FIITJEE
ALL INDIA TEST SERIES
JEE (Advanced)-2022
PART TEST – I
PAPER –2
TEST DATE: 21-11-2021

ANSWERS, HINTS & SOLUTIONS

Physics

PART – I

Section – A

1. A, B, C

Sol. $60 \times 72 + 90 \times 90 = 150$ v

$v = \frac{12420}{150} = 82.8$ km/hr = 23 m/s

Force on the person = $\frac{|\Delta \vec{P}_{person}|}{\Delta t} = \frac{\sqrt{(60\times5)^2 + \{60\times(23-20)\}^2}}{0.1}$

$= \frac{60\sqrt{25+9}}{0.1} = 60 \times 5.83 \times 10 = 3500$ N

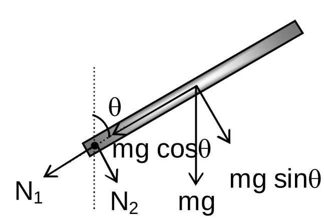
Force on the trolley = $\frac{90\times(25-23)}{0.1} = \frac{90\times2}{0.1}$ =1800 N

2. A, C, D

Sol. Using conservation of energy

$mg\frac{L}{2}(1-\cos\theta) = \frac{1}{2}\frac{mL^2}{3}\omega^2$

$\omega^2 = \frac{3g(1-\cos\theta)}{L}$



$N_1 \rightarrow$ normal reaction at hinge along rod

$mg\cos\theta + N_1 = \frac{3mg(1-\cos\theta)}{L} \times \frac{L}{2}$

$N_1 + mg\cos\theta = \frac{3}{2}mg(1-\cos\theta)$

$N_1 = \frac{3}{2}mg - \frac{5}{2}mg\cos\theta$ …(i)

For $N_1 = 0$, $\cos\theta = \frac{3}{5}$

$(mg\sin\theta)\frac{L}{2} = \frac{mL^2}{3}\alpha \Rightarrow \alpha = \frac{3g\sin\theta}{2L}$

For centre of mass

$a_t = \frac{L}{2}\alpha = \frac{3}{4}g\sin\theta$

$mg\sin\theta + N_2 = \frac{3}{4}mg\sin\theta$

$N_2 = -\frac{mg}{4}\sin\theta$

For any θ, N_2 is always negative

$N_2 \rightarrow$ normal reaction at hinge perpendicular to rod

3. C, D

Sol. Graph shows that particles P and Q are starting from rest while their initial positions may be different. Also impulse is change in linear momentum so it will depend on the mass of P and Q.

4. B, D

Sol. If spring is initially in relaxed state.

$\frac{1}{2}kx^2 = \frac{1}{2}mv^2$

$x = \left(\sqrt{\frac{m}{k}}\right)v = \sqrt{\frac{2}{50}} \times 5 = 1\,m$

5. A, B, C

Sol. $F_x = -\frac{\partial U}{\partial x} = -6N$

$F_y = -\frac{\partial U}{\partial y} = 8N$

$\Rightarrow \vec{F} = -6\hat{i} + 8\hat{j}$

$\vec{a} = \frac{\vec{F}}{m} = (-3\hat{i} + 4\hat{j}) = \text{constant}$

$\vec{v} = \vec{u} + \vec{a}t = \left(2\hat{i} - 3\hat{j}\right) + \left(-3\hat{i} + 4\hat{j}\right)t$

$= (2 - 3t)\hat{i} + (-3 + 4t)\hat{j}$

For any time t, v_x and v_y are not zero simultaneously

$(x\hat{i} + y\hat{j}) = (2\hat{i} - 3\hat{j})t + \frac{1}{2}(-3\hat{i} + 4\hat{j})t^2 = \left(2t - \frac{3}{2}t^2\right)\hat{i} + (-3t + 2t^2)\hat{j}$

for x = 0, $t = \frac{4}{3}$sec

for y = 0, $t = \frac{3}{2}$sec

angle between velocity and acceleration at t = 1 sec

$\cos\theta = \frac{\vec{v}\cdot\vec{a}}{|\vec{v}||\vec{a}|} = \frac{(-\hat{i} + \hat{j})(-3\hat{i} + 4\hat{j})}{\sqrt{2}\times 5} = \frac{3 + 4}{\sqrt{2}\times 5} = \left(\frac{7}{5\sqrt{2}}\right)$

