FIITJEE
ALL INDIA TEST SERIES
JEE (Advanced)-2022
PART TEST – I
PAPER –1
TEST DATE: 21-11-2021

Time Allotted: 3 Hours **Maximum Marks: 180**

General Instructions:

- The test consists of total 57 questions.
- Each subject (PCM) has 19 questions.
- This question paper contains **Three Parts.**
- **Part-I** is Physics, **Part-II** is Chemistry and **Part-III** is Mathematics.
- Each **Part** is further divided into **Three Sections**: **Section-A, Section-B & Section-C**.

  Section – A *(01 – 04, 20 – 23, 39 – 42)*: This section contains **TWELVE (12)** questions. Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.

  Section – A *(05 –10, 24 – 29, 43 – 48)*: This section contains **EIGHTEEN (18)** questions. Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).

  Section – B *(11 – 13, 30 – 32, 49 – 51)*: This section contains **NINE (09)** questions. The answer to each question is a **NON-NEGATIVE INTEGER.**

  Section – C *(14 – 19, 33 – 38, 52 – 57)*: This section contains **NINE (09)** question stems. There are **TWO (02)** questions corresponding to each question stem. The answer to each question is a **NUMERICAL VALUE.** If the numerical value has more than two decimal places, truncate/round-off the value to **TWO** decimal places.

MARKING SCHEME

Section – A (Single Correct): Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+3	If ONLY the correct option is chosen.
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	–1	In all other cases.

Section – A (One or More than One Correct): Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+4	If only (all) the correct option(s) is (are) chosen;
Partial Marks	:	+3	If all the four options are correct but ONLY three options are chosen;
Partial marks	:	+2	if three or more options are correct but ONLY two options are chosen and both of which are correct;
Partial Marks	:	+1	If two or more options are correct but ONLY one option is chosen and it is a correct option;
Zero Marks	:	0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	:	–2	In all other cases.

Section – B: Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+4	If ONLY the correct integer is entered;
Zero Marks	:	0	In all other cases.

Section – C: Answer to each question will be evaluated according to the following marking scheme:

Full Marks	:	+2	If ONLY the correct numerical value is entered at the designated place;
Zero Marks	:	0	In all other cases.

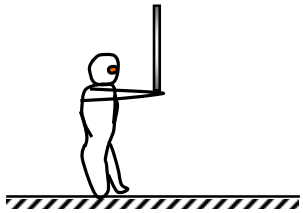
Physics

PART – I

Section – A (Maximum Marks: 12)

*This section contains **FOUR (04)** questions. Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.*

1. A vertical uniform rod of length ℓ is projected into the air with a vertical velocity v_0. It is given just enough angular impulse so that the rod rotates by an angle of 2π before landing at the same horizontal level of projection. The initial horizontal velocity of the bottom of the rod is



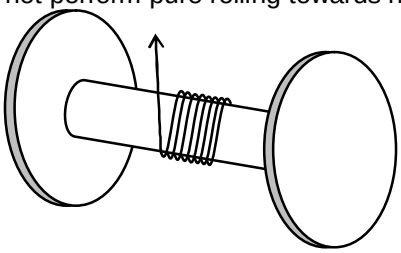
(A) $\dfrac{2v_0^2}{\pi\sqrt{\ell g}}$

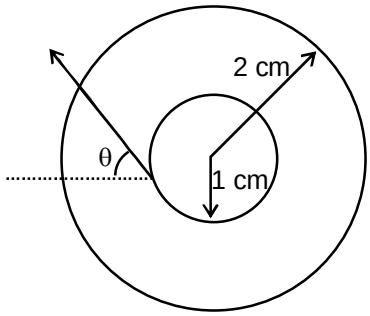
(B) $\sqrt{\ell g}$

(C) $\dfrac{\pi\ell g}{2v_0}$

(D) $\dfrac{2v_0^2}{\sqrt{\ell g}}$

2. The toy in the figure consists of a solid cylinder of radius 1 cm attached to the centre of two discs of radius 2cm. It is placed on a sufficiently rough horizontal surface where it can roll without slipping. A thread is wrapped around the central cylinder, when the thread is pulled at the angle $\theta = 90°$ to the horizontal, the toy rolls to the right. Then the maximum value of θ for which it will not perform pure rolling towards right when pulling on the thread.





