FIITJEE

ALL INDIA TEST SERIES

FULL TEST – XIII

JEE (Main)-2021

TEST DATE: 27-06-2021

Time Allotted: 3 Hours **Maximum Marks: 300**

General Instructions:

- The test consists of total 90 questions.
- Each subject (PCM) has 30 questions.
- This question paper contains **Three Parts.**
- **Part-A** is Physics, **Part-B** is Chemistry and **Part-C** is Mathematics.
- Each part has only two sections: **Section-A and Section-B**.
- **Section – A** : Attempt all questions.
- **Section – B :** Do any five questions out of 10 Questions.

  Section-A (01 – 20, 31 – 50, 61 – 80) *contains 60 multiple choice questions which have* ***only one correct answer****. Each question carries* ***+4 marks*** *for correct answer and* ***–1 mark*** *for wrong answer.*

  Section-B (21 – 30, 51 – 60, 81 – 90) *contains 30 Numerical answer type questions with answer XXXXX.XX and each question carries* ***+4 marks*** *for correct answer. There is no negative marking.*

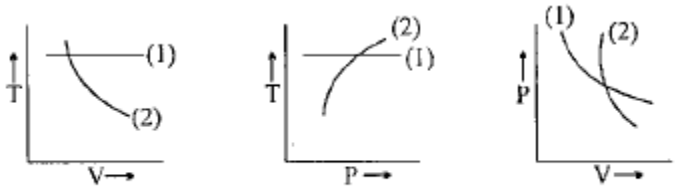
Physics

PART – A

SECTION – A
(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

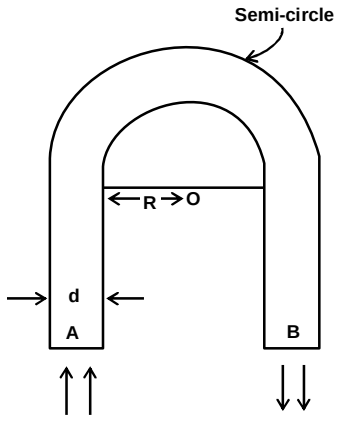
1. A 200 W and a 100 W bulb, both meant for operation at 220 V are connected to a 220 V supply.
   (A) Total power consumed by them will be 150 W, if they are in series.
   (B) Total power consumed by them will be (200/3) W, if they are in series.
   (C) Total power consumed by them will be 150 W, if they are in parallel.
   (D) Total power consumed by them will be (200/3) W, if they are in parallel.

2. 

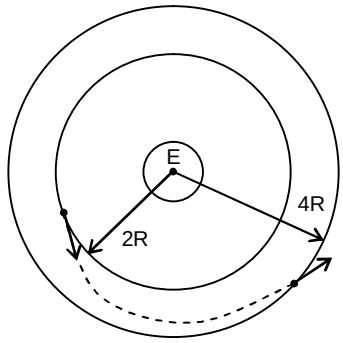
I II III

   If (1) represents isothermal and (2) represents adiabatic, which of the graphs given above in respect of an ideal gas are correct?
   (A) (I) and (II)
   (B) (II) and (III)
   (C) (I), (II) and (III)
   (D) Only (III)

3. A glass rod (μ = 1.5) of square cross-section is bent into the shape shown in the figure. A parallel beam of light falls perpendicularly on the plane flat surface A. Referring to the diagram, d is the width of a side and R is the radius of inner semi-circle. The maximum value of ratio (d/R) so that all light entering the glass through surface A emerge from the glass through surface B is
   (A) 1/2
   (B) 2/3
   (C) 3/2
   (D) 2/1

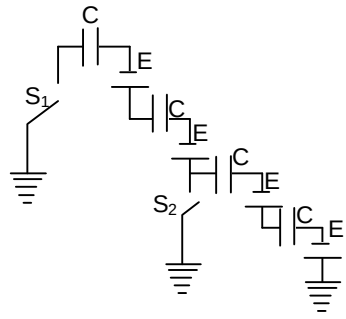


4. An artificial satellite of mass m orbiting the earth in a circular orbit of radius twice the earth's radius R. It is to be taken in another circular orbit of radius 4R. The transfer is accomplished through an elliptical orbit as shown in the figure. The energy needed to complete the job is calculated to be equal to $\frac{1}{K}\left(\frac{GMm}{R}\right)$, where M is the mass of the earth. Find the value of K



(A) 2
(B) 4
(C) 6
(D) 8

5. In the given circuit, all the capacitors are initially uncharged. After closing the switch S_1 for a long time suddenly S_2 is also closed and kept closed for a long time. Total heat produced after closing S_2 will be :

