FIITJEE

ALL INDIA TEST SERIES

PART TEST – I

JEE (Main)-2022

TEST DATE: 20-11-2021

Time Allotted: 3 Hours **Maximum Marks: 300**

General Instructions:

- The test consists of total 90 questions.
- Each subject (PCM) has 30 questions.
- This question paper contains **Three Parts.**
- **Part-A** is Physics, **Part-B** is Chemistry and **Part-C** is Mathematics.
- Each part has only two sections: **Section-A and Section-B**.
- **Section – A** : Attempt all questions.
- **Section – B :** Do any five questions out of 10 Questions.

  Section-A (01 – 20, 31 – 50, 61 – 80) *contains 60 multiple choice questions which have* ***only one correct answer****. Each question carries* ***+4 marks*** *for correct answer and* ***–1 mark*** *for wrong answer.*

  Section-B (21 – 30, 51 – 60, 81 – 90) *contains 30 Numerical answer type questions with answer XXXXX.XX and each question carries* ***+4 marks*** *for correct answer. There is no negative marking.*

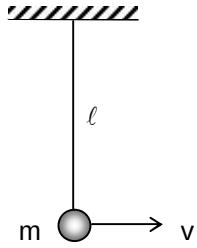
Physics

PART – A

SECTION – A
(One Options Correct Type)

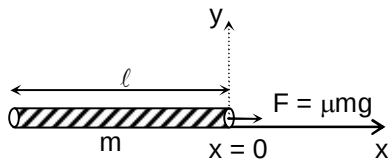
This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

1. A simple pendulum of length 'ℓ' is attached to a point in a horizontal roof as shown in the figure. The bob of mass 'm' is given an initial horizontal velocity $v = \sqrt{5g\ell}$. The coefficient of restitution between the bob and the horizontal roof is $e = \frac{1}{2}$. After how many collisions, the bob will never come in contact with the horizontal roof?



(A) 1
(B) 2
(C) 4
(D) none of these

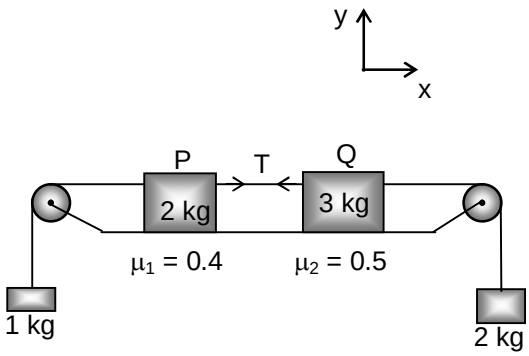
2. A uniform rope of mass m and length 'ℓ' lies on a horizontal floor as shown in the figure. Surface is smooth for x < 0, and has coefficient of friction 'μ' for x > 0. If a constant horizontal force F = μmg starts acting on the rope as shown in the figure, find the speed of the rope when its rear end crosses x = 0.



(A) zero
(B) $\sqrt{\frac{\mu g\ell}{2}}$
(C) $\frac{\sqrt{\mu g\ell}}{2}$
(D) $\sqrt{\mu g\ell}$

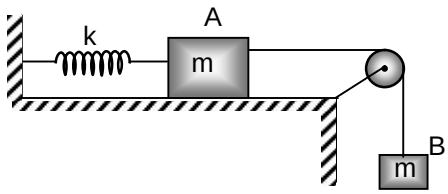
3. A particle moves on an elliptical path with its position vector is given by $\vec{r} = a\cos\omega t\hat{i} + b\sin\omega t\hat{j}$. The angular acceleration of particle is
(A) zero
(B) $-a\omega^2\cos\omega t\hat{i}$
(C) $-b\omega^2\sin\omega t\hat{j}$
(D) $-\omega^2\vec{r}$

4. In the mass pulley system shown in the figure, frictional force on 2 kg block P is f_1 and on 3 kg block Q is f_2. If T is tension in the string connecting the blocks P and Q, then which of the following options is correct?



(A) $\vec{f}_1 = -8N\hat{i}, \vec{f}_2 = -15N\hat{i}, T = 2N$
(B) $\vec{f}_1 = 8N\hat{i}, \vec{f}_2 = 15N\hat{i}, T = 2N$
(C) $\vec{f}_1 = -5N\hat{i}, \vec{f}_2 = -15N\hat{i}, T = 5N$
(D) $\vec{f}_1 = 5N\hat{i}, \vec{f}_2 = -15N\hat{i}, T = 5N$

5. The system is held with the spring at its relaxed length and then released. Find the maximum elongation of spring if coefficient of friction between the block A and the horizontal surface is $\frac{1}{4}$. (Take g = acceleration due to gravity and k = stiffness of spring, $m_A = m_B = m$)



