FIITJEE

ALL INDIA TEST SERIES

PART TEST – I

JEE (Advanced)-2021

PAPER – 2

TEST DATE: 06-12-2020

ANSWERS, HINTS & SOLUTIONS

Physics

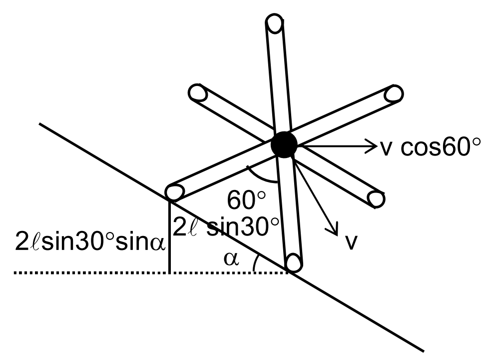
PART – I

SECTION – A

1. A, C, D

Sol. When the pivot point is changed the speed of central point changes suddenly and the kinetic energy is lost. Velocity component perpendicular to new spoke survives.
Average velocity becomes constant when the loss
in K.E. due to changing contact point = gain in P.E.

$$\frac{1}{2}mv_0^2\left(1-\cos^2\frac{\pi}{3}\right)=mg\ell.\ 2\sin\frac{\pi}{6}\sin\alpha$$



2. C, D

Sol. The force on the particle is not the central force.

3. A, B, C, D

Sol. $a_x + a_y = a_1$ …(i)

$a_y - a_x = a_2$ …(ii)

$a_{cm(x)} = 0$

$2a_2 = a_1 + a_x$ …(iii)

4. A, B

Sol. $L = mv_\theta r$ …(i)

$\frac{dL}{dt} = -\alpha v_\theta r$

$\Rightarrow \frac{dL}{dt} = -\frac{\alpha L}{m}$ from (i)

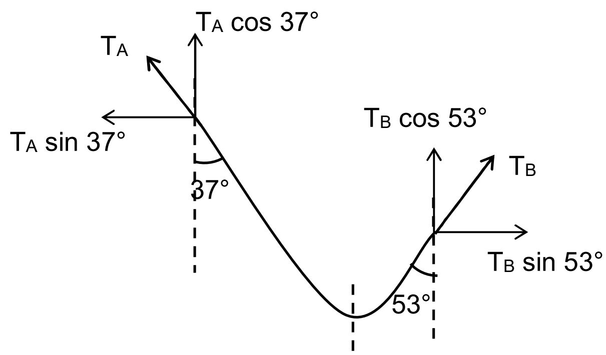
$\Rightarrow \int_{L_0}^{L} \frac{dL}{L} = -\frac{\alpha}{m}\int_0^t dt$

$L = L_0 e^{-\frac{\alpha t}{m}}$

5. A, C, D

Sol. $3T_A = 4T_B$

$\frac{4T_A}{5} + \frac{3T_B}{5} = mg \Rightarrow T_B = \frac{3}{5}mg$



6. A, B, C

Sol. $Ta_1 + \frac{T}{2}a_2 + \frac{T}{4}a_3 + ... = 0$

$\left(\frac{T}{m} - g\right) + \frac{1}{2}\left(\frac{T}{2m} - g\right) + \frac{1}{4}\left(\frac{T}{4m} - g\right) + ... = 0$

$\frac{T}{m}\left(1 + \frac{1}{4} + \frac{1}{16} + ...\right) = g\left(1 + \frac{1}{2} + \frac{1}{4} + ...\right)$

