FIITJEE

ALL INDIA TEST SERIES

PART TEST – II

JEE (Advanced)-2021

PAPER –1

TEST DATE: 13-12-2020

ANSWERS, HINTS & SOLUTIONS

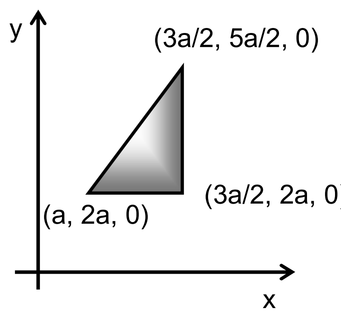
Physics

PART – I

SECTION – A

1. C

Sol. If a point charge is at $\left(a, 2a, \frac{a}{2}\right)$ then given surface is $\frac{1}{8}$th of a square surface of side 'a'



2. D

Sol. In the steady state current through each bulb is

$$i_0 = \frac{400}{100+100} = 2\text{ A}$$

Now, when the switch is opened, the direction of current in bulb B_1 is reversed.

3. B

Sol. Initially

$P_1V_1 = nRT_1$ …(1)

and

$$P_2 = P_0 + \frac{mg}{A} = \frac{7}{3}\times 10^5 \text{Pa} \qquad \ldots(2)$$

$P_2V_2 = nRT_2$ …(3)

So $\frac{P_2}{P_1} = \frac{T_2}{T_1}$

T_2 = 700 K = 427°C

4. C

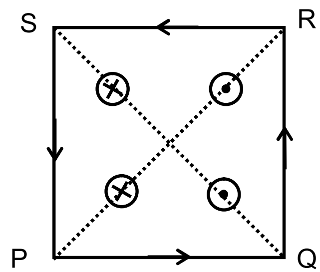
Sol. Let $\int_P^Q \vec{B}_A \cdot d\vec{\ell} = -9\mu_0 = x$

$\int_R^S \vec{B}_B \cdot d\vec{\ell} = 5\mu_0 = y$

Now, apply amperes low in loop PQRS

$$2\left[\left(x - \frac{y}{2}\right) + (-2x + y) + (-8x + 4y) + (4x - 2y)\right] = \mu_0(5i_0)$$

$i_0 = 23\,\text{amp}$



5. D

Sol. $d\phi = B.h.dr$

$$\phi = \int_{R_1}^{R_2} \frac{\mu_0 N}{4\pi}.\frac{2i}{r} h\,dr$$

$$= \frac{\mu_0 N h i}{2\pi} \ln\frac{R_2}{R_1}$$

$$= \left(\frac{\mu_0 Nh}{2\pi}\ln\frac{R_2}{R_1}\right) i_0 \cos\omega t$$

$$\text{emf} = \frac{d\phi}{dt}$$

$$(\text{emf})_{max} = \frac{\mu_0 h N \omega i_0}{2\pi}\ln\frac{R_2}{R_1}$$

6. B

Sol. At the state of impending motion, friction force has maximum value.

$mg\sin\theta - qE\cos\theta - f_f = ma_0$

$a_0 = r\alpha$

On solving 3qE = mg

7. A, D

Sol. $i^2R_1t = m_1c_1(T_1 - T_0)$

$$t_0 = \frac{c_1 d_1 S_1^2 (T_1 - T_0)}{i^2\rho_1} \approx 0.095\,\text{sec}$$

Also $i^2R_2t = m_2c_2(\Delta T)$

$$\frac{R_1}{R_2} = \frac{m_1c_1(T_1 - T_0)}{m_2c_2\Delta T}$$

$$\Delta T = \frac{c_1 d_1 S_1^2 \rho_2}{c_2 d_2 S_2^2 \rho_1}\left(T_1 - T_0\right) \approx 0.106\,^\circ C$$

8. B, C, D

Sol. Since the particle is having acceleration along x-axis so the path of the particle is a non uniform helix.

