FIITJEE

ALL INDIA TEST SERIES

PART TEST – I

JEE (Main)-2021

TEST DATE: 05-12-2020

ANSWERS, HINTS & SOLUTIONS

Physics PART – I

SECTION – A

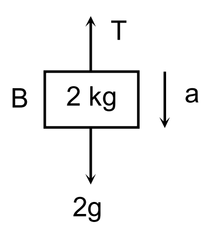
1. D

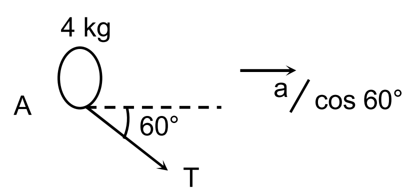
Sol. For block B, $2g - T = 2a$ …(1)

For ring A, $T\cos 60^\circ = \dfrac{4a}{\cos 60^\circ}$ …(2)

Solving (1) & (2)

$$\frac{a}{\cos 60^\circ} = \frac{2g}{9}$$





2. B

Sol. From conservation of mechanical energy

$$mgh = mg\ell\sin\theta = \frac{mv^2}{2} + \frac{1}{2}\frac{mR^2}{2}\frac{v^2}{R^2}$$

$$\Rightarrow \quad v = \sqrt{\frac{4g\ell\sin\theta}{3}}$$

$\therefore \quad L = mvR + I_{CM}\,\omega$

$$= mvR + \frac{mR^2}{2}.\frac{v}{R} = \frac{3}{2}mvR$$

$$\therefore \quad L = \sqrt{3m^2R^2g\ell\sin\theta}$$

3. C

Sol. Velocity of B just before the string is taut is $v_1 = \sqrt{u^2 - 2gL} = \sqrt{4gL}$

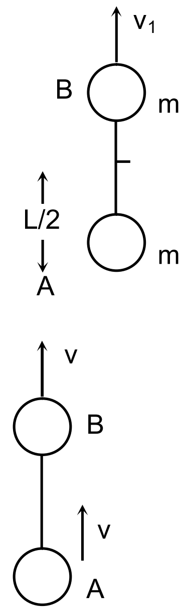
After string is taut, both the bodies will move up with the same speed v.

$\therefore \quad v = \frac{mv_1}{m+m}$

$\Rightarrow \quad v = \sqrt{gL}$

Rise in the centre of mass = $\frac{v^2}{2g} = \frac{L}{2}$

$\therefore$ Maximum height of centre of mass = $\frac{L}{2} + \frac{L}{2} = L$



4. C

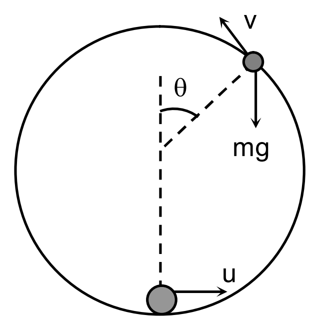
Sol. $mg\cos\theta = \frac{mv^2}{R}$ ...(1)

using conservation of energy,

$\frac{1}{2}mu^2 = mgR(1+\cos\theta) + \frac{mv^2}{2}$

$\Rightarrow \quad \frac{1}{2}mu^2 = mgR(1+\cos\theta) + \frac{mgR\cos\theta}{2}$

Solving we get, $\theta = 53°$



5. A

Sol. Acceleration of each block is $a = \frac{4g - \mu(4g)}{4+4}$ = 4 m/s²

$\therefore \quad a_{CM} = 2\sqrt{2}$ m/s²

6. A

Sol. $(\lambda\pi Rg)\left(\frac{2R}{\pi}\right) = (\lambda Lg).\frac{L}{2}$

$\Rightarrow \quad \frac{R}{L} = \frac{1}{2}$

7. A

Sol. Let y = height of point B above the ground.

$\therefore \quad \frac{v\sin 53° - gt}{v\cos 53° - a_0 t} = 1$ …(1)

$v\cos 53° t + x = \frac{1}{2}a_0 t^2$ …(2)

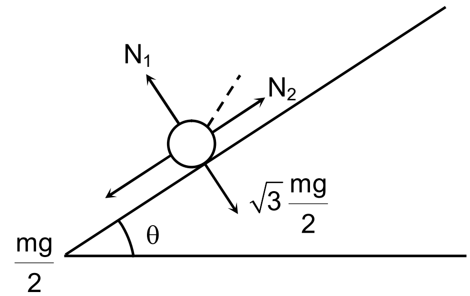
And $y = \left(v\sin 53^\circ\right)t - \frac{1}{2}gt^2$ …(3)

Solving (1), (2) & (3); we get

$a_0 = \frac{130}{15}$

8. B

Sol. Applying COME at P & Q



$2Mg = \frac{Mv^2}{2}$ …(1)

$N_1 = Mg\frac{\sqrt{3}}{2}$

$N_2 - \frac{Mg}{2} = \frac{Mv^2}{R} = 2Mg$ [from (1)]

$\therefore \quad N_2 = \frac{5}{2}Mg$

$\therefore$ Contact force = $\sqrt{N_1^2 + N_2^2} = \sqrt{7}\,Mg$.

9. A

Sol. $F.x = \frac{m\ell^2}{3}\alpha \Rightarrow \alpha = \frac{3Fx}{m\ell^2}$

$\therefore \quad a_{CM} = \frac{\alpha\ell}{2} = \frac{3Fx}{m\ell^2}.\frac{\ell}{2} = \frac{3Fx}{2m\ell}$

Hence (A) is correct answer.