$T = \frac{3mg}{2}$

$\therefore \quad a_1 = \frac{g}{2}$

SECTION – B

7. 1

Sol. $\left[< \omega > = \frac{\Delta\theta}{\Delta t} \right]$

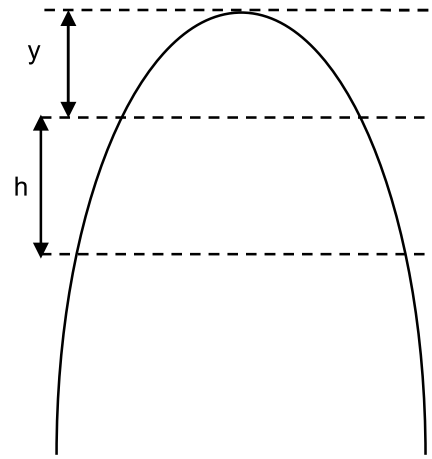
8. 1

Sol. Writing equation of motion in y-direction

$$y = \frac{1}{2}g\left(\frac{T_1}{2}\right)^2$$

$$h + y = \frac{1}{2}g\left(\frac{T_2}{2}\right)^2$$

$$g = \frac{8h}{T_2^2 - T_1^2}$$

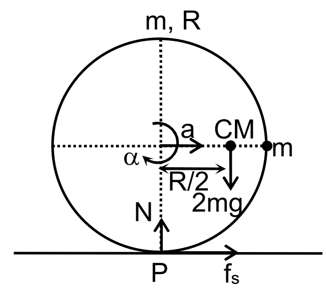


9. 3

Sol. For a pure rolling motion

a = αR

Using conservation of energy

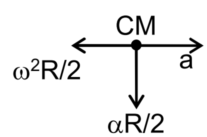


$$mg\frac{R}{2} = \frac{1}{2}I_P\omega^2$$

$$mg\frac{R}{2} = \frac{1}{2}\times 4mR^2\omega^2$$

$$\Rightarrow \omega = \sqrt{\frac{g}{4R}} \quad \ldots(i)$$

$$\tau_P = I_P\alpha$$



$$2m(g + \omega^2R)\frac{R}{2} = 4mR^2\alpha$$

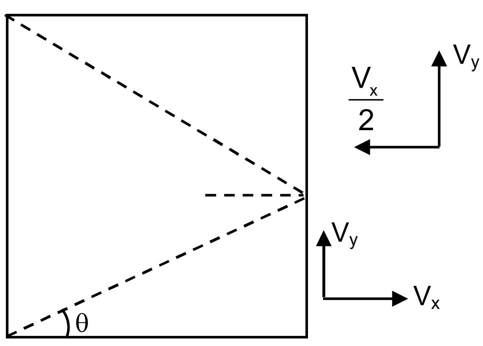
$$2m\left(g + \frac{g}{4}\right)\frac{R}{2} = 4mR^2\alpha \Rightarrow \alpha = \frac{5g}{16R} \quad \ldots(ii)$$

$$f_s = 2m\left(a - \frac{\omega^2R}{2}\right)$$

$$f_s = 2m\left(\frac{5g}{16} - \frac{g}{8}\right), \quad f_s = \frac{3mg}{8}$$

10. 3

Sol.



11. 4

Sol. Tangential component of force = $V_0r^2\sin\theta$

Magnitude of the x-component of force = $V_0r^2\sin^2\theta$.

12. 7

Sol. $\frac{4}{3}\pi R^3\rho_1,\frac{dv}{dt}+v\frac{d}{dt}\left(\frac{4}{3}\pi R^3\rho_1\right)=\frac{4}{3}\pi R^3\rho_1 g.$

$R\frac{dv}{dt}+3v\frac{dR}{dt}=Rg$ …(i)

Also, $\pi R^2 v\rho_2=\frac{dm}{dt}$

$v=\frac{4\rho_1}{\rho_2}\cdot\frac{dR}{dt}$ …(ii)

After a long time when acceleration becomes constant v = at will satisfy our differential equation.

$v=at$ $v=\frac{4\rho_1}{\rho_2}\frac{dR}{dt}$

$R=\frac{at^2\rho_2}{8\rho_1}$

From equation (i) and (ii)

$\frac{at^2\rho_2 a}{8\rho_1}+\frac{3\rho_2}{4\rho_1}(at)^2=\frac{at^2\rho_2 g}{8\rho_1}$

$\frac{a}{8}+\frac{3a}{4}=\frac{g}{8}$

$a=\frac{g}{7}$

SECTION – C

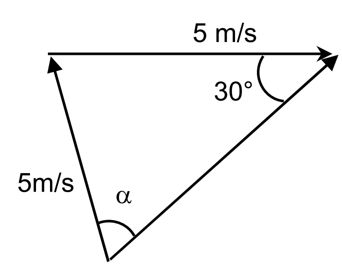
13. 00100.00

Sol. $5V^2 = 50 \times 10^3$

$V = 10^2\ ms^{-1}$

14. 00060.00

Sol. $\frac{5}{\sin\alpha}=\frac{5}{\sin 30^\circ} \Rightarrow \alpha = 30^\circ$