9. B, D

Sol.
$$\left\{\begin{aligned} & P_i\left(A\frac{\ell}{2}\right) = 2RT_i \\ & P_f\left(A\left(\frac{\ell}{2}+x\right)\right) = 2RT_f \end{aligned}\right\} \quad \ldots(1)$$

$$\left\{\begin{aligned} & P_i A = K\frac{\ell}{2} \\ & P_f A = K\left(\frac{\ell}{2}+x\right) \end{aligned}\right\} \quad \ldots(2)$$

Now

$$\left\{\begin{aligned} & K\left(\frac{\ell}{2}\right)^2 = 2RT_i \\ & K\left(\frac{\ell}{2}+x\right)^2 = 2RT_f \end{aligned}\right\} \quad \ldots(3)$$

Work done by gas,

dw = pdv = (ky)dy

$$W = K\int_{\ell/2}^{(\ell/2+x)} y\,dy = \frac{K}{2}\left[\left(\frac{\ell}{2}+x\right)^2 - \left(\frac{\ell}{2}\right)^2\right] \quad \ldots(4)$$

Change in internal energy

$$\Delta U = nC_V\,\Delta T = 2.\frac{5}{2}R\left(T_f - T_\ell\right) \quad \ldots(5)$$

$$\Delta Q = W + \Delta U = 6R\,\Delta T$$

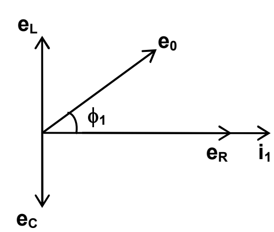
$$C = \frac{\Delta Q}{\Delta T} = 6R$$

10. C, D

Sol. Charges are non-uniformly distributed on the outer surface of the conductor due to point charge ‘q’.

11. A, C

Sol. Phasor diagram of Box-1



$$\tan\phi_1 = \frac{100-70}{40} = \frac{3}{4}$$

$$z_1 = \sqrt{(x_L - x_C)^2 + R^2} = 50\Omega$$

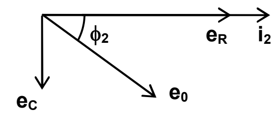
$$i_1 = \frac{200}{50} = 4\Omega$$

Phasor Diagram of Box-2

$\tan\phi_2 = \frac{40}{30} = \frac{4}{3}$

$z_2 = \sqrt{x_C^2 + R^2} = 50\Omega$

$i_2 = \frac{200}{50} = 4\text{ amp}$



12. A, B, C, D

Sol. Let a charge 'q' flown from capacitor C_1 to C_2 at time 't'. Now for KVL.

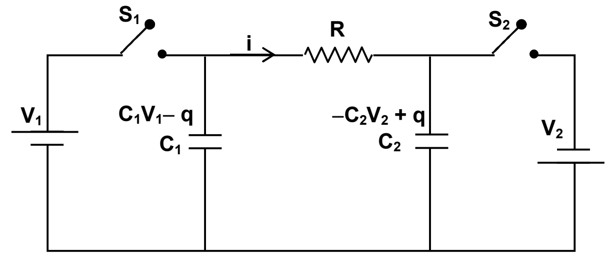
$\frac{C_1V_1 - q}{C_1} - iR + \frac{C_2V_2 - q}{C_2} = 0$ …(i)

Let $V_1 + V_2 = V$

$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$

After solving

$q = CV\left(1 - e^{-\frac{t}{RC}}\right)$



SECTION – C

13. 00108.75

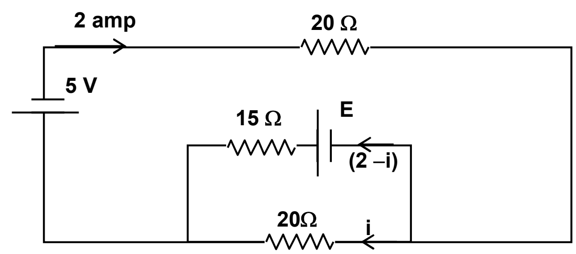
Sol. Circuit can be redrawn as shown.