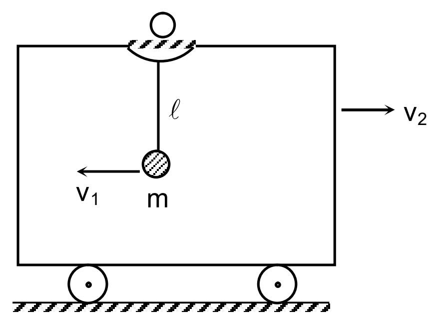
10. D

Sol. $mv_1 = 6mv_2$ …(1)

$mg\ell = \frac{1}{2}mv_1^2 + \frac{1}{2}6mv_2^2$ …(2)

From (1) & (2)

$v_{rel} = v_1 + v_2 = \sqrt{\frac{7}{3}g\ell}$



11. C

Sol. Only z-component of velocity will change.

$\vec{v}_f = 15\hat{i} - 20\hat{j} + 10\hat{k}$.

12. D

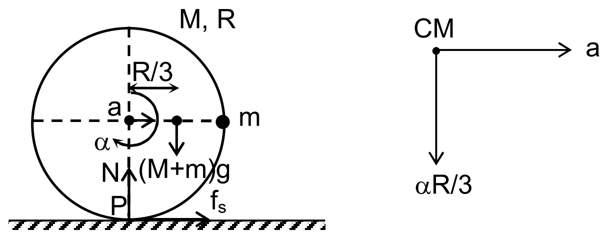
Sol. $r_{CM} = \frac{mR + 0}{M + m} = \frac{R}{3}$ (M = 2m)

For rolling without slipping

a = αR

$I_P = \frac{3MR^2}{2} + m(R\sqrt{2})^2$

$I_P = 3mR^2 + 2mR^2 = 5mR^2$



Now, $\tau_P = I_P\alpha$

$$(M+m)g\frac{R}{3} = 5mR^2\alpha$$

$$mgR = 5mR^2\alpha \qquad \Rightarrow \alpha = \frac{g}{5R}$$

$$f_s = (M+m)a$$

$$f_s = \frac{3mg}{5} = \frac{3\times 0.5\times 10}{5} = 3\ \text{N}$$

$f_s = 3N$

13. A

Sol. $a_t = t \Rightarrow v = \frac{t^2}{2}$

$$a_h = \frac{v^2}{R} = \frac{t^4}{4(2)} = \frac{t^4}{8}$$

$\because\ a_t = a_4$

$\Rightarrow\ t = \frac{t^4}{8}$

$\therefore$ t = 2 sec

14. D

Sol. $$2m\left(\frac{(2m)+(2m)}{m+2m}\right)g = \mu(2m)(g)$$

$\Rightarrow\ \frac{4}{3} = \mu$

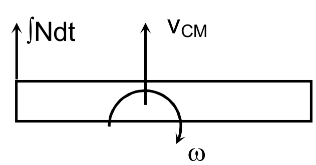
15. D

Sol. Since collision is elastic;

$$v = v_{CM} + \frac{\omega L}{2} \qquad \ldots(1)$$

$$\int Ndt = M(v_{CM} + v) \qquad \ldots(2)$$

$$\frac{L}{2}\int Ndt = \frac{ML^2}{12}\omega \qquad \ldots(3)$$



From (1), (2) & (3),

$$\omega = \frac{3v}{L}$$

16. B

Sol. From graph,

U = 4 sin (πx) + 4

$\therefore\ F = -\frac{dU}{dx} = -4\pi\cos\pi x$

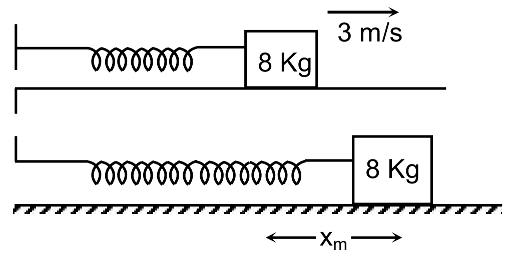
$\therefore\ F_{max} = 4\times 3.14 = 12.56$

17. C

Sol. w.r.t free end of spring initial and final stage are shown.
From conservation of mechanical energy.

$$\frac{1}{2}mv^2 = \frac{1}{2}Kx_m^2$$

$$\therefore \quad x_m = \sqrt{\frac{m}{k}}.v = 6\,cm$$



18. D

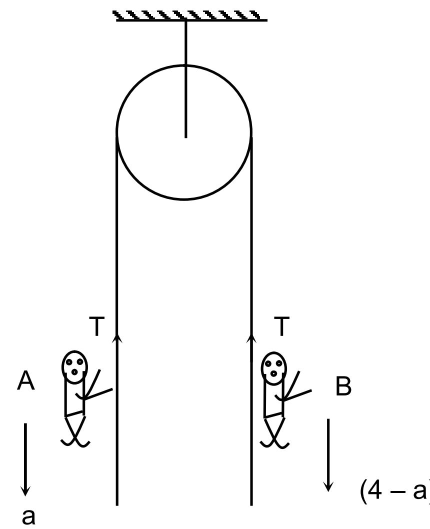
Sol. a = 2 sin πt

$$\Rightarrow \quad v = \frac{2}{\pi}(1 - \cos \pi t)$$

$$\therefore \quad \Delta x = \int_0^{t_0} \frac{2}{\pi}(1 - \cos \pi t)dt$$

19. B

Sol. 10 g – T = 10 (4 – a) …(1)
For A 6g – T = 6a …(2)
From (1) and (2)
a = 0
$\therefore$ T = 60 N



