FIITJEE ALL INDIA TEST SERIES JEE (Advanced)-2022 <u>PART TEST – I</u> PAPER –2 <u>TEST DATE: 21-11-2021</u>

Time Allotted: 3 Hours

Maximum Marks: 180

General Instructions:

- The test consists of total 57 questions.
- Each subject (PCM) has 19 questions.
- This question paper contains **Three Parts**.
- Part-I is Physics, Part-II is Chemistry and Part-III is Mathematics.
- Each Part is further divided into Three Sections: Section-A, Section-B & Section-C.

Section – A (01 –06, 20 – 25, 39 – 44): This section contains EIGHTEEN (18) questions. Each question has FOUR options. ONE OR MORE THAN ONE of these four option(s) is(are) correct answer(s).

Section – A (07 – 10, 26 – 29, 45 – 48): This section contains SIX (06) paragraphs. Based on each paragraph, there are TWO (02) questions. Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.

Section – B (11 – 13, 30 – 32, 49 – 51): This section contains NINE (09) questions. The answer to each question is a NON-NEGATIVE INTEGER.

Section – C (14 – 19, 33 – 38, 52 – 57): This section contains NINE (09) question stems. There are TWO (02) questions corresponding to each question stem. The answer to each question is a NUMERICAL VALUE. If the numerical value has more than two decimal places, truncate/round-off the value to TWO decimal places.

			MARKING SCHEME	
Section – A (One or More than One Correct): Answer to each question will be evaluated according to the following				
marking scheme:				
Full Marks	:	+4	If only (all) the correct option(s) is (are) chosen;	
Partial Marks	:	+3	If all the four options are correct but ONLY three options are chosen;	
Partial marks	:	+2	if three or more options are correct but ONLY two options are chosen and both of which are correct;	
Partial Marks	:	+1	If two or more options are correct but ONLY one option is chosen and it is a correct option;	
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);	
Negative Marks	:	-2	In all other cases.	
Section – A (Single Correct): Answer to each question will be evaluated according to the following marking scheme:				
Full Marks	:	+3	If ONLY the correct option is chosen.	
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);	
Negative Marks	:	-1	In all other cases.	
Section – B: Answer to each question will be evaluated according to the following marking scheme:				
Full Marks	:	+4	If ONLY the correct integer is entered;	
Zero Marks	:	0	In all other cases.	
Section – C: Answer to each question will be evaluated according to the following marking scheme:				
Full Marks	:	+2	If ONLY the correct numerical value is entered at the designated place;	
Zero Marks	:	0	In all other cases.	

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Physics

PART – I

Section – A (Maximum Marks: 24)

This section contains SIX (06) question. Each question has FOUR options (A), (B), (C) and (D). ONE OR MOER THAN ONE of these four option(s) is (are) correct answer(s).

- 1. A person of mass 60 kg is standing on the door of a bus moving with a velocity 72 km/hr. A trolley of mass 90 kg is moving on the smooth rails along side the bus with a velocity 90 km/hr in the direction of motion of the bus. Person jumps on the trolley with a velocity 18 km/hr with respect to the bus in a direction perpendicular to the velocity of trolley. (Assuming person comes to rest on the trolley in 0.1 sec after coming in contact with it)
 - Speed of the trolley is 82.8 km/hr after person comes at rest on it. (A)
 - (B) Force on the person during his contact with the trolley in time interval 0.1 sec is nearly 3500 N.
 - Force on the trolley during person's contact with the trolley in time interval 0.1 sec is (C) 1800 N.
 - (D) Force on the person during his contact with the trolley in time interval 0.1 sec is 1800 N.
- 2. A rod of length L and mass m is hinged at its one end at point O. It can rotate freely in the vertical plane. It is given an angular velocity ω_0 when it is in the vertical position. Rod just completes the full circle. (θ is angle made by the rod with the vertically up direction)
- Α
- (A) Magnitude of normal reaction at hinge versus θ graph is a continuous graph.
- Normal reaction at hinge is along the rod at $\theta = \cos^{-1} \left(\frac{3}{5} \right)$ (B)
- Normal reaction at hinge is perpendicular to the rod at $\theta = \cos^{-1} \left(\frac{3}{5} \right)$ (C)
- (D) Direction of normal reaction at hinge continuously changes with θ .
- 3. Two particles P and Q are moving on a straight line and their Velocity velocity versus time graph are shown in the figure.