$-5 - 40 - 20i = 0$

$i = -\frac{9}{4}$ …(i)

$E - 15(2 - i) + 20i = 0$

$E = 108.75$



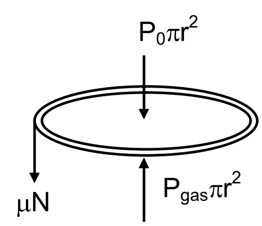
14. 00121.20

Sol. F.B.D. of cork

$\mu N + P_0\pi r^2 = 4P_0\pi r^2$

$N = \frac{3P_0\pi r^2}{\mu}$

So $\frac{N}{2\pi r} = \frac{3}{2}\frac{P_0 r}{\mu} = 121.2 \times 10^3$ N/m



15. 00001.00

Sol. induced emf, $\varepsilon = Bbv_T$

$i = \frac{Bbv_T}{R}$

In equilibrium

$mg = ibB$

$v_T = \frac{mgR}{B^2b^2}$

16. 00110.88

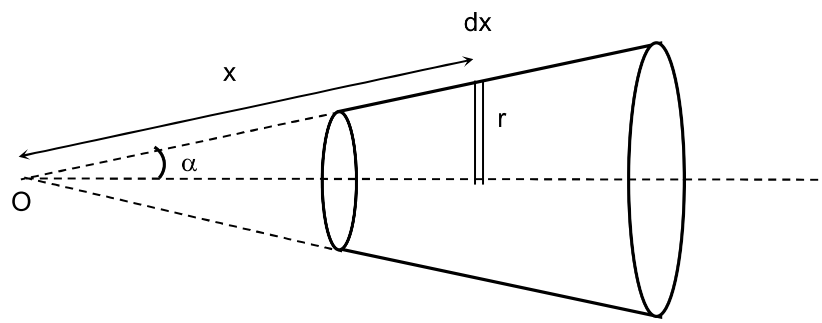
Sol. Consider a strip of length dx at x from apex along the slant length

$$di = \left(\frac{N}{\ell_0}dx\right)i \qquad \text{…(i)}$$

$$dB = \frac{\mu_0}{2}\frac{(di)r^2}{x^3} \qquad \text{…(ii)}$$

$$\sin\alpha = \frac{r}{x} = \frac{b-a}{\ell_0} \qquad \text{…(iii)}$$

So, $B = \frac{\mu_0 Ni(b-a)^2}{2\ell_0^3}\ell n\left(\frac{b}{a}\right) = 110.88\times10^{-6}$ Tesla



17. 00504.20

Sol. At equilibrium

$$\sigma A\left(T^4 - T_1^4\right) = \sigma A T_1^4$$

$$T_1^4 = \frac{T^4}{2}$$

18. 00000.02

Sol. $\alpha = \frac{1}{R}\frac{dR}{dt} = \frac{a+2bt+3ct^2}{1+at+bt^2+ct^3} = 0.02\ \left(°C\right)^{-1}$

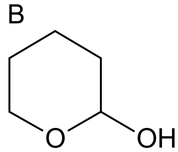
Chemistry PART – II

SECTION – A

19. A
Sol. Beckmann rearrangement.

20. C
Sol. Guanidine is the most basic compound among the given compounds. It's conjugate acid is most stable due to resonance.

21. C
Sol. Bayer Villiger oxidation.

22. B
Sol. 

is hemiacetal. It gives positive Fehling solution test.

23. C
Sol. Hydroboration oxidation is a *syn* addition.

24. C
Sol. $NaBH_4$ reduces ketones but does not reduce esters.

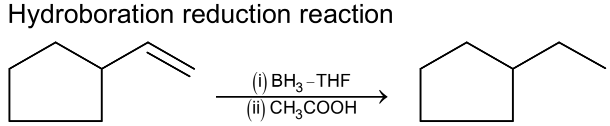
25. A, B, D
Sol. Cumulenes with even number of double bond do not show geometrical isomerism.