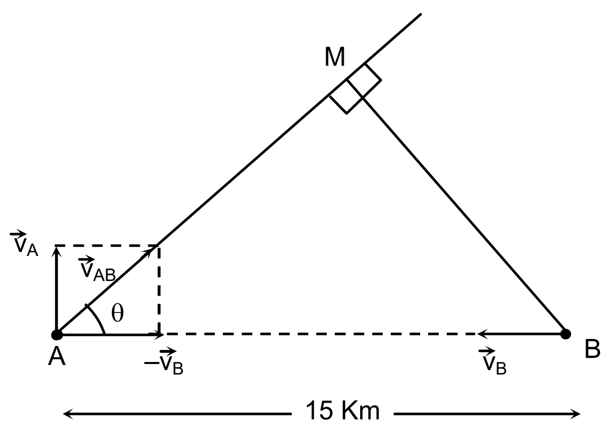
20. B

Sol. $|\vec{V}_{AB}|$ = 50 m/s

AM = (15000) cosθ

$$= (15000)\left(\frac{30}{50}\right) = 9000\text{ m}$$

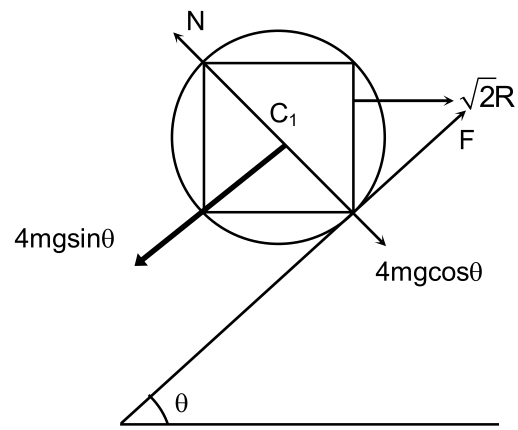
$$\therefore \quad t = \frac{9000\,m}{50\,m/s} = 180\text{ sec.}$$



SECTION – B

21. 3

Sol. $I = 4\left[\frac{m\left(\sqrt{2}R\right)^2}{12} + m\left(\frac{R}{\sqrt{2}}\right)^2\right] = \frac{8}{3}mR^2$



For translatory motion;

$4mg\sin\theta - f = 4ma$ …(1)

For rotational motion; $fR = I\alpha$

$\Rightarrow$ $fR = \frac{8}{3}mR^2\alpha$ …(2)

Condition for Rolling without slipping;

$a = R\alpha$ …(3)

From (1), (2) & (3);

$f = \frac{8mg\sin\theta}{5} \le \mu_S N$

Here; $N = 4\,mg\cos\theta$

$\Rightarrow$ $\mu_S \ge \frac{2}{5}\tan\theta$

$\therefore$ $\mu_S = \frac{2}{5}\tan 37^\circ$

$= \frac{2}{5} \times \frac{3}{4} = \frac{3}{10}$

$\therefore$ N = 3

22. 4

Sol. Momentum of A at time 't' is

$P_A = m_A v_A = m_A\left(16 - \mu g t\right)$

And $P_B = m_B v_B = m_B\left(\frac{\mu m_A g}{m_B}\right)t$

$\because$ $P_A = P_B$

$\Rightarrow$ $16 - \mu gt = \mu gt$

$\Rightarrow$ $\mu gt = 8$

$\therefore$ $t = \frac{8}{(0.2)(10)} = 4$ sec

$\therefore$ t = 4 sec.

SECTION – C

23. 00030.00

Sol. Let v_1 and v_2 are initial and final speed of stone respectively

$\therefore$ $v_2^2 = v_1^2 - 2gh$ …(1)

Also; $v_1\cos 60^\circ = v_2\cos 30^\circ$ …(2)

From (1) & (2)

$v_1 = \sqrt{3gh} = \sqrt{3 \times 10 \times 30}$ = 30 m/s

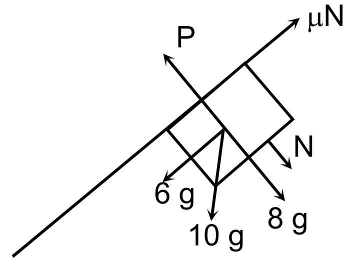
24. 00200.00

Sol. $P = N + 8g$

$\mu N = 6g$

$\Rightarrow \mu(P - 8g) = 6g$

$\Rightarrow P = 8g + \frac{6g}{\mu} = 200\,N$



25. 00320.00

Sol. Collision takes place at t = 4 sec. and 80 m below. In next 4 sec. it will fall 240 m, before striking the ground.

$\therefore$ H = 240 m + 80 m

H = 320 m

Chemistry

PART – II

SECTION – A

26. D

Sol. BeH_2 is covalent and polymeric hydride.

27. B

28. D

Sol. H_3BO_3 is water soluble.

