FIITJEE

ALL INDIA TEST SERIES

PART TEST – III

JEE (Main)-2021

TEST DATE: 19-12-2020

ANSWERS, HINTS & SOLUTIONS

Physics PART – I

SECTION – A

1. C

Sol. $\lambda_{\alpha+\beta} = \lambda_\alpha + \lambda_B$

$$\Rightarrow \frac{\ell n2}{T_{\frac{1}{2}(\alpha+\beta)}} = \frac{\ell n2}{T_{\frac{1}{2}(\alpha)}} + \frac{\ell n2}{T_{\frac{1}{2}(B)}}$$

$$T_{\frac{1}{2}(\alpha+\beta)} = \frac{T_\alpha T_\beta}{T_\alpha + T_\beta} = 20 \text{ years}$$

$\therefore$ one fourth of sample will remain after 2 half life = 40 years

2. B

Sol. $$T = 2\pi\sqrt{\frac{I_P}{mg\ell_{cm}}}$$

$$T = 2\pi\sqrt{\frac{2/3\ m\ell^2}{2mg\left(\frac{\ell}{2\sqrt{2}}\right)}}$$

$$T = 2\pi\sqrt{\frac{2\sqrt{2}\ell}{3g}}$$

3. B

Sol. 5 VSD = 4 MSD

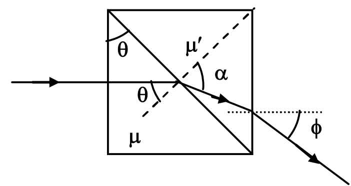
$\therefore$ 1 VSD = 4/5 MSD

L.C = 1 MSD – 1 VSD

$= \frac{1}{5}$MSD = 1 mm

For screw gage (LC)= $\frac{\text{pitch}}{100}$ = 0.01 mm

4. A



Sol. $\mu \sin\theta = \mu' \sin\alpha$

$\Rightarrow \mu\theta = \mu'\alpha$ ...(i)

$\mu \sin(\alpha - \theta) = \sin\phi$

$\Rightarrow \mu'(\alpha - \theta) = \phi$ ...(ii)

From (i) and (ii)

$\mu\theta - \mu'\theta = \phi$

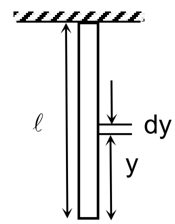
$\Rightarrow (\mu - \mu') = \frac{\phi}{\theta}$

5. A

Sol. $I = \frac{1}{2}\varepsilon_0 E_0^2 c$

$= 3.3 \ W/m^2$

6. B



Sol. At a distance y from the free end, the speed of the wave is

$v = \sqrt{\frac{\lambda y g}{\lambda}} = \sqrt{yg}$

$t = \int dt = \int_0^\ell \frac{dy}{\sqrt{gy}}$

$t = 2\sqrt{\frac{\ell}{g}}$ ...(i)

Also, $v_0 = \sqrt{\frac{\ell g}{2}}$

$\Rightarrow \ell = \frac{2v_0^2}{g}$ ...(ii)

From equation (i) and (ii)

$t = \frac{2\sqrt{2}v_0}{g}$

7. B

Sol. At time t,

length of tube = $\ell - vt$

$\therefore f = \frac{c}{4(\ell - vt)}$

Fundamental frequency

$f_0 = \frac{c}{4\ell} \left[\because \ell = \frac{\lambda}{4}\right]$

At time t, length of tube = $\ell - vt$

$\therefore\ f = \dfrac{c}{4(\ell - vt)}$

Or, $\dfrac{df}{dt} = \dfrac{cv}{4(\ell - vt)^2}$

$\because\ \ \ell >> vt$

$\Rightarrow \dfrac{df}{dt} = \dfrac{cv}{4\ell^2}$

8. A

Sol. Volume of cylinder = V

$\rho\left(\dfrac{3}{4}V\right)g = mg$

$V = \dfrac{4m}{3\rho}$

In reference frame of vessel

$mg - \dfrac{\rho}{4}Vg_{eff} - mA = \dfrac{mA}{3}$ $\qquad [\because g_{eff} = g - A]$

$\Rightarrow mg - \dfrac{\rho}{4}\left(\dfrac{4m}{3\rho}\right)(g - A) - mA = \dfrac{mA}{3}$