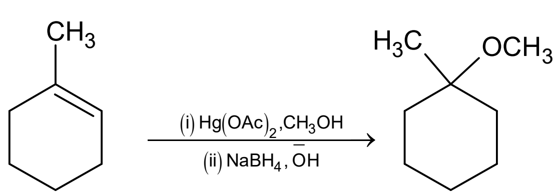
26. B, C, D
Sol. Compound (P) is Lactose.

27. B, C, D
Sol. Neoprene is a polymer of chloroprene.

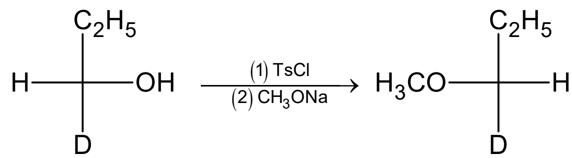
28. A, B
Sol. $-SO_3H$ and $-CN$ are metadirecting and deactivating groups.

29. A, C
Sol. (B) Hydroboration reduction reaction



(D)


30. A, D

Sol. (B) 

(C) When alcohol reacts with $SOCl_2$ in presence of pyridine inversion takes place.

SECTION – C

31. 00049.20

Sol. $[\alpha] = \dfrac{\theta}{\ell \times c} = \dfrac{24.6}{\frac{2}{10} \times \frac{25}{10}}$

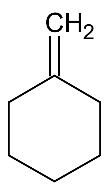
= 49.20

32. 00096.47

Sol. Percentage yield of 2-Bromopropane $= \dfrac{164}{170} \times 100$

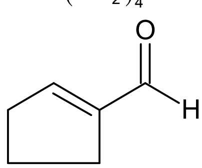
= 96.47

33. 00012.50

Sol. Product (R) is 

34. 00019.20

Sol. $A = OHC - (CH_2)_4 - CHO$

$B =$ 

MW of B = 96

$\therefore \dfrac{x}{5} = \dfrac{96}{5} = 19.20$

35. 00001.50

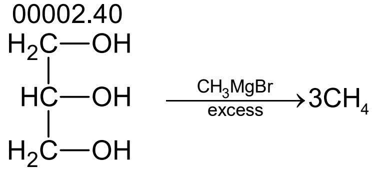
Sol. $6HCHO + 4NH_3 \longrightarrow \underset{\text{Urotropine}}{(CH_2)_6 N_4} + 6H_2O$

Number of carbon atom = x = 6

Number of nitrogen atom = y = 4

$\therefore \dfrac{x}{y} = \dfrac{6}{4} = 1.5$

36. 00002.40

Sol. 

Weight of CH_4 produced $= \dfrac{48}{92} \times 4.6$ = 2.4 grams

Mathematics PART – III

SECTION – A

37. C

Sol. Area of hexagon = 2 · Area of ΔABC

= 2 · 6 · Area of ΔBMD

$= 12 \cdot \frac{1}{2} \times BM \cdot MD \cdot \sin\frac{\pi}{6}$

= 3 · 6 · 2 = 36 sq. units

38. B

Sol. Equation of ellipse can be written as $\frac{\left(\frac{x+y+1}{\sqrt{2}}\right)^2}{2} + \frac{\left(\frac{x-y-3}{\sqrt{2}}\right)^2}{1} = 1$ which is similar to

$\frac{x^2}{2} + \frac{y^2}{1} = 1$ whose director circle is $x^2 + y^2 = 3$

Let $P \equiv \left(\sqrt{3}\cos\theta, \sqrt{3}\sin\theta\right)$ equation of chord of contact is $x\sqrt{3}\cos\theta + 2y\sqrt{3}\sin\theta - 2 = 0$

It touches the circle $x^2 + y^2 = r^2 \Rightarrow r = \frac{|0+0-2|}{\sqrt{3\cos^2\theta + 12\sin^2\theta}} = \frac{2}{\sqrt{12 - 9\cos^2\theta}}$

$\Rightarrow r_{max} = \frac{2}{\sqrt{3}}$ and $r_{min} = \frac{2}{\sqrt{12}} \Rightarrow \frac{r_{max}}{r_{min}} = 2$

39. B

Sol. We have $\left|\sin^{-1}\left(a^2-3\right)\right| + \cos^{-1}\left(a^2-3\right) = \frac{\pi}{2}$; for $a^2 \geq 1$