- Both particles start from the same position. (A)
- (B) At any given time Q is always ahead of P.
- (C) In given time interval Q travels more distance than P.
- Impulse on P may be greater than that on Q in a given time interval. (D)

4. One end of a spring having spring constant k = 50 N/m is attached to a block of mass 2 kg. Free end of the spring moves with a constant velocity 5 m/s. Assuming surfaces to be smooth and block is at rest at t = 0. (Initially the spring is nondeformed)



- (A) Block will finally move with a velocity 5 m/s.
- (B) Maximum elongation in the spring may be 1 m.
- (C) Speed of the block will always vary between zero and 5 m/s.
- (D) Maximum speed of the block may be greater than 5 m/s.
- 5. A particle is moving in a conservative field and its potential energy is given as U = (6x 8y) J, where x and y co-ordinates are in meters. Particle has initial velocity $(2\hat{i} - 3\hat{j})m/s$ at origin. (Given: mass of the particle is 2 kg)
 - (A) Speed of the particle will not be zero for any time t.
 - (B) At t = 1 sec, the angle between velocity and acceleration of particle is $\cos^{-1}\left(\frac{7}{5\sqrt{2}}\right)$

(C) When the particle crosses x axis,
$$x = -\frac{3}{8}m$$

- (D) Particle never has its velocity perpendicular to acceleration.
- 6. Two particles A and B each of mass m are attached to free ends of inextensible string of length L. Whole arrangement is lying on a smooth horizontal surface. Particle A is a given velocity $v_0(\hat{i} + \hat{j})$ m/s, while particle B is given a velocity $v_0(\frac{1}{2}\hat{i} + \frac{3}{2}\hat{j})$ m/s. (Take z-axis normal to horizontal plane) (A) String will become taut at $t = \frac{2L}{v_0}$
 - (B) Just after string becomes taut, velocity of particle A will be $\frac{\sqrt{41}}{4}v_0$ m/s
 - (C) Tension in the string just after it becomes taut is $\frac{mv_0^2}{8l}$
 - (D) There will be loss of energy when string becomes taut.

Section – A (Maximum Marks: 12)

This section contains **TWO (02) paragraphs.** Based on each paragraph, there are **TWO (02)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.

Paragraph for Question Nos. 07 and 08

A small body of mass m is rigidly fixed to the inside of a thin rigid hoop of same mass m and radius R placed on a rough horizontal surface. When the body is getting in the lowest position, the velocity of the centre of the hoop is $v_0 = \sqrt{5gR}$ as shown in the figure. The hoop rolls without slipping on the rough horizontal surface.



y

L

х

В

7. When the body will get to the left end of the horizontal diameter of the hoop, the frictional force acting on the hoop due to the horizontal surface is

4

- (A) $\frac{\text{mg}}{2}$ (B) $\frac{3\text{mg}}{4}$ (C) $\frac{3\text{mg}}{2}$ (D) $\frac{7\text{mg}}{2}$
- 8. When the body will get to the left end of the horizontal diameter of the hoop, the normal force acting on the hoop due to the horizontal surface is

(A)	mg 4
(B)	$\frac{3mg}{4}$
(C)	<u>5mg</u> 4
(D)	$\frac{7mg}{4}$

Paragraph for Question Nos. 09 and 10

A hemispherical body B of mass 2m is placed on a rough horizontal surface with coefficient of friction $\mu = 0.5$. A ball 'A' of mass m moving with a velocity $v_0 = 15$ m/s at an angle $\theta = 37^{\circ}$ from the horizontal surface collides with the body B at point P with coefficient of restitution e = 0.6 as shown in the figure. There is no friction between the ball A and the body B. (Take m = 1 kg)



- 9. The velocity of the hemispherical body B just after collision is
 - (A) 3 m/s
 - (B) 5 m/s
 - (C) 8 m/s
 - (D) 10 m/s
- 10. The impulse due to frictional force of the horizontal surface on the hemispherical body B during collision is
 - (A) 20 N-s
 - (B) 16 N-s
 - (C) 12 N-s
 - (D) 6 N-s