At, $t = \frac{3}{2}$sec, $x = 2\times\frac{3}{2} - \frac{3}{2}\left(\frac{3}{2}\right)^2 = 3 - \frac{27}{8} = -\frac{3}{8}\,m$

For velocity to be perpendicular to acceleration

$\vec{a}\cdot\vec{v} = 0$

$\left(-3\hat{i}+4\hat{j}\right)\cdot\left[(2-3t)\hat{i}+(-3+4t)\hat{j}\right] = 0$

$-3(2-3t)+4(-3+4t) = 0$

$-6+9t-12+16t = 0$

$25t = 18$

$t = \dfrac{18}{25}$

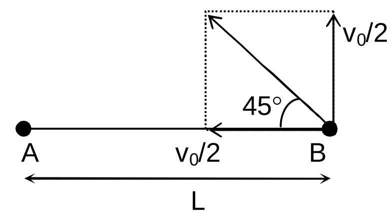
6. A, B, C, D

Sol. Velocity of particle B with respect to particle A at t = 0

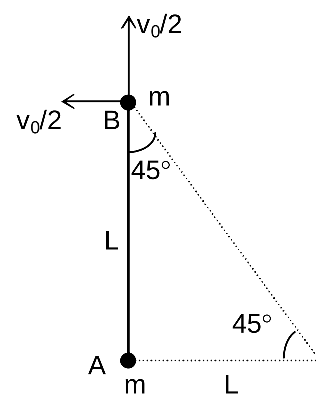
$v_{B/A} = v_{B/G} - v_{A/G}$

$v_0\left(\dfrac{1}{2}\hat{i}+\dfrac{3}{2}\hat{j}\right) - v_0\left(\hat{i}+\hat{j}\right)$

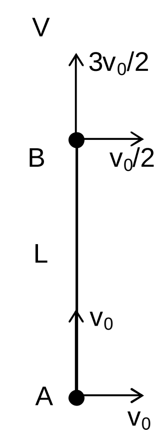
$= -\dfrac{v_0}{2}\hat{i}+\dfrac{v_0}{2}\hat{j}$



Just before string becomes taut



In the frame of particle A



In ground frame

Just after string becomes taut.
Conserving momentum of A and B along length of string

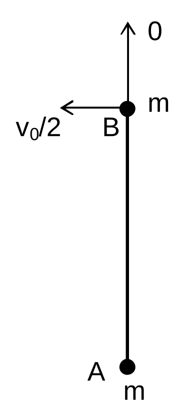
$2mv = mv_0 + \dfrac{3}{2}mv_0 = \dfrac{5}{2}mv_0$

$\Rightarrow v = \dfrac{5}{4}v_0$

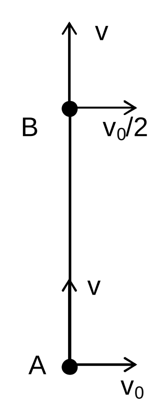
Also string will become taut at $t = \dfrac{L\sqrt{2}}{\dfrac{v_0}{2}\sqrt{2}} = \dfrac{2L}{v_0}$

In the frame of A, motion of particle B will be circular

$T + ma_A = \dfrac{m\left(\dfrac{v_0}{2}\right)^2}{L}$, $T = \dfrac{m\left(v_0\right)^2}{8L}$



In the frame of particle A



In ground frame

Section – A

7. A

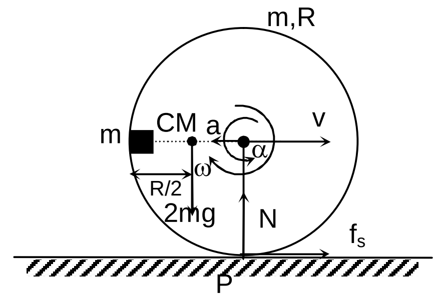
8. C

Sol. (Q.7-8)