$\Rightarrow \sin^{-1}\left(a^2-3\right) \geq 0 \Rightarrow 0 \leq a^2 - 3 \leq 1$

$\Rightarrow$ a has two integral values

40. D

Sol. In a triangle ABC, cos A < 1 $\Rightarrow \frac{b^2+c^2-a^2}{2bc} < 1 \Rightarrow b^2 + c^2 - a^2 < 2bc$ ..... (1)

Similarly $c^2 + a^2 - b^2 < 2ca$ ..... (2)

$a^2 + b^2 - c^2 < 2ab$ ..... (3)

$\Rightarrow a^2 + b^2 + c^2 < 2(ab + bc + ca) \Rightarrow \frac{ab+bc+ca}{a^2+b^2+c^2} > \frac{1}{2}$

Also, $\frac{ab+bc+ca}{a^2+b^2+c^2} \leq 1 \Rightarrow \frac{1}{2} < \cos\theta \leq 1 \Rightarrow \theta \in \left[0, \frac{\pi}{3}\right)$

41. D

Sol. $A\left(x_1, \sqrt{7}x_1\right)$ and $B\left(x_2, \sqrt{7}x_2\right)$ satisfy the curve $\sqrt{2}x^2 + \sqrt{3}y + \sqrt{5}x + \sqrt{8} = 0$

$\Rightarrow \sqrt{2}x^2 + \left(\sqrt{21} + \sqrt{5}\right)x + \sqrt{8} = 0 \Rightarrow x_1x_2 = 2$

Length of tangent $OP = \sqrt{OA \cdot OB} = \sqrt{8x_1x_2} = 4$

42. D

Sol. R has two positions

$\Rightarrow$ Q and T have two positions

$\Rightarrow$ QT has two values 2 and 10

As PQ_1R_1 and PQ_1S_1 are congruent

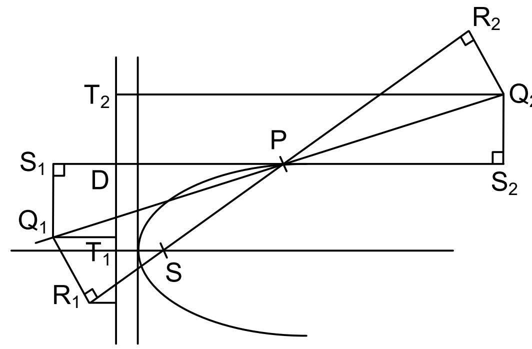
$\Rightarrow PR_1 = PS_1$ also PS = PD

$\Rightarrow Q_1T_1 = DS_1 = 2$

Similarly Δs PQ_2R_2

and PQ_2S_2 are also congruent

$\Rightarrow Q_2T_2 = 10$



43. B, D

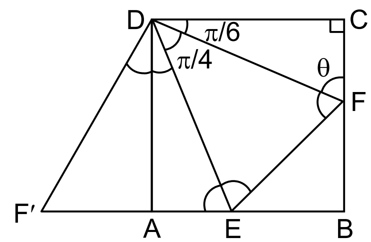
Sol. As FD = 2FC $\Rightarrow \frac{FC}{FD} = \frac{1}{2} \Rightarrow \cos\alpha = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

Paste DCF such that C coincides with A and F goes to F′

Clearly triangles DEF′ and DEF are congruent

$\Rightarrow \angle DEF = \angle DEF' = \frac{5\pi}{12}$ and $\angle DFE = \frac{\pi}{3}$

Clearly D is excentre of Δ FBE $\Rightarrow$ Ex-radius = CD = 1 unit



44. A, C

Sol. Point of tangency is the mid-point of QR and hence lies on hyperbola $4x^2 + 8xy - y^2 + \lambda = 0$

$\Rightarrow \lambda = -4$

Let the circumcircle of ΔOQR be $x^2 + y^2 - 2ax - 2by = 0$

Homogenizing and comparing with pair of asymptotes, we get a – b = 1

45. B, C, D

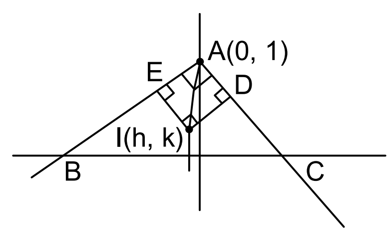
Sol. Two of the three lines represented are perpendicular to each other and intersecting at (0, 1) which is orthocentre a fixed point.