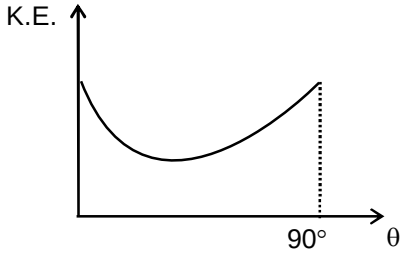
(A) $\theta = 30°$

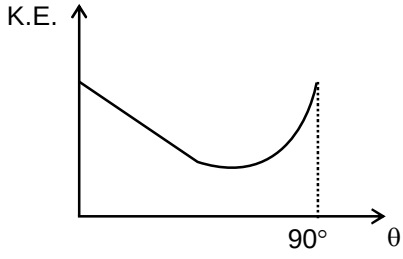
(B) $\theta = 45°$

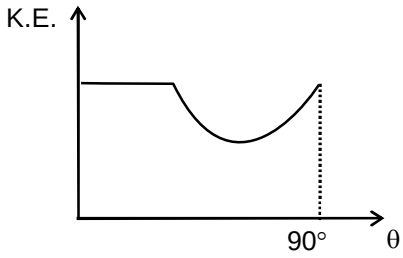
(C) $\theta = 60°$

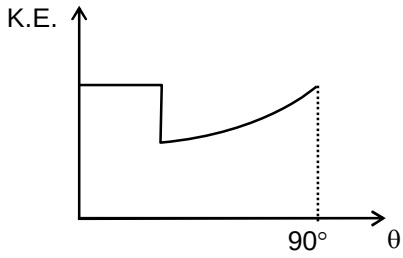
(D) $\theta = \tan^{-1}\left(\dfrac{1}{2}\right)$

3. A solid ball is released from rest down the incline at various inclination angle θ but through a fixed vertical height h. The coefficient of static and kinetic friction are both equal to μ. Which of the following graph best represents the total kinetic energy of the ball at the bottom of the incline as a function of the angle of the incline?

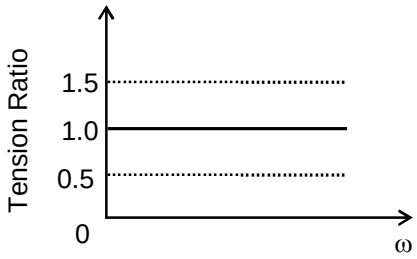
(A) 

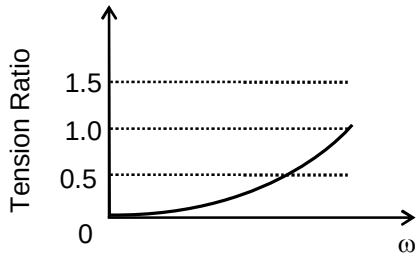
(B) 

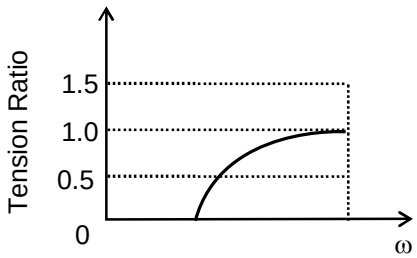
(C) 

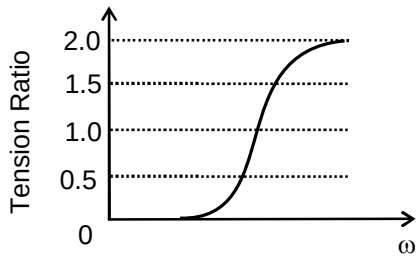
(D) 

4. A vertical pole has two massless strings each of length ℓ, attached to the vertical pole at a distance L apart. The other ends of the strings are attached to a mass M. The mass is rotated around the pole with angular speed ω. Which of the following graphs best gives the ratio of the tension in the bottom string to the tension in the top string as a function of ω.