29. C

Sol. Polarization increases acidity of oxides increases

$\therefore$ The correct order of acidic nature is

$Na_2O < MgO < Al_2O_3 < SiO_2$

$\therefore$ Na_2O is most basic oxide

30. D

Sol. Number of unpaired electron in N_2^+ and N_2^- is same.

$\therefore$ Both have same magnetic moment.

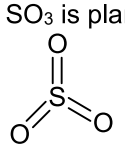
31. C

Sol. (A) NH_3 is polar with all equal bond angle

(B) NH_3 is non – planar molecule with all bond angle same

(C) BF_3 is planar with all bond angle same while NH_3 is non – planar with all bond angle same

(D) SO_3 is planar molecule with 3π-bond



32. B

Sol. Minimum energy required to excite a hydrogen atom from ground state is 10.2 eV.

33. B

Sol. $$K_c = \frac{[C][D]^3}{[A]^2[B]} = \frac{\left(\frac{2}{V}\right)\times\left(\frac{2}{V}\right)^3}{\left(\frac{2}{V}\right)^2\left(\frac{2}{V}\right)}$$

$$2 = \frac{2}{V} \qquad \therefore V = 1\,\text{litre}$$

34. B

Sol. $K_{sp} = 32 \times 10^{-12}$ $\quad CaF_2 \rightleftharpoons Ca^{2+} + 2F^-$

$K_{sp} = 4s^3$

$\therefore 4s^3 = 32 \times 10^{-12}$

$\therefore S = 2 \times 10^{-4}$

$\therefore [F^-] = 2s = 2 \times 2 \times 10^{-4} = 4 \times 10^{-4}$

35. C

Sol.
$$A \longrightarrow 2B$$
$$t = 0 \quad C \qquad 0$$
$$t = 20 \quad C - x \qquad 2x$$
At t = 20 min

C – x = 2x

$\therefore$ x = C/3

$$\therefore [A]_t \text{ at } t = 20 \text{ minutes} = C - \frac{C}{3} = \frac{2C}{3}$$
$$K = \frac{2.303}{20}\log\frac{C}{2C/3} = \frac{2.303}{20}\log\left(\frac{3}{2}\right)$$
$\therefore$ time for 75% completion of reaction = $2t_{1/2}$
$$= 2 \times \frac{0.693}{K} = \frac{2 \times 2.303 \log 2}{\frac{2.303}{20} \times \log(1.5)}$$
$$t = \frac{2 \times 20 \times \log 2}{\log(1.5)} = 68.38 \text{ minutes}$$

36. B

Sol. $_{92}U^{238} \longrightarrow {}_{82}Pb^{206}$

Number of α - particle emitted = $\frac{238 - 206}{4} = 8$

$$_{92}U^{238} \xrightarrow{-8\alpha} {}_{76}X^{206} \xrightarrow{-6\beta} {}_{82}Pb^{206}$$

Moles of $_{92}U^{238}$ decayed in $2t_{1/2}$ = $0.1 \times \frac{3}{4}$

$\therefore$ Total number of β - particle emitted = $0.1 \times \frac{3}{4} \times 6 = 0.45$ moles

37. B

Sol. 3rd ionization energy of Mg would be very high.

38. C

Sol. Equation (iv) can be obtained by

(ii) + 2 (i) – 3/2 (iii)

$$\therefore K = \frac{K_2 . K_1^2}{K_3^{3/2}}$$

39. C

Sol.
$$\begin{array}{lllll} CH_3COONa & + HCl & \longrightarrow & CH_3COOH & + NaCl \\ 0.1V & 0.2V & & 0 & \\ 0 & 0.1V & & 0.1V & \end{array}$$
Resulting solution is mixture of S.A. and W.A., so cannot act as buffer.

40. B

Sol. Balance reaction:
$$3BaCl_2 + 2Na_3PO_4 \longrightarrow Ba_3(PO_4)_2 + 6NaCl$$
$\therefore$ 3/2 mole of $BaCl_2$ needed

41. C

42. D
Sol. IE_1 of P > IE_1 of S

43. B
Sol. Number of neutrons is same for isotone.
$_{32}Ge^{76}$ $n = 76 - 32 = 44$
$_{38}As^{77}$ $n = 77 - 33 = 44$

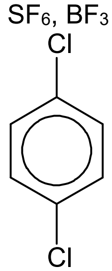
44. C
Sol. $\mu = 2 \quad \therefore K_{45} = 4\,K_{25}$
$$t_{1/2}\left(25^0 C\right) = \frac{0.693}{K_{25}} = 10 \text{ min}$$
$$\therefore K_{25} = \frac{0.693}{10}\text{min}^{-1}$$
$$\therefore K_{45} = \frac{0.693}{10} \times 4 \text{ min}^{-1}$$
$$\therefore t_{1/2}\left(45^0 C\right) = \frac{0.693}{0.693 \times 4/10} = 2.5 \text{ min}$$
$\therefore$ t = 2.5 × 60 = 150 seconds

45. D
Sol. Addition of inert gas of constant pressure causes backward reaction if $\Delta n_g < 0$.

SECTION – B

46. 7
Sol.
$$\frac{\bar{\nu}_{Li^{2+}}}{\bar{\nu}_H} = \frac{R_H (3)^2 \left(\frac{1}{3^2} - \frac{1}{4^2}\right)}{R_H (1)^2 \left(\frac{1}{4^2} - \frac{1}{\infty^2}\right)}$$
$$= \frac{9 \times \frac{(16-9)}{(16 \times 9)}}{\frac{1}{16}} = 7$$

47. 4
Sol. SF_6, BF_3 PCl_3F_2



has zero-dipole moment.

