FIITJEE
ALL INDIA TEST SERIES
JEE (Advanced)-2022
PART TEST – I
PAPER –1
TEST DATE: 21-11-2021

ANSWERS, HINTS & SOLUTIONS

Physics

PART – I

Section – A

1. C

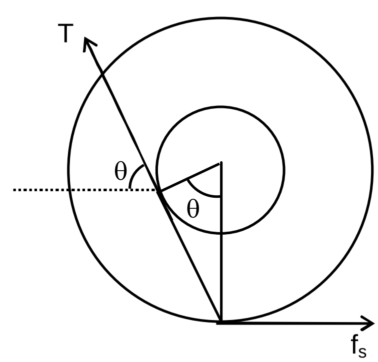
Sol. Time of flight $= \frac{2v_0}{g}$

So, $2\pi = \omega \frac{2v_0}{g}$ (ω is the angular velocity of rod)

So, $v_x = \omega \frac{\ell}{2} = \frac{\pi \ell g}{2v_0}$

2. C

Sol. $\cos\theta = \frac{1}{2} \Rightarrow \theta = 60°$



3. C

Sol. By observation

4. C

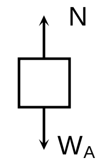
Sol. For smaller ω, tension in the lower string is zero. So, the tension ratio is zero. For a larger ω, tension in both the strings will tend to the same value. So, the ratio will tend to 1.

Section – A

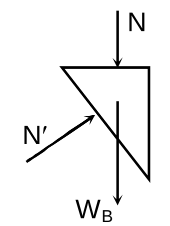
5. A, B, D

Sol. So, acceleration of A is always vertically downward and acceleration of B is along the incline more than g sinθ. Further we can also say that there is no horizontal force on the system. So, $m_B a_{Bx} + m_C a_{Cx} = 0$

F.B.D of A



F.B.D of B



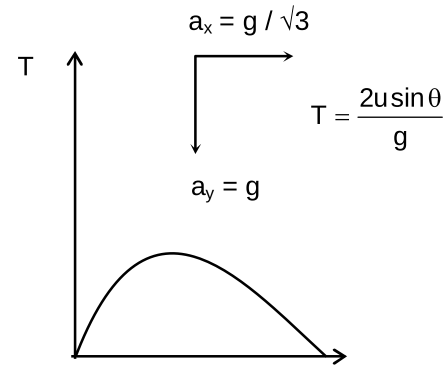
6. B, D

Sol. $R = u\cos\theta\frac{2u\sin\theta}{g} + \frac{1}{2}\frac{g}{\sqrt{3}}\left(\frac{2u\sin\theta}{g}\right)^2$

$= \frac{2u^2}{g}\left[\cos\theta\sin\theta + \frac{1}{\sqrt{3}}\sin^2\theta\right]$

$\frac{dR}{d\theta} = 0, \ \tan 2\theta = -\sqrt{3}$

$\theta = 60°$



7. A, C, D

Sol. $U = cmr^2$

$F = -\frac{dU}{dr} = -2cmr$

So, $\frac{mv^2}{r} = 2cmr$

So, mechanical energy = $cmR^2 + cmR^2 = 2cmR^2$

So, $\omega = \frac{v}{R} = \sqrt{2c}$

Acceleration of a particle $\vec{a} = -2c\vec{r}$

So, it will perform oscillatory motion about O.

8. B, C, D

Sol. $I_1 = \frac{1}{2}MR^2 + \frac{M}{12}(2R)^2 = \frac{5}{6}MR^2$

$I_2 = \frac{5}{6}MR^2 + MR^2 = \frac{11}{6}MR^2$

I_1 and I_2 are related by parallel axis theorem.

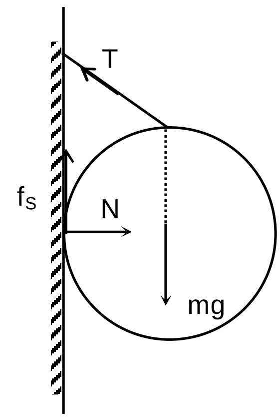
9. A, C, D

Sol. $f_s = N$ …(i)

$f_S \le \mu_S N$ …(ii)

