FIITJEE

ALL INDIA TEST SERIES

PART TEST – III

JEE (Advanced)-2021

PAPER – 2

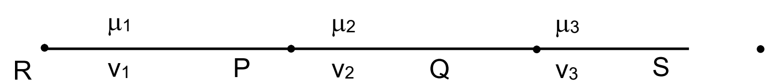
TEST DATE: 20-12-2020

ANSWERS, HINTS & SOLUTIONS

Physics PART – I

SECTION – A

1. A, B, C, D



Sol. $v_1 = \sqrt{\frac{T}{0.1/2}} = \sqrt{20T}$

$v_2 = \sqrt{\frac{T}{0.2/3}} = \sqrt{15T}$

$v_3 = \sqrt{\frac{T}{0.15/4}} = \sqrt{\frac{80T}{3}}$

For RP and PQ $v_1 > v_2$

So, reflection is from denser medium so phase difference between the wave-1 and wave-2 is π.

$\frac{A_3}{A_1} = \frac{2v_2}{v_2 + v_1} = \frac{2\sqrt{3}}{2+\sqrt{3}}$

$\frac{A_4}{A_3} = \frac{v_3 - v_2}{v_3 + v_2} = \frac{1}{7}$

2. A, D

Sol. $K_{maximum} = \frac{1}{2}\int_0^{\ell}(\lambda dx)\omega^2(A\sin kx)^2$

So, $E_k = \frac{1}{4}mA^2\omega^2$

also $<E_k> = \frac{E_k}{2} = \frac{1}{8}mA^2\omega^2$

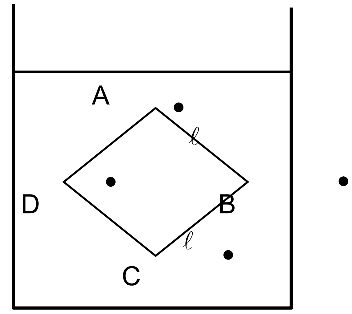
3. A, B, D

Sol. According to Pascal's law

$P_D = P_B$.

Because B and D lie on the same horizontal plane.

Let ℓ be the side of the square, then the vertical distance between A and C is $\ell\sqrt{2}$.



$$\therefore \quad P_B = P_D = P_A + \frac{P_C - P_A}{2} = \frac{P_A + P_C}{2}$$

similarly, $P_B = P_D = P_A + \rho g \frac{\ell}{\sqrt{2}}$

since, $\rho g \ell = \frac{P_C - P_A}{\sqrt{2}}$

$$\therefore \quad P_B = P_D = P_A + \frac{P_C - P_A}{2} = \frac{P_A + P_C}{2}.$$

4. B, C, D

Sol. The weight will be same

5. B, C

Sol. For the vernier callipers, 1MSD = 1/8 cm. 5VSD = 4MSD

$\Rightarrow$ 1VSD = $\frac{4}{5}\text{MSD} = \frac{4}{5} \times \frac{1}{8}\text{cm} = \frac{1}{10}\text{cm}$

Since, LC of the vernier callipers = 1MSD − 1VSD = $\frac{1}{40}$cm = 0.025 cm

Pitch of the screw gauge = 2 × 0.025 cm = 0.05 cm

LC of the screw gauge = $\frac{0.05}{100}\text{cm} = 0.005\text{ mm}$

LC of the liner scale of the screw gauge = 0.05 cm

Pitch = 0.05 × 2 cm = 0.1 cm

So, LC of the screw gauge = $\frac{0.1}{100}\text{cm} = 0.01\text{ mm}$

6. B, D

Sol. $\frac{1}{2}mv^2 + \left(-\frac{2GMm}{L}\right) = 0$

$\Rightarrow v = 2\sqrt{\frac{GM}{L}}$

Gravitational field is conservative, in which total mechanical energy (KE + PE) remains conserved. Hence, the kinetic energy imparted to the mass m is gradually reduced and gets converted into its potential energy, so that at every point of its flight the total mechanical energy remains constant.