For rolling without slipping,

v = ωR and a = αR

Using conservation of energy



$$mv_0^2 = mv^2 + \frac{1}{2}m(v\sqrt{2})^2 + mgR$$

$$mv_0^2 = 2mv^2 + mgR$$

$$5mgR = 2mv^2 + mgR$$

$$v = \sqrt{2gR} \text{ and } \omega = \frac{v}{R} = \sqrt{\frac{2g}{R}} \quad \ldots(i)$$

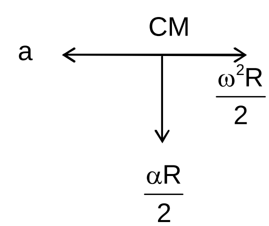
$$I_P = 2mR^2 + m\left(R\sqrt{2}\right)^2 = 4mR^2$$

$$\tau_P = I_P\alpha$$

$$2m(g+\omega^2R)\frac{R}{2} = 4mR^2\alpha$$

$$m(g+2g)R = 4mR^2\alpha$$

$$\Rightarrow \quad \alpha = \frac{3g}{4R} \quad \ldots(ii)$$



$$f_s = 2m\left(\frac{\omega^2R}{2} - a\right)$$

$$f_s = 2m\left(\frac{\omega^2R}{2} - \alpha R\right)$$

$$f_s = 2m\left(g - \frac{3g}{4}\right)$$

$$f_s = \frac{mg}{2}$$

Also, $2mg - N = \frac{2m\alpha R}{2}$

$$2mg - N = m\alpha R$$

$$2mg - N = \frac{3mg}{4}$$

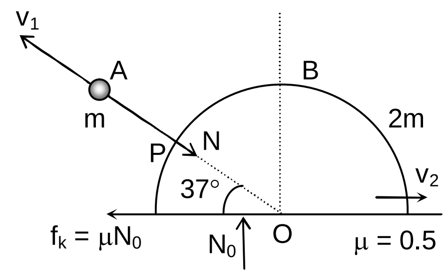
$$\Rightarrow N = 2mg - \frac{3mg}{4}$$

$$\Rightarrow N = \frac{5mg}{4}$$

9. B

10. D

Sol. (Q.9-10)



N_0 = normal force on the body B due to the horizontal surface

$\int N_0 dt = \int N \sin 37° dt$

$\int N_0 dt = \frac{3}{5}\int Ndt$ …(i)

$\int f_k dt = \mu \int N_0 dt = 0.5 \times \frac{3}{5}\int Ndt$

$\int f_k dt = 0.3\int Ndt$ …(ii)

$\int Ndt = m(v_1 + v_0)$ …(iii)

$\int N\cos 37° dt - \int f_k dt = 2mv_2$

$0.8\int Ndt - 0.3\int Ndt = 2mv_2$

$0.5\int Ndt = 2mv_2 \Rightarrow \int Ndt = 4mv_2$ …(iv)

From equation (iii) and (iv)

$m(v_1 + v_0) = 4mv_2$

$4v_2 - v_1 = v_0$ …(v)

$0.6 = e = \frac{v_2 \cos 37° + v_1}{v_0}$

$0.8v_2 + v_1 = 0.6v_0$ …(vi)

Solving equation (v) and (vi), we get

$4.8v_2 = 1.6v_0$

$v_2 = \frac{v_0}{3} = 5 \text{ m/s} \Rightarrow v_2 = 5 \text{ m/s}$

From equation (v),

$v_1 = 4v_2 - v_0 = 20 - 15 = 5 \text{ m/s}$

$v_1 = 5$ m/s

From equation (iii)

$\int f_k dt = 0.3\int Ndt = 0.3 \times 20 = 6$ N-S

The impulse due to frictional force of the horizontal surface on the hemispherical body B during collision is

$\int f_k dt = 6$ N-s

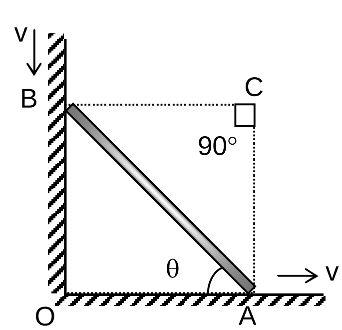
Section – B

11. 1

Sol. At given instant OA = OB

$\Rightarrow$ OACB will be square and point C will be ICOR

Angular momentum about C



$L_C = I_C\omega = \left[\frac{mL^2}{12} + m\left(\frac{L}{2}\right)^2\right]\omega$