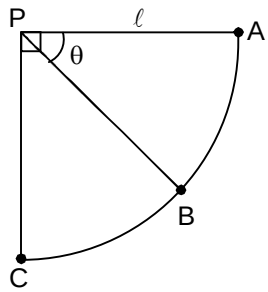
(A) $\frac{3mg}{k}$

(B) $\frac{3mg}{2k}$

(C) $\frac{2mg}{3k}$

(D) $\frac{2mg}{k}$

6. One end of a light string is tied to a bob and other end is connected to a fixed point P. The ball is released from rest with string horizontal and just taut. The ball moves in a vertical circular path as shown in figure. If velocity of the ball at B and C are $\vec{v}_B$ and $\vec{v}_C$ related as $2|\vec{v}_B| = |\vec{v}_C|$, then the value of sin θ is



(A) $\frac{1}{4}$

(B) $\frac{2}{3}$

(C) $\frac{1}{2}$

(D) $\frac{1}{3}$

7. If $\vec{P} = \hat{i} + \hat{j}$, and vector component of $\vec{Q}$ along $\vec{P}$ is $\frac{\hat{i} + \hat{j}}{\sqrt{2}}$, then which of the following cannot be $\vec{Q}$?

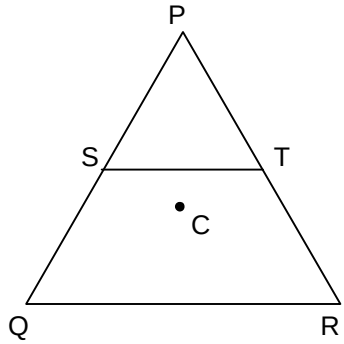
(A) $\hat{i} + \hat{j}$

(B) $\frac{\hat{i} + \hat{j}}{\sqrt{2}}$

(C) $\frac{3\hat{i} - \hat{j}}{\sqrt{2}}$

(D) $\sqrt{2}(\hat{i} - \hat{k})$

8. PQR is a plane lamina of the shape of an equilateral triangle. S and T are the mid points of PQ and PR respectively and C is the centroid of lamina. Moment of inertia of the lamina about an axis passing through C and perpendicular to plane PQR is I_0. If PST is removed, then the moment of inertia of the remaining part about the same axis is



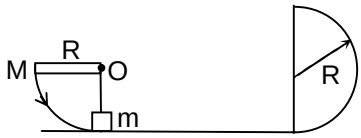
(A) $\frac{5\,I_0}{16}$

(B) $\frac{11\,I_0}{16}$

(C) $\frac{I_0}{16}$

(D) $\frac{15\,I_0}{16}$

9. A rod of length 'R' and mass 'M' is free to rotate about a horizontal axis passing through 'O'. It is released from horizontal position as shown in figure. At the lowest point the rod hits the block of mass 'm' and stops. If the block just completes the vertical circle after collision, then find $\frac{M}{m}$.



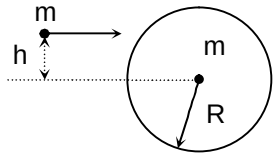
(A) $\sqrt{5}$

(B) $\sqrt{10}$

(C) $\sqrt{15}$

(D) $\sqrt{20}$

10. A particle of mass m collides elastically with a smooth horizontal circular disc of same mass 'm' and radius 'R' placed on a smooth horizontal surface with the plane of the disc lying over the horizontal surface. If $h = \frac{R}{2}$, then angle of deviation of particle of mass m after collision in degree will be



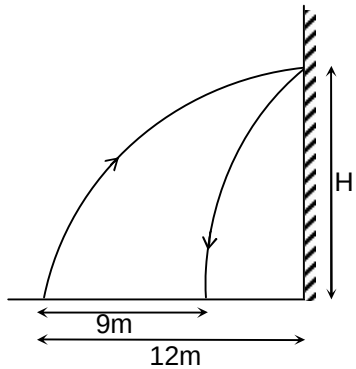
(A) 30°

(B) 60°

(C) 90°

(D) 120°

11. A ball is projected from ground with some initial speed at an angle 37° with the horizontal. There is a smooth vertical wall at a distance of 12m from the point of projection. The collision of the ball with the wall is elastic. After collision, the ball lands on the ground at a distance 9m from the point of projection. Find the height 'H' at which the ball hits the wall.



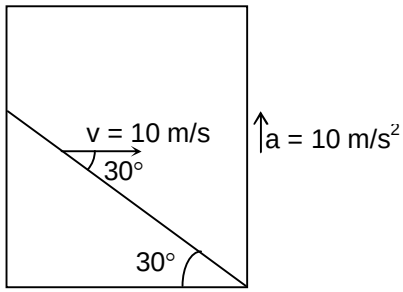
(A) 1.2 m

(B) 2 m

(C) 1.8 m

(D) 3.6 m

12. A particle is thrown with a velocity 10 m/s with respect to a moving inclined plane as shown in the figure, going up with an acceleration a = 10 m/s^2. Find the time after which it lands on the inclined plane. (Take g = 10 m/s^2)