(A) 

(B) 

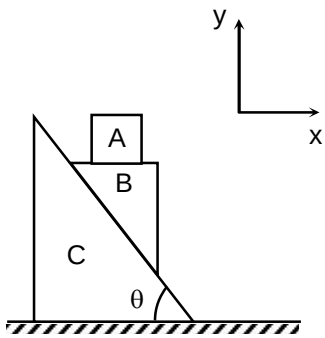
(C) 

(D) 

Section – A (Maximum Marks: 24)

*This section contains **SIX (06)** question. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MOER THAN ONE** of these four option(s) is (are) correct answer(s).*

5. In the figure shown all the surfaces are smooth. All the blocks A, B and C are movable. x-y plane is vertical plane as shown in the figure. The system is released from rest at the shown instant. Then choose the correct opiton(s).



(A) Acceleration of block A relative to ground is along negative y-direction.
(B) Acceleration of block A relative to block B is along negative x-direction.
(C) Acceleration of block B with respect to ground is g sinθ along the incline direction.
(D) Acceleration of block B and block C along horizontal must be equal in magnitude if their masses are equal.

6. A ball of mass m is projected from a level ground with a velocity u making an angle θ with the horizontal. There is a horizontal wind blowing in the direction of motion of the ball. Due to wind the ball experiences a constant horizontal force of $\frac{mg}{\sqrt{3}}$ in the direction of its motion. Then choose the correct option(s).

(A) Trajectory of the ball with respect to ground will not be parabolic.
(B) For horizontal range of the ball to be maximum, $\theta = 60°$
(C) For horizontal range of the ball to be maximum, $\theta = \frac{1}{2}\tan^{-1}(-2)$
(D) Maximum possible horizontal range = $\frac{\sqrt{3}u^2}{g}$.

7. A particle of mass m is moving under the action of a central force. Its potential energy function is given by $U = cm|\vec{r}|^2$, where c is a positive constant and $\vec{r}$ is the position vector of the particle with respect to the centre of attraction O. Choose the correct option(s).

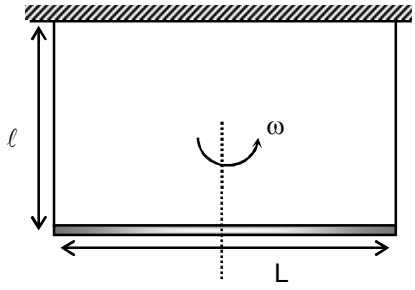
(A) Mechanical energy of the particle moving in a circle of radius R and centered at O is $2\ cmR^2$
(B) Angular velocity of the line joining the point P (performing circular motion about O) and point O is $\sqrt{c}$.
(C) If the particle is released at any $\vec{r}$, then it will perform oscillatory motion about point O.
(D) If the particle is released at a distance R from point O, then its speed at O will be $R\sqrt{2c}$.

8. The moment of inertia of a hollow cylinder of mass M, length 2R and radius R about an axis passing through the centre of mass and perpendicular to the axis of the cylinder is I_1 and about an axis passing through one end of the cylinder and perpendicular to the axis of the cylinder is I_2. Then choose the correct option(s).

(A) $\frac{I_1}{I_2} = \frac{1}{4}$
(B) $I_1 + I_2 = \frac{8}{3}MR^2$
(C) $I_2 - I_1 = MR^2$
(D) I_1 is the minimum among all the axes parallel to I_1

9. A heavy uniform sphere is suspended from a string whose end is attached to a vertical wall. The point at which the string is attached to the sphere lies on the same vertical line as the center of the sphere. If the sphere is in equilibrium, then choose the correct option(s).
   (A) The wall can not be smooth for the sphere to remain in equilibrium.
   (B) Coefficient of static friction between the wall and the sphere can be less than 1.
   (C) Coefficient of static friction between the wall and the sphere can be equal to 1.
   (D) Coefficient of static friction between the wall and the sphere can be greater than 1.