$\Rightarrow A = \dfrac{2g}{3}$

9. A

Sol. A satellite will appear motionless when its period of revolution is same as that of earth that is T = 24 hr. Let r be the radius of orbit from the centre of earth

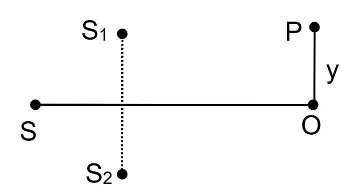
Then, $m\omega^2 r = \dfrac{GMm}{r^2}$

$\left(\dfrac{2\pi}{T}\right)^2 r^3 = GM \quad \Rightarrow r = \left(\dfrac{GMT^2}{4\pi^2}\right)^{1/3}$

$\Rightarrow r = \left(\dfrac{gR^2T^2}{4\pi^2}\right)^{1/3} \left[\because GM = gR^2\right]$

10. B

Sol. Let the intensity of individual wave be I, then



$I_0 = 4I$

$\Rightarrow I = \dfrac{I_0}{4}$

$\Rightarrow \Delta x$ at P = d sin θ

$\Rightarrow \Delta x \simeq d\tan\theta = \dfrac{yd}{D}$

$\Delta x = \dfrac{d}{D} \times \dfrac{\beta}{4} = \dfrac{d}{D}\dfrac{D\lambda}{4d} = \dfrac{\lambda}{4}$ $\qquad \left[\because \beta = \dfrac{D\lambda}{d}\right]$

$\Delta\phi = \dfrac{2\pi}{\lambda} \times \dfrac{\lambda}{4} = \dfrac{\pi}{2}$

$I_P = I + I + 2\sqrt{I^2}\cos\dfrac{\pi}{2} = 2I$

$I_P = \dfrac{I_0}{2}$

11. C

Sol. $5 = \frac{320}{3}\left[\frac{330+V_0}{330-10}\right]-\frac{320}{3}$

$\Rightarrow V_0 = 5m/s$

$\therefore t = \frac{90}{15} = 6\,sec$

12. B

Sol. $K = h\nu - w$

$K' = 2h\nu - w$

$= 2(h\nu - w) + w$

$= 2K + w$

Thus, the kinetic energy of photoelectrons will be more than double when incident frequency is doubled.

13. C

Sol. For narrow single slit diffraction width of central maxima is $\frac{\lambda}{b}$, where $\lambda = \frac{h}{mv}$ and b is slit width.

14. C

Sol. The velocity vector is perpendicular to initial velocity vector at $t = \frac{u}{g\sin\theta}$ and at this instant its speed is $v = u\cot\theta$

Since $\lambda = \frac{h}{mv}$

$\Rightarrow \lambda = \frac{h}{mu\cot\theta}$

$\Rightarrow \lambda = \frac{h}{mu}\tan\theta$

15. A

Sol. $U = eV$

$= eV_0 \log_e\left(\frac{r}{r_0}\right)$

$|f| = \left(-\frac{dU}{dr}\right) = \frac{eV_0}{r}$

This force will provide the necessary centripetal force

$\Rightarrow \frac{mv^2}{r} = \frac{eV_0}{r}$

$\Rightarrow v = \sqrt{\frac{eV_0}{m}}$ …(i)

Also, $mvr = \frac{nh}{2\pi}$ …(ii)

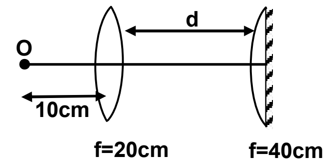
Dividing (ii) by (i)

$mr = \left(\frac{nh}{2\pi}\right)\sqrt{\frac{m}{eV_0}}$

$\Rightarrow r \propto n$

16. C

Sol. The silvered lens can be replaced by a mirror of focal length given as



For plano convex lens

$\frac{1}{f} = \frac{1}{40} = \left(\frac{\mu - 1}{R}\right)$ …(i)

For lens,

$\frac{1}{v_1} - \frac{1}{(-10)} = \frac{1}{20}$

$\Rightarrow v_1 = -20\text{cm}$

Distance of image from plano-convex is d + 20

For left surface of plano-convex lens

$\frac{\mu}{\infty} - \frac{1}{-(d+20)} = \frac{\mu - 1}{R}$

$\Rightarrow$ d = 20 cm

17. A

Sol. $A_R = \sqrt{(\sqrt{3})^2 + (1)^2} = 2$

$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3}$

$\therefore$ y = $2\sin\left(\omega t + \frac{\pi}{3}\right)$

$\therefore \frac{d^2y}{dt^2} = a = -2\omega^2 \sin\left(\omega t + \frac{\pi}{3}\right)$