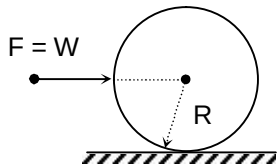
(A) $\frac{1}{2}$ sec

(B) $\frac{2}{\sqrt{3}}$ sec

(C) $\sqrt{3}$ sec

(D) $\frac{1}{\sqrt{3}}$ sec

13. For a thin non-uniform disc of radius R, whose area density of mass, σ as a function of r from its centre is given as $\sigma = \sigma_0\left(1-\frac{r}{R}\right)$. The disc is placed on a rough horizontal surface and a constant horizontal force F = W, where W is weight of disc is applied as shown in figure. If the disc starts rolling without slipping, then the linear acceleration of the disc is



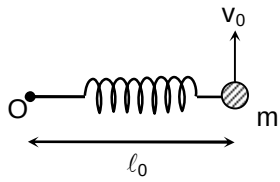
(A) $\frac{5g}{13}$

(B) $\frac{10g}{13}$

(C) $\frac{15g}{13}$

(D) $\frac{20g}{13}$

14. One end of an ideal spring is fixed at point O and other end is attached to small mass m. Mass is given an initial velocity v_0 perpendicular to its length on a smooth horizontal surface. If the maximum elongation of the spring is $\frac{\ell_0}{4}$, where ℓ_0 is natural length then find the spring constant.



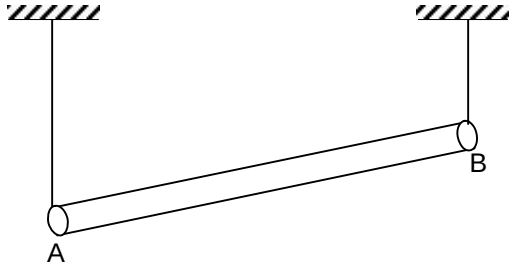
(A) $\frac{144}{25}\frac{mv_0^2}{\ell_0^2}$

(B) $144\frac{mv_0^2}{\ell_0^2}$

(C) $\frac{25}{144}\frac{mv_0^2}{\ell_0^2}$

(D) $\frac{1}{144}\frac{mv_0^2}{\ell_0^2}$

15. Two vertical strings support a uniform rod AB inclined at some angle from horizontal as shown in the figure. Which of the following options is correct just after string at right end B is cut?



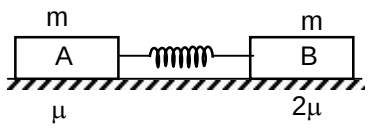
(A) Initial acceleration of A is vertical and initial acceleration of centre of mass of the rod is vertical

(B) Initial acceleration of A is vertical and initial acceleration of centre of mass of the rod is horizontal.

(C) Initial acceleration of A is horizontal and initial acceleration of centre of mass of the rod is vertical.

(D) Initial acceleration of A is horizontal and initial acceleration of centre of mass of the rod is horizontal.

16. Two blocks of same mass 'm' are joined with an ideal spring of force constant k and kept on a rough horizontal surface as shown in the figure. The spring is initially unstretched and the coefficient of friction between the blocks and the horizontal surface are μ and 2μ respectively. What should be the maximum speed given to block 'A' such that block 'B' does not move.



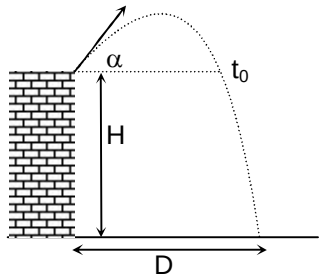
(A) $2\mu g\sqrt{\frac{2m}{k}}$

(B) $2\mu g\sqrt{\frac{m}{2k}}$

(C) $\mu g\sqrt{\frac{2m}{k}}$

(D) $\mu g\sqrt{\frac{m}{k}}$

17. A projectile is launched at an angle 'α' above the horizontal of a roof of height 'H' above the ground. After a time 't_0' has elapsed since the launch, the projectile passes the level of roof top moving downward. It eventually lands on the ground at a horizontal distance D from its launch site. Value of tan α, in terms of H, t_0 and the acceleration due to gravity (g) is:



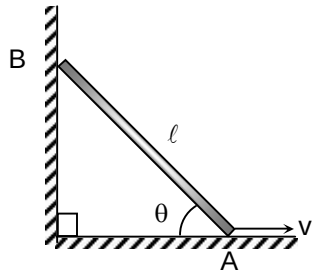
(A) $\tan\alpha = \frac{t_0^2 g\left[1+\sqrt{1+\frac{8H}{gt_0^2}}\right]}{2D}$

(B) $\tan\alpha = \frac{t_0^2 g\left[1+\sqrt{1+\frac{8H}{gt_0^2}}\right]}{4D}$

(C) $\tan\alpha = \frac{t_0^2 g\left[1+\sqrt{1+\frac{4H}{gt_0^2}}\right]}{2D}$

(D) $\tan\alpha = \dfrac{t_0^2 g\left[1+\sqrt{1+\dfrac{4H}{gt_0^2}}\right]}{4D}$

18. One end of the rigid rod (point A) is moving with a constant velocity v in horizontal direction. If the rod makes an angle θ with the horizontal, then acceleration of point 'B' is



(A) $\dfrac{v^2}{\ell \sin^2\theta\cos\theta}$

(B) $\dfrac{v^2}{\ell \cos^2\theta\sin\theta}$

(C) $\dfrac{v^2}{\ell \sin^3\theta}$

(D) $\dfrac{v^2}{\ell \cos^3\theta}$

19. A circular disc of radius R and thickness $\dfrac{R}{6}$ has moment of inertia I about an axis passing through its centre and perpendicular to its plane. It is melted and recasted into solid sphere. What is the moment of inertia of the sphere about an axis passing through its diameter?