10. A rod of mass m = 2kg and length L = 1m is suspended form the ceiling with the help of two light inextensible cords each of the length $\ell = \frac{1}{2}$m, so that the rod is horizontal. The rod is given an angular velocity ω = 2 rad/s about its central vertical axis as shown in the figure. Then just immediately after the rod is given the angular velocity.



   (A) Angular velocity ω' of the cord is 2 rad/s
   (B) Angular velocity ω' of the cord is 4 rad/s
   (C) Increment in tension in each cord is 2N
   (D) Increment in tension in each cord is 1 N

Section – B (Maximum Marks: 12)

*This section contains **THREE (03)** questions. The answer to each question is a **NON-NEGATIVE INTEGER.***

11. A 1500 watt motor is used to pump the water to a vertical height of 2 meters out of a flooded basement through a cylindrical pipe. The water is ejected through the end of the pipe at a speed of 2.5 m/s. Ignoring friction and assuming that all of the energy of the motor goes to the water, then the radius(in cm) of the pipe is (Take g = 10 m/s^2, density of water = 1000 kg/m^3 and π = 3.20)

12. A 60 kg boy carries a launcher loaded with a 15 kg ball. The launcher is having negligible mass. The boy can fire the ball with a relative speed of 10 m/s. The boy is skating on an ice rink (can be assumed smooth) with a velocity of 4 m/s with respect to ground. The boy wants to fire the ball in such a way that his velocity turns through the largest possible angle. Find this maximum angle in degrees.

13. Cart of mass m moving with a velocity 12 m/s to the right collides perfectly elastically with a cart of mass 4 kg that is originally at rest. After the collision, the cart of mass m moves to the left with a velocity of 6 m/s. Assuming the collision to be head on, what is the velocity (in m/s) of the centre of mass of the two carts before the collision?

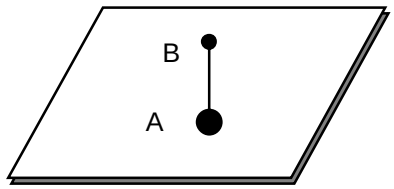
Section – C (Maximum Marks: 12)

*This section contains **THREE (03)** question stems. There are **TWO (02)** questions corresponding to each question stem. The answer to each question is a **NUMERICAL VALUE.** If the numerical value has more than two decimal places, truncate/round-off the value to **TWO** decimal places.*

Question Stem for Question Nos. 14 and 15

Question Stem

An unsymmetrical dumbell formed by two small spherical masses A of 3 kg and B of 1 kg connected by a massless rod of length ℓ = 1 meter placed vertically on a smooth horizontal surface as shown in the figure. The original orientation of the dumbell is vertical. Now, the mass B is given a negligible horizontal impulse. (Take g = 9.8 m/s^2)



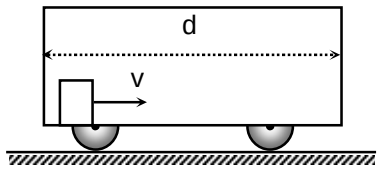
14. Horizontal displacement (in meter) of mass B just before it hits the ground is …….

15. Speed v (in m/s) of mass B just before it hits the ground is …….

Question Stem for Question Nos. 16 and 17

Question Stem

In a stationary wagon of mass M = 32 kg and length d = 1m, a small block of mass m = 1 kg is projected along its length with a velocity v = 10 m/s towards right end from left end as shown in the figure. If coefficient of restitution of the collision is $e = \frac{1}{2}$. All contact surfaces are smooth and each collision is instantaneous.