$a_{max} = -2\omega^2 = g$

For which mass just breaks off the plank $\omega = \sqrt{\frac{g}{2}}$

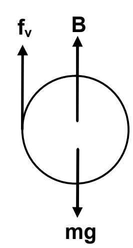
This will happen for the first time when

$\omega t + \frac{\pi}{3} = \frac{\pi}{2}$ or $\omega t = \frac{\pi}{6}$

$\Rightarrow t = \frac{\pi}{6\omega} = \frac{\pi}{6}\sqrt{\frac{2}{g}}$

18. A

Sol. $f_v + B = mg$



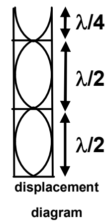
$6\pi\eta r v_T + m'g = mg$

$\Rightarrow V_T = \frac{(m - m')g}{6\pi\eta r}$

$\Rightarrow V_T \propto \frac{(m - m')}{r}$

19. D

Sol. $\frac{5\lambda}{4} = 42 + (0.3 \times 10)$



$\Rightarrow \frac{5\lambda}{4} = 45\text{cm}$

$\Rightarrow \lambda = 36\text{cm}$

$\therefore \ \Delta P = (\Delta P_0)\sin kx$

$= \Delta P_0 \sin\left(\frac{2\pi}{36} \times 24\right)$

$\Delta P = \Delta P_0 \frac{\sqrt{3}}{2}$

20. B

Sol. The cut-off wavelength when V = V_1 =10 kV is

$\lambda_1 = \frac{hc}{eV_1} = 1243.125 \times 10^{-13}\text{m}$

The cut-off wavelength when V = V_2 = 20 kV is

$\lambda_2 = \frac{hc}{eV_2} = 621.56 \times 10^{-13}\text{m}$

The wavelength corresponding to k_α line is $\frac{1}{\lambda} = \frac{3R}{4}(z-1)^2$

Given that $(\lambda - \lambda_2) = 3(\lambda - \lambda_1)$

Putting all above

z = 29

SECTION – B

21. 5

Sol. $M_0 = \frac{1.5^\circ}{0.25^\circ} = \frac{f_0}{f_e}$

and $f_0 + f_e = 35$

22. 5

Sol. Applying conservation of mechanical energy,

$\Delta K = \Delta U$

$\frac{1}{2}m_0 v^2 = U_B - U_A = m_0(V_B - V_A)$

or $v = \sqrt{2(V_B - V_A)}$ …(i)

potential at A

$V_A =$ potential due to complete sphere – potential due to cavity

$= -\frac{1.5\ GM}{R} - \left[\frac{-Gm}{R/2}\right]$

Here, $m = \frac{4}{3}\pi\left(\frac{R}{2}\right)^3 \rho = \frac{\pi\rho R^3}{\sigma}$

And $M = \frac{4}{3}\pi R^3 \rho$

Substituting m and M

$$V_A = -\frac{5}{3}\pi G\rho R^2$$

Potential at B,

$$V_B = \frac{-GM}{R^3}\left[1.5R^2 - 0.5\left(\frac{R}{2}\right)^2\right] + \frac{1.5Gm}{R/2}$$

$$= \frac{-4}{3}\pi G\rho R^2$$

$$\therefore \; V_B - V_A = \frac{1}{3}\pi G\rho R^2$$

So, from equ. (i)

$$v = \sqrt{\frac{2}{3}\pi G\rho R^2}$$

SECTION – C

23. 00002.72

Sol. $T = 2\pi\sqrt{\frac{L}{g}}$

or $T^2 = 4\pi^2\left(\frac{L}{g}\right)$

$$\Rightarrow g = \frac{4\pi^2 L}{T^2}$$

$$\Rightarrow \frac{\Delta g}{g} = \frac{\Delta L}{L} + \frac{2\Delta T}{T}$$

% error in g = $\frac{\Delta g}{g} \times 100$

$$= \left(\frac{\Delta L}{L} + \frac{2\Delta T}{T}\right) \times 100$$

$$= \left(\frac{\frac{1}{10}}{20} + 2 \times \frac{1}{90}\right) \times 100$$

=2.72%

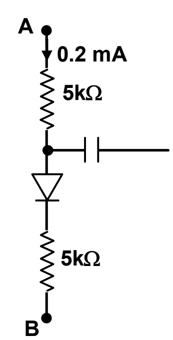
24. 00002.30

Sol. In the middle right of the circuit the capacitor behaves like an open circuit for d.c. 0.2 mA current, so current will flow from A and B only. Let potential across A and B is V, so by Kirchoff's loop law.