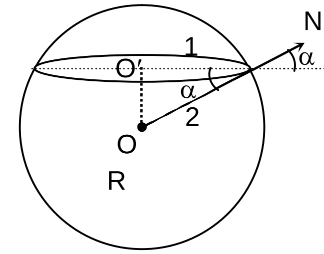
Angular momentum about O

$L_O = \left|\left[\frac{mL^2}{12} - m\left(\frac{L}{2}\right)^2\right]\right|\omega = \frac{mL^2}{6}\omega$

$$= \frac{mL^2}{6}\frac{\left(2\frac{v}{\sqrt{2}}\right)}{L} = \frac{\sqrt{2}}{6}(mLv) = 1 \text{ kg-m}^2\text{/sec}$$

12. 8

Sol. $\cos\alpha = \frac{1}{2} \Rightarrow \alpha = 60°$



Consider small length dL of loop which makes angle $d\theta$ at O′

Let λ is mass per unit length of string

$\Rightarrow dm = \lambda.d\ell = \lambda.rd\theta$ …(i)

$N.\sin\alpha = (dm)g$ …(ii)

$2T\sin\frac{d\theta}{2} - N\cos\alpha = (dm)r\omega^2$

$2T\sin\frac{d\theta}{2} = N\cos\alpha + (dm)r\omega^2$

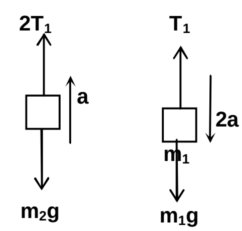
$2T\cdot\frac{d\theta}{2} = \frac{(dm)g}{\sin\alpha}\cos\alpha + (dm)r\omega^2$

$T = \frac{(\lambda\cdot rd\theta)g}{d\theta\cdot\sin\alpha}\cos\alpha + \frac{(\lambda.r.d\theta)}{d\theta}\cdot r\omega^2 = \lambda rg\cot\alpha + \lambda r^2\omega^2 = \frac{16}{3}$

$\Rightarrow n = 8$

13. 4

Sol. $m_1g - T_1 = 2m_1a$ …(i)



$2T_1 - m_2g = m_2a$ …(ii)

$2m_1g - m_2g = (4m_1 + m_2)a$

$a = \left(\frac{2m_1 - m_2}{4m_1 + m_2}\right)g = \left(\frac{2\times1-1}{4\times1+1}\right)\times10 = 2 \text{ m/s}^2$

$a_A = 4 \text{ m/s}^2$

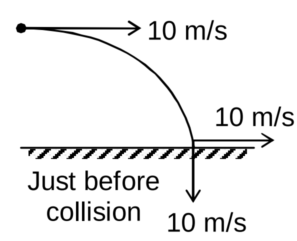
Section – C

14. 125.00

15. 8.75

Sol. (Q.14-15)

Vertical component of velocity of ball just before impact = $\sqrt{2gh} = \sqrt{2\times10\times5} = 10 \text{ m/s}$

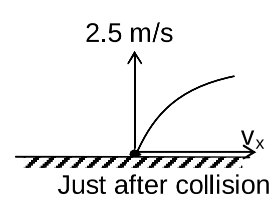


After impact

$N\cdot\Delta t = 1\times2.5 - (-1\times10)$

$N\cdot\Delta t = 12.5$ ...(i)

$-(\mu N)\Delta t = 1\times v_x - 1\times10$ …(ii)



$v_x = 10 - 0.1\times12.5$

$= 10 - 1.25 = 8.75$ m/s

Putting value of Δt in (i)

$N\times0.1 = 12.5$

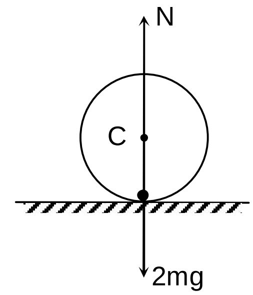
N = 125 newton

16. 40.00

17. 2.24

Sol. (Q.16-17)