(A) $\dfrac{2I}{5}$

(B) $\dfrac{I}{5}$

(C) $\dfrac{I}{10}$

(D) $\dfrac{3I}{5}$

20. From the top of a cliff at point A, a stone is projected vertically upward. When the stone is at a distance h below A, its velocity is double of what it was at a height h above A. What is the greatest height attained by the stone above the point A?

(A) $\dfrac{2h}{3}$

(B) $\dfrac{5h}{3}$

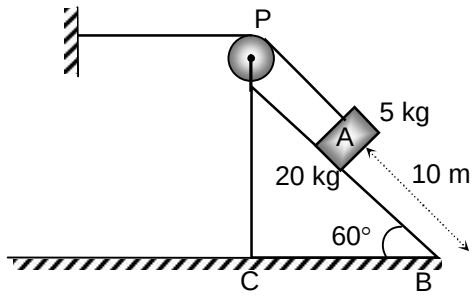
(C) $\dfrac{10h}{3}$

(D) 5h

SECTION – B
(Numerical Answer Type)

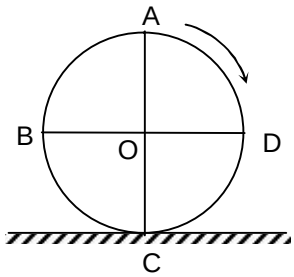
This section contains **10** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. XXXXX.XX).

21. The system is released from rest and the wedge slides to the left on a smooth horizontal surface and block A moves on the inclined surface as shown in the figure. Find the net displacement (in meter) of the block A relative to ground as it reaches the bottom of the inclined surface.



22. A block of mass 1 kg is moving with a constant acceleration 1 m/s^2 on a rough horizontal surface. The coefficient of friction between the block and the plane is $\mu = 0.1$. If initial velocity of the block is zero, then what is the power delivered in watt by the external agent at a time t = 2 sec from the beginning? (Take g = 10 m/s^2)

23. A wheel rolls on a plane surface without slipping. At a certain instant velocity and acceleration of center 'O' are 1 m/s and 3 m/s^2. Radius of the wheel is 0.5 m, BD is horizontal and AC is vertical. If at that instant ratio of acceleration of point 'C' and that of point 'A' on the wheel is $\sqrt{N}$. Find N.

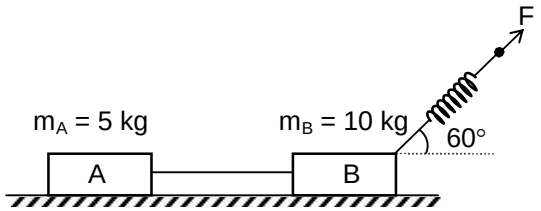


24. A system of two balls of mass m and 2m attached by a spring is released in earth's gravity by compressing the spring by some amount. Acceleration of mass 2m at a certain moment during the fall is 2 m/s^2 in vertically upward direction. What is the magnitude of acceleration (in m/s^2) of m at that instant? (Take g = 10 m/s^2)

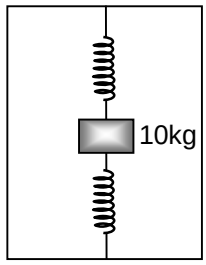
25. A particle is projected along the inner surface of a smooth vertical tube of radius R, its velocity at the lowest point is $\sqrt{\frac{95Rg}{25}}$. It will leave the circle at an angular distance from the highest point in degrees will be

26. A particle is projected from horizontal surface with angle of projection 53° from horizontal. If the smallest radius of curvature of its path is 90 m. Its radius of curvature at time t = 1sec after the projection is $n\sqrt{2}$ meter. Find the value of n. (Take g = 10 m/s^2)

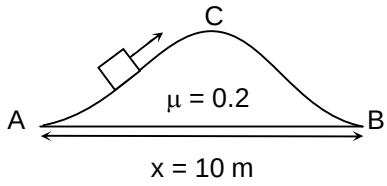
27. Block 'B' is pulled by using a light string, which has a light spring attached to it as shown in figure. If the pulling force F = 30 N, then find the tension (in newton) in the string connecting the two blocks. There is no friction anywhere. (Take g = 10 m/s^2)



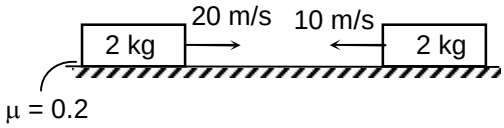
28. A block of mass 10 kg is suspended inside a lift with the help of two identical vertical springs, as shown in the figure. When the lift is at rest, tension in the lower spring is 10 N and it is stretched. What is the magnitude of the acceleration (in m/s^2) of the lift so that to make the tension in the lower spring equal to zero? (Take $g = 10\ m/s^2$)