Section – B (Maximum Marks: 12)

This section contains **THREE (03)** questions. The answer to each question is a **NON-NEGATIVE INTEGER.**

11. A rod of length L and mass m is sliding with one end on the horizontal surface and the other end on the vertical wall as shown in the figure. At the given instant, speed of end A and end B are same. Angular momentum (in kg-m²/sec) of rod about point O at the given instant is (Given L = 3m, m = 1kg, $v = \sqrt{2}$ m/s)





12. A loop of radius 1 m is made from string and this loop is slid on the surface of sphere of radius 2m. Sphere along with loop rotates about its axis with a constant angular velocity ω rad/sec. Assuming that loop

does not slip on the sphere. If tension in the loop (in S.I. units) is $\frac{2n}{3}$.

Find n. (There is no friction between the loop and spherical surface).

[Given mass per unit length of the loop is $\frac{1}{\sqrt{3}}$ kg/m, $\omega^2 = 2\sqrt{3}$

 $(rad/sec)^2$ and g = 10 m/s²]

13. Assuming all parts of the string to be vertical and pulleys P_1 and P_2 to be massless and frictionless. Find the acceleration (in m/s²) of point A of the string. Assuming string to be massless and inextensible. (Take $m_1 = m_2 = 1$ kg, g = 10 m/s²)

Section – C (Maximum Marks: 12)

This section contains **THREE (03)** question stems. There are **TWO (02)** questions corresponding to each question stem. The answer to each question is a **NUMERICAL VALUE.** If the numerical value has more than two decimal places, truncate/round-off the value to **TWO** decimal places.

Question Stem for Question Nos. 14 and 15

Question Stem

A ball of mass 1 kg is projected horizontally with a velocity 10 m/s from a height of 5 m from ground. After hitting the ground, the ball has its vertical component of velocity 2.5 m/s. The co-efficient of friction between the ball and the ground is 0.1 and the ball remains in contact with the ground for 0.1 sec. Average normal reaction during collision (contact force between the ball and the ground in vertical direction) is N newton and the horizontal component of velocity of the ball just after the impact of ball with ground for first time is v_x m/s. (Take g = 10 m/s²)

14. The value of N is

15. The value of v_x is

Question Stem for Question Nos. 16 and 17

Question Stem

A particle of mass m is attached to inner portion of periphery of a ring of radius R = 1m and mass m. System is slightly disturbed when the particle is at the highest point of the ring. Normal reaction between the ring and the ground is N and angular velocity of system is ω rad/s. (Given m = 1kg, g = 10 m/s²). There is no slipping between the ring and the ground at any instant.



- 16. The value of N (in newton) when the particle is at lowest position of the ring is
- 17. The value of ω when the particle is at level of the centre of ring is.....

Question Stem for Question Nos. 18 and 19

Question Stem

Two particles P and Q each of mass 2 kg are projected along the surface AC and AD respectively with the same speed 10 m/s. Assuming that particles negotiate the points C and D smoothly. The velocity of particle P is v_P m/s and the velocity of particle Q is v_Q m/s, when they reach at point B. (Take g = 10 m/s² and AB is horizontal and coefficient of friction for the path ACB and the path ADB is 0.2 between the path and the particle)



- 18. The value of v_P is
- 19. The value of the ratio $\left(\frac{v_{P}}{v_{Q}}\right)$ is

Chemistry

PART – II

Section – A (Maximum Marks: 24)

This section contains **SIX (06)** question. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MOER THAN ONE** of these four option(s) is (are) correct answer(s).