Clearly circumcentre lies on x-axis

We have $AI = k\sqrt{2}$

$\Rightarrow (h - 0)^2 + (k - 1)^2 = (k\sqrt{2})^2$

$\Rightarrow x^2 - y^2 - 2y + 1 = 0$



46. A, C

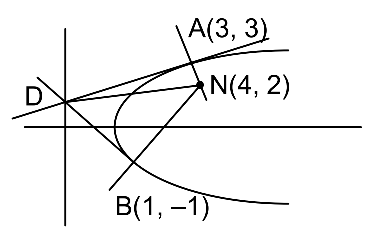
Sol. Clearly AN $\perp$ BN $\Rightarrow$ AB is a focal chord

$\Rightarrow D \equiv (0, 0)$

Directrix will be perpendicular to DN

$\Rightarrow$ Slope of directrix = –2. Focus will be foot of perpendicular from D on focal chord 2x – y – 3 = 0

$\Rightarrow S \equiv \left(\frac{6}{5}, -\frac{3}{5}\right)$



47. A, B, C

Sol. With centre at $(2, \sqrt{3})$ equation of circle is $(x-2)^2 + (y-\sqrt{3})^2 = r^2$

Let (x_2, y_2) and (x_3, y_3) be two rational points

$\Rightarrow x_2^2 + y_2^2 - 4x_2 - 2\sqrt{3}y_2 + 7 - r^2 = 0$ ..... (1)

and $x_3^2 + y_3^2 - 4x_3 - 2\sqrt{3}y_3 + 7 - r^2 = 0$ ..... (2)

Subtracting (1) and (2), we get

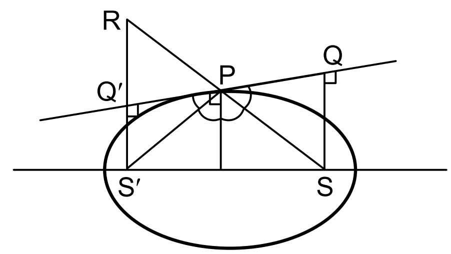
$(x_2 - x_3)(x_2 + x_3 - 4) + (y_2 - y_3)(y_2 + y_3 - 2\sqrt{3}) = 0$

For two rational points $\Rightarrow y_2 - y_3 = 0 \Rightarrow$ slope PQ = 0 for only one rational point

$\Rightarrow x_2 = x_3 \Rightarrow x_1 = 2$

48. B, C

Sol. Clearly Δs PSQ and PS′Q′ are similar



$\Rightarrow \frac{SP}{S'P} = \frac{SQ}{S'Q'} = 2 \Rightarrow SP = 8$

Also, S′P = PR = 4 $\Rightarrow$ SR = 12

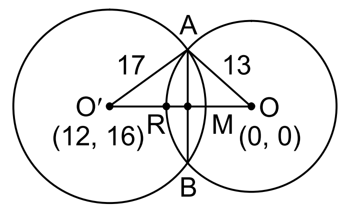
Also, SQ.S′Q′ = $b^2 = a^2 - a^2e^2$

$\Rightarrow \frac{SQ^2}{2} = 36\left(1 - \frac{1}{4}\right) = 27 \Rightarrow SQ = 3\sqrt{6}$

SECTION – C

49. 00065.72

Sol. Let M be the mid-point of AB



$\Rightarrow AM^2 = 17^2 - x^2 = 13^2 - (20 - x)^2$

$\Rightarrow x = 13$

$\Rightarrow AB = 4\sqrt{30}$, clearly when C lies at R

Area is maximum $\Rightarrow$ Maximum area = $\frac{1}{2} \times 4\sqrt{30} \times 6$