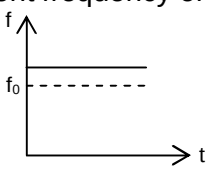


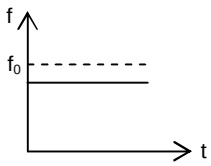
(A) $4C\varepsilon^2$
(B) $\frac{1}{2}C\varepsilon^2$
(C) $2C\varepsilon^2$
(D) 0

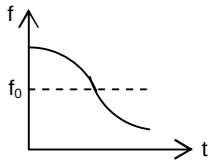
6. The efficiency of a Carnot cycle is 1/6. If on reducing the temperature of the sink by 65^0C, the efficiency becomes 1/3, the initial temperatures between which the cycle is working are
(A) 390 K, 325 K
(B) 780 K, 325 K
(C) 390 K, 162 K
(D) 300 K, 100 K

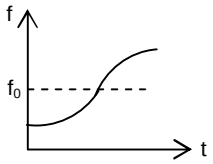
7. The angular frequency of a spring block system is ω_o. This system is suspended from the ceiling of an elevator moving downwards with a constant speed V_o. The block is at rest relative to the elevator. Lift is suddenly stopped. Assuming the downwards as positive direction, choose the wrong statement:
(A) The amplitude of the block is V_o/ω_o.
(B) The initial phase of the block is π.
(C) The equation of motion for the block is $\frac{V_o}{w_o}\text{Sin}(\omega_o t)$.
(D) The maximum speed of the block is V_o.

8. Source and observer both start moving simultaneously from origin one along x-axis and the other along y-axis with speed of source = 2 (speed of observer). The graph between the apparent frequency observed by observer (f) and time (t) would be (dotted lines represent frequency of source f_0)

(A) 

(B) 

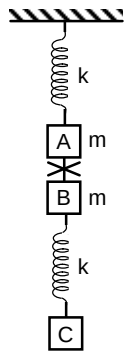
(C) 

(D) 

9. A composite string is made up by joining two strings of different masses per unit length, μ and 4μ respectively. The composite string is under the same tension. A transverse wave pulse: y = (6 mm) sin (5t + 40x), where 't' is in seconds and 'x' in meters, is sent along the lighter string towards the joint. The joint is at x = 0. The equation of the wave pulse reflected from the joint is
(A) (2 mm) sin(5t – 40x)
(B) (4 mm) sin(40x – 5t)
(C) –(2 mm) sin(5t – 40x)
(D) (2 mm) sin(5t – 10x)

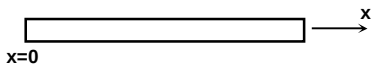
10. In the displacement method, magnification for two positions of the lens are 2 and 0.5 and the distance between the two positions of the lens is 30 cm. If the focal length of lens is
(A) 15 cm
(B) 20 cm
(C) 25 cm
(D) 30 cm

11. A spring block system as shown in figure is in equilibrium. The spring connecting blocks A and B is cut. The mass of all the three blocks is m and spring constant of both the springs is k. The amplitude of resulting oscillation of block A after cutting the string is



(A) $\frac{mg}{k}$
(B) $\frac{2mg}{k}$
(C) $\frac{3mg}{k}$
(D) $\frac{4mg}{k}$

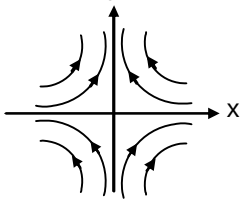
12. If along a uniform rod of length ℓ carrying current I, the voltage V changes with position x along the length of the rod such that dV/dx = – k, where k is a positive number, then the resistance of the rod is

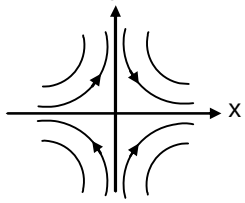


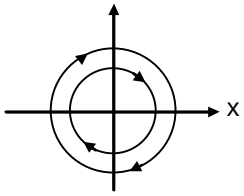
(A) $k\ell/2I$
(B) $k\ell/I$
(C) $I/k\ell$
(D) $k\ell I$

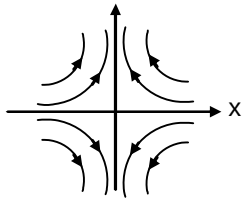
13. The resistivity of pure silicon is 2300 Ω-m and the mobilities of electrons and holes in it are 0.135 and 0.048 m^2/V-s respectively. The resistivity of a specimen of silicon doped with 10^{19} atoms of phosphorus per meter will be·
(A) 4.6 Ω m
(B) 4 Ω m
(C) 4.4 Ω m
(D) 0 Ωm