- 20. The correct statement (s) among the following is/are (A) Nitrogen atom in $N(CH_3)_2$ is sp³ hybridized
 - (B) Nitrogen atom in $N(SiH_3)_3$ is sp² hybridized
 - (C) $N(SiH_3)_3$ is less basic than $N(CH_3)_3$ hybridized
 - (D) The shape of $N(SiH_3)_2$ is pyramidal

21. The disproportionation reaction (s) among the following is/are

- $(A) \qquad 2H_2O_2 \longrightarrow 2H_2O + O_2$
- (B) $4H_3PO_4 \xrightarrow{\Delta} 3H_3PO_4 + PH_3$
- (C) $3CI_2 + 6NaOH \longrightarrow 5NaCI + NaCIO_3 + 3H_2O$

(D)
$$2Pb(NO_3)_2 \xrightarrow{\Delta} 2PbO + 4NO_2 + O_2$$

22. Consider the following equilibrium in a vessel

$$NH_4HS(s) \Longrightarrow NH_3(g) + H_2S(g)$$

The incorrect statement(s) among the following is/are

- (A) Addition of catalyst at constant temperature increases the value of K_c
- (B) Addition of catalyst at constant temperature increases the value of K_P
- (C) Addition of NH_4HS at equilibrium increases the value of K_c
- (D) Addition of inert gas at constant volume shifts the equilibrium in the forward direction
- 23. The correct statement (s) among the following is/are
 - (A) A catalyst can change the value of ΔH of a reaction
 - (B) A positive catalyst decreases the activation energy of a reaction
 - (C) At equilibrium the rate of forward reaction is equal to the rate of backward reaction
 - (D) Molecularity and order of reaction both are equal for elementary reaction
- 24. The correct statement(s) among the following is/are
 - (A) LiHCO_3 does not exist in the solid state
 - (B) $LiNO_3$ on decomposition by heating produces N_2 gas
 - (C) Lithium reacts with nitrogen to form Li₃N
 - (D) Lithium is the strongest reducing agent among alkali metals in aqueous solution
- 25. The molecule (s) in which the central atom is sp³d hybridized is/are
 - (A) BrF_5
 - (B) PCI₅
 - (C) CIF₃
 - (D) $XeO_{2}F_{2}$

Section – A (Maximum Marks: 12)

8

This section contains **TWO (02) paragraphs.** Based on each paragraph, there are **TWO (02)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.

Paragraph for Question Nos. 26 and 27

Hydrolysis of salt is the reaction of cation or anion or both of a salt with water. Salts of strong acid and strong base do not undergo hydrolysis. Salts of weak acid and strong base undergo anionic hydrolysis and salts of strong acid and weak base undergo cationic hydrolysis. Salts of weak acid and weak base undergo both cationic and anionic hydrolysis.

26. The pH of a solution obtained by mixing 100 ml of 0.2 M CH_3COOH with 100 ml of 0.2 M NaOH

is $(pK_a \text{ of } CH_3COOH = 4.74)$

- (A) 4.74
- (B) 8.87
- (C) 9.26
- (D) 8.57

27. The pH of 0.005 M $(NH_4)_2 SO_4$ solution is

 $(pK_{b} \text{ of } NH_{4}OH = 4.74)$

- (A) 9.26 (B) 8.87
- (C) 4.73
- (D) 5.63

Paragraph for Question Nos. 28 and 29

According to valence shell electron pair repulsion theory the shape of the molecule depends upon the repulsion between the electron pairs present in the valence shell. The repulsion between two lone pairs is greater than the repulsion between a lone pair and a bond pair, which in turn is greater than the repulsion between two bond pairs. The magnitude of repulsion between bonding pairs of electrons depends on the electronegativity difference between the central atom and the other atoms.

- 28. The shape of which of the molecule is pyramidal?
 - (A) SF₄
 - (B) XeO₃
 - (C) XeF₄
 - (D) BF₃

29. Which of the following pair of species have identical shapes?

- (A) I_3^- , XeF₂
- (B) CIF_3, BF_3
- (C) PCI_5, BrF_5
- (D) CH_4, XeF_4

Section – B (Maximum Marks: 12)

This section contains **THREE (03)** questions. The answer to each question is a **NON-NEGATIVE INTEGER.**

- 30. Among the following the total number of species which are paramagnetic is $C_2, O_2, S_2, O_2^+, N_2, N_2^-, CN^-, B_2, NO_2$
- 31. What is the maximum number of electrons in a sub-shell that can have quantum number n = 3 and $\ell = 1$?
- 32. A compound of vanadium has a magnetic moment of 1.73 B.M. If the vanadium ion in the compound is present as V^{x+}, then the value of x is

Section – C (Maximum Marks: 12)

This section contains **THREE (03)** question stems. There are **TWO (02)** questions corresponding to each question stem. The answer to each question is a **NUMERICAL VALUE.** If the numerical value has more than two decimal places, truncate/round-off the value to **TWO** decimal places.