SECTION – C

48. 00002.00

Sol. Balanced reaction is

$$Cu + 4HNO_3 \longrightarrow Cu(NO_3)_2 + 2NO_2 + 2H_2O$$

$$\therefore \text{Ratio} = \frac{4}{2} = 2$$

49. 00003.25

Sol. (i) when 80 ml NaOH is added:

CH_3COOH	$+ NaOH$	$\longrightarrow$	$CH_3COO^-Na^+$	$+ H_2O$
100×0.1	80×0.1		0	
$= 10$ m mol	$= 8$ m mol			
2 m mol			8 m mol	

$$pH = pK_a + \log\frac{8/180}{2/180} = 5 + \log 4$$

$$= 5 + 2 \times 0.3 = 5.6$$

pOH = 14 – 5.6 = 8.4

(ii) When 100 ml NaOH is added:

CH_3COOH	$+ NaOH$	$\longrightarrow$	$CH_3COO^-Na^+$	$+ H_2O$
100×0.1	100×0.1		0	
$= 10$ m mol	$= 10$ m mol			
0	0		10 m mol	

$$pH = 7 + \frac{1}{2}\left[pK_a + \log\left(\frac{10}{200}\right)\right] = 7 + \frac{1}{2}(5 - 1.3) = 8.85$$

pOH = 14 – 8.85 = 5.15

$\therefore$ pOH (i) – pOH (ii) = 8.4 – 5.15 = 3.25

50. 00000.01

Sol.

HA	$+ NaOH$	$\longrightarrow$	NaA	$+ H_2O$
$0.1 \times V$	$0.1 \times V$		0	
0	0		0.1 V	

$$[NaA] = \frac{0.1\,V}{2V} = 0.05\text{ M}$$

K_b (HA) = 5×10^{-10}

$$\therefore K_a = \frac{10^{-14}}{5 \times 10^{-10}} = 2 \times 10^{-5}$$

$$h = \sqrt{\frac{K_w}{K_a.c}} = \sqrt{\frac{10^{-14}}{2 \times 10^{-5} \times 0.05}} = 10^{-4}$$

% hydrolysis = $10^{-4} \times 100 = 0.01$ %

Mathematics

PART – III

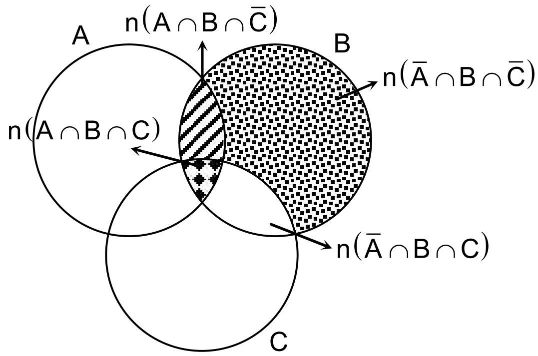
SECTION – A

51. D

Sol. So $n(B) = n(\bar{A}\cap B\cap\bar{C}) + n(A\cap B\cap\bar{C}) + n(A\cap B\cap C) + n(\bar{A}\cap B\cap C)$

230 = 100 + 50 + 10 + $n(\bar{A}\cap B\cap C)$

$n(\bar{A}\cap B\cap C) = 70$



52. C

Sol. $$\lim_{t\to\infty}\frac{\int_{1/3}^{\cot 1}[\cot^{-1}x]dx + \int_{\cot 1}^{t}[\cot^{-1}x]dx}{\int_{1/2}^{1}\left[1+\frac{1}{x}\right]dx + \int_{1}^{t}\left[1+\frac{1}{x}\right]dx} = \lim_{t\to\infty}\frac{\int_{1/3}^{\cot 1}dx + \int_{\cot 1}^{t}0\,dx}{\int_{1/2}^{1}2\cdot dx + \int_{1}^{t}dx}$$

$$= \lim_{t\to\infty}\frac{\left(\cot 1-\frac{1}{3}\right)+0}{2\left(1-\frac{1}{2}\right)+(t-1)} = \lim_{t\to\infty}\frac{\cot 1-\frac{1}{3}}{1+t-1} = \frac{\cot 1-\frac{1}{3}}{t} = 0$$

53. D

Sol. f(|x|) = |3 – 2|x||, then $-1 \le f(|x|) \le 1$

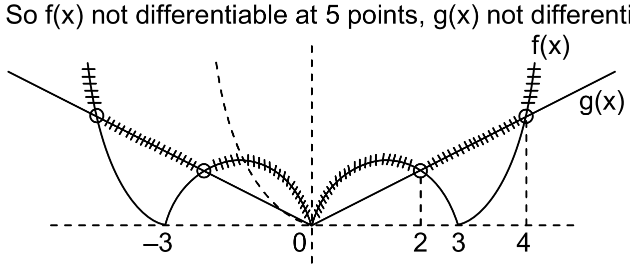
$\Rightarrow |3-2|x|| \le 1 \Rightarrow -1 \le 3-2|x| \le 1 \Rightarrow -1 \le 2|x|-3 \le 1$