$V_{AB} = (5000 \times 0.2 \times 10^{-3} + 0.3 + 5000 \times 0.2 \times 10^{-3})$

$V_{AB} = 1\text{ V} + 0.3\text{ V} + 1\text{V}$

$V_{AB} = 2.3\text{ V}$



25. 00000.03

Sol. $\beta = 10\log\left(\frac{I}{I_0}\right)$

$60 = 10\log\left(\frac{I}{I_0}\right)$

$I = 10^{-6}\ W/m^2$

$I = \frac{\Delta p^2}{2\rho v}$

$\Rightarrow \Delta p = \sqrt{10^{-6} \times 2 \times \frac{15}{11} \times 330}$

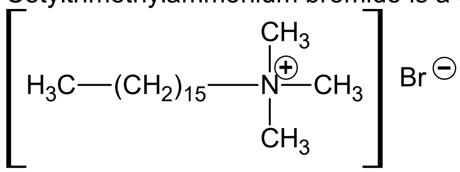
$\Rightarrow \Delta p = 0.03\ N/m^2$

Chemistry PART – II

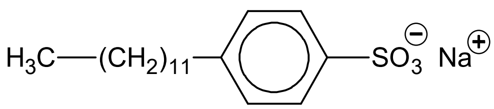
SECTION – A

26. D

Sol. Cetyltrimethylammonium bromide is a cationic detergent



Sodium dodecylbenzene sulphonate and sodium lauryl sulphate both are anionic detergents.



$H_3C—(CH_2)_{10}—CH_2O\ SO_3^{\ominus}\ Na^{\oplus}$

Sodium stearate $(C_{17}H_{35}COONa)$ is not a detergent.

27. B

Sol. (A) Ni – Mond process
(B) Pig iron can be moulded into a variety of shapes.
(C) Wrought iron is purest form of iron containing very low carbon content.
(D) Blister copper has blistered appearance due to evolution of SO_2(g).

28. A

Sol. Preferential adsorption of ions from solution is responsible for charge on colloid particles, and the charge is responsible for stability of colloid.

29. C

Sol. Correct order of increasing field strength of ligands according to spectro-chemical series is

$SCN^- < F^- < C_2O_4^{-2} < CN^-$

30. C

Sol. (A) Acetone + chloroform shows –ve deviation from Raoult's law.
(B) Chloroethane + Bromoethane is an example of ideal solution.
(C) Ethanol + acetone shows +ve deviation from Raoult's law
(D) Benzene + Toluene is an example of ideal solution.

31. C

Sol. For adiabatic free-expansion of an ideal gas:

$q = 0,\ W = 0,\ \therefore \Delta U = 0,\ \therefore \Delta T = 0$

But free-expansion is an irreversible process

$$\therefore \Delta S_{sys} \neq \frac{q_{sys}}{T} \neq \frac{0}{T} \neq 0$$

In fact $\Delta S_{sys} = +ve$

32. B

Sol. $\Delta T_b = T_b s - T_b o$

$= 373.52 - 373\ K = 0.52\ K$

$\Delta T_b = K_b \times m$

$0.52 = 0.52 \times \frac{W_{solute} \times 1000}{150 \times W_{solvent}}$

$\frac{W_{solute}}{W_{solvent}} = 0.15$

Now $\frac{w}{W}\% = \frac{W_{solute}}{W_{solute} + W_{solvent}} \times 100$

$= \frac{W_{solute}}{W_{solute} + \frac{W_{solute}}{0.15}} \times 100$

$\simeq 13\%$

33. D

Sol. Option (A) is correct.
Option (B) is correct.
Option (C) is correct.
Option (D) is incorrect.
Rhombic sulphur transforms to monoclinic sulphur when heated above 369 K.