Question Stem for Question Nos. 33 and 34

Question Stem

Solid Na_2SO_4 is slowly added to a solution which is 0.1 M in Ca^{2+} and 0.1 M in Ba^{2+} . The minimum concentration of SO_4^{2-} required to start the precipitation of $CaSO_4$ is $x \times 10^{-4}$ M. The concentration of Ba^{2+} when $CaSO_4$ begins to precipitate is $y \times 10^{-6}$ M.

$$\begin{split} & \textbf{K}_{\text{SP}} \ \textbf{CaSO}_4 = 2.4 \times 10^{-5} \\ & \textbf{K}_{\text{SP}} \ \textbf{BaSO}_4 = 9 \times 10^{-10} \end{split}$$

- 33. The value of x is_____
- 34. The value of y is_____

Question Stem for Question Nos. 35 and 36

Question Stem

When NO and NO₂ are mixed the following equilibria are readily obtained

 $2NO_2(g) \rightleftharpoons N_2O_4(g) K_P = 6.8 \text{ atm}^{-1}$

 $NO(g) + NO_2(g) \Longrightarrow N_2O_3(g) K_{P'}$

In an experiment when NO and NO₂ are mixed in the ratio 1 : 2, the final total pressure was found to be 5 atm and the partial pressure of N_2O_4 at equilibrium was 1.7 atm.

35. The K_{P} for the reaction

 $NO(g) + NO_2(g) \Longrightarrow N_2O_3(g)$ is_____

36. The equilibrium partial pressure of N_2O_3 in atm is_____

Question Stem for Question Nos. 37 and 38

10

Question Stem

20 ml of $\frac{M}{10}$ CH₃COOH solution is titrated with $\frac{M}{10}$ solution of NaOH. When 16 ml of NaOH solution is added to the CH₃COOH solution a buffer solution was formed and when 20 ml of NaOH solution was added complete neutralization took place pK_a of CH₃COOH is 4.74. (log2 = 0.3, log 5 = 0.7)

- 37. What is the pH of the solution when 20 ml of $\frac{M}{10}$ CH₃COOH is mixed with 16 ml of $\frac{M}{10}$ NaOH solution?
- 38. What is the pH of the solution when 20 ml of $\frac{M}{10}$ CH₃COOH is mixed with 20 ml of $\frac{M}{10}$ NaOH solution?

Mathematics PART-III

Section – A (Maximum Marks: 24)

This section contains **SIX (06)** question. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MOER THAN ONE** of these four option(s) is (are) correct answer(s).

39. Let $y_k(x) = \int \frac{x^2}{x^4 + kx^2 + 1} dx$ be family of curves and $S = \{y_k(x) | k \in R\}$, then which of the following family of curves belong(s) to the set S2

ollowing family of curves belong(s) to the set S =
$$1 |x^2 - x - 1| = 1 |x^2 - \sqrt{5}x + 1|$$

(A)
$$\frac{1}{4}\ln\left|\frac{x^2-x-1}{x^2+x-1}\right| + \frac{1}{4\sqrt{5}}\ln\left|\frac{x^2-\sqrt{5}x+1}{x^2+\sqrt{5}x+1}\right| + c$$

(B)
$$\frac{-x}{2x^2-2} + \frac{1}{4} \ln \left| \frac{x-1}{x+1} \right| + c$$

(C)
$$\frac{1}{2\sqrt{2}} \tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right) + \frac{1}{4\sqrt{2}} \ln\left|\frac{x^2-\sqrt{2}x+1}{x^2+\sqrt{2}x+1}\right| + \frac{1}{4\sqrt{2}} \ln\left|\frac{x^2-\sqrt{2}x+1}{x^2+\sqrt{2}x+1}\right| + \frac{1}{4\sqrt{2}} \ln\left|\frac{x^2-1}{x^2+\sqrt{2}x+1}\right| + \frac{1}{4\sqrt{2}} \ln\left|\frac{x^2$$