When the particle is at the lowest position of the ring



$2mgR = \frac{1}{2} I_0 \omega^2$

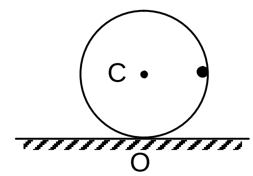
$2mgR = \frac{1}{2} \times (2mR^2)\omega^2$

$\Rightarrow \ \omega = \sqrt{2g/R}$

$N - 2mg = 2m \times \frac{(R\omega^2)}{2}$

$N = 2mg + mR\omega^2$

$= 2 \times 1 \times 10 + 1 \times 1 \times 20 = 40$ N



Also, $2mg \times \frac{R}{2} = \frac{1}{2}\left[2mR^2 + 2mR^2\right]\omega^2$

$2gR = 4R^2\omega^2$

$\omega = \sqrt{\frac{g}{2R}} = 2.24 \text{ rad/s}$

18. 8.94

19. 1.00

Sol. (Q.18-19)

Work done by the friction force in both cases will be equal and given by

$\Delta W_{(friction)} = -\mu mgx = -0.2 \times 2 \times 10 \times 5 = -20$ J

Using work energy theorem

$\frac{1}{2} \times 2 \times v_P^2 = \frac{1}{2} \times 2 \times (10)^2 - 20$

$v_P^2 == 100 - 20 = 80$

$v_P = \sqrt{80} = 8.94$ m/s

Chemistry

PART – II

Section – A

20. A, B, C

Sol. In $N(SiH_3)_3$ there is $p\pi - d\pi$ bond.

21. A, B, C

Sol. Decomposition of $Pb(NO_3)_2$ is not a disproportionation reaction.

22. A, B, C, D

Sol. Addition of catalyst does not change the value of K_P and K_C.

23. B, C, D

Sol. A catalyst does not change the value of ΔH of a reaction.

24. A, C, D

Sol. $LiNO_3$ on decomposition produces NO_2.

25. B, C, D

Sol. In BrF_5 the central atom is sp^3d^2 hybridized.

Section – A

26. B

Sol. $pH = 7 + \frac{1}{2}pK_a + \frac{1}{2}\log C$

$= 7 + \frac{4.74}{2} + \frac{1}{2}\log 10^{-1}$

= 8.87

27. D

Sol. $\left[NH_4^+\right] = 2 \times 0.005 = 0.01\,M$

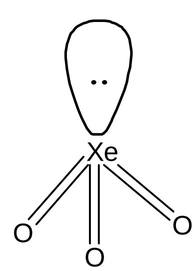
$pH = 7 - \frac{1}{2}pK_b - \frac{1}{2}\log C$

$= 7 - \frac{4.74}{2} - \frac{1}{2}\log 10^{-2}$

= 5.63

28. B

Sol. The shape of XeO_3 is pyramidal



29. A

Sol. I_3^- and XeF_2 have linear shape.

Section – B

30. 6

Sol. The following species are paramagnetic

$O_2, S_2, O_2^+, N_2^-, B_2, NO_2$

31. 6

Sol. For 3p sub-shell $n = 3$ and $\ell = 1$.

3p sub-shell contains maximum 6 electrons.

32. 4

Sol. $\mu = \sqrt{n(n+2)}$ BM

$= \sqrt{n(n+2)} = 1.73$

$\therefore n = 1$

Vanadium ion is present as V^{4+}

$\therefore x = 4$

Section – C

33. 2.40

Sol. $\left[SO_4^{2-}\right] = \dfrac{K_{SP}\ CaSO_4}{\left[Ca^{2+}\right]}$

$= \dfrac{2.4\times10^{-5}}{0.1}$

$= 2.4\times10^{-4}$ M

$\therefore x = 2.40$

34. 3.75

Sol. $\left[Ba^{2+}\right] = \dfrac{K_{SP}\ BaSO_4}{\left[SO_4^{2-}\right]}$

$= \dfrac{9\times10^{-10}}{2.4\times10^{-4}}$

$= 3.75\times10^{-6}$

$\therefore y = 3.75$

35. 3.09

36. 1.70

Sol. (Q. 35 and 36)

