I = \int_1^e (\sin x)\left(\frac{1}{x}\right) dx + \int_1^e (x\ln x - x)\sin x\, dx$. Applying integration by parts, we get

$I = \sin x \ln x\Big|_1^e - \int_1^e \cos x \ln x\, dx - (x\ln x - x)\cos x\Big|_1^e + \int_1^e \ln x \cdot \cos x\, dx = \sin e - \cos 1$

68. C

Sol. $f'(0) = \lim_{h\to 0} \frac{f(h) - f(0)}{h}$; $\lim_{h\to 0} \frac{h^a \sin\left(\frac{1}{h}\right)}{h} = \lim_{h\to 0} = \lim\ h^{a-1}\sin\left(\frac{1}{h}\right)$

This limit will not exist it a – 1 $\le$ 0 or a $\le$ 1

Now, $\lim_{x\to 0} f(x) = \lim_{x\to 0} x^a \sin\left(\frac{1}{x}\right) = 0$ if a > 0 $\Rightarrow$ a $\in$ (0, 1]

69. A

Sol. $(xy + 1)(3x^2 y\,dx - x^3\,dy) = (x^6 + y^2)(x\,dy + y\,dx)$

$$\Rightarrow (xy+1)d\left(\frac{x^3}{y}\right) = \left(\left(\frac{x^3}{y}\right)^2 + 1\right)d(xy)$$

70. A

Sol. $\sin^4 x = \sin^2 x - \frac{1}{4}\sin^2 2x$

$$\Rightarrow S_n = \left(\sin^2 x - \frac{1}{4}\sin^2 2x\right) + \left(\frac{1}{4}\sin^2 2x - \frac{1}{4^2}\sin^2 4x\right) + ... + \left(\frac{1}{4^n}\sin^2 2^n x - \frac{1}{4^{n+1}}\sin^2 2^{n+1}x\right)$$

$$\Rightarrow S_n = \sin^2 x - \frac{1}{4^{n+1}}\sin^2 2^{n+1}x \Rightarrow \lim_{n\to\infty} S_n = \sin^2 x$$

71. B

Sol. f(8) = f(4 + 4) = f(16) = f(8 + 8) = f(64) = f(16 + 4) = f(20) = f(5 + 4) = f(9)

72. D

Sol. $\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \frac{x f(x)}{x} = \lim\limits_{x\to\infty} \left(f(x) + x f'(x)\right) = a \Rightarrow \lim\limits_{x\to\infty} x f'(x) = 0$

Now, $\lim\limits_{x\to\infty} x f'(x) = \frac{x^2 f'(x)}{x}$; $\lim\limits_{x\to\infty} \left(2xf'(x) + x^2 f''(x)\right) = 0 \Rightarrow \lim\limits_{x\to\infty} x^2 f''(x) = 0$

73. C

Sol. From LMVT, there exists atleast (n – 1) points where $f'(x) = m$, therefore, there exists atleast (n – 2) points where $f''(x) = 0$ (using Roll's theorem)

74. B

Sol. This is equivalent to $p \vee q$

75. B

Sol. It is equal to $\int_0^1 \left[\frac{x^3}{4} - x\left(f(x) - \frac{x}{2}\right)^2\right]dx$ which is less than $\int_0^1 \frac{x^3}{4}dx = \frac{1}{16}$

76. C

Sol. $\left[-\sin^2 x\right] = 0$ or –1 but $\sec^{-1}(0)$ is not defined. Hence, $y = \sec^{-1}\left[-\sin^2 x\right] = \pi$

Now, $\pi = \frac{16 - x^2}{4} \Rightarrow x = \pm 2\sqrt{4 - \pi}$

Hence, required area = $\int_{-2\sqrt{4-\pi}}^{2\sqrt{4-\pi}} \left(\frac{16 - x^2}{4} - \pi\right)dx = \frac{8}{3}(4 - \pi)^{\frac{3}{2}}$

77. B

Sol. Analyse by graph

78. C

Sol. Using $\int_0^a f(x)dx = \int_0^a f(a - x)dx$, we get I = –I $\Rightarrow$ I = 0

79. B

Sol. Consider $f(x) = (\tan^{-1}x)^2 + \frac{2}{\sqrt{x^2+1}}$, then f′(x) > 0 ∀ x ∈ (0, ∞)