= $12\sqrt{30}$ sq. units = 65.72

50. 00001.34

Sol. Let the equation of hyperbola be $\frac{x^2}{A^2} - \frac{y^2}{B^2} = 1$ ..... (1)

Eccentricity (e) of the ellipse $e = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$

As hyperbola intersects the ellipse orthogonally and passes through vertices

$\Rightarrow$ Their foci are coincident $\Rightarrow Ae_H = 3 \Rightarrow B^2 = A^2e_H^2 - A^2 = 9 - A^2$

Again hyperbola must pass through (5, 4) the vertex of rectangle

$\Rightarrow \frac{25}{A^2} - \frac{16}{9 - A^2} = 1 \Rightarrow A^4 - 50A^2 + 225 = 0 \Rightarrow A^2 = 5$ only

$\Rightarrow$ Equation of hyperbola is $\frac{x^2}{5} - \frac{y^2}{4} = 1,\ e = \sqrt{1 + \frac{4}{5}} = 1.34$

51. 00001.41

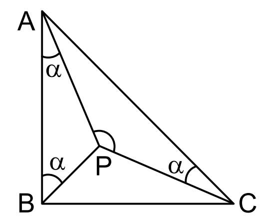
Sol. $S = \tan^{-1}\left(\frac{1}{2\sqrt{2}}\right) + \tan^{-1}\left(\frac{1}{4\sqrt{2}}\right) + \tan^{-1}\left(\frac{1}{7\sqrt{2}}\right) + ...$

$= \tan^{-1}\left(\frac{\left(\frac{2}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)}{1 + \frac{2}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}}}\right) + \tan^{-1}\left[\frac{\frac{3}{\sqrt{2}} - \frac{2}{\sqrt{2}}}{1 + \frac{3}{\sqrt{2}} \cdot \frac{2}{\sqrt{2}}}\right] + ... \infty$ terms

$= \frac{\pi}{2} - \tan^{-1}\frac{1}{\sqrt{2}} = \frac{\pi}{2} - \cot^{-1}\sqrt{2} = \tan^{-1}\left(\sqrt{2}\right)$

52. 00001.50

Sol. Applying sin rule in Δs PAB and PAC, we get



$\frac{PA}{\sin\alpha} = \frac{AB}{\sin 2\alpha}$ ..... (1)

$\frac{PA}{\sin\alpha} = \frac{AC}{\sin\frac{3\pi}{4}}$ ..... (2)

$\Rightarrow \sin 2\alpha = \frac{1}{2} \Rightarrow \alpha = \frac{\pi}{12}$. Maximum value of $f(\theta) = \sqrt{\sin^2 2\alpha + \text{cosec}^2 3\alpha} = \sqrt{\frac{1}{4} + 2} = \frac{3}{2}$

53. 04319.69

Sol. $\sum_{q=1}^{10}\sum_{r=1}^{10}\sum_{p=1}^{10} p\tan^{-1}\left(\frac{q}{r}\right) = \sum_{q=1}^{10}\sum_{r=1}^{10}\tan^{-1}\left(\frac{q}{r}\right)\times\frac{10\times 11}{2} = 55.[25\pi] = 4319.69$

54. 00014.50

Sol. Let P = sin 3°.sin 9°.sin 15° ..... sin 81°.sin 87°

Q = sin 6°.sin 12°.sin 18° ..... sin 84°

PQ = sin 3°.sin 6°.sin 9°.sin 12° ..... sin 81°.sin 84°.sin 87°

$= \frac{1}{2^{14}}\cdot(2\sin 3°\cdot\cos 3°)\cdot(2\sin 6°\cdot\cos 6°)(2\sin 9\cdot\cos 9°).....(2\sin 42°\cdot\cos 42°)\cdot\sin 45°$

$= \frac{1}{2^{14.5}}\cdot Q$

$\Rightarrow P = \frac{1}{2^{14.5}}$

$\Rightarrow x = 14.50$