$2NO_2(g) \rightleftharpoons N_2O_4(g)$

$2P - x - 2y \qquad y$

$NO(g) + NO_2(g) \rightleftharpoons N_2O_3(g)$

$P - x \qquad 2P - x - 2y \qquad x$

$K_P = \frac{P_{N_2O_4}}{\left(P_{NO_2}\right)^2}, P_{NO_2} = \sqrt{\frac{1.7}{6.8}} = 0.5$

$P_{NO_2} + P_{N_2O_4} + P_{NO} + P_{N_2O_3} = 5$

$0.5 + 1.7 + P - x + x = 5$

$P = 2.8$ atm,

$2P - x - 2y = 0.5$

$x = 1.7$ atm

$\therefore P_{N_2O_3} = 1.7 \text{ atm}, P_{NO} = P - x = 2.8 - 1.7 = 1.1$

$K_P' = \frac{P_{N_2O_3}}{\left(P_{NO}\right)\left(P_{NO_2}\right)} = \frac{1.7}{1.1 \times 0.5} = 3.09$

37. 5.34

Sol. $pH = pK_a + \log\frac{\left[CH_3COO^-\right]}{\left[CH_3COOH\right]}$

$pH = 4.74 + \log\frac{1.6}{0.4}$

pH = 5.34

38. 8.72

Sol. $pH = 7 + \frac{1}{2}pK_a + \frac{1}{2}\log C$

$= 7 + \frac{4.74}{2} + \frac{1}{2}\log\frac{2}{40}$

= 8.72

Mathematics

PART – III

Section – A

39. A, B, C, D

Sol. $I=\int \frac{x^2}{x^4+kx^2+1}dx=\frac{1}{2}\left[\int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}+k}dx+\int \frac{1-\frac{1}{x^2}}{x^2+\frac{1}{x^2}+k}dx\right]$

Let $x-\frac{1}{x}=t$ + Let $x+\frac{1}{x}=s$

$\frac{1}{2}\left[\int \frac{dt}{t^2+k+2}+\int \frac{ds}{s^2+k-2}\right]$

If k = –3 ⇒ $I=\frac{1}{2}\left[\frac{1}{2}\ln\left(\frac{x-\frac{1}{x}-1}{x-\frac{1}{x}+1}\right)+\frac{1}{2\sqrt{5}}\ln\left(\frac{x+\frac{1}{x}-\sqrt{5}}{x+\frac{1}{x}+\sqrt{5}}\right)\right]+c$

If k = –2 ⇒ $I=\frac{1}{2}\left[\frac{-1}{x-\frac{1}{x}}+\frac{1}{4}\ln\left(\frac{x+\frac{1}{x}-2}{x+\frac{1}{x}+2}\right)\right]+c$

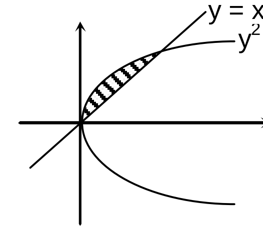
If k = 0 ⇒ $I=\frac{1}{2}\left[\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x-\frac{1}{x}}{\sqrt{2}}\right)+\frac{1}{2\sqrt{2}}\ln\left(\frac{x+\frac{1}{x}-\sqrt{2}}{x+\frac{1}{x}+\sqrt{2}}\right)\right]+c$

If k = 2 ⇒ $I=\frac{1}{2}\left[\frac{1}{2}\tan^{-1}\left(\frac{x-\frac{1}{x}}{2}\right)-\frac{1}{x+\frac{1}{x}}\right]+c$

40. A, C, D

Sol. $A(t)=\int_0^{2\sqrt{a}\cot\theta}\sqrt{4ax}-x\tan\theta\,dx=\frac{8a^2}{3}\left|\cot^3\theta\right|$

Minimum area bounded by y cot θ = x – a and P is when θ is $\frac{\pi}{2}$, then area is $\frac{8a^2}{3}$



41. A, B, C

Sol. $f(x)=x+x\underbrace{\int_0^1 t^2 f(t)dt}_{a}+x^2\underbrace{\int_0^1 tf(t)dt}_{b}$

⇒ f(x) = (a + 1)x + bx^2 where $a=\int_0^1(a+1)t^3+bt^4dt$ and $b=\int_0^1(a+1)t^2+bt^3dt$