$N \le \mu_S N$

So, $\mu_s \ge 1$



10. A, C

Sol. $\frac{L}{2}d\theta = \ell d\theta'$

So, $\frac{d\theta'}{dt} = \frac{L}{2\ell}\omega = 2$ rad/s

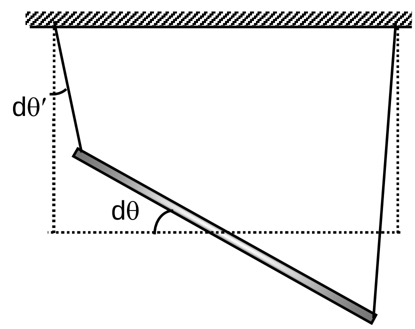
$\omega' = 2$ rad/s

$dy = \frac{\ell}{2}(d\theta')^2 = \frac{1}{2}a_{CM}(dt)^2$

So, $a_{CM} = \frac{L^2\omega^2}{4\ell}$

So, $2\Delta T = ma_{CM}$

$\Rightarrow \Delta T = \frac{mL^2\omega^2}{8\ell} = 2N$



Section – B

11. 9

Sol. Mass of water pumped in time dt is $dm = \rho\pi r^2 v dt$

So, $Pdt = dm\left(gh + \frac{v^2}{2}\right) = \rho\pi r^2 v dt\left(gh + \frac{v^2}{2}\right)$

So, $P = \rho\pi r^2 v\left(gh + \frac{v^2}{2}\right)$

So, $r = \sqrt{\frac{P}{\rho\pi v\left(gh + \frac{v^2}{2}\right)}} = 9$ cm

12. 30

Sol. Velocity of centre of mass of the system = 4 m/s. With respect to the centre of mass frame, net momentum is zero.

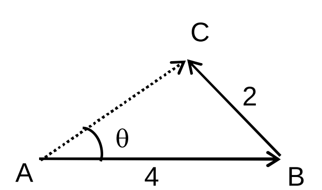
So, –60 V + 15(10 – v) = 0 $\Rightarrow$ v = 2 m/s

So velocity triangle for the boy

$\overrightarrow{AB} = \vec{v}_{Cg}$, $\overrightarrow{BC} = \vec{v}_{bC}$

For θ_{max}, $\angle ACB = 90°$

So, $\theta_{max} = \sin^{-1}\left(\frac{2}{4}\right) = 30°$



13. 3
Sol. In the centre of mass frame, change in the velocity of the moving cart
= 12 –(–6) = 18 m/s
So, in the centre of mass frame velocity before the collision = 9 m/s
So, $v_{cm} = 12 - 9 = 3$ m/s

Section – C

14. 0.75

15. 1.40
Sol. (Q.14-15)
Position of centre of mass from mass B is $\frac{3\times1}{1+3} = 0.75$ m
So, horizontal displacement of B = 0.75 m
From conservation of mechanical energy
$mg\ell = \frac{1}{2}mv^2$ (velocity of mass A will be zero at this instant)
So, $v = \sqrt{2g\ell} = 1.40$ m/s

16. 0.23

17. 3.10
Sol. (Q.16-17)
Relative velocity of the block after two collisions = $\frac{v}{4}$ towards right.
From conservation of momentum
$mv = m\left(\frac{v}{4} + v_w\right) + 32mv_w$
So, $v_w = \frac{v}{44} = \frac{10}{44} = 0.23$ m/s
Velocity of the wagon, when the block is at rest with respect to ground = $\frac{v}{32}$ m/s
So, time taken = $\frac{d}{v} + \frac{d}{v/2} + \frac{d}{v/4} + \frac{d}{v/8} + \frac{d}{v/16} = \frac{31d}{v} = 3.10$ sec

18. 41.87

19. 2.62
Sol. (Q.18-19)
Pressure $P = \frac{\rho Qv}{A}$, Q = volume flow rate
$= \frac{4}{3}\pi\rho n r^3 v^2 = 41.87$ N/m^2
So, new pressure $P' = \frac{P}{16} = 2.62$ N/m^2

Chemistry

PART – II

Section – A

20. D

Sol. Addition of inert gas at constant volume does not affect the equilibrium.

21. C

Sol. $\frac{k_1}{k_2} = \frac{[O_2][O]}{[O_3]}$

$[O] = \frac{k_1}{k_2}\frac{[O_3]}{[O_2]}$

$\text{Rate} = k_3[O][O_3]$

$\text{Rate} = \frac{k_3.k_1}{k_2}\frac{[O_3]}{[O_2]}[O_3]$

$\text{Rate} = \frac{k_1.k_3}{k_2}\frac{[O_3]^2}{[O_2]}$

22. D

Sol. $\frac{hc}{\lambda} = \frac{hc}{\lambda_o} + \frac{1}{2}mv^2$

$V = \left[\frac{2hc}{m}\left(\frac{\lambda_o - \lambda}{\lambda_o\lambda}\right)\right]^{1/2}$

23. A

Sol. Pyrosilicate contains the unit $(Si_2O_7)^{6-}$. $Sc_2[Si_2O_7]$ is a pyrosilicate.