16. Velocity (in m/s) of the wagon after two collisions is …….

17. Time (in sec) after the projection of the block when the block comes at rest with respect to ground is …….

Question Stem for Question Nos. 18 and 19

Question Stem

Raindrops with a number density of n drops per cubic meter and radius r hits the ground perfectly inelastically with a speed v = 10 m/s. (Given that π = 3.14, r = 1 mm, n = 10^5 and water density = 10^3 kg/m^3)

18. The pressure (in N/m^2) applied by the raindrops on the ground is ……..

19. If the number density is doubled, the drop radius is halved and the speed of raindrops is also halved, then new pressure (in N/m^2) will be …….

Chemistry

PART – II

Section – A (Maximum Marks: 12)

*This section contains **FOUR (04)** questions. Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.*

20. Consider the following equilibrium in a vessel at 25°C

$SO_2Cl_2(g) \rightleftharpoons SO_2(g) + Cl_2(g)$

Which of the following statement is correct?

(A) Addition of helium gas at constant volume increases the concentration of SO_2.

(B) Addition of helium gas at constant volume increases the concentration of SO_2Cl_2.

(C) Increase in pressure increases the value of K_P for the reaction.

(D) Addition of helium gas at constant volume does not change the concentration of SO_2Cl_2

21. The chemical reaction $2O_3 \longrightarrow 3O_2$ proceeds as

$O_3 \underset{k_2}{\overset{k_1}{\rightleftharpoons}} O_2 + O(\text{fast})$

$O_3 + O \xrightarrow{k_3} 2O_2(\text{slow})$

The rate law expression will be

(A) $\text{Rate} = k_1[O][O_3]$

(B) $\text{Rate} = k_1[O_3]$

(C) $\text{Rate} = \dfrac{k_1.k_3}{k_2}\dfrac{[O_3]^2}{[O_2]}$

(D) $\text{Rate} = k_3\dfrac{[O_3]^2}{[O_2]}$

22. If λ_o is the threshold wavelength for photoelectric emission from metal surface, λ is the wavelength of light falling in the metal surface and m is the mass of electron, then the maximum speed of ejected electron is given by

(A) $\left[\dfrac{2h}{m}(\lambda_o - \lambda)\right]^{1/2}$

(B) $\left[\dfrac{2hc}{m}(\lambda_o - \lambda)\right]^{1/2}$

(C) $\left[\dfrac{2hc}{m}\left(\dfrac{1}{\lambda_o} - \dfrac{1}{\lambda}\right)\right]^{1/2}$

(D) $\left[\dfrac{2hc}{m}\left(\dfrac{\lambda_o - \lambda}{\lambda_o\lambda}\right)\right]^{1/2}$

23. Which of the following is a pyrosilicate?

(A) $Sc_2[Si_2O_7]$

(B) $Ca_3[Si_3O_9]$

(C) $Zn_2[SiO_4]$

(D) $Mg_2[SiO_4]$

Section – A (Maximum Marks: 24)

*This section contains **SIX (06)** question. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MOER THAN ONE** of these four option(s) is (are) correct answer(s).*

24. The incorrect order for the given properties among the following is/are
    (A) $LiH < NaH < KH$ Thermal stability
    (B) $MgSO_4 < CaSO_4 < SrSO_4$ Solubility
    (C) $MgO < CaO < SrO$ Basic nature
    (D) $Li_2CO_3 < Na_2CO_3 < K_2CO_3$ Thermal stability

25. According to molecular orbital theory the correct statement among the following is/are
    (A) O_2^+ is more stable than O_2
    (B) N_2^+ is more stable than N_2
    (C) O_2^{2-} is paramagnetic
    (D) N_2^{2-} is paramagnetic

26. The ion(s) which contain O – O bond is/are
    (A) $S_2O_8^{2-}$
    (B) $S_4O_6^{2-}$
    (C) SO_5^{2-}
    (D) $S_2O_7^{2-}$

27. The correct statement(s) among the following is/are
    (A) Number of B – O – B bonds in borax is 5.
    (B) Formula of borax is $Na_2\left[B_4O_5(OH)_4\right].8H_2O$.
    (C) BF_3 is a stronger Lewis acid than BI_3.
    (D) $B(OH)_3$ behaves as a strong acid in the presence of glycerol.