14. Electric potential is varying with x and y as $V = 2\left(x^2 - y^2\right)$. The corresponding field pattern is:

(A)


(B)


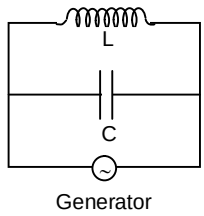
(C)


(D)


15. A particle is describing circular motion about a point O under an attractive force which is proportional to its mass and inversely proportional to the square of its distance from O. If it strikes a stationary particle at a point on its path and both stick together the path of the composite particle is:
(A) the same circle
(B) an ellipse outside the circle
(C) an ellipse inside the circle
(D) a hyperbola

16. Two polaroids are placed in the path of unpolarized beam of intensity I_0 such that no light is emitted from the second polaroid. If a third Polaroid whose polarization axis makes an angle θ with the polarization axis of first Polaroid, is placed between these polaroids then the intensity of light emerging from the last polaroid will be
(A) $\left(\frac{I_0}{8}\right)\sin^2 2\theta$
(B) $\left(\frac{I_0}{4}\right)\sin^2 2\theta$
(C) $\left(\frac{I_0}{2}\right)\cos^4 \theta$
(D) $I_0 \cos^4 \theta$

17. For the circuit shown in the figure, the current through the inductor is 0.6 A, while the current through the capacitor is 0.4 A. The current drawn from the generator is

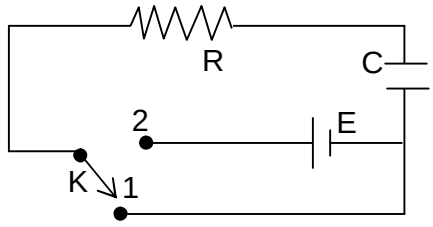


(A) 1.0 A
(B) 0.4 A
(C) 0.6 A
(D) 0.2 A

18. A particle starts from rest and travels a distance s with uniform acceleration, then it travels a distance 2s with uniform speed finally it travels a distance 3s with uniform retardation and comes to rest. if the complete motion of the particle is a straight line then the ratio of its average velocity to maximum velocity is
(A) 6/7
(B) 4/5
(C) 3/5
(D) 2/5

19. The first overtone of an open organ pipe beats with the first overtone of a closed organ pipe with a beat frequency of 2.2 Hz. The fundamental frequency of the closed organ pipe is 110 Hz. The lengths of the pipes are
(A) 1.1 m, 0.9 m
(B) 1.2 m, 0.8 m
(C) 1.0067 m, 0.9937 m
(D) 2.0067 m, 0.9937 m

20. In the shown circuit involving a resistor of resistance R Ω, capacitor of capacitance C farad and an ideal cell of emf E volt, the capacitor is initially uncharged and the key is in position 1. At t = 0 second the key is pushed to position 2 for t_0 = RC seconds and then key is pushed bank to position 1 for t_0 = RC seconds. This process is repeated again and again. Assume the time taken to push key from position 1 to 2 and vice versa to be negligible.

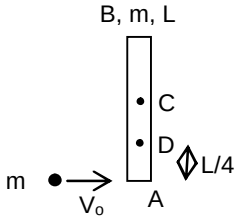


The charge on capacitor at t = 2RC second is
(A) CE
(B) $CE\left(1-\frac{1}{e}\right)$
(C) $CE\left(\frac{1}{e}-\frac{1}{e^2}\right)$
(D) $CE\left(1-\frac{1}{e}+\frac{1}{e^2}\right)$

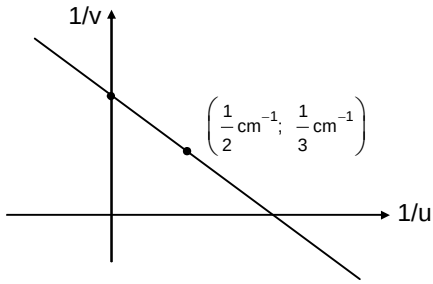
SECTION – B
(Numerical Answer Type)

This section contains **10** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. XXXXX.XX).