$\Rightarrow f\left(\frac{1}{e}\right) < f(1) < f(e) \Rightarrow I_2 < I_1 < I_2$

80. B

Sol. From the given information, we get the differential equation (y – 1)dy + (x – 1)dx = 0
On solving, we get the particular solution $(x-1)^2 + (y-1)^2 = 25$

SECTION – B

81. 00001.13

Sol. On solving the given equation after differentiating, we get

$f(x) = 3\cos x - 2\cos^2 x = -2\left[\cos x - \frac{3}{4}\right]^2 + \frac{9}{8}$

$f(x)_{max} = \frac{9}{8} = 1.125 = 1.13$

82. 00003.00

Sol. Considering [2x] = 0, 1, 2, 3, 4 and 5 and corresponding [y] = 5, 4, 3, 2, 1, 0, we get six squares each of area = 0.5 sq. units
Hence, total area = 6 × 0.5 = 3 sq. units

83. 00000.50

Sol. $I = \int_0^1 \frac{\tan^{-1}x}{\tan^{-1}x + \tan^{-1}(1-x)}dx$; $2I = \int_0^1 1dx \Rightarrow I = \frac{1}{2} = 0.5$

84. 00056.00

Sol. f(1) = 0 and f(2) = 30 ⇒ $\int_0^2 f(x)dx + \int_0^{30} f^{-1}(y)dy = \int_0^1 f(x)dx + \int_1^2 f(x)dx + \int_1^2 tf'(t)dt$

$= (x^4 + x^2 - 6x)_0^1 + \int_1^2 f(x)dx + (tf(t))_1^2 - \int_1^2 f(t)dt = 56$

85. 02010.00

Sol. $I = \int \frac{\sin^{2009}x\cos x - \cos^{2009}x \cdot \sin x}{\sin^{2010}x + \cos^{2010}x}dx$, put $\sin^{2010}x + \cos^{2010}x = t$

$\Rightarrow I = \frac{1}{2010}\int\frac{dt}{t} = \frac{1}{2010}\log|\sin^{2010}x + \cos^{2010}x| + c \Rightarrow k = 2010$

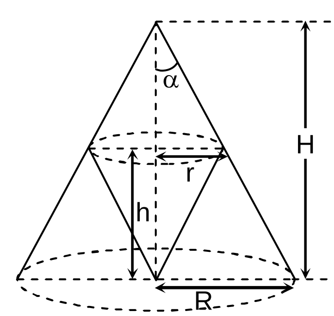
86. 00003.00

Sol. Here, $\frac{r}{R} = \frac{H-h}{H} \Rightarrow r = \frac{R(H-h)}{H}$

Volume = $\frac{1}{3}\pi\frac{R^2(H-h)^2}{H^2}\cdot h$

Now, $\frac{dv}{dh} = 0 \Rightarrow h = \frac{H}{3}$

$\Rightarrow$ For maximum volume $\frac{H}{h} = 3$



87. 00003.00

Sol. f(1) = f(2) also f′(x) = 0 at $x = \frac{4}{3}$ on solving, we get m = –5, n = 8 ; m + n = 3

88. 00013.00

Sol. $\left(\frac{dy}{dx}\right)_{x_1y_1} = 3x_1^2 + 1 \Rightarrow 3x_1^2 + 1 = \frac{y_1}{x_1} \Rightarrow 3x_1^3 + x_1 = x_1^3 + x_1 + 16 \Rightarrow x_1 = 2$ and $y_1 = 26 \Rightarrow m = \frac{y_1}{x_1} = 13$

89. 00025.00

Sol. On simplification, this becomes $\lim_{x\to 1} \frac{50\left(1 - x^{50}\right)}{\left(1 - x^{50}\right)\left(1 + x^{50}\right)} = \lim_{x\to 1} \frac{50}{1 + x^{50}} = 25$

90. 00028.00

Sol. $f(x) = ax^7 + bx^5 + cx^3 + dx + \frac{1}{x} + 15$; $f(-x) = -ax^7 - bx^5 - cx^3 - dx - \frac{1}{x} + 15$

$\Rightarrow f(x) + f(-x) = 30 \Rightarrow f(5) + f(-5) = 30 \Rightarrow f(-5) = 28$