SECTION – B

7. 5

Sol. $A\left(\frac{dh}{dt}\right) = Q - av$ (where Q = 100 cm^3s^{-1})

$= Q - a\sqrt{2gh}$

At steady state Q = $a\sqrt{2gh}$

100 = 1 $\sqrt{2\times1000\times h}$

h = 10/2 $\Rightarrow$ h = 5 cm

8. 5

Sol. $V_T = \frac{2}{9}\times\frac{r^2(\rho-\rho_L)g}{\eta}$

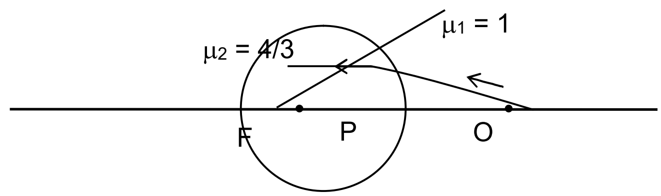
$\Rightarrow \quad V_T = \frac{2}{9}\times\frac{(3\times10^{-3})^2\times1260\times10}{1.260}$

$V_T = 2\times10^{-2}$ m/s $\Rightarrow V_T$ = 2 cm/s

t = $\frac{10cm}{V_T} \Rightarrow$ t = 5 sec

9. 7

Sol. $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$



$\Rightarrow \frac{4}{3v}-\frac{1}{-R}=\frac{4/3-1}{R}$

$\Rightarrow \quad \frac{4}{3v}=\frac{1}{3R}-\frac{1}{R}$

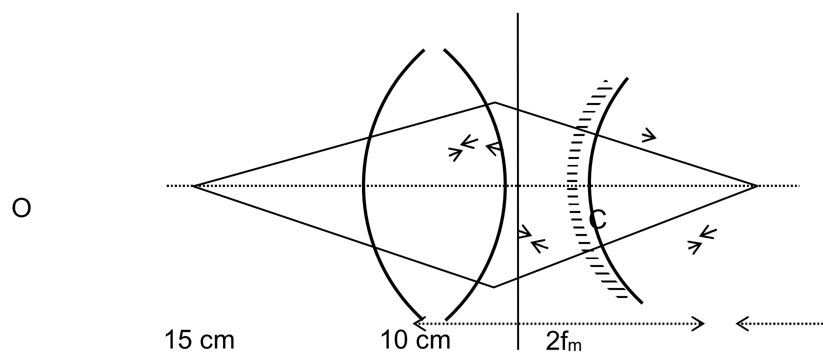
$\frac{4}{3v}=\frac{1-3}{3R}\Rightarrow v=-\frac{4R}{2} \Rightarrow$ v = –2R

Distance of image from centre = 3R

10. 2

Sol. To retrace the path after reflection from mirror, rays after refraction from the lens must be incident towards the centre of curvature of mirror

For the image by lens, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$



$\Rightarrow \frac{1}{10+2f_m}-\frac{1}{-15}=\frac{1}{10}$

$\frac{1}{10+2f_m}=\frac{1}{10}-\frac{1}{15} \Rightarrow \frac{1}{10+2f_m}=\frac{3-2}{30}$

$10+2f_m = 30 \Rightarrow f_m$ = 10 cm

11. 8

Sol. $w=3\pi=\frac{2\pi}{T} \therefore T=\frac{2}{3}$sec

y = A sin (ωt – kx)

kx = π/8 $\Rightarrow \frac{2\pi}{\lambda}=\frac{\pi}{8}$

So, λ = 16 cm

$v=\frac{\lambda}{T}=\frac{16}{2/3}=24$ cm/sec

12. 6

Sol. $A = A_0e^{-\lambda t}$

$100 = 141e^{-3\lambda}$

$e^{3\lambda} = 1.41$

$e^{3\lambda} = \sqrt{2}$

Half life = $\ell n2/\lambda$ = 6 days

SECTION – C

13. 00001.60

Sol. $\frac{dN_A}{dt} = -\lambda_1 N_A$, $\frac{dN_B}{dt} = 2\lambda_1 N_A - \lambda_2 N_B$

For N_B to be maximum, $\frac{dN_B}{dt} = 0$

$\Rightarrow 2\lambda_1 N_A = \lambda_2 N_{B_{max}} \Rightarrow N_{B_{max}} = \frac{2\lambda_1}{\lambda_2} N_A$

$\Rightarrow N_{B_{max}} = \frac{2\lambda_1}{\lambda_2} N_0 e^{-\lambda_1 t} = 1.60$

14. 00003.00

Sol. Apply snells' law on various surface one by one:

$1 \sin 90° = \mu_1 \sin r_1 \Rightarrow \sin r_1 = \frac{1}{\sqrt{2}} \Rightarrow 45°$

$\mu_1 \cos r_1 = \mu_2 \sin r_2 \Rightarrow \sin r_2 = \frac{1}{\mu_2}$

$\mu_2 \cos r_2 = \mu_3 \sin r_3 \Rightarrow \sin r_3 = \frac{\mu_2\sqrt{1-\sin^2 r_2}}{\sqrt{3}} = \frac{\sqrt{\mu_2^2 - 1}}{\sqrt{3}}$

$\mu_3 \cos r_3 = 1$

$\sin^2 r_3 + \cos^2 r_3 = 1$

$\Rightarrow \quad \frac{\mu_2^2 - 1}{3} + \frac{1}{3} = 1 \Rightarrow \mu_2^2 = 3$

$\Rightarrow \mu_2 = \sqrt{3}$

15. 00000.50

Sol. f = 25 cm

$\frac{1}{25} = \frac{1}{v} + \frac{1}{-25} \Rightarrow v = \frac{25}{2}$ cm

$\Rightarrow m = \frac{-v}{u} = \frac{1}{2}$

16. 00002.40

Sol. $(\pi r^2)\sqrt{2gy} = \pi x^2\left(-\frac{dy}{dt}\right)$

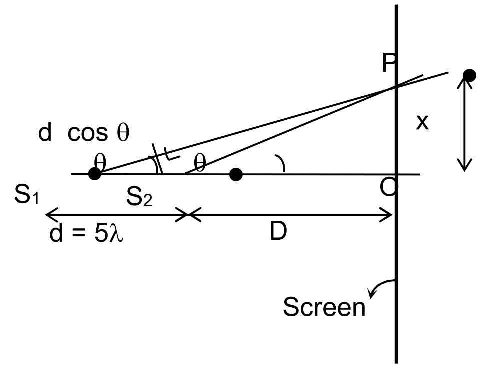
$\Rightarrow (r^2)\sqrt{2gy} = x^2\lambda$

$\Rightarrow y \alpha x^4$

$\Rightarrow$ n = 4

17. 00001.20

Sol. d cos θ = nλ (for maxima)



$\cos\theta = \frac{n\lambda}{d}$

$\cos\theta = \frac{n}{5}$

For the minimum value of x

n = 4

$\cos\theta = \frac{4}{5} \Rightarrow \theta = 37°$

$\tan\theta = \frac{x}{D}$

$\frac{3}{4} = \frac{x}{D}$

$x = \frac{3D}{4} = \frac{3\times 1.6}{4} = 1.20\text{m}$

x = 1.20 m

18. 00003.70

Sol. $E(t) = A^2 e^{-\alpha t}$

Given: $\alpha = 0.2\ s^{-1}$, $\frac{dA}{A} = 1.25\%, \frac{dt}{t} = 1.50\%$

$\Rightarrow \frac{dE}{E} = \pm 2\frac{dA}{A} \pm \alpha\frac{dt}{t}\cdot t$

= ± 2(1.25) ± 0.2(1.5) (4)

= ± 2.50 ± 1.20 = ± 3.70%

Chemistry

PART – II

SECTION – A

19. A, B, C
Sol. Facts

20. A, D
Sol. Triatomic linear gas molecule $= \frac{1}{2}KT \times 5$

$= \frac{5}{2}KT$

Triatomic angular gas molecule $= \frac{1}{2}KT \times 6$

= 3 KT

Monoatomic gas molecule $= \frac{1}{2}KT \times 3$

$= \frac{3}{2}KT$

Ratio $= \frac{5/2\ KT}{3/2\ KT} = \frac{5}{3}$

Ratio $= \frac{3KT}{3/2\ KT} = 2$

Ans. $\frac{5}{3}$ and 2

21. A, B, C
Sol. Here $\lambda = 3.5 \times 10^{-5}$ m

$\sigma = 0.25 \times 10^{-9}$ nm

T = 300 K

$$\lambda = \frac{1}{\sqrt{2}\pi\sigma^2 N^*}$$

N^* = Number of molecules per unit volume of gas.