34. A

Sol. Option (A) is incorrect.
Actually catalyst participates in the reaction.
Option (B), (C) and (D) are correct.

35. A

Sol. 10 M $C_2H_5OH(aq.)$ solution means 10 mol/L.

Moles of solute $(C_2H_5OH) = 10$ mole

Mass of solute $(C_2H_5OH) = 10 \times 46 = 460$ g

Mass of solution $= 1000\,mL \times 1\frac{g}{mL} = 1000\,g$

Mass of solvent = 1000 g – 460 g = 540 g

Moles of solvent $(H_2O) = \frac{540}{18} = 30$ mole

$x_{solute} = \frac{10}{40} = \frac{1}{4}$

$x_{solvent} = \frac{30}{40} = \frac{3}{4}$

$P_{solution} = P^o_{C_2H_5OH} \times x_{C_2H_5OH} + P^o_{H_2O} \times x_{H_2O}$

$= 40 \times \frac{1}{4} + 20 \times \frac{3}{4}$

= 10 + 15 = 25 mm of Hg.

36. C

Sol. The material obtained from bottom of blast furnace after smelting during metallurgy of Cu, which is further subjected for bessemerization is called Matte.

37. B

Sol. Let us assume x mL of O_3 in given sample (25 mL) of ozonised oxygen.
Now O_3 present in the sample reacts with KI to liberate I_2 according to following reaction

$O_3 + 2KI + H_2O \longrightarrow O_2 + I_2 + 2KOH$ … (1)

The I_2 liberated in above reaction [Eq. (1)] further reacts with $Na_2S_2O_3$ according to the following reaction

$2Na_2\overset{+2}{S}_2O_3 + I_2 \longrightarrow 2NaI + Na_2\overset{+2.5}{S}_4O_6$ … (2)

n-factor of $Na_2S_2O_3$ is above reaction = 1

$\therefore$ N of $Na_2S_2O_3$ = M of $Na_2S_2O_3$

According to reaction (2)

$$\text{m moles of } I_2 = \frac{\text{m moles of } Na_2S_2O_3}{2} = \frac{0.08 \times 15}{2} = 0.6 \text{ m mole}$$

Now according to reaction (1)

m moles of O_3 reacted = m moles of I_2 formed = 0.6 m mole

Thus, volume of O_3 at NTP = 0.6 × 22.4 = 13.44 mL

38. B

Sol.
1. $\left[Co(NO_2)_6\right]^{-3}$
2. $\left[Co(NO_2)_5(ONO)\right]^{-3}$
3. $\left[Co(NO_2)_4(ONO)_2\right]^{-3}$ → cis, trans
4. $\left[Co(NO_2)_3(ONO)_3\right]^{-3}$ → facial, meridonial
5. $\left[Co(NO_2)_2(ONO)_4\right]^{-3}$ → cis, trans
6. $\left[Co(NO_2)(ONO)_5\right]^{-3}$
7. $\left[Co(ONO)_6\right]^{-3}$

Thus there are total 10 isomers for the given complex.

39. A

Sol. $C_2H_6(g) + \frac{7}{2}\overset{0}{O}_2 \longrightarrow 2C\overset{-2}{O}_2(g) + 3H_2O(\ell)$

Moles of electrons transferred in above net cell reaction = 14

$\Delta G^o = -nFE^o_{cell}$

$-1229.41 \times 10^3 = -14 \times 96500 \times E^o_{cell}$

$$\therefore E^o_{cell} = \frac{1229.41 \times 10^3}{14 \times 96500} = +0.91 \text{ V}$$

40. B

Sol. For the reaction

$2O_3(g) \longrightarrow 3O_2(g)$

$\Delta H = -ve$ and $\Delta S = +ve$

$\therefore \Delta G = \Delta H - T\Delta S$

$\therefore \Delta G = -ve$

41. A

Sol. Rate of effusion is given by $r \propto A$ (assuming all other conditions to be same) where A = area of orifice.

$$\frac{\text{rate of effusion from vessel A}}{\text{rate of effusion from vessel B}} = \frac{\pi r^2}{\ell^2}$$

Given : $\ell = r$

$$\therefore \frac{\text{Rate of effusion from vessel A}}{\text{Rate of effusion from vessel B}} = \frac{\pi r^2}{r^2} = \frac{\pi}{1}$$

42. A

Sol. More the –ve deviation from ideality, stronger the inter-molecular attraction force among gas molecules and easier to liquefy the gas.