15. 00000.50

Sol. $\frac{1}{2}mv_2^2=\frac{1}{2}mv_1^2+mg\ell$

$v_{2y}^2=v_{1y}^2+2g\ell$ …(i)

$ev_{2y}=v_{1y}$

$$v_{1x} = v_{2x} = \sqrt{\frac{g\ell}{2}\left(\frac{1-e}{1+e}\right)} = 0.50 \quad m/s$$

16. 00008.66

Sol. $\frac{R}{2}\sqrt{3} = 10 \times \frac{1.732}{2} = 8.66\ cm$

17. 00011.42

Sol. Centre of mass should not cross the corner

$$d = \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \frac{1}{10}\right) \times 10\,cm = 11.42\ cm$$

18. 00031.00

Sol. $I \propto h^5$

Chemistry PART – II

SECTION – A

19. A, B, C

Sol. $Na_2O \xrightarrow{400^{o}C} Na_2O_2 + Na$

$Na_2O_2 + O_2 \xrightarrow{450^{o}C} NaO_2$

$Na_2O_2 + H_2O \longrightarrow 2NaOH + H_2O_2$

20. A, B, C

Sol. $NaOH + P_4 \longrightarrow PH_3 + NaH_2PO_2$

$S_8 + NaOH \longrightarrow Na_2S + Na_2SO_3$

$Cl_2 + NaOH \longrightarrow NaCl + NaClO_3$

$B + NaOH \longrightarrow Na_3BO_3 + H_2$

21. A, B, D

22. A, B, C

Sol. (D) four oxygen atoms are shared.

23. A, B, C

Sol. $Al_4C_3 + H_2O \longrightarrow Al(OH)_3 + CH_4$

$CaC_2 + H_2O \longrightarrow Ca(OH)_2 + C_2H_2$

24. A, B

SECTION – B

25. 9

Sol. $5U(SO_4)_2 + 2H_2O + 2KMnO_4 \longrightarrow 2MnSO_4 + 5UO_2SO_4 + 2H_2SO_4 + K_2SO_4$

26. 8

Sol. 100 ml water contain = 0.192 mg

1000 ml water contain = 1.92 mg

Milli mole of $Mg^{+2} = \frac{1.92}{24} = 0.08$

Hence milli mole of $CaCO_3$ = 0.08

Hence mass of $CaCO_3$ with 1L = 0.08 × 100 = 8 mg

Hardness will be = 8 ppm.

27. 3

Sol. $60 = \frac{hc}{\lambda_1}\left(\frac{n_1}{t_1}\right)$

$80 = \frac{hc}{\lambda_2}\left(\frac{n_2}{t_2}\right)$

$\lambda_1 = \frac{1}{R_H}$

$\lambda_2 = \frac{1}{4R_H}$

$\frac{(n_1/t_1)}{n_2/t_2} = 3$

28. 6

Sol.
$A + B \rightleftharpoons C + D$

$0.4 + x \quad 0.4 + x \quad 2 - x \quad 2 - x \quad K_c = 16$

x = 0.08

$[B]_{eq} = \frac{(0.4 + 0.08)}{0.08} = 6$

29. 4

Sol. $_{92}U^{238} \longrightarrow {}_{82}Pb^{206} + 8\alpha + 6\beta$

$\left[4 - 4\left(\frac{1}{2}\right)^n\right] \times 8 \times 6.02 \times 10^{23} = 18.066 \times 10^{24}$

n = 4

30. 4

Sol. $5.35 = 4.75 + \log\frac{[CH_3COO^{-}]}{[CH_3COOH]}$

$\frac{[CH_3COO^{-}]}{[CH_3COOH]} = 4$

SECTION – C

31. 00025.02

Sol. $\% = \frac{1.2 \times 3.336 \times 10^{-30}}{1.6 \times 10^{-19} \times 10^{-10}} \times 100\%$