28. The correct order for the given properties among the following is/are
    (A) $NF_3 > NH_3$ (Dipole moment)
    (B) $NH_3 > PH_3$ (Bond angle)
    (C) $O_2 > O_2^+$ (Bond order)
    (D) Cl – Cl > F – F (Bond energy)

29. The hydroxide (s) which is/are soluble in excess of NaOH is/are
    (A) $Al(OH)_3$
    (B) $Zn(OH)_2$
    (C) $Fe(OH)_3$
    (D) $Sn(OH)_2$

Section – B (Maximum Marks: 12)

*This section contains **THREE (03)** questions. The answer to each question is a **NON-NEGATIVE INTEGER.***

30. Pure PCl_5 is introduced into an evacuated chamber comes to equilibrium at 25^oC and 1 atm. The equilibrium mixture contains 40% chlorine by volume K_P for the reaction

$PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$

is $x \times 10^{-1}$ atm. . The value of x is

31. The speed of an electron in an orbit of hydrogen atom is 4.376×10^5 m / sec. The number of waves made by the electron in one complete revolution in this orbit is

32. In a closed flask at 400 K solid $(NH_4)_2S$ was taken. At equilibrium a constant pressure of 3 atm was found in the flask. What is the K_P of the reaction

$(NH_4)_2S(s) \rightleftharpoons 2NH_3(g) + H_2S(g)$

at 400K is

Section – C (Maximum Marks: 12)

*This section contains **THREE (03)** question stems. There are **TWO (02)** questions corresponding to each question stem. The answer to each question is a **NUMERICAL VALUE.** If the numerical value has more than two decimal places, truncate/round-off the value to **TWO** decimal places.*

Question Stem for Question Nos. 33 and 34

Question Stem

100 ml of 0.1 M H_3PO_4 solution is titrated with 0.5 M NaOH solution till the second equivalent point. At second equivalent point the pH of the solution was found to be x. after second equivalent point 10 ml of 0.5 M HCl solution is added to the resulting solution and the pH of the solution after adding HCl was found to be y.

Dissociation constants K_{a_1}, K_{a_2} and K_{a_3} of H_3PO_4 are $10^{-3}, 10^{-8}$ and 10^{-13} respectively.

33. The value of x is_____

34. The value of y is_____

Question Stem for Question Nos. 35 and 36

Question Stem

$PCl_5(g)$ when heated in a sealed tube at 500 K it undergoes decomposition as

$PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$

The equilibrium constant K_P is 98.52 at 500 K. Vapour density of equilibrium mixture is 60.

$(R 0.0821 \text{ L atm } K^{-1} \text{ mol}^{-1})$

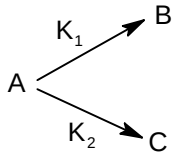
35. Percentage dissociation of PCl_5 is_____

36. Equilibrium constant K_C for the reaction is_____

Question Stem for Question Nos. 37 and 38

Question Stem

Substance 'A' undergoes first order reaction by two parallel paths forming products B and C in two paths respectively as follows



The percentage yield of B is 10% and that of C is 90%. The rate constant for the disappearance of A is 1.4×10^{-4} sec^{-1}. The rate constant (k_1) for the formation of B is $x\times10^{-5}$ sec^{-1} and the rate constant (k_2) for the formation of C is $y\times10^{-4}$ sec^{-1}.

37. The value of x is_____

38. The value of y is_____

Mathematics

PART – III

Section – A (Maximum Marks: 12)

*This section contains **FOUR (04)** questions. Each question has **FOUR** options. **ONLY ONE** of these four options is the correct answer.*

39. Consider two functions $f_1(x)$ and $f_2(x)$ satisfying $f_1(x)=\begin{cases}\sqrt{x} & ; \ \sqrt{x}\in Z\\ 1+f_1(x+1) & ; \ \text{otherwise}\end{cases}$ and $f_2(x)=\begin{cases}\sqrt{x} & ; \ \sqrt{x}\in Z\\ 2+f_2(x+1) & ; \ \text{otherwise}\end{cases}$, if for an integer n, $\frac{f_1(n)}{f_2(n)}=\frac{4}{7}$ (Z is set of integer), then n is

(A) 2
(B) 16
(C) 258
(D) 129

40. Let f(x) be a real valued function such that $f(x)=\lim\limits_{n\to\infty}\left(1+\frac{x}{n^2}\right)\left(1+\frac{2x}{n^2}\right).....\left(1+\frac{nx}{n^2}\right)\forall\ x\in R^+$ and $n\in N$, then which one is CORRECT?