Section – A

24. A, B

Sol. The correct order are

LiH > NaH > KH	Thermal stability
$MgSO_4 > CaSO_4 > SrSO_4$	Solubility

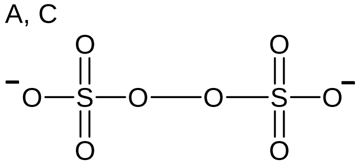
25. A, D

Sol. N_2^+ is less stable than N_2

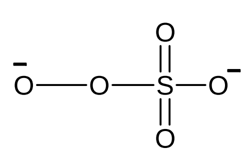
O_2^{2-} is diamagnetic

26. A, C

Sol.



$(S_2O_8^{2-})$



(SO_5^{2-})

27. A, B, D
Sol. BF_3 is a weaker Lewis acid than BI_3.

28. B, D
Sol. $NH_3 > NF_3$ (Dipole moment)
$O_2^+ > O_2$ (Bond order)

29. A, B, D
Sol. $Al(OH)_3 + NaOH \longrightarrow NaAlO_2 + 2H_2O$
$Zn(OH)_2 + 2NaOH \longrightarrow Na_2ZnO_2 + 2H_2O$
$Sn(OH)_2 + 2NaOH \longrightarrow Na_2SnO_2 + 2H_2O$

Section – B

30. 8
Sol. $P_{Cl_2} = 0.4$ atm, $P_{PCl_3} = 0.4$ atm
$P_{PCl_5} = 0.2$ atm
$$K_P = \frac{P_{PCl_3} \times P_{Cl_2}}{P_{PCl_5}}$$
$$K_P = \frac{0.4 \times 0.4}{0.2} = 0.8 \text{ atm}$$
$K_P = 8 \times 10^{-1}$ atm
$\therefore x = 8$.

31. 5
Sol. $$V = \frac{2.188 \times 10^6}{n} \text{ m/sec}$$
$$4.376 \times 10^5 = \frac{2.188 \times 10^6}{n}$$
$n = 5$
Number of waves made by an electron in one complete revolution = $n = 5$.

32. 4
Sol. $(NH_4)_2S(s) \rightleftharpoons 2NH_3(g) + H_2S(g)$
2P P
2P + P = 3 atm
$\therefore P = 1$ atm
$P_{NH_3} = 2$ atm and $P_{H_2S} = 1$ atm
$K_P = (P_{NH_3})^2 \times (P_{H_2S})$
$K_P = (2)^2 \times 1 = 4$

Section – C

33. 10.50
Sol. $$pH = \frac{pK_{a_2} + pK_{a_3}}{2} = \frac{8+13}{2} = 10.5$$

34. 8.00

Sol.

HPO_4^{2-}	+ H^+	$\rightleftharpoons$ $H_2PO_4^-$
10 millimoles	5 millimoles	0
5 millimoles	0	5 millimoles

$$pH = pK_{a_2} + \log\frac{5}{5}$$

pH = 8 + log1 = 8

35. 73.75

Sol. $\alpha = \frac{D-d}{d(n-1)}$

$$\frac{104.25-60}{60(2-1)} = 0.7375$$

Percentage dissociation = 73.75

36. 2.40

Sol. $K_P = K_C(RT)^{\Delta n}$

$$98.52 = K_C(0.0821\times500)$$

$$K_C = 2.40$$

37. 1.40

Sol. $k_1 = k\times\frac{10}{100}$

$$= 1.4\times10^{-4}\times\frac{10}{100}$$

$$= 1.4\times10^{-5}\ \text{sec}^{-1}$$

$$\therefore x = 1.40$$

38. 1.26

Sol. $k_2 = k\times\frac{90}{100}$

$$= 1.4\times10^{-4}\times\frac{90}{100}$$

$$= 1.26\times10^{-4}\ \text{sec}^{-1}$$

$$\therefore y = 1.26$$

Mathematics

PART – III

Section – A

39. C

Sol. Let $n = k^2 + m$

$f_1(n) = 3k + 2 - m \rightarrow \dfrac{3k+2-m}{5x+6-m} = \dfrac{4}{7}$

$f_2(n) = 5k + 6 - m$

So, $3m = k - 10 \Rightarrow$ for min k, m = 2

$\Rightarrow k = 16 \Rightarrow n = 258$

40. B

Sol. For $1 \le k \le n$

$$\left(1+\frac{x}{n}\right)^{k/n} \le \left(1+\frac{kx}{n^2}\right) \le \left(1+\frac{x}{n^2}\right)^k$$