29. A block of mass 1 kg is slowly pulled along the curved path ACB by a tangential force as shown in the figure. If coefficient of friction between the curved path and the block is 0.2, then find the work done (in Joule) against the frictional force when the block is moved from A to B. (Take $g = 10\ m/s^2$)



30. Two blocks are initially far apart and are approaching each other with the velocities as shown in the figure. If coefficient of friction for both the blocks is 0.2, then find the distance (in meter) covered by the centre of mass of the system before coming to rest permanently. (Take $g = 10\ m/s^2$)



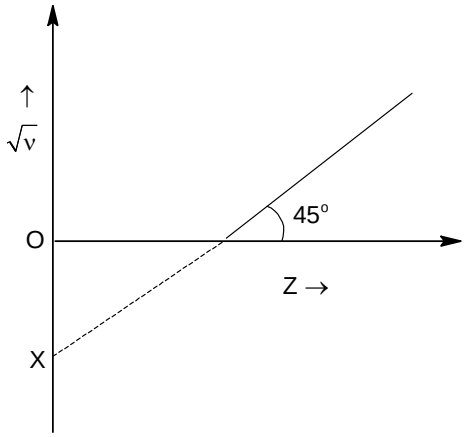
Chemistry

PART – B

SECTION – A
(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

31. Consider the graph between $\sqrt{\nu}$ and Z. If OX = 1 and Z = 52 then the frequency ν of the element is



(A) 2500 Hz
(B) 2501 Hz
(C) 2600 Hz
(D) 2601 Hz

32. $K_2Cr_2O_7 + HCl \longrightarrow xKCl + yCrCl_3 + zCl_2 + wH_2O$
Then the value of x + y + z is…….
(A) 7
(B) 8
(C) 6
(D) 9

33. The increasing order of bond angles in OF_2, OCl_2, ClO_2 and H_2O is
(A) $OF_2 < H_2O < OCl_2 < ClO_2$
(B) $H_2O < OF_2 < ClO_2 < OCl_2$
(C) $OF_2 < OCl_2 < H_2O < ClO_2$
(D) $ClO_2 < OCl_2 < OF_2 < ClO_2 < H_2O$

34. The pH of pure water at 25°C and 35°C are 7 and 6 respectively. Then heat of formation of H^+ and OH^- ions from H_2O in Kcal mol^{-1} is $\left(R = 2 \text{ cal } mol^{-1} k^{-1}\right)$
(A) +84.55
(B) – 84.55
(C) +184.55
(D) – 184.55

35. $2SO_3(g) \rightleftharpoons 2SO_2(g) + O_2(g)$

At equilibrium 700 cc of gaseous mixture contains 100 cc of gas O_2 at 10 atm and 300 K. The K_P of the reaction is

(A) $\frac{41}{7}$

(B) $\frac{1}{30}$

(C) $\frac{5}{14}$

(D) $\frac{27}{9}$

36. Choose the correct statement about saline hydrides

(A) have high melting point
(B) formed by transition metals
(C) poor reducing agents
(D) stable towards water and alcohols

37. H_2O_2 acts neither as an oxidant nor as a reductant in the following reactions:

(A) $MnO_4^- + H_2O_2 + H^+ \longrightarrow$

(B) $PbS + H_2O_2 \longrightarrow$

(C) $Na_2CO_3 + H_2O_2 \longrightarrow$

(D) $SO_2 + H_2O_2 \longrightarrow$

38. The correct order of IE_1 values of Group (13) elements is

(A) B > Al > Ga > In > Tl
(B) B > Tl > Ga > Al > In
(C) B > Ga > Al > Tl > In
(D) B > Al > Tl > Ga > In

39. Which of the following is not formed when Pb_3O_4 reacts with conc. HNO_3 ?

(A) $Pb(NO_3)_2$

(B) PbO_2

(C) PbO

(D) H_2O

40. Which of the following statement is incorrect?

(A) In pyrosilicates, one oxygen atom per tetrahedron is shared.
(B) Pyrosilicates shows structural similarity with $Na_4P_2O_7$.
(C) The constituent unit of cyclic silicates is $(SiO_3)_n^{2n-}$.
(D) Beryl is an example of orthosilicates.