$$\therefore N^* = \frac{1}{\sqrt{2}\pi\sigma^2\lambda}$$

$$= \frac{1}{(1.414)(3.14)(0.25 \times 10^{-9})^2 (3.5 \times 10^{-5})}$$

$= 1.0296 \times 10^{23}\ m^{-3}$

$= 1.03 \times 10^{23}\ m^{-3}$

$P = N^*\ KT = (1.03 \times 10^{23})(1.3806 \times 10^{-23}) \times (300)$

$= 4.26 \times 10^2$ Pa

22. A, B, D
Sol. Density remains same in Frenkel defects.

23. A, B, C, D
Sol. Facts

24. A, B, C, D
Sol. Facts

SECTION – B

25. 5
Sol. $\Delta T_f = i\,K_f \times \text{molality}$

$0.383 = i \times 1.86 \times \frac{1.1/267.4}{100/1000}$

$i = 5$

26. 9
Sol. $FeS_2O_3 + Cl_2 + H_2O \longrightarrow Fe^{3+} + SO_4^{2-}$

$Fe^{2+} \longrightarrow Fe^{2+} + e^-$

$S_2O_3^{2-} \longrightarrow SO_4^{2-} + 8e^-$

So, x = 9.

27. 2
Sol. $P_4 + NaOH \longrightarrow \overset{-3}{P}H_3 + NaH_2\overset{+1}{P}O_2$

$\therefore (-3) + (+1) = -2$

28. 5
Sol. $AgI, PbI_2, AgBr \rightarrow \text{Yellow}$

$Hg_2I_2 \rightarrow \text{Brown}$

$HgI_2 \rightarrow \text{Red}$

$AgCl, PbCl_2, Hg_2Cl_2 \rightarrow \text{White}$

29. 4
Sol. $\text{Density} = \frac{Z \times M}{a^3\,N_{av}}$

Z = Rank of unit cell

$\therefore Z = \frac{\text{density} \times a^3 \times N_{av}}{M}$

$= \frac{2.70 \times (4.04 \times 10^{-8})^3 \times 6.022 \times 10^{23}}{27.0}$

$= 3.97 \approx 4$

30. 5
Sol. $d = \frac{4 \times 6.023\ A \times 10^{-3}}{(6.023 \times 10^{23})(2A^{1/3} \times 10^{-9})^3}$

SECTION – C

31. 00003.79

Sol. $\frac{P^o - P_s}{P_s} = \frac{\Delta W_{solvent}}{\Delta W_{solution}}$

$= \frac{n_{solute}}{N_{solvent}}$

$\therefore \frac{0.4}{2.6} = \frac{6 / M_{solute}}{185 / 18}$

$M_{solute} = \frac{6 \times 18}{185} \times \frac{2.6}{0.4}$

= 3.7945

$\simeq 3.79$

32. 00001.08

Sol. $\frac{P^o - P_s}{P_s} = \frac{W_{solute}}{W_{solvent}} \times \frac{M_{solvent}}{M_{solute}}$

$1000 \times \left(\frac{100 - 95.85}{95.85}\right) = \left(\frac{w \times 78}{m \times w}\right) \times 1000$

$\left(\frac{4.15}{95.85} \times 1000\right) \times \frac{1}{78} = \text{molality}$

Molality = 0.555

$\simeq 0.56$

$0.60 = K_f \times 0.555$

$K_f = \frac{0.60}{0.555} = 1.08$

$\simeq 1.08$

33. 00001.73

Sol. Specific conductance of AgCl = Specific conductance of solution – specific conductance of H_2O

$= (3.32 - 1.5) \times 10^{-6} = 1.82 \times 10^{-6}$

For saturated solution of sparingly soluble salt $\lambda_{eq} = \lambda_{eq}^{\infty}$

$\therefore \lambda_{eq}^{\infty} = \frac{1000 \times 1.82 \times 10^{-6}}{S} = 138.3$