$\therefore$ Correct order of liquefiability $H_2 < N_2 < CH_4 < CO_2$

43. D

Sol. $\underbrace{A(\ell)}_{2\,\text{mole}} + \underbrace{B(\ell)}_{2\,\text{mole}}$

$P_s = P_A^o \times x_A + P_B^o \times x_B$

$$1 = P_A^o \times \frac{1}{2} + P_B^o \times \frac{1}{2} \qquad \dots (1)$$

Now, $\underbrace{A(\ell)}_{1\,\text{mole}} + \underbrace{B(\ell)}_{3\,\text{mole}} + \underbrace{C(\ell)}_{4\,\text{mole}}$

$P_s' = P_A^o \times x_A + P_B^o \times x_B + P_C^o \times x_C$

$$1 = P_A^o \times \frac{1}{8} + P_B^o \times \frac{3}{8} + 0.8 \times \frac{4}{8}$$

$$0.6 = P_A^o \times \frac{1}{8} + P_B^o \times \frac{3}{8} \qquad \dots (2)$$

Solving equations (1) and (2), we get

$P_A^o = 0.6$ atm

$P_B^o = 1.4$ atm

$$\therefore \frac{P_A^o}{P_B^o} = \frac{0.6}{1.4} = \frac{3}{7}$$

44. A

Sol. As_2S_3 is negatively charged colloid while $Fe(OH)_3, TiO_2$ and basic dyestuff are positively charged colloids.

45. C

Sol. (A) $Fe^{+3} + K_4\left[Fe(CN)_6\right] \longrightarrow Fe_4\left[Fe(CN)_6\right]_3$
Potassium ferrocyanide — Prussian blue (Blue colour)

(B) $Fe^{+2} + K_3\left[Fe(CN)_6\right] \longrightarrow Fe_3\left[Fe(CN)_6\right]_2$
Potassium ferricyanide — Turnbull blue (Blue colour)

(C) $Fe^{+2} + K_4\left[Fe(CN)_6\right] \longrightarrow \underbrace{K_2Fe\left[Fe(CN)_6\right]}_{\text{White precipitate}}$

(D) $Cu^{+2} + K_4\left[Fe(CN)_6\right] \longrightarrow Cu_2\left[Fe(CN)_6\right]$
Red – Brown precipitate

$\therefore$ Only (C) option is incorrect.

SECTION – B

46. 6

Sol. Chalcopyrites - $CuFeS_2$
Bauxite - $Al_2O_3.2H_2O$
Limonite - $Fe_2O_3.3H_2O$
Pyrolusite - MnO_2
Rutile - TiO_2
Casseterite - SnO_2
Calamine - $ZnCO_3$
Cuprite - Cu_2O
Chalcocite = Cu_2S
Argentite = Ag_2S
Cinnabar = HgS
Siderite - $FeCO_3$
Thus, there are six oxide ores.

47. 6

Sol.

	Hybridization	Bond angle
$H_2\ddot{\underline{O}}:$	sp^3 Bent (V – shape)	104.5°
$H_2\ddot{\underline{S}}:$	sp^3 Bent (V – shape)	92°
$:\ddot{\underline{C}}lF_3$	sp^3d (Bent T-shape)	Equatorial – axial = 87.5° Axial – axial = 175°
$\underline{I}F_7$	sp^3d^3 pentagonal bipyramidal	Equatorial – equatorial = 72° Equatorial – axial = 90° Axial – axial = 180°
$\underline{X}eF_2$	sp^3d Linear	180°

SF_6	sp^3d^2 Octahedral	90^o and 180^o
$[TeF_5]^{\ominus}$	sp^3d^2 Square pyramidal	$< 79^o$
XeF_4	sp^3d^2 Square planar	90^o and 180^o
O_3	sp^2 (Bent V-shape)	117^o
SO_2	sp^2 (Bent V-shape)	$119^\circ 30'$
SO_3	sp^2 Trigonal planar	120^o

SECTION – C

48. 00001.50

Sol. For adiabatic reversible process

$T.V^{\gamma-1} = \text{constant}$ … (1)

Now according to question

$TV^{\frac{1}{2}} = \text{constant}$ … (2)

Comparing equation (1) and (2)