(D)
$$\frac{1}{4} \tan^{-1} \left(\frac{x^2 - 1}{2x} \right) - \frac{x}{2x^2 + 2} + c$$

- 40. Let $P: y^2 = 4ax$ and $L: y = x \tan \theta$. Let A(t) represents area bounded by P and L where θ varies with t and is equal to $2\pi t$ (t is time in seconds), then which of the following statements is/are true for the function A(t)
 - (A) It is a periodic function
 - (B) Fundamental period of the function in 2π

(C) Minimum area bounded by y cot
$$\theta = x - a$$
 and P is $\frac{8a^2}{3}$

(D) There exists at least four roots of the equation
$$A(t) = \frac{8a^2}{3}$$
 in $t \in (0, 1)$

41. Let f(x) be a differentiable function satisfying $f(x) = x + \int_{0}^{1} (xt^{2} + x^{2}t)f(t)dt$ and g(x) be a

polynomial satisfying $g(x) + g\left(\frac{1}{x}\right) = g(x) \cdot g\left(\frac{1}{x}\right)$, g(2) = 9, then which of the following is/are

FALSE?

- (A) y = f(x) and y = g(x) intersect at only two points
- (B) y = f(x) is even function
- (C) y = g(x) is odd function
- (D) f(x) and g(x) do not have a global minima $\forall x \in R \{0\}$

42. Which of the following statements is/are true?

- (A) If f(x) is differentiable at x = a, then f'(x) is always continuous at x = a
- (B) If f(x) and g(x) are discontinuous at x = a and x = f(a) respectively, then g(f(x)) can be continuous at x = a

(C) If
$$\lim_{x \to a} |f(x)| = b$$
 and $\lim_{x \to b} |g(x)| = c$, then $\lim_{x \to a} |g(f(x))| = c$
(D) If $\int_{a}^{b} f(x) dx = \int_{a}^{b} g(x) dx = 0$, then the equation $f(x) = g(x)$ may not have any solution in $x \in [a, b]$

4

43. Let
$$I_n = \int_0^1 \frac{x^n}{\sqrt{1+x^2}} dx$$
, $n \in N$, then $I_n = a_n + b_n I_{n-2}$, where a_n and b_n are functions of 'n', then
(A) $a_1 + b_2 = \sqrt{2} - \frac{1}{2}$
(B) $2a_2 + b_3 = \sqrt{2} - \frac{2}{3}$
(C) $3a_4 - b_4 = \sqrt{2} + \frac{1}{2}$
(D) $a_2 - 3b_5 = \frac{1}{\sqrt{2}} - 21$
44. Let $I = \int_1^2 \frac{\sin x}{x^2} dx$, then
(A) $I < \ln 2$
(B) $I > \ln 2 - \frac{1}{4}$
(C) $I > \frac{1}{2\sqrt{2}}$

Section – A (Maximum Marks: 12)

This section contains **TWO (02) paragraphs.** Based on each paragraph, there are **TWO (02)** questions. Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.

Paragraph for Question Nos. 45 and 46

A tank contains 100 litre of salt solution which has 100 grams of salt at time t = 0. It has an inlet valve through which salt solution of 10 gm/litre concentration enters the tank at the rate of 10 ltr/hr. Tank also has an outflow valve through which salt solution in tank exists at a rate of 5 ltr/hr. The contents of the tank are kept thoroughly mixed at all times.

The tank can only hold upto 200 litre of contents, the rest exits out through a spill over valve

45. Amount of salt (in grams) in tank after 20 hours is

(D) $I > \frac{2}{\pi} \left[ln\left(\frac{\pi^2}{2}\right) + \left(\frac{2-\pi}{2}\right) \right]$