(A) $f'(1) > f(1)$
(B) $f(x) < e^x\ \forall\ x\in(0,\infty)$
(C) $f(c)<\frac{1}{2}$ for some c, $c\in(0,\infty)$
(D) $f(x) < x^2\ \forall\ x\in(0,\infty)$

41. Let $f(x): Z\to Z$ be a function that satisfies $f(x)+f(y)=f(x+y)-xy$ and if $f(7)=21$, then (Z is a set of integers)

(A) f(x) is onto and one-one
(B) f(x) is into and many one
(C) f(x) is into and one-one
(D) f(x) is onto and many one

42. Consider a real triplet (x_1, x_2, x_3) satisfying $\left(2^{2x_1}+1\right)\left(2^{2x_2}+2\right)\left(2^{2x_3}+8\right)-\left(2^{x_1+x_2+x_3+5}\right)=0$, then which of the following is TRUE?

(A) $x_1 < 0$
(B) $x_2 > 1$
(C) $x_1+x_2<\frac{3}{4}$
(D) $x_1+x_2+x_3 > 3$

Section – A (Maximum Marks: 24)

*This section contains **SIX (06)** question. Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MOER THAN ONE** of these four option(s) is (are) correct answer(s).*

43. Let $f:[0,\infty)\to R$ be a function satisfying $f(x)\ e^{f(x)}=x\ \forall\ x\in[0,\infty)$, then which of the following statements is(are) TRUE?

(A) $f'(x)\ge 0\ \forall\ x\in(0,\infty)$
(B) $\lim\limits_{x\to\infty} f(x)=0$
(C) $\lim\limits_{x\to\infty}\frac{f(x)}{\ln x}=1$
(D) $\lim\limits_{x\to\infty}\frac{f(x)}{\ln x}=0$

44. Let f : (0, ∞) → R, $f(x)=\left(\frac{x+1}{x}\right)^{\left(\frac{x+1}{2}\right)}$ and $g(x)=\frac{f'(x)}{f(x)}$, then

(A) g(x) is increasing in (0, ∞)

(B) $\lim\limits_{x\to\infty} g(x)=0$

(C) $\lim\limits_{x\to\infty} f(x)=\sqrt{e}$

(D) $\lim\limits_{x\to\infty} g(x)=\sqrt{e}$

45. For any natural number n, let $\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\frac{1}{r^2}=\frac{\pi^2}{6}$, then which of the following statements is(are) TRUE?

(A) $\int\limits_0^1 \frac{\ln(1+x)}{x}dx=\frac{\pi^2}{12}$

(B) $\int\limits_0^1 \frac{\ln(1-x)}{x}dx=-\frac{\pi^2}{6}$

(C) $\sum\limits_{k=1}^{\infty}\int\limits_0^1 x^k \ln x\,dx=1-\frac{\pi^2}{6}$

(D) $\int\limits_0^1 \frac{\ln(1-x^2)}{x}dx=\frac{\pi^2}{12}$

46. Let A_1 and A_2 are two real numbers such that $A_1=\int\limits_0^{\pi/4}\sin(\tan x)dx$ and $A_2=\int\limits_0^{\pi/3}\tan(\sin x)dx$, then for A_1 and A_2 which of the following statements is(are) TRUE?