$$\left(1+\frac{x}{n}\right)^{\frac{n+1}{2}} \le P_n \le \left(1+\frac{x}{n^2}\right)^{\frac{n(n+1)}{2}}$$

$$e^{x/2} \le \lim_{n\to\infty} P_n \le e^{x/2} \Rightarrow f(x) = e^{x/2}$$

41. B

Sol. Put y = 1

$f(x) - f(x+1) = -f(1) - x$

Put x = 1, 2, ... x

$f(x) = f(1)(x) + \dfrac{x(x-1)}{2}$ as f(7) = 21

$\Rightarrow f(1) = 0 \Rightarrow f(x) = \dfrac{x(x-1)}{2}$

42. C

Sol. Let $f(t) = \ln(2^t + 1) \Rightarrow f''(t) > 0 \Rightarrow f(t)$ is concave up

Let $2^{x_1} = a$, $2^{x_2 - \frac{1}{2}} = b$, $2^{x_3 - \frac{3}{2}} = c$

$(a^2 + 1) \ge 2a$, $(b^2 + 1) \ge 2b$, $(c^2 + 1) \ge 2c$

$(a^2 + 1)(b^2 + 1)(c^2 + 1) \ge 2^3 \cdot 2^{x_1+x_2+x_3-2}$

$\Rightarrow (x_1, x_2, x_3) \equiv \left(0, \dfrac{1}{2}, \dfrac{3}{2}\right)$

Section – A

43. A, C

Sol. Let f(x) is decreasing $x_2 > x_1 \Rightarrow f(x_2) < f(x_1)$

$\Rightarrow e^{-f(x_2)} < e^{-f(x_1)} \Rightarrow f(x_2) < f(x_1)$

$\Rightarrow$ f(x) is increasing

Now $x \to \infty \Rightarrow f(x) \to \infty$

$$\frac{\ln x}{f(x)} = 1 + \frac{\ln f(x)}{f(x)} \Rightarrow \lim_{x\to\infty} \frac{\ln x}{f(x)} = 1$$

44. A, B, C

Sol. $\frac{f'(x)}{f(x)} = \left(\frac{-2x-1}{2x(x+1)} + \ln\left(1+\frac{1}{x}\right)\right) = g(x)$

$\Rightarrow g'(x) = \frac{1}{2(x^2+x)^2} > 0$

$\lim_{x\to\infty} f(x) = e^{\left(\frac{x+1}{2}\right)\left(\frac{1}{x}\right)} = e^{1/2}$

45. A, B, C

Sol. $\int_0^1 \frac{\ln(1+x)}{x}dx = -\sum_{n=1}^{\infty}\int_0^1 \frac{x^{n-1}}{n}dn = -\sum_{n=1}^{\infty}\frac{1}{n^2}$

$\int_0^1 \frac{\ln(1-x)}{x}dx = \sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^2} = \frac{\pi^2}{12}$

$\sum_{n=1}^{\infty}\int_0^1 x^n \ln x = -\sum_{k=1}^{\infty}\frac{1}{(k+1)^2} = 1-\frac{\pi^2}{6}$

46. B, C, D

Sol. $\begin{cases} \sin(\tan x) \ge x & x \in \left(0, \frac{\pi}{4}\right) \\ \tan(\sin x) \ge x & x \in \left(\frac{\pi}{3}, 0\right) \\ \sin(\tan x) \ge \sin x \\ \tan(\sin x) \le \tan x \end{cases}$

47. A, D

Sol. If a, b ≥ 0

$\frac{a+b}{2} \le \sqrt{\frac{a^2+b^2}{2}} \le \frac{a+b}{\sqrt{2}}$

$g'(x) \ge 0 \Rightarrow \int_0^1 \frac{1+g'(x)}{2}dx \le \int_0^1 \sqrt{\frac{1+g'(x)^2}{2}}dx \le \int_0^1 \frac{1+g'(x)}{\sqrt{2}}dx$