41. Which of the following represents the amphoteric oxides only?

(A) Cr_2O_3, PbO, BeO, CrO_3

(B) $Cr_2O_3, B_2O_3, SnO, PbO$

(C) Cr_2O_3, BeO, SnO, SnO_2

(D) ZnO, Al_2O_3, PbO, CrO

42. Which of the following pairs, the two species are not isostructural?

(A) CO_3^{2-}, NO_3^-

(B) $PCl_4^+, SiCl_4$

(C) PF_5, BrF_5

(D) AlF_6^{3-}, SF_6

43. Consider the following reaction at equilibrium

$H_2S \rightleftharpoons H^+ + HS^-$

Now some dil. HCl is added to the aqueous solution of H_2S without any change in temperature, then

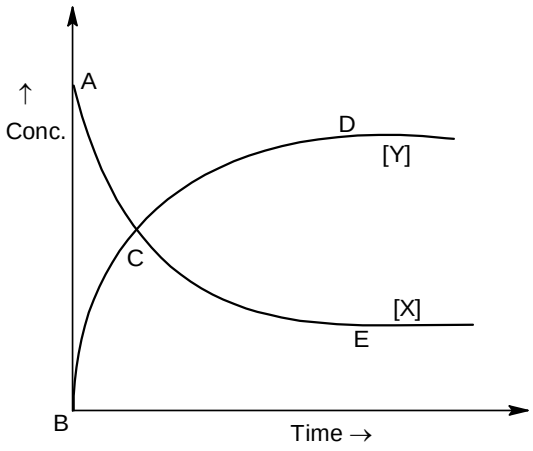
(A) Equilibrium constant will change

(B) Conc. of HS^- will increase

(C) Conc. of H_2S will decrease

(D) Conc. of HS^- will decrease

44. $X \rightleftharpoons 2Y$, $K_C = 4$ and the plots of the reaction are given below:



I. Q_C is maximum at A.

II. Reaction is in forward direction when [X] = [Y] = 0.1 M

III. $K_C = Q_C$ when point D or E is reached

Select the correct option regarding I., II., and III.

(A) I, II

(B) II, III

(C) II only

(D) III only

45. Consider the following mechanism

$X_2 \rightleftharpoons 2X \qquad (K_{eq} = K)$

$X + Y_2 \xrightarrow{K_1} Z + Y \qquad (K_1 = \text{rate constant})$

$Y + X_2 \xrightarrow{K_2} Z + X \qquad (K_2 = \text{rate constant})$

For the overall reaction

$X_2 + Y_2 \longrightarrow 2Z$

Assuming that the equilibrium is rapidly established and $\frac{d[Y]}{dt} = 0$ for [Y], the correct rate expression for [Z] is

(A) $2K_1K^{1/2}[X_2]^{1/2}[Y_2]$

(B) $2K^{1/2}[X_2]^{1/2}$

(C) $2K_1[Y_2]$

(D) $2K_1K^{1/2}[X_2]^{1/2}[Y_2]^{1/2}$

46. $A_2(g) \longrightarrow B(g) + \frac{1}{2}C(g)$

Shows increase in pressure from 100 mm Hg to 120 mm Hg in 5 mins. What is the rate of disappearance of $A_2(g)$

(A) 10 mm min^{-1}

(B) 6 mm min^{-1}

(C) 8 mm min^{-1}

(D) 12 mm min^{-1}

47. $CaSO_4.2H_2O \xrightarrow[\Delta]{200^oC} (Y)$

$\Delta \downarrow 120^oC$ (X)

$\downarrow$ Very strong heating (Z)[a gaseous compound]

Select the correct statement of the following:

(A) (X) is dead burnt plaster.

(B) (Y) is plaster of paris.

(C) (Z) is a gas having irritating smell.

(D) (Z) is an alkaline gas.

48. Choose the correct order of thermal stability

(A) $K_2CO_3 < MgCO_3 < CaCO_3 < BeCO_3$

(B) $BeCO_3 < CaCO_3 < K_2CO_3 < MgCO_3$

(C) $BeCO_3 < MgCO_3 < CaCO_3 < K_2CO_3$

(D) $CaCO_3 < MgCO_3 < BeCO_3 < K_2CO_3$

49. Choose the incorrect statement of the following

(A) There is a change in hybridization of P when PCl_5 vapours undergoes solidification.

(B) There is a change in hybridization of Si when SiF_4 vapours are passed in HF (aq.).

(C) The hybridization of Be in $BeCl_2(s)$ is sp^3.

(D) The species $[SiCl_6]^{2-}$ and $[SiF_6]^{2-}$ both do not exist.

50. Which of the following is not paramagnetic?

(A) NO

(B) CO

(C) MnO_4^{2-}

(D) B_2

SECTION – B
(Numerical Answer Type)

This section contains **10** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. XXXXX.XX).

51. Element (X) has K-shell configuration $(n-5)s^2$ and total number of electrons in outermost, penultimate and pre-penultimate shell are 2, 8 and 23 respectively, then the number of unpaired electrons in the ground state of (X) is_____

52. If 'x' is the number of species having bond order 2.5 among the following then the value of $\frac{x}{4}$ is.....

$N_2^+, N_2^-, O_2^+, O_2^-, NO, CN$

53. The number of water molecules of crystallization present in $Na_2B_4O_7.10H_2O$ is

54. The sum of number of five membered and six membered rings present in C_{60} (the fullerene) is 'T' then the value of $\frac{T}{10}$ is.....