$\gamma - 1 = \frac{1}{2}$

$\gamma = 1.50$

49. 00000.62

Sol. $2H_2O_2(aq.) \longrightarrow 2H_2O(\ell) + O_2(g)$

Moles of electrons transferred during above reaction = 2

$\Delta G^o = -nFE^o_{cell}$

$-120 \times 10^3 = -2 \times 96500 \times E^o_{cell}$

$E^o_{cell} = \frac{120 \times 10^3}{2 \times 96500} = 0.62\ V$

50. 00004.43

Sol. $\pi = iCST$ … (1)

$\alpha = \frac{i-1}{n-1}$

$0.8 = \frac{i-1}{2-1}$

$i = 1.8$

$0.745\% \frac{w}{V}$ KCl solution means that 0.745 g solute (KCl) is present in 100 mL solution.

$C = \frac{0.745}{74.5} \times \frac{1000}{100} = 0.1\,M$

Putting values of *i* and C in eq. (1)

$\pi = 1.8 \times 0.1 \times 0.082 \times 300$

= 4.43 atm

Mathematics PART – III

SECTION – A

51. D

Sol. $^nC_0 + {}^nC_3 + {}^nC_6 + ... = \dfrac{2^n + (-1)^n 2}{3}$, n = 3λ, λ ∈ integer

$= \dfrac{2^n + (-1)^{n+1}}{3}$, n ≠ 3λ, λ ∈ integer

⇒ n = 10

52. C

Sol. $\left|\dfrac{1+\sin\frac{3\pi}{4}+i\cos\frac{3\pi}{4}}{1-\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}}\right|^6 = \left(\dfrac{1+\sin^2\frac{3\pi}{4}+\cos^2\frac{3\pi}{4}+2\sin\frac{3\pi}{4}}{1+\cos^2\frac{3\pi}{4}+\sin^2\frac{3\pi}{4}-2\cos\frac{3\pi}{4}}\right)^3 = \left(\dfrac{1+\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}}\right)^3 = 1$

53. B

Sol. Required probability = $\dfrac{1}{5}\times\dfrac{4}{5}+\dfrac{1}{5}\times\dfrac{3}{5}+\dfrac{1}{5}\times\dfrac{2}{5}+\dfrac{1}{5}\times\dfrac{1}{5}=\dfrac{10}{25}=\dfrac{2}{5}$

54. A

Sol. $A\cdot A^T = \begin{bmatrix} a^2+b^2 & ac+bd \\ ac+bd & c^2+d^2 \end{bmatrix}$ ⇒ $a^2 + b^2 + c^2 + d^2 = 6$

$\Rightarrow \dfrac{\frac{a^2}{2}+\frac{a^2}{2}+b^2+\frac{c^2}{2}+\frac{c^2}{2}+d^2}{6} \ge \left(\dfrac{a^4b^2c^4d^2}{16}\right)^{\frac{1}{6}}$

$\Rightarrow \dfrac{a^4b^2c^4d^2}{16} \le 1 \Rightarrow a^2bc^2d \le 4$

55. C

Sol. $f(x) = \begin{vmatrix} \frac{x}{\sin x} & \frac{2x}{\tan x} & \frac{xe^x}{e^x-1} \\ \frac{3\sin x}{x} & \frac{1-\cos x}{x^2} & \frac{2\tan x}{x} \\ 2 & 3x & 1 \end{vmatrix} \Rightarrow \lim_{x\to 0} f(x) = \begin{vmatrix} 1 & 2 & 1 \\ 3 & \frac{1}{2} & 2 \\ 2 & 0 & 1 \end{vmatrix} = \dfrac{3}{2}$

56. B

Sol. $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a} = -10$

$\Rightarrow \vec{b} = 2\vec{a}$ and $\vec{c} = -4\vec{a}$

$\Rightarrow |2\vec{a}+3\vec{b}+4\vec{c}| = |-8\vec{a}| = 8$

57. B

Sol. Let $f(x) = x^2 - 2kx + k^2 - 1$

⇒ f(1) < 0 and f(k) < 0

⇒ k ∈ (0, 2)

58. B

Sol. $(10+24i)^{\frac{1}{2}}-(10-24i)^{\frac{1}{2}}=\pm 4\sqrt{2}\,i$

59. A

Sol. Let $\frac{x-1}{3}=\frac{y}{2}=\frac{z}{1}=\lambda$ and $\frac{x}{1}=\frac{y-1}{2}=\frac{z}{3}=\mu$