$= \frac{3 \times 3.336 \times 10}{4}$

= 25.02%

Hence, x = 25.02

32. 00005.60

Sol. $pH = 5 + \log\frac{80}{20}$

$= 5 + \log 4 = 5.6$

33. 00001.20

Sol. $K_4[Fe(CN)_6] + 6H_2SO_4 + 6H_2O \xrightarrow{\Delta} 2K_2SO_4 + FeSO_4 + 3(NH_4)_2SO_4 + 6CO\uparrow$

$5CO + I_2O_5 \longrightarrow 5CO_2 + I_2$

34. 00000.48

Sol. $N_2O_3(g) \rightleftharpoons NO_2(g) + NO$

$2-x \qquad x-2y \qquad x$

$2NO_2(g) \rightleftharpoons N_2O_4(g)$

$x-2y \qquad y$

Given x = 1.5

$$2.5 = \frac{(1.5-2y)(1.5)}{0.5}$$

35. 00000.69

36. 00000.28

Sol. $2A \rightleftharpoons 2B + 3C + D$

$\qquad 0.24 \quad 0.36 \quad 0.12$

$X_A = (1-0.72) = 0.28$

Mathematics PART – III

SECTION – A

37. B, C

Sol. $I_1 = \int_0^{512} \frac{\{x^{1/3}\}}{\sqrt[3]{x^2}} dx$. Let $x^{1/3} = t \Rightarrow \frac{dt}{dx} = \frac{1}{3}x^{-2/3}$

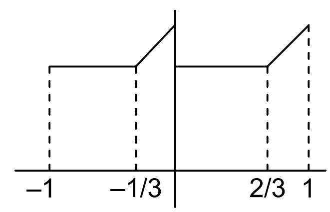
$\Rightarrow I_1 = 3\int_0^8 \{t\}\,dt = 3 \times 8 \times \int_0^1 \{t\}\,dt = 24 \times \frac{1}{2} = 12$

$I_2 = \int_0^2 x^2 \{x^3\} dx$. Let $x^3 = t \Rightarrow 3x^2\,dx = dt$

$= \frac{1}{3}\int_0^8 \{t\}\,dt = \frac{1}{3} \times 8 \times \int_0^1 \{t\}\,dt = \frac{8}{3} \times \frac{1}{2} = \frac{4}{3}$

38. A, C

Sol. $f(x) = \begin{cases} -6x & -1 < x < -\frac{2}{3} \\ 4 & -\frac{2}{3} \le x \le \frac{2}{3} \\ 6x & \frac{2}{3} < x < 1 \end{cases}$



$g(x) = \{x\}$

$\therefore f(g(x)) = \begin{cases} 4 & 0 \le \{x\} \le \frac{2}{3} \\ 6\{x\} & \frac{2}{3} < \{x\} < 1 \end{cases}$ $\therefore c = 1$ and $d = 3$

39. A, B, C, D

Sol. (A) $\int_0^1 x \tan x\,dx \ge \int_0^1 x\left(x + \frac{x^3}{3}\right) dx \ge \frac{2}{5}$

(B) $\int_0^1 x^2 \cos x \le \int_0^1 x^2 dx \le \frac{1}{3}$

(C) $\int_0^1 x^3 \sin x \ge \int_0^1 x^3\left(x - \frac{x^3}{3!}\right) \ge \frac{37}{210}$

(D) x sin x is an even function

$\therefore \int_{-1}^0 x \sin x = \int_0^1 x \sin x \ge \int_0^1 x\left(x - \frac{x^3}{3!}\right) dx \ge \frac{3}{10}$

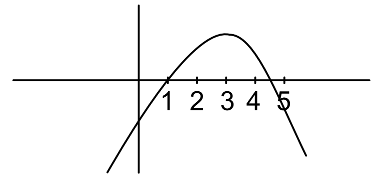
40. A, C

Sol. $\frac{1990}{80} < \frac{f(2020)}{80} < \frac{2050}{80} \Rightarrow \left[\frac{f(2020)}{80}\right] = 24$ or 25