$2 \le 2|x| \le 4,\ 1 \le |x| \le 2$

$x \in [-2, -1] \cup [1, 2]$

54. A

Sol. So f(x) not differentiable at 5 points, g(x) not differentiable at 7 points



55. C

Sol. $f''(x) = p^2 (3)^{px} \cdot (\log_e 3)^2 + q^2 (3)^{qx} (\log_e 3)^2$

$f'(x) = p(3)^{px} (\log_e 3) + q(3)^{qx} (\log_e 3)$

So, $p^2 \cdot 3^{px} + q^2 (3)^{qx} - 2p3^{px} - 2q(3)^{qx} - 15(3)^{px} - 15(3)^{qx} = 0$

$(p^2 - 2p - 15)(3)^{px} + (q^2 - 2q - 15)(3)^{qx} = 0$

So, $p^2 - 2p - 15 = 0$, $q^2 - 2q - 15 = 0$

$(p - 5)(p + 3) = 0$, $(q - 5)(q + 3) = 0$

$p = 5, -3$, $q = 5, -3$

Since, $p \neq q$, $p = 5$, $q = -3$

$p = -3$, $q = 5$

$|p + q| = 2$

56. C

Sol. Let $g(x) = \frac{ax^5}{5} + \frac{bx^3}{3} + \frac{cx^2}{2} + dx + e$

$g(0) = e$

$g(2) = \frac{32a}{5} + \frac{8b}{3} + \frac{4c}{2} + 2d + e$

$g(2) = \frac{192a + 80b + 60c + 60d}{30} + e$

$g(2) = \frac{16(12a + 5b) + 60(c + d)}{30} + e$

$g(2) = e$

So $g'(x) = ax^4 + bx^2 + cx + d = 0$ has at least one root on (0, 2)

57. D

Sol. Let points be $P(x_1, y_1)$, $Q(x_2, y_2)$, $R(x_3, y_3)$

So, $y_1 = 2x_1^3 - 4x_1 + 2$ ….. (1)

$y_1 = x_1^3 + 2x_1 - 1$

$2y_1 = 2x_1^3 + 4x_1 - 2$ ….. (2)

$y_1 = 2x_1^3 - 4x_1 + 2$ ….. (1)

$y_1 = 8x_1 - 4$, $y_2 = 8x_2 - 4$

$y_2 - y_1 = 8x_2 - 4 - 8x_1 + 4 = 8(x_2 - x_1)$

$\frac{y_2 - y_1}{x_2 - x_1} = 8$

58. B

Sol. Let p(x) is a polynomial of degree n

So, 3n = 2 + 2n

n = 2, so p(x) is a quadratic equation

$p(x) = ax^2 + bx + c$, $p(0) = -3$, $c = -3$

$a + b + c = 2$, $a + b = 5$

$p'(x) = 2ax + b$, $2a \cdot \frac{1}{3} + b = 0, b = -\frac{2a}{3}$

$a - \frac{2a}{3} = 5$, $a = 15$, $b = -10$, $c = -3$

$p(x) = 15x^2 - 10x - 3$

$p(3) = 15 \times 9 - 30 - 3 = 135 - 33 = 102$

59. C

Sol. $\lambda = \sum_{n=1}^{\infty} \frac{(12)^n}{4^n\left(1-\left(\frac{3}{4}\right)^n\right)\cdot 3^n\left(4\cdot\left(\frac{4}{3}\right)^n-3\right)} = \sum_{n=1}^{\infty} \frac{(12)^n}{(12)^n\left(1-\left(\frac{3}{4}\right)^n\right)\left(\frac{4}{3}\right)^n\left(4-3\cdot\left(\frac{3}{4}\right)^n\right)}$

$= \sum_{n=1}^{\infty} \frac{\left(\frac{3}{4}\right)^n}{\left(1-\left(\frac{3}{4}\right)^n\right)\left(4-3\left(\frac{3}{4}\right)^n\right)} = \sum_{n=1}^{\infty} \frac{\left(4-3\left(\frac{3}{4}\right)^n\right)-4\left(1-\left(\frac{3}{4}\right)^n\right)}{\left(1-\left(\frac{3}{4}\right)^n\right)\left(4-3\left(\frac{3}{4}\right)^n\right)} = \sum_{n=1}^{\infty} \frac{1}{\left(1-\left(\frac{3}{4}\right)^n\right)} - \frac{4}{4-3\left(\frac{3}{4}\right)^n}$

$= \left(\frac{1}{1-\frac{3}{4}} - \frac{4}{4-3\left(\frac{3}{4}\right)}\right) + \left(\frac{1}{1-\left(\frac{3}{4}\right)^2} - \frac{4}{4-3\left(\frac{3}{4}\right)^2}\right) + \ldots + \left(\frac{1}{\left(1-\left(\frac{3}{4}\right)^n\right)} - \frac{4}{4-3\left(\frac{3}{4}\right)^n}\right)$

$= \left(\frac{1}{1-\frac{3}{4}} - \frac{4}{4-3\left(\frac{3}{4}\right)^n}\right) = 4-1 = 3$, $\lambda = 3$

60. D

Sol. $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{-xh+f(h)}{h} = \lim_{h\to 0} -x + \frac{f(h)}{h}$