(A) $A_1<\frac{\sqrt{2}-1}{\sqrt{2}}$

(B) $A_2\geq\frac{\pi^2}{18}$

(C) $A_1\geq\frac{\pi^2}{32}$

(D) $A_2\leq\ln 2$

47. Let y = g(x) be a real valued function and g′(x) is continuous function such that g′(x) ≥ 0 and g(0) = 0, g(1) = 1 and if area bounded by $y=\sqrt{1+(g'(x))^2}$, x = 0, x = 1 and x-axis is S, then which of the following statements is(are) TRUE?

(A) S < 2

(B) S > 2

(C) $S<\frac{1}{\sqrt{2}}$

(D) $S\leq\sqrt{2}$

48. Let y = f(x) be a real valued differentiable function $\forall\ x \in R$, such that $\frac{f'(x)}{f(x)} \neq -1$ and $(f'(x))^3 + x^3 + 3xf(x)f'(x) = (f(x))^3\ \forall\ x \in R$, then which of the following statements is(are) TRUE?

(A) if f(–1) = 0, then $f(x) = (x + 1)e^k$

(B) if f(–1) = 1, then $f(x) = (x + 1) - e^{(x-1)}$

(C) $\frac{f'(c) - f(c)}{c} = -1$ for some $c \in R$

(D) $\frac{f'(c) - f(c)}{c} = e$ for some $c \in R$

Section – B (Maximum Marks: 12)

*This section contains **THREE (03)** questions. The answer to each question is a **NON-NEGATIVE INTEGER.***

49. If $x_1, x_2, x_3, ..., x_n$ are n positive real numbers which satisfies the equation $\{x\} + \frac{[x]}{25} = 5$, then the value of $\frac{1}{3}\sum_{i=1}^{n}\left(x_i - \left[x_i\right]\right)$_____ (where [.] denotes greatest integer function and {.} is fractional part function)

50. Let $f(n) = 1 - 4\sin^2\frac{\pi}{3\cdot 2^n}$ be a function and $T_k = \prod_{n=2}^{k} f(n)$, then value of $\lim_{k\to\infty}\frac{4}{T_k}$ is _____

51. Let f(x) be a real valued function such that $f(x) = \frac{2x-1}{x-2}\ \forall\ x \in (2, \infty)$ and $g(x) = \frac{x^2+1}{x} + \frac{(f(x))^2+1}{f(x)}\ \forall\ x > 2$, then minimum value of g(x) is _____

Section – C (Maximum Marks: 12)

*This section contains **THREE (03)** question stems. There are **TWO (02)** questions corresponding to each question stem. The answer to each question is a **NUMERICAL VALUE.** If the numerical value has more than two decimal places, truncate/round-off the value to **TWO** decimal places.*

Question Stem for Question Nos. 52 and 53

Question Stem

Let $f(x) = x^2 + ax + b$ is a function $a, b \in R$, $a^2 > 4b$ and $g(x) = x^2 + 2x - 1$ such that $f(g(x_i)) = 0\ \forall\ i \in \{1, 2, 3, 4\}$ and $x_i < x_i + 1\ \forall\ i = 1, 2, 3$ and x_1, x_2, x_3, x_4 are for an Arithmetic Progression, then

52. The maximum value of (a – b) is _____

53. The values of |a| when (a – b) at its maximum value _____

Question Stem for Question Nos. 54 and 55

Question Stem

Let $f(x) = x^2 - kx$ and $g(x) = x^3 - kx$, $k \in R$ are two real valued function such that for a rational number α, $f(\alpha)$ and $g(\alpha)$ both are rational number, then

54. Least positive value of k is _____

55. Least positive integral value of k is _____

Question Stem for Question Nos. 56 and 57

Question Stem

For a real valued function $f(x, k) = \lim_{n\to\infty}\left(\frac{k}{(x)^{1/n}+k-1}\right)^n \ \forall\ x \in (0, 1), k > 1, n \in N$ and let

$T(k) = \int_0^1 f(x,k)\cdot(\ln x)^2\,dx$, then

56. The value of $\int_0^1 f\left(x, \frac{5}{2}\right)dx$ is _____

57. The value of T(3) is _____