$1 \le \frac{1}{\sqrt{2}}S \le \sqrt{2} \Rightarrow \sqrt{2} \le S \le 2$

48. B, C

Sol. $(f'(x))^3 + (-f(x))^3 + x^3 = -3x\, f(x)\, f'(x)$

$\Rightarrow f'(x) = -f(x) = x$ or $f'(x) - f(x) + x = 0$ as $f'(x) \ne -f(x)$ so $f'(x) - f(x) = -x$

$\frac{d}{du}\left(e^{-x}f(x)\right) = -xe^{-x} \Rightarrow e^{-x}f(x) = x^{-x}e + e^{-x} + c$

$f(x) = (x + 1) + ce^x$

Section – B

49. 8

Sol. $\{x\} \in [0, 1) \Rightarrow [x] \in (100, 125]$

$\Rightarrow \sum\{x_i\} = 24$

50. 6

Sol. $f(n) = 4\cos^2\frac{\pi}{3\cdot 2^n} - 3 = \dfrac{\cos\frac{\pi}{2^n}}{\cos 3\cdot\frac{\pi}{2^n}}$ as sin 2θ = 2 sin θ cos θ

$\Rightarrow f(n) = \dfrac{\sin\left(\frac{\pi}{2^{n-1}}\right)\Big/2\sin\left(\frac{\pi}{2^n}\right)}{\sin\left(\frac{3\pi}{2^{n-1}}\right)\Big/2\sin\left(\frac{3\pi}{2^n}\right)}$; $f(n) = \dfrac{\sin\left(\frac{\pi}{2^{n-1}}\right)}{\sin\left(\frac{\pi}{2^n}\right)} \times \dfrac{\sin\left(\frac{3\pi}{2^n}\right)}{\sin\left(\frac{3\pi}{2^{n-1}}\right)}$

$T_k = \dfrac{\sin\left(\frac{\pi}{2}\right)}{\sin\left(\frac{\pi}{6}\right)} \cdot \dfrac{\sin\left(\frac{\pi}{3\cdot 2^k}\right)}{\sin\left(\frac{\pi}{2^k}\right)} \Rightarrow \lim_{k\to\infty} T_k = \frac{2}{3}$

51. 8

Sol. $g(x) = \dfrac{2(x^2-1)^2}{(x-2)x(2x-1)}$

$g'(x) = \dfrac{4(x-1)^2(x^4-5x^3+6x^2-5x+1)}{(x^2-2)x^2(2x-1)^2}$

$g'(x) = 0 \Rightarrow x = \pm 1,\ 2 \pm \sqrt{3}$

$\Rightarrow g(2+\sqrt{3}) = 8$

Section – C

52. 2.78

53. 1.56

Sol. (Q.52 – 53)

Let four root p – 3q, p – q, p + q, p + 3q

$\Rightarrow p = -1$ and $q = \sqrt{\frac{4-a}{10}}$; $b = \dfrac{9a^2+128a-256}{100}$

$(a-b) = \dfrac{25}{9} - \dfrac{9\left(a+\frac{14}{9}\right)^2}{9}$

54. 1.33

55. 2.00

Sol. (Q.54 – 55)

Let $x = a + \sqrt{b}$, $b \ge 0$

$\left(a+\sqrt{b}\right)^2 - k\left(a+\sqrt{b}\right) = Q_1 \Rightarrow 2a - k = 0$

$\left(a+\sqrt{b}\right)^3 - k\left(a+\sqrt{b}\right) = Q_2 \Rightarrow 3a^2 + b - k = 0$

$\Rightarrow b = k - \frac{3}{4}k^2$, $b \ge 0$, $k \in (-\infty, 0] \cup \left[\frac{4}{3}, \infty\right)$

56. 1.67

57. 6.75

Sol. (Q.56 – 57)

$$f(x, k) = e^{\lim_{n\to\infty}\frac{n\cdot(1-x^{1/n})}{(x^{1/n}+k-1)}} = e^{\ln(x^{-1/x})} = x^{-1/k}$$

$$\int_0^1 x^{-2/5}dx = \frac{5}{3}$$

$$T(k) = \int_0^1 x^{-1/k}(\ln x)^2\,dx \text{ (apply integration by parts)}$$

$$T(k) = \frac{2k^3}{(k-1)^3} \Rightarrow T(3) = \frac{27}{4}$$