$f'(x) = -x + 2$

$f(x) = 2x - \frac{x^2}{2} + c$

$f(5) - f(3) = 10 - \frac{25}{2} + c - 6 + \frac{9}{2} - c = 4 - \frac{16}{2} = 4 - 8 = -4$

61. A

Sol. $a(x)^n = -3x^5 - bx^2 - c$

It will have infinite solutions if a = –3, n = 5, b = 0, c = 0

a + b + c + n = 2

62. C

Sol. $\int \frac{9\cos 2x + 12\cos x - 6\sin x}{(2+3\sin x)(4+3\cos x)}dx = \int \frac{3\cos x(4+3\cos x)+(2+3\sin x)(-3\sin x)}{(2+3\sin x)(4+3\cos x)}dx$

$= \int \frac{3\cos x}{2+3\sin x}dx + \int \frac{-3\sin x}{4+3\cos x}dx$ = log(2 + 3 sin x) + log (4 + 3 cos x) + c

= log((2 + 3 sin x)(4 + 3 cos x)) + c

63. A

Sol. $\int \frac{x^4+3}{\sqrt{x^4(x^4+x^2-3)}}dx = \int \frac{x^4+3}{x^2\sqrt{x^2\left(x^2-\frac{3}{x^2}+1\right)}}dx$

$= \int \frac{x+\frac{3}{x^3}}{\sqrt{x^2-\frac{3}{x^2}+1}}dx$, Let $t^2 = x^2 - \frac{3}{x^2} + 1$

$2t\,dt = \left(2x + \frac{6}{x^3}\right)dx$, $t\,dt = \left(x + \frac{3}{x^3}\right)dx$

$= \int \frac{t\,dt}{t} = t + c = \sqrt{x^2 - \frac{3}{x^2} + 1} + c = \frac{\sqrt{x^4+x^2-3}}{x} + c$

64. C

Sol. $\int x^x \cdot \log_e x\left(x\left(1+\log_e x\right)\right)dx = \int \left(x^x\left(1+\log_e x\right)\right)\left(x\log_e x\right)dx$ (II, I)

$= x\log_e x \cdot x^x - \int\left(1\ \log_e x + x\cdot\frac{1}{x}\right)x^x dx = x^{x+1}\cdot\log_e x - x^x + c$

$= x^x\left(\log_e x^x - 1\right) + c = x^x \log_e\left(\frac{x^x}{e}\right) + c$

65. B

Sol. $f(2x)\,f'(-2x) = f(-2x)f'(2x)$, $f'(2x)f(-2x) - f(2x)f'(-2x) = 0$

$\frac{d}{dx}\left(f(2x)f(-2x)\right) = 0$, $f(2x)f(-2x) = c$ at $x = 0$, $f^2(0) = c = 16$

$f(2x)f(-2x) = 16$

$$I = \int_{-24}^{24}\frac{dx}{4+f(2x)}dx \quad \text{..... (1)}$$

$$I = \int_{-24}^{24}\frac{dx}{4+f(-2x)}dx \quad \text{..... (2)}$$

Adding equation (1) and (2), we get

$$2I = \int_{-24}^{24}\left(\frac{1}{4+f(2x)} + \frac{1}{4+f(-2x)}\right)dx$$

$$2I = \int_{-24}^{24}\frac{8+f(2x)+f(-2x)}{16+4\left(f(2x)+f(-2x)\right)+f(2x)f(-2x)}dx$$

$$2I = \int_{-24}^{24}\frac{8+f(2x)+f(-2x)}{4\left(8+f(2x)+f(-2x)\right)}dx = \int_{-24}^{24}\frac{dx}{4}$$

$$2I = \frac{1}{4}\times 48 = 12 \ \therefore\ I = 6$$

66. C

Sol. $\int_0^{\pi/8}\left[\sec x + 2\left[\tan x + 3[\cos x] + 12[\sin x]\right]\right]dx$

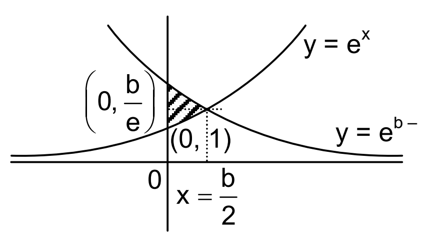
$= \int_0^{\pi/8}[\sec x + 2[\tan x] + 6[\cos x] + 24[\sin x]]dx$

$= \int_0^{\pi/8}[\sec x]dx + 2\int_0^{\pi/8}[\tan x]dx + 6\int_0^{\pi/8}[\cos x]dx + 24\int_0^{\pi/8}[\sin x]dx$

$= \int_0^{\pi/8}[\sec x]dx = \int_0^{\pi/8}dx = \frac{\pi}{8}$

67. B

Sol. $S_1 : y = e^x$, $S_2 : y = e^{b-x}$, $e^x = e^{b-x}$, $x = \frac{b}{2}$



Area enclosed $A = \int_0^{b/2}\left(e^{b-x} - e^x\right)dx = \left(e^{b/2}-1\right)^2$

$\lim_{b\to 0}\frac{3}{4}\cdot\frac{\left(e^{b/2}-1\right)^2}{4\left(\frac{b^2}{4}\right)} = \frac{3}{16}$, $\lim_{b\to 0}\left(\frac{e^{b/2}-1}{\frac{b}{2}}\right)^2 = \frac{3}{16}$