55. Hydrolysis of RX by dil. OH^- ions takes place simultaneously by S_N2 and S_N1 path. A plot of $-\frac{1}{[RX]}\frac{d[RX]}{dt}$ vs. $[OH^-]$ is a straight line of slope 4×10^3 $mol^{-1}\,L\,s^{-1}$ and intercept is equal to $2\times10^2\,s^{-1}$. The initial rate of disappearance of RX when reaction starts with [RX] = 1 M and $[OH^-] = 0.1\,M$ is..... $mol\,L^{-1}s^{-1}$

56. The first order reaction of di-tertbutyl peroxide (D.T.B.P.) to acetone and ethane is given by equation

$C_8H_{18}O_2(g) \longrightarrow 2CH_3COCH_3(g) + C_2H_6(g)$

At 300 K, the following data was obtained.

Time 'minutes'	0	2	4
P_{total} 'mm Hg'	1000	2000	2500

After how many minutes from start of reaction the partial pressure of acetone will become 1000 mm Hg?

57. The sum of number of nodal planes in all the atomic orbitals having principal quantum number n = 3 is_____

58. $C_2H_5OH + I_2 + OH^- \longrightarrow CHI_3 + HCOO^- + H_2O + I^-$

If the coefficient of OH^-, I_2 and I^- in the balanced equation are x, y and z respectively then the value of $\left(\frac{x+z}{y}\right)$ is _____.

59. The pH of 0.004 M hydrazine solution is 9.7, then pK_b of hydrazine is....
(log2 = 0.30)

60. At 900 K and 1 atm SO_3 (g) dissociates as follows

$$SO_3 \rightleftharpoons SO_2(g) + \frac{1}{2}O_2(g)$$

density of equilibrium mixture is 0.925 gL^{-1}. Then degree of dissociation α is....%

$\left(R = 0.08 \text{ L atm } k^{-1} \text{ mol}^{-1}\right)$

Mathematics

PART – C

SECTION – A
(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

61. $\lim\limits_{x\to\infty}\left(\frac{x^{2013}}{e^{3x}}+\left(\cos\frac{2}{x}\right)^{x^2}\right)$ is

(A) 0
(B) e^3
(C) e^2
(D) e^{-2}

62. Let A = {1, 2, 3}, then the relation R = {(1, 1), (1, 2), (2, 1)} on A is

(A) reflexive
(B) transitive
(C) symmetric
(D) none of these

63. $\int\frac{x^3-x^2+x-1}{x^5+1}dx$ is

(A) $\frac{1}{5}\ln\left(\frac{1+x^5}{(1+x^3)^5}\right)+c$

(B) $\frac{1}{5}\ln\left(\frac{1+x^5}{1+x^3}\right)+c$

(C) $\frac{1}{5}\ln\left(\frac{1+x^5}{(1+x)^5}\right)+c$

(D) $\frac{1}{5}\ln\left(\frac{1+x^5}{1+x}\right)+c$

64. If $|a_2| > |a_1 + a_3|$, then atleast one root of the equation $a_1x^4 + a_2x^3 + a_3 = 0$ lies in the interval

(A) (–2, 0)
(B) (–1, 1)
(C) (0, 2)
(D) (1, 3)

65. Let $f(x) = x^3 + x + 1$ and let g(x) be it's inverse function, then equation of the tangent to y = g(x) at x = 3 is

(A) x – 4y + 1 = 0
(B) x + 4y – 1 = 0
(C) 4x – y + 1 = 0
(D) 4x + y – 1 = 0

66. The value of $\int_0^1 \frac{\ln(x+1)}{1+x^2}dx$ is

(A) $\frac{\pi \ln 2}{4}$

(B) $\frac{\pi \ln 2}{8}$

(C) $\pi \ln 2$

(D) 0

67. The value of $\int_1^e \left(\left(\frac{1}{x} - x + x\ln x\right)\sin x\right)dx$ is

(A) sin e – cos 1

(B) cos e – sin 1

(C) sin e + cos 1

(D) cos e + sin 1

68. If $f(x) = \begin{cases} x^a \sin\left(\frac{1}{x}\right) & \forall\ x \neq 0 \\ 0 & \forall\ x = 0 \end{cases}$ is continuous but not differentiable at x = 0, then

(A) $a \in (-1, 0)$

(B) $a \in (0, 2]$

(C) $a \in (0, 1]$

(D) $a \in [1, 2)$

69. The family of curve whose differential equation is $(xy + 1)(3x^2 y\,dx - x^3\,dy) = (x^6 + y^2)(x\,dy + y\,dx)$ is

(A) $\tan^{-1}\left(\frac{x^3}{y}\right) = \ln(1+xy) + c$

(B) $\sin^{-1}\left(\frac{x^3}{y}\right) = \ln(1-xy) + c$

(C) $\sin^{-1} x^3 = y \ln(1 + xy) + c$

(D) none of these

70. The value of $\lim_{n\to\infty}\left(\sin^4 x + \frac{1}{4}\sin^4 2x + ...\frac{1}{4^n}\sin^4\left(2^n x\right)\right)$ is

(A) $\sin^2 x$

(B) $\sin^4 x$

(C) $\cos^2 x$

(D) none of these

71. Let $f : N \to N$ where N is set of natural numbers be a function such that $f(x + y) = f(xy)\ \forall\ x \geq 4$ and $y \geq 4$, then