Now put x = 2020 we get f(2020) = 1920 or 2000

41. B, C, D

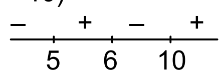
Sol. $f(x)=\begin{cases}-(x-1)(x-5) & x\le 3\\ -4(x-2)(x-4) & x>3\end{cases}$



Lowest y intercept will be of tangent made on (1, 0) and highest y intercept will be of tangent made on (4, 0) tangent at (1, 0) is y = 4x – 4 $\therefore$ m_2 = 4, b_2 = –4 and tangent at (4, 0) is y = –8x + 32 $\therefore$ m_1 = –8, b_1 = 32

42. A, D

Sol. $F'(x) = f(x) = (x-2)^2(x-5)(x-6)(x-10)$



F′(x) changes sign from negative to positive at 5 and 10

$\therefore$ F(x) has local minima at x > 5, 10, F(x) has local maxima at x = 6

SECTION – B

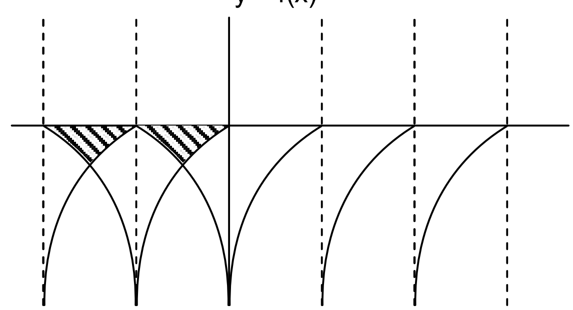
43. 0

Sol. $f(8-x) = f(8+x) \Rightarrow f'(8-x) = -f'(8+x)$

$I=\int_{-8}^{8} f'(8+x)x^2e^{x^2}dx = -\int_{-8}^{8} f'(8-x)x^2e^{x^2}dx = -I \Rightarrow I = 0$

44. 6

Sol. Area = $2\times 2\times \int_{-1/2}^{0} \ln\{x\}dx$



$=\left|4\times\int_{1/2}^{1}(\ln x dx)\right|$

$=\left|4(x\ln x - x)\right|_{1/2}^{1}$

$=\left|4\left(-1-\left(\frac{1}{2}\ln\frac{1}{2}-\frac{1}{2}\right)\right)\right|$

$=4\left(\frac{1}{2}+\frac{1}{2}\ln\frac{1}{2}\right)=2(1-\ln 2)$

= 2 – 2 ln 2 = 2 – ln 4 $\therefore$ k = 2 and p = 4

45. 2

Sol. $f(x)=\begin{cases}0 & 0\le 9x^2<1\\ \frac{\ln 2-\cos 1}{2} & 9x^2=1\\ -\cos 9x^2 & 9x^2>1\end{cases}$ discontinuous at $x=\pm\frac{1}{3}$

46. 7

Sol. Comparing the coefficient of x^n both sides we get $b_n - b_{n-1} = a_n$

Also, $b_0 = a_0$ and $b_1 = a_0 + a_1 \Rightarrow b_0 = 1$, $a_1 = 2$

$\therefore$ $b_3 = a_0 + a_1 + a_2 = 1 + 2 + 4 = 7$

Area bounded by $y=\frac{7}{1+x^2}$

x-axis, y-axis and $x=\frac{\pi}{3}$ will be $\int_0^{\sqrt{3}}\frac{7}{1+x^2}dx=\frac{7\pi}{3}$

47. 6

Sol. $x\frac{dy}{dx} - 2y = 2 - 4\ln x \Rightarrow \frac{dy}{dx} - \frac{2}{x}y = \frac{2-4\ln x}{x}$

$I.F = e^{-\int \frac{2}{x}dx} = \frac{1}{x^2}$ $\therefore$ $y\frac{1}{x^2} = \int \frac{2-4\ln x}{x^3} \Rightarrow \frac{y}{x^2} = -\frac{1}{x^2} - 4\left(\frac{\ln x\, x^{-2}}{-2} + \frac{x^{-2}}{-4}\right) + c$

$\Rightarrow y = cx^2 + 2\ln x$ $\because$ it passes through (1, 3) $\therefore$ c = 3 $\therefore$ $y = 3x^2 + 2\ln x$