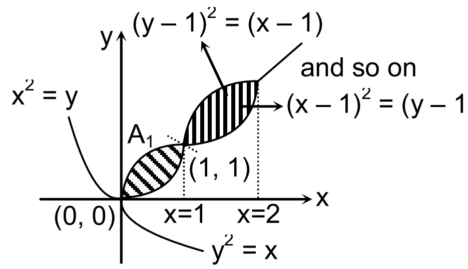
68. A

Sol. $A_0 = \int_0^1 \left(\sqrt{x} - x^2\right)dx$

$= \left|\frac{2}{3}x^{\frac{3}{2}} - \frac{x^3}{3}\right|_0^1 = \frac{2}{2} - \frac{1}{3} = \frac{1}{3}$

$A_0 = A_1 = A_2 = A_2 = \ldots = A_{20}$

So, sum = $21 \times \frac{1}{3} = 7$ sq. units



69. A

Sol. Reflexive a R a ⇒ |a – a| ≤ 1, true
Symmetric 1 R 2 and 2 R 1 but 1 ≠ 2 but not anti-symmetric
Transitive 1 R 2 and 2 R 3 but 1 R 3 (not related)

70. A

Sol. $(p \vee q) \wedge (\sim p \wedge \sim q)$

p	q	~p	~q	p ∨ q	~p ∧ ~q	(p ∨ q) ∧ (~p ∧ ~ q)
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	F

So, it is contradiction

p	q	~p	p ∧ q	(p ∧ q) ∨ ~p
T	T	F	T	T
T	F	F	F	F
F	T	T	F	T
F	F	T	F	T

$(\sim p \vee q)$

p	q	~p	~p ∨ q
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

So, true

SECTION – B

71. 4

Sol. $f^2(x) = \frac{f(x) + xf'(x) - 1}{x^2}$, $x^2f^2(x) + 1 = f(x) + xf'(x)$

$\frac{f(x) + xf'(x)}{1 + (xf(x))^2} = 1$, $\int \frac{f(x) + xf'(x)}{1 + (xf(x))^2}dx = \int dx + c$

$\tan^{-1}(xf(x) = x + c$, $\tan^{-1}\left(\frac{\pi}{4}f\left(\frac{\pi}{4}\right)\right) = \frac{\pi}{4} + c$, c = 0

$xf(x) = \tan x$, $f(x) = \frac{\tan x}{x}$ = $\lim_{x \to 0} 1 + f(x) + f^2(x) + f^3(x) = 4$

72. 5

Sol. Let y = f(x), so x = $f^{-1}(y)$ = g(y), $\frac{dy}{dx} = f'(x)$, dy = f′(x) dx

$= \frac{1}{17}\left(\int_2^7 f(x)dx + \int_2^7 xf'(x)dx\right) = \frac{1}{17}\left(\int_2^7 \left(f(x) + xf'(x)\right)dx\right) = \frac{1}{17}\left|xf(x)\right|_2^7$

$= \frac{1}{17}\times\left(7f(7) - 2f(2)\right) = \frac{1}{17}\times\left(7\times13 - 2\times3\right) = \left(\frac{91-6}{17}\right) = \frac{85}{17} = 5$

SECTION – C

73. 00002.35

Sol. $\int_0^x tf(t)dt = x^2 f(x)$, xf(x) = 2xf(x) + $x^2f'(x)$

Let y = f(x), xy = 2xy + $x^2\frac{dy}{dx}$, $x^2\frac{dy}{dx} = -xy$, $\frac{dy}{dx} = -\frac{y}{x}$

$\int\frac{dy}{y} = -\int\frac{dx}{x} + \log c$, log y = –log x + log c

log xy = log c, xy = c, c = 6, xy = 6, $f(x) = \frac{6}{x}$

f(6) = 1, f(8) = $f(8) = \frac{6}{8} = \frac{3}{4} = 0.75$, $f(10) = \frac{6}{10} = \frac{3}{5} = 0.6$

f(6) + f(8) + f(10) = 1 + 0.75 + 0.60 = 2.35

74. 00080.80

Sol. $(a-1)^n = {}^nC_0a^n - {}^nC_1a^{n-1} + {}^nC_2a^{n-2} + ... + (-1)^{n-1}\,{}^nC_{n-1}a + (-1)^n\,{}^nC_n$

$(a-1)^n - (-1)^n = a\left({}^nC_0a^{n-1} + {}^nC_1a^{n-2} + {}^nC_2a^{n-3} + ... + (-1)^{n-1}\,{}^nC_n\right)$

$\frac{(a-1)^n - (-1)^n}{a} = f(n)$

$f(2020) + f(2021) = \frac{(a-1)^{2020} - (-1)^{2020}}{a} + \frac{(a-1)^{2021} - (-1)^{2021}}{a}$

$= (a-1)^{2020} = (7)^{\frac{2020}{25}} = (7)^{80.80}$ $\therefore \lambda = 80.80$

75. 00002.67

Sol. $y^2 = x^4 - 2x^2 + 1 = (x^2-1)^2$

$y = \pm(x^2-1)$

$y = x^2 - 1$

$y = 1 - x^2$

$= 4\int_0^1 (1-x^2)dx = 4\left[x - \frac{x^3}{3}\right]_0^1$

$= 4\left(1 - \frac{1}{3}\right) = 4\times\frac{2}{3} = \frac{8}{3} = 2.66$