(A) f(4) = f(5)

(B) f(8) = f(9)

(C) f(7) = f(8)

(D) f(5) = f(6)

72. Let f : R $\to$ R be a twice differentiable function such that $\lim_{x\to\infty} f(x) = a\,(a \neq 0)$, then $\lim_{x\to\infty} x^2 f''(x)$ is equal to
(A) a
(B) 2a
(C) $a^2 + a$
(D) none of these

73. A continuous and differentiable function y = f(x) is such that it's graph cuts line y = mx + c at n distinct points. Then the minimum number of points at which f″(x) = 0 is
(A) n – 1
(B) n – 3
(C) n – 2
(D) can't say

74. The Boolean expression $(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$ is equivalent to
(A) $p \wedge q$
(B) $p \vee q$
(C) $p \vee \sim q$
(D) $\sim p \wedge q$

75. Let f : [0, 1] $\to$ R be a continuous function, then which of the following is the maximum value of $\int_0^1 x^2 f(x)\,dx - \int_0^1 x(f(x))^2\,dx$ for all such functions
(A) $\frac{1}{12}$
(B) $\frac{1}{16}$
(C) $\frac{1}{8}$
(D) $\frac{1}{20}$

76. The area of the region bounded by the curve $y = \frac{16 - x^2}{4}$ and $y = \sec^{-1}[-\sin^2 x]$ (where [.] denotes greatest integer function) is
(A) $\frac{1}{3}(4-\pi)^{\frac{3}{2}}$
(B) $8(4-\pi)^{\frac{3}{2}}$
(C) $\frac{8}{3}(4-\pi)^{\frac{3}{2}}$
(D) $\frac{8}{3}(4-\pi)^{\frac{1}{2}}$

77. At how many points in the interval (0, 2) : $f(x) = x^2[2x] - x[x^2]$ is discontinuous (where [x] represents greatest integer $\leq x$)
(A) 5
(B) 4
(C) 3
(D) infinite

78. The value of $\int_0^1 \sqrt[3]{2x^3 - 3x^2 - x + 1}\,dx$ is equal to

(A) $5\sqrt{7}$
(B) 1
(C) 0
(D) $2\sqrt{2}$

79. Let $I_1 = \frac{\pi^2}{4} + \sqrt{2}$, $I_2 = \left(\tan^{-1}\left(\frac{1}{e}\right)\right)^2 + \frac{2e}{\sqrt{e^2+1}}$, $I_3 = \left(\tan^{-1} e\right)^2 + \frac{2}{\sqrt{e^2+1}}$, then which of the following is true?

(A) $I_1 < I_2 < I_3$
(B) $I_2 < I_1 < I_3$
(C) $I_1 < I_3 < I_2$
(D) $I_3 < I_2 < I_1$

80. The equation of the curve passing through the point (5, 4) if the sum of reciprocal of the intercepts of the normal drawn at any point P(x, y) on it is

(A) $x^2 + y^2 = 41$
(B) $(x - 1)^2 + (y - 1)^2 = 25$
(C) $(x - 1)^2 + (y - 2)^2 = 20$
(D) none of these

SECTION – B
(Numerical Answer Type)

This section contains **10** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. XXXXX.XX).

81. Let f be a function defined on the interval [0, 2π] such that $\int_0^x (f'(t) - \sin 2t)\,dt = \int_x^0 f(t)\tan t\,dt$ and f(0) = 1. Then the maximum value of f(x) is

82. Area bounded by the curve [2x] + [y] = 5, x, y > 0 (where [.] denotes greatest integer function) is equal to

83. The value of $\int_0^1 \frac{\tan^{-1} x}{\cot^{-1}(1 - x + x^2)}\,dx$ is equal to

84. Let y = f(x) = $4x^3 + 2x - 6$, then the value of $\int_0^2 f(x)\,dx + \int_0^{30} f^{-1}(y)\,dy$ is equal to

85. If $I = \int \frac{1 - \cot^{2008} x}{\tan x + \cot^{2009} x}\,dx = \frac{1}{k}\log\left|\sin^k x + \cos^k x\right| + c$, then the value of k is equal to

86. From a given solid cone of height H, another inverted cone is carved whose height is h such that it's volume is maximum. Then the ratio of $\frac{H}{h}$ is

87. Let $f(x) = x(x^2 + mx + n) + 2$ for all $x \in R$ and $m, n \in R$. If Roll's theorem hold for f(x) at $x = \frac{4}{3}$ in $x \in [1, 2]$, then $(m + n)$ is equal to

88. If the slope of line through the origin which is tangent to the curve $y = x^3 + x + 16$ is m, then the value of m is equal to

89. The value of $\lim_{x\to 1} \frac{100}{1-x^{100}} - \frac{50}{1-x^{50}}$ is equal to

90. Suppose that f(x) is a function of this form $f(x) = \frac{ax^8 + bx^6 + cx^4 + dx^2 + 15x + 1}{x}$ $\forall$ $x \neq 0$. If $f(5) = 2$, then the value of $f(-5)$ is equal to