$\therefore S = 13.159 \times 10^{-6}$

$\simeq 13.16 \times 10^{-6}$

$K_{sp} = S^2 = 173.1856 \times 10^{-12}$

$\simeq 1.73 \times 10^{-10}$

34. 00005.08

Sol. At anode $H_2 \longrightarrow 2H^+ + 2e^-$

At cathode $Hg_2Cl_2 + 2e^- \longrightarrow 2Hg + 2Cl^-$

$H_2 + Hg_2Cl_2 \longrightarrow 2H^+ + 2Hg + 2Cl^-$

$E_{cell} = E_{cell}^o - \frac{0.059}{2} \log \frac{[Cl^-]^2 [H^+]^2}{p_{H_2}}$

$$0.58 = 0.28 - \frac{0.059}{2}\log\frac{\left[H^+\right]^2(1)^2}{1}$$

$$0.58 = 0.28 + 0.059\ pH$$

$$\therefore pH = 5.0847 \simeq 5.08$$

35. 00057.39

Sol. $\Delta S^o = \sum S^o_{product} - \sum S_{reactant}$

$\Delta S^o = 2(238.9) - [151.3 + 221.0]$

= 477.8 – 372.3

= 105.5

$\Delta G^o = \Delta H^o - T\Delta S^o$

$\Delta G^o = 25.7 \times 10^3 - (298 \times 105.5)$

= - 5739

$\therefore \frac{5739}{100} = 57.39$

36. 00185.01

Sol. For BCC structure

$2r^+ + 2r^- = \sqrt{3}a$

$2r_{A^+} + 2 \times 235 = \sqrt{3} \times 485$

$\therefore r_{A^+} = 185.01$ pm

Mathematics PART – III

SECTION – A

37. B, D

Sol. Let $f(x) = (1 + x)^{2000}$

then $f(1) + \omega f(\omega) + \omega^2 f(\omega^2) = 3({}^{2000}C_2 + {}^{2000}C_5 + {}^{2000}C_8 + {}^{2000}C_{11} \,.....)$

$\Rightarrow {}^{2000}C_2 + {}^{2000}C_5 + {}^{2000}C_8 + ... = \frac{2^{2000} - 1}{3} \quad \left\{\text{where } \omega = -\frac{1}{2} + \frac{i\sqrt{3}}{2}\right\}$

38. C, D

Sol. **Method-1:** Let $f(x) = 1 - x + x^2 - x^3 \,.....\, - x^{17}$

$f(x) = \frac{1 - x^{18}}{1 + x}$

$f(x) = f(y-1) = \frac{1-(y-1)^{18}}{y} \Rightarrow f(y-1) = \frac{1-(y-1)^{18}}{y}$

a_2 = coefficient of y^3 in $(1 - (y - 1)^{18})$ = ${}^{18}C_3$

a_1 = coefficient of y^2 in $(1 - (y - 1)^{18}$ = $-{}^{18}C_2$

Method-2: $f(x) = f(y - 1) = 1 - (y - 1) + (y - 1)^2 - (y - 1)^3$

$f(y - 1) = 1 + (1 - y) + (1 - y)^2 + (1 - y)^3 \,.....\, (1 - y)^{17}$

So, $a_2 = {}^2C_2 + {}^3C_2 + {}^4C_2 \,.....\, {}^{17}C_2 = {}^{18}C_3$

39. C, D

Sol. $p(x) = ax^2 + (a - 1)x + (a - 2)$ has root α and β, then $|\alpha - \beta| < 2$

$\Rightarrow \sqrt{\left(-\left(\frac{a-1}{a}\right)\right)^2 - 4\left(\frac{a-2}{a}\right)} < 2 \Rightarrow \frac{-7a^2 + 6a + 1}{a^2} < 0$

$\Rightarrow a \in \left(-\infty, -\frac{1}{7}\right) \cup (1, \infty)$ and discriminant D > 0

$\Rightarrow (a - 1)^2 - 4a(a - 2) > 0 \Rightarrow a \in \left(1 - \frac{2}{\sqrt{3}}, 0\right) \cup \left(0, 1 + \frac{2}{\sqrt{3}}\right)$

So, the answer is $\left(1 - \frac{2}{\sqrt{3}}, -\frac{1}{7}\right) \cup \left(1,\ 1 + \frac{2}{\sqrt{3}}\right)$

40. A, B

Sol. If there are 2k occurrence of a's, those can occur in ${}^nC_{2k}$ places and remaining position can be filled by 2^{n-2k} ways

So, answer is $\sum_{k=0} {}^nC_{2k} 2^{n-2k} = \frac{1}{2}(3^n + 1) \Rightarrow f(n) = \frac{1}{2}(3^n + 1)$