$\Rightarrow$ A $\equiv$ (3λ + 1, 2λ, λ) and B $\equiv$ (μ, 2μ + 1, 3μ)

If AB is minimum $\Rightarrow$ $\lambda=\mu=-\frac{1}{4}$

60. C

Sol. $\frac{r^2+r+1}{(r-1)(r)(r+1)(r+2)}=\frac{r}{(r-1)(r+1)}-\frac{r+1}{r(r+2)}$

61. B

Sol. 88200 = $2^3 . 3^2 . 5^2 . 7^2$

$\Rightarrow \alpha$ = 27 and β = 15

$\Rightarrow \alpha - \beta$ = 12

62. A

Sol. k = 14×3^6

63. C

Sol. Required probability = $1-\frac{^6C_1}{2^5}-\frac{^6C_1}{2^5}+{}^6C_2\cdot{}^2C_1\frac{1}{2^5}\cdot\frac{1}{2^4}=\frac{175}{216}$

64. D

Sol. ω is root of $z^{117}=1$

$\Rightarrow |z-\omega|_{min} = 0$

65. B

Sol. Equation of plane is 2x + y – 3z = 7

66. D

Sol. As $\frac{x+\frac{y}{2}+\frac{y}{2}+\frac{z}{2}+\frac{z}{2}}{5}\geq\left(\frac{xy^2z^2}{16}\right)^{\frac{1}{5}}$

$\Rightarrow xy^2z^2 \leq 512$

67. C

Sol. $|\vec{r}-\vec{a}|$ is maximum if $\vec{r}=\hat{i}+2\hat{j}-\hat{k}$

$\Rightarrow |\vec{r}-\vec{a}|_{max}=\sqrt{3}$

68. B

Sol. Required ways = ${}^8C_5\cdot{}^{10}C_7-{}^4C_1\cdot{}^7C_5\cdot{}^9C_7+{}^4C_2\cdot{}^6C_5\cdot{}^8C_7-{}^4C_3\cdot{}^5C_5\cdot{}^7C_7=3980$

69. C

Sol. $A = 3A^T - 4I \Rightarrow A^T = 3A - 4I \Rightarrow A = 9A - 16I \Rightarrow A = 2I \Rightarrow |A| = 2^3 = 8$

70. B

Sol. Sum of roots = 0 $\Rightarrow$ a = ± 2
Also, product of roots < 0
$\Rightarrow$ a = 2

SECTION – B

71. 0

Sol. $\vec{r} \times \vec{a} = \vec{b} \times \vec{r} \Rightarrow \vec{r} \times (\vec{a} + \vec{b}) = 0 \Rightarrow \vec{r} = \lambda(\vec{a} + \vec{b})$

$\Rightarrow \vec{r} = \pm \frac{(\hat{i} - \hat{j})}{\sqrt{2}}$, As $|\vec{r}| = 1 \Rightarrow \vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 0$

72. 6

Sol. $k = {}^8C_5 \times 5! + {}^3C_1 \cdot {}^7C_3 \cdot \frac{5!}{2!} + {}^3C_2 \cdot {}^6C_2 \cdot \frac{5!}{2! \cdot 2!} = 13560$

$\Rightarrow \frac{k}{565} = 24$

SECTION – C

73. 00001.00

Sol. Let $x^2 + 14x + 53 = k^2$, $k \in$ integer
$\Rightarrow x^2 + 14x + 53 - k^2 = 0$ for some $x \in$ integer
$\Rightarrow (14)^2 - 4(53 - k^2) = l^2$, $l \in$ integer
$\Rightarrow k^2 = 4 \Rightarrow x = -7$

74. 00003.00

Sol. $\frac{13}{2}\left[2a_1 + 12d\right] = 52a_1$ (d is common difference)

$\Rightarrow a_1 = 2d \Rightarrow \frac{a_{17}}{a_5} = 3$

75. 00006.00

Sol. Equation of plane is $x - 2y + z - 1 = 0$

$\Rightarrow k = \left|\frac{2 + 2 + 3 - 1}{\sqrt{1^2 + 2^2 + 1^2}}\right| = \sqrt{6}$

$\Rightarrow k^2 = 6$