$\Rightarrow f(x) = \frac{180x}{119} + \frac{80x^2}{119}$ and g(x) = 1 + x^3

42. B, D

Sol. (A) Not always true of $f(x) = \begin{bmatrix} x^2 \sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{bmatrix}$

(B) e.g. f(x) = [x], g(x) = {x}, x = 0

(C) $\lim_{x \to -b} g(x)$ may not be equal to $\lim_{x \to b} g(x)$

(D) f(x) – g(x) needs to be continuous for a solution to always exist

43. A, B

Sol. $\int_0^1 \frac{x^n}{\sqrt{1+x^2}} dx = \int_0^1 x^{n-1} \frac{x}{\sqrt{1+x^2}} = \left| x^{n-1}\sqrt{1+x^2} \right|_0^1 - \int (n-1)x^{n-2}\sqrt{1+x^2} dx$

$= \sqrt{2} - (n-1)\int_0^1 \frac{x^{n-2}+x^n}{\sqrt{1+x^2}} dx = \frac{\sqrt{2}}{n} - \frac{(n-1)I_{n-2}}{n} = \frac{\sqrt{2}}{n} + \left(\frac{1}{n} - 1\right) I_{n-2}$

$\Rightarrow a_n = \frac{\sqrt{2}}{n},\ b_n = \frac{1}{n} - 1$

44. A, B, C, D

Sol. (A) $\int_1^2 \frac{\sin x}{x^2} < \int_1^2 \frac{x}{x^2} dx = \ln 2$

(B) $\int_1^2 \frac{\sin x}{x^2} dx > \int_1^2 \frac{1}{x} - \frac{x}{6} dx = \ln 2 - \frac{1}{4}$

(C) $\int_1^2 \frac{\sin x}{x^2} dx > \int_1^2 \frac{1}{\sqrt{2}x^2} dx = \frac{1}{2\sqrt{2}}$

(D) $\int_1^2 \frac{\sin x}{x^2} dx > \int_1^{\pi} \frac{\frac{2x}{\pi}}{x^2} dx + \int_{\pi}^2 \frac{2 - \frac{2}{\pi}(x-\pi)}{x^2} dx$

$= \frac{2}{\pi}\ln\pi - \left[\frac{2}{\pi}\ln\left(\frac{2}{\pi}\right) + 2\left[\frac{1}{2} - \frac{1}{\pi}\right]\right] = \frac{2}{\pi}\ln\left(\frac{\pi^2}{2}\right) - \left(\frac{\pi-2}{\pi}\right)$

Section – A

45. D

46. A

Sol. (Q.45 – 46)

Let x be amount of salt of time t in the tank. So concentration of salt at time t is $\frac{x}{100+5t}$

$\Rightarrow \frac{dx}{dt} = 100 - \frac{5x}{100+5t} \Rightarrow \frac{dx}{dt} + \frac{x}{20+t} = 100$

$\Rightarrow x(20 + t) = \int 100(20+t)\, dt \Rightarrow x(20+t) = \frac{100(20+t)^2}{2} + c$

At t = 0, x = 100 $\Rightarrow$ c = –18000 $\Rightarrow$ $x = 50(20+t) - \frac{18000}{20+t}$

At t = 20 $\Rightarrow$ $x = 50 \cdot 40 - \frac{1800}{4} = 1550$

Let y be amount of salt in table after at time t (t > 20), then $\frac{dy}{dt} = 100 - \frac{10y}{200}$

$\Rightarrow \int \frac{dy}{2000-y} = \int \frac{dt}{20} \Rightarrow -\ln(2000-y) = \frac{t}{20} + c'$

At t = 20, y = 1550 $\Rightarrow$ c′ = 1 – ln(450) $\Rightarrow$ $\frac{450}{2000-y} = e^{\frac{t}{20}+1}$

$\Rightarrow y = 2000 - 450e^{\left(-\frac{t}{20}-1\right)}$

47. B

Sol. From graph of the functions, we can observe $n(A_1) = 4040$, $n(B_2) = 2022$