41. C, D

Sol. The perpendicular line to both given line will be $\frac{x+1}{-2} = \frac{y+2}{0} = \frac{z-3}{1}$

A point can be taken on this line $(-2t - 1, -2, t + 3)$

So, perpendicular distance is $2\sqrt{3}$ from $x + y + z - 3 = 0$, then $t = -9, 3$

Corresponding points are (17, –2, –6), (–7, –2, 6)

42. A, D

Sol. $x + y + z + \omega = 20$ where $0 \le x, y, z, \omega \le 20$

Total number of integers = ${}^{20+4-1}C_{4-1} = {}^{23}C_3$

If all are odd integer then number of integer = ${}^{11}C_3$ and if x is prime number greater than 10 then x can be 11, 13, 17, 19 corresponding number of integers are

${}^{11}C_2 + {}^{9}C_2 + {}^{5}C_2 + {}^{3}C_2$

SECTION – B

43. 2

Sol. Let $2^x = a$ and $3^x = b$, then $\frac{a^3 + b^3}{a^2b + ab^2} = \frac{7}{6}$

$\frac{a^2 + b^2 - ab}{ab} = \frac{7}{6}$, $6a^2 - 13ab + 6b^2 = 0$, $(2a - 3b)(3a - 2b) = 0$

$\Rightarrow 2a = 3b$ or $3a = 2b \Rightarrow 2^{x+1} = 3^{x+1}$ or $2^{x-1} = 3^{x-1} \Rightarrow x = -1$ or $x = 1$

44. 2

Sol. $|z_1 + 1| + |z_2 + 1| + |z_1z_2 + 1|$

$\ge |z_1 + 1| + |z_1z_2 + 1 - (z_2 + 1)|$

$\ge |z_1 + 1| + |z_1z_2 - z_2|$

$\ge |z_1 + 1| + |z_1 - 1|$ (as $|z_2| = 1$)

$\ge |(z_1 + 1) - (z_1 - 1)|$

≥ 2

45. 1

Sol. $\frac{1+z+z^2}{1-z+z^2} = 1 + \frac{2z}{1-z+z^2} \in R$ if $\frac{2z}{1-z+z^2} \in R$

So, $\frac{z}{1-z+z^2} \in R$, then $\frac{1-z+z^2}{z} \in R$ also

$\Rightarrow \frac{1}{z} - 1 + z$ also belongs to R $\Rightarrow \frac{1}{z} + z = \frac{1}{\bar{z}} + \bar{z}$

$\Rightarrow (z - \bar{z}) - \frac{1}{z} - \frac{1}{\bar{z}} = 0 \Rightarrow (z - \bar{z}) - \left(\frac{(z-\bar{z})}{z \cdot \bar{z}}\right) = 0$, $(z - \bar{z})\{z \cdot \bar{z} - 1\} = 0$

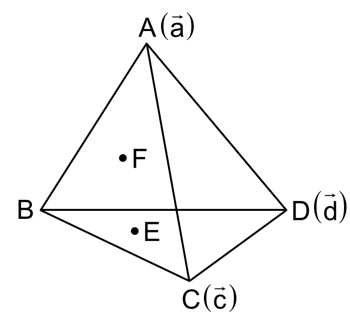
So, if $z - \bar{z} \ne 0$ (as z is not a real number)

$z\bar{z} = 1$

$|z|^2 = 1 \Rightarrow |z| = 1$

46. 9

Sol. Let B vertex as origin



$\vec{E} = \frac{\vec{c} + \vec{d}}{3}$

$\vec{F} = \frac{\vec{a} + \vec{c}}{3}$, then volume of tetrahedron

ABCD is = $\frac{1}{6}[\vec{a}\ \vec{c}\ \vec{d}]$ ..... (1)

Volume of tetrahedron BCEF is = $\frac{1}{6}\left[\vec{c}\ \ \frac{\vec{a}+\vec{d}}{3}\ \ \frac{\vec{a}+\vec{c}}{3}\right]$

= $\frac{1}{6}\times\frac{1}{9}\left[\vec{a}\quad\vec{c}\quad\vec{d}\right] \Rightarrow \frac{\text{Volume of tetrahedron ABCD}}{\text{Volume of tetrahedron BCEF}} = 9$