- (A) 2000
- (B) 1850
- (C) 1750
- (D) 1550

46. Amount of salt (in grams) in tank at t = $20 \left(\ln \frac{10}{e} \right)$ will be

- (A) 1955
- (B) 1895
- (C) 1925
- (D) none of these

Paragraph for Question Nos. 47 and 48

13

Let
$$f(x) = \sum_{r=1}^{2021} \frac{r}{rx^2 - 1}$$
, $F(x) = \int f(x) dx$, $g(x) = \sum_{r=1}^{2022} \tan\left(\frac{\pi rx}{2022}\right)$, $G(x) = \int g(x) dx$
 $A_r = \{x \mid f(x) = r, r \in \mathbb{R}, x \in (-1, 1)\}$, $B_r = \{y \mid g(y) = r, r \in \mathbb{R}, x \in (-1, 1)\}$

- 47. Value of $n(A_1) + n(B_2)$ is
 - (A) 6060
 - (B) 6062
 - (C) 6064
 - (D) none of these
- 48. If m_1 , m_2 be number of local minima of functions F, G and n_1 , n_2 be number of local maxima of F, G respectively in $x \in (-1, 1)$, then value of $m_1 + m_2 n_1 n_2$ equals
 - (A) –2
 - (B) 0
 - (C) 2
 - (D) none of these

Section – B (Maximum Marks: 12)

This section contains **THREE (03)** questions. The answer to each question is a **NON-NEGATIVE INTEGER.**

49. Let
$$\int_{0}^{3} f(x) dx = 4$$
, $\int_{3}^{6} f(x-1) dx = 5$ and $\int_{0}^{1} f(x+2) dx = -1$, then $\int_{0}^{5/2} f(2x) dx$ equals _____

50. Number of points where $f(x) = [\{x\} + |x + \sin x|] \operatorname{sgn}(x^2 - 1), x \in (-6, 6)$ is discontinuous is _____ (where [.] denotes greatest integer function and $\{.\}$ is fractional part function, $\operatorname{sgn}(x)$ is signum function)

51. Let y(x) be solution of the differential equation $\frac{dy}{dx} = \frac{x^2y^3 + 2y^3 - 2xy^2}{y^2x^3 + 2x^2y - 2x^3}$. If y(1) = 1 and the

solution is $y = xe^{k_1\left(\frac{x-y}{xy}\right)^{k_2}}$, then $k_1 + k_2$ equals _____

Section – C (Maximum Marks: 12)

This section contains **THREE (03)** question stems. There are **TWO (02)** questions corresponding to each question stem. The answer to each question is a **NUMERICAL VALUE.** If the numerical value has more than two decimal places, truncate/round-off the value to **TWO** decimal places.

Question Stem for Question Nos. 52 and 53

Question Stem

 $\text{Let } f_{1}(x) = (1+x)^{1/x}, x > 0 \text{ and } f_{n}(x) = \left(\frac{f_{n-1}(x)}{e}\right)^{1/x} \ \forall \ n \in N-\{1\}. \ \text{Let } \lim_{x \to 0^{+}} \frac{1}{f_{n}(x)} = I_{n}, \ \lim_{x \to 0^{+}} f_{n}(x) = m_{n} \text{ and } I_{n}(x) = I_{n}, \ \lim_{x \to 0^{+}} f_{n}(x) = I_{n}(x) = I_{n}(x)$

 $S = \{m_i \mid j \in N\}$. Let n(S) denote number of elements in set S

52. Value of l₂ is _____

53. Value of n(S) is _____

Question Stem for Question Nos. 54 and 55

14

Question Stem

Tangents PA, PB are drawn from any point P to the ellipse E: $\frac{x^2}{4} + y^2 = 1$ such that the angle between

the tangents is fixed angle θ . Let R = {(x, y) | (x, y) lies in region bounded by PA, PB and E} S = {(x, y) | (x, y) lies in region bounded by PA, PB and E if P varies so as to keep θ fixed} If point P is (2, 1), then

- 54. Area of region R is _____
- 55. Area of region S is _____

Question Stem for Question Nos. 56 and 57

Question Stem

Let S = {1, 2, 3, 4, 5}. A, B and C be non-null subsets of S. Let f : A \rightarrow B, g : B \rightarrow C and 'h' be the composite function defined from A to C such that h(x) = g(f(x)) and h(x) is bijective function. Let P(A, B, C) = $\sum h(a)$

$$= \sum_{a \in A} h($$

56. Number of possible functions h(x) is _____

57. Number of possible functions h(x) if P is divisible by 3 is _____