48. D

Sol. $m_1 = 2020$ $m_2 = 2021$
$n_1 = 2020$ $n_2 = 0$

Section – B

49. 5

Sol. $\int_3^6 f(x-1)dx = \int_2^5 f(x)dx$, $\int_0^1 f(x+2)dx = \int_2^3 f(x)dx$

$\int_0^{5/2} f(2x)dx = \frac{1}{2}\int_0^5 f(x)dx = \frac{1}{2}\left[\int_0^3 f(x)dx + \int_2^5 f(x)dx - \int_2^3 f(x)dx\right] = 5$

50. 19

Sol. f(x) = ({x} + [|x + sin x|]) sgn(x^2 – 1)
|x + sin x| will have 11 points of discontinuity {x} will have 11 points of discontinuity of which x = 0 is common in both and f(x) is continuous at x = ± 1

51. 1

Sol. $(x^3y^2 + 2x^2y - 2x^3)dy = (x^2y^3 + 2y^3 - 2xy^2)dx$
$\Rightarrow x^3y^2\,dy - x^2y^3\,dx = 2y(y^2\,dx - x^2\,dy) + 2x(x^2\,dy - y^2\,dx)$
$\Rightarrow y^2x^3\,dy - x^2y^3\,dx = 2(y-x)(y^2\,dx - x^2\,dy)$

$\Rightarrow \frac{dy}{y} - \frac{dx}{x} = 2\left(\frac{1}{x} - \frac{1}{y}\right)\left(\frac{dx}{x^2} - \frac{dy}{y^2}\right)$

$\Rightarrow d(\ln y) - d(\ln x) = -2\left(\frac{1}{y} - \frac{1}{x}\right)d\left(\frac{1}{y} - \frac{1}{x}\right)$

$\Rightarrow \ln y - \ln x = -\left(\frac{1}{y} - \frac{1}{x}\right)^2 + c$, satisfy (1, 1) $\Rightarrow$ c = 0

Section – C

52. 1.65

Sol. Let $P = \lim_{x\to 0^+}\left(\frac{e}{(1+x)^{1/x}}\right)^{1/x} \Rightarrow \ln P = \lim_{x\to 0^+}\frac{1}{x}\ln\left(\frac{e}{(1+x)^{1/x}}\right)$

$\Rightarrow \ln P = \lim_{x\to 0^+}\frac{x-\ln(1+x)}{x^2} = \frac{1}{2} \Rightarrow P = \sqrt{e}$

53. 3.00

Sol. $\lim_{x\to 0^+} f_2(x) = \sqrt{e} \Rightarrow \lim_{x\to 0^+} f_n(n) = 0 \ \forall\ n > 2$

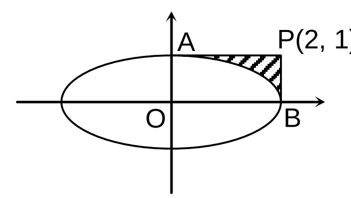
54. 0.43

Sol. Area of region R

= Area of OAPB – Area of OAB

= $2 - \frac{\pi}{4}\cdot 2$

~ 0.43



55. 9.42

Sol. Area of region S = $5\pi - 2\pi = 3\pi$

56. 126525.00

Sol. For h(x) to be bijective n(A) = n(B) = n(C)

Number of possible function

= $\left({}^5C_1\right)^3 + \left({}^5C_2\right)^3(2!)^2 + \left({}^5C_3\right)^3(3!)^2 + \left({}^5C_4\right)^3(4!)^2 + \left({}^5C_5\right)^3(5!)^2 = 126525$

57. 44825.00

Sol. Let n(A) = n(B) = n(C) = d

If d = 1 $\rightarrow {}^5C_1{}^5C_1\cdot 1 = 25$

If d = 2 $\rightarrow {}^5C_2{}^5C_2\cdot 4\times(2!)^2 = 1600$

If d = 3 $\rightarrow {}^5C_3{}^5C_3\cdot 4\times(3!)^2 = 14400$

If d = 4 $\rightarrow {}^5C_4{}^5C_4\cdot 1\times(4!)^2 = 14400$

If d = 5 $\rightarrow {}^5C_5\cdot{}^5C_5\cdot{}^5C_5\cdot(5!)^2 = 14400$

Total = 44825