48. 1

Sol. $I_1 + I_2 = \int_0^{\pi/2} 1 \cdot dx = \frac{\pi}{2}$

$I_1 - I_2 = \int_0^{\pi/2} \frac{\sin^2 x - \cos^2 x + \sin x - \cos x}{1 + \sin x + \cos x} dx = \int_0^{\pi/2} (\sin x - \cos x) dx = 0$

$\therefore I_1 = I_2 = \frac{\pi}{4}$

SECTION – C

49. 00000.59

Sol. $I_1 = \int_0^{\pi/2} (\cos x)^{\sqrt{2}} \cos x\, dx = (\cos x)^{\sqrt{2}} \sin x \Big|_0^{\frac{\pi}{2}} + \sqrt{2}\int_0^{\pi/2} (\cos x)^{\sqrt{2}-1} \sin^2 x\, dx$

$\Rightarrow I_1 = \sqrt{2} I_2 - \sqrt{2} I_1 \Rightarrow \frac{I_1}{I_2} = \frac{\sqrt{2}}{1+\sqrt{2}} = 2 - \sqrt{2} = 2 - 1.41 = 0.59$

50. 00000.20

Sol. $f'(x) = k(x-1)(x-3)(x-2)^2$ and $k > 0 \Rightarrow f(x) = k\int (x^2 - 4x + 3)(x^2 - 4x + 4) dx$

$= k\int x^4 + 16x^2 - 8x^3 + 7(x^2 - 4x) + 12\, dx = k\left(\frac{x^5}{5} - 2x^4 + \frac{23x^3}{3} - 14x^2 + 12x\right) + c$

$\because$ f(0) = 2 $\Rightarrow$ c = 2 $\therefore$ $f(1) = \frac{88}{15} \Rightarrow \frac{88}{15} = k\left(\frac{1}{5} - 2 + \frac{23}{3} - 14 + 12\right) + c$

$\Rightarrow \frac{58}{15} = k \times \frac{58}{15} \Rightarrow k = 1$ $\therefore$ $a_5 = \frac{1}{5}$

51. 00000.07

Sol. Let $t^3 = x \Rightarrow \lim_{x \to 0} \frac{\sin x - x\cos x + x^5}{x(e^{3x} - 1 - 3x)}$

Now use expansions of sin x, cos x and e^{3x}

52. 00007.00

Sol. $I = \int \frac{x+2}{\sqrt{(x+2)^4 - 4(x+2)^2 + 4}}$. Let $(x+2)^2 = t \Rightarrow \frac{dt}{dx} = 2(x+2) \Rightarrow \frac{dt}{2} = (x+2)dx$

$\Rightarrow I = \frac{1}{2}\int \frac{1}{\sqrt{t^2 - 4t + 4}} dt \Rightarrow I = \frac{1}{2}\int \frac{1}{t-2} dt = \frac{1}{2}\ln|t-2| + c$

$= \frac{1}{2}\ln\left|(x+2)^2 - 2\right| + c \;\therefore\; f(x) = x^2 + 4x + 2$

53. 00008.00

Sol. I.F = e^x $\therefore$ solution is $f(x)e^x = \int 4x\sin 2x dx \Rightarrow f(x)e^x = 4\left(\frac{-x\cos 2x}{2} + \frac{\sin 2x}{4}\right)$

$\Rightarrow f(x) = (-2x\cos 2x + \sin 2x)e^{-x}$, $f\left(\frac{\pi}{4}\right) = e^{-\frac{\pi}{4}}$

$\therefore\; f\left(\frac{5\pi}{4}\right) = e^{-\frac{5\pi}{4}}$

$f\left(\frac{9\pi}{4}\right) = e^{-\frac{9\pi}{4}}$

$e^{-\frac{\pi}{4}}\left(\frac{e^{-8\pi} - 1}{e^{-\pi} - 1}\right) = \sum_{n=0}^{7} f\left(\frac{(4n+1)\pi}{4}\right)$

54. 00008.50

Sol. $\frac{17}{4} - f(x) = -\frac{17}{4} + g(x) \Rightarrow f(x) + g(x) = \frac{17}{2}\ \forall\ x \in R$