47. 1

Sol. The line of intersection of plane 2x + 3y + 4z – 1 = 0 and x + y + z – 3 = 0 is $\frac{x-8}{1} = \frac{y+5}{-2} = \frac{z-0}{1}$
and the line of intersection of plane 2x + 3y + 4z – 1 = 0 and x + y + z + 3 = 0 is
$\frac{x+10}{1} = \frac{y-7}{-2} = \frac{z-0}{1}$
Shortest distance will be $\sqrt{174}$

48. 0

Sol. If $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$
So, p(x) is a constant function

SECTION – C

49. 00022.50

Sol. Total words are $\frac{10!}{2!2!2!}$

The words contains the word "IIT" are $\frac{8!}{2!}$

$\Rightarrow$ Probability = $\dfrac{\frac{8!}{2!}}{\frac{10!}{2!2!2!}} = \frac{2}{45}$

50. 00600.00

Sol. $\Delta(\Delta A) = 1$
$\Rightarrow \Delta A = \{d, d + 1, d + 2, d + 3, ...\}$
$A = \{a_1, a_1 + d, a_1 + d + (d + 1), a_1 + d + (d + 1) + (d + 2) .....\}$
$\Rightarrow$ n^{th} term of sequence A is
$T_n = a_1 + \{d + (d + 1) + (d + 2) ..... (n – 1) \text{ term}\}$
$a_n = a_1 + (n – 1)d + \frac{(n-2)(n-1)}{2}$
Clearly a_n is a quadratic polynomial with leading coefficient is $\frac{1}{2}$ and $a_{51} = a_{41} = 0$

$\Rightarrow a_n = \frac{1}{2}(n-51)(n-41)$

Clearly now, $a_{11} = \frac{1}{2}\times 40\times 30 = 600$

51. 02019.00

Sol. $T_r = \frac{r+2}{r!+(r+1)!+(r+2)!} = \frac{(r+2)}{r!\{1+(r+1)+(r+1)(r+2)\}} = \frac{(r+2)}{r!(r+2)^2} = \frac{1}{r!\,(r+2)}$

$= \frac{r+1}{(r+2)!} = \frac{(r+2)-1}{(r+2)!}$

$$T_r = \frac{1}{(r+1)!} - \frac{1}{(r+2)!}$$

$$T_1 = \frac{1}{2!} - \frac{1}{3!}$$

$$\vdots$$

$$T_n = \frac{1}{(n+1)!} - \frac{1}{(n+2)!}$$

$$\text{Sum} = \frac{1}{2!} - \frac{1}{(n+2)!}$$

$$\Rightarrow \frac{1}{2!} - \frac{1}{(2021)!}$$

52. 00000.50

Sol. The 5 digit number with product of digit is 25 consist two 5 and three 1

Therefore number of ways 'a' number can be formed is $\frac{5!}{2!3!}$ ways

The 'b' number consist three 1, one 3 and one 5

So 'b' number $\frac{5!}{3!}$ ways can be formed

53. 00201.00

Sol. $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 12$

where each $x_i = 0, 2, 4, 6$ where i = 1, 2, 3, 4, 5

and $x_6 = 2, 4, 6$, this is equivalent to find coefficient of x^{12} in $(x^0 + x^2 + x^4 + x^6)^5 (x^2 + x^4 + x^6)$

$\Rightarrow$ Coefficient of x^{12} in $\left(\frac{1-x^8}{1-x^2}\right)^5 \left(\frac{x^2(1-x^6)}{1-x^2}\right)$

$\Rightarrow$ Coefficient of x^{10} in $(1 + x^2)^5 (1 + x^4)^5 (1 + x^2 + x^4)$

$\Rightarrow$ Coefficient of x^{10} in $(1+x^2)^5(1+x^4)^5 + x^2(1+x^2)^5(1+x^4)^5 + x^4(1+x^2)^5(1+x^4)^5$

= 201

54. 00024.00

Sol. By characteristic matrix

$|A - \lambda \mathbf{I}| = 0$

$\Rightarrow A^3 - 7A^2 + 10A - \mathbf{I} = 0$

$\Rightarrow A^{-1} = A^2 - 7A + 10\mathbf{I}$

$\Rightarrow A^{-2} = A - 7\mathbf{I} + 10A^{-1}$

$\Rightarrow A^{-2} = 10A^2 - 69A + 93\mathbf{I}$