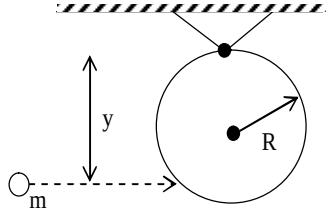
21. A thin, uniform rod of mass M and length L is at rest on a smooth horizontal surface. A particle of same mass M collides with the rod at one end perpendicular to its length with velocity $V_o = \frac{\pi L}{3}$ and sticks to it. C is the middle point of the rod and D is a point at a distance of L/4 from end A. The time taken by the rod turn through 90^o after collisions



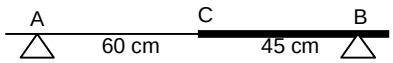
22. $\frac{1}{v}$ vs $\frac{1}{u}$ graph for a spherical mirror is shown in the figure. The focal length of the mirror will be



23. A small ball of mass m hits the cylinder which is hinged at top and free to rotate in vertical plane as shown in figure. Mass of cylinder is M. Liner momentum of system (ball + cylinder) will remain conserved if y=nR where n is



24. A steel wire of length 60 cm and area of cross section $10^{-6}m^2$ is joined with an aluminium wire of length 45 cm and area of cross section $3\times10^{-6}m^2$. The composite string is stretched by a tension of 80 N. Density of steel is 7800 kg m^{-3} and that of aluminium is 2600 kg m^{-3}. The minimum frequency of turning fork, which can produce standing wave in it with node at joint is



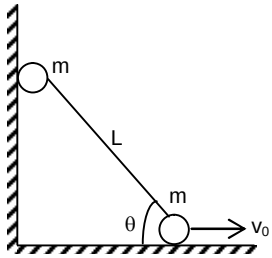
25. A body (solid sphere) of mass m makes an elastic collision with another identical body at rest. Just after collision the angle between the velocity vector of one body with the initial line of motion is 15° then the angle between velocity vector of the other body with the initial line of motion is in radian is (Take π = 3.14)

26. The image of an object, formed by a plano-convex lens at a distance of 8 m behind the lens, is real and is one-fourth the size of the object. The wavelength of light inside the lens is $\frac{2}{3}$ times the wavelength in free space. The radius of the curved surface of the lens is

27. A spherical object of mass 1 kg and radius 1 m is falling vertically downward inside a viscous liquid in a gravity free space. At a certain instant the velocity of the sphere is 2 m/s. If the coefficient of viscosity of the liquid is $\frac{1}{18\pi}$ N-S/m^2, then velocity of ball will become 0.5 m/s after a time

28. A uniform rod of length L and mass M is pivoted freely at one end and released as shown. The tangential acceleration of the free end when the rod is horizontal are (g = 9.8 m/s^2).

29. In the shown diagram two point masses m are joined by a massless rod of length 5.5 m. The lower end of the mass system is dragged with constant velocity v_0 = 3 m/s towards rightwards. The magnitude of velocity of centre of mass when $\theta = 37^0$, will be



30. Two radioactive elements R and S disintegrate as

$R \longrightarrow P + \alpha$; $\lambda_R = 4.5 \times 10^{-3}$ years^{-1}

$S \longrightarrow Q + \beta$; $\lambda_S = 3 \times 10^{-3}$ years^{-1}

Starting with number of atoms of R and S in the ratio of 5 : 1, this ratio after the lapse of three half lives of R will be

Chemistry

PART – B

SECTION – A
(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

31. What is the oxidation number of iodine in KI_3?
(A) –1
(B) +1
(C) $-\frac{1}{3}$
(D) $+\frac{1}{3}$

32. $Cu^{2+} + 2e^- \longrightarrow Cu, \quad E^o = +0.34\,V$
$Cu^{2+} + e^- \longrightarrow Cu^+, \quad E^o = +0.15\,V$
The equilibrium constant for the reaction $Cu + Cu^{2+} \longrightarrow 2Cu^+$ at 25°C is
(A) 2.9×10^{-8}
(B) 3.72×10^{-7}
(C) 2.9×10^{-5}
(D) 3.72×10^{-5}

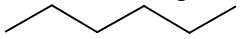
33. At high pressure the van der Waal's equation for one mole of a real gas reduces to
(A) $PV = RT - \frac{a}{V}$
(B) PV = RT + Pb
(C) $\left(P + \frac{a}{V^2}\right)V = RT$
(D) PV = RT

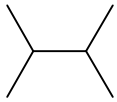
34. 100 mL of a sample of hard water requires 25.1 mL of 0.02 N H_2SO_4 for complete reaction. The hardness of water (density 1 g/mL) is:
(A) 200 ppm
(B) 250 ppm
(C) 251 ppm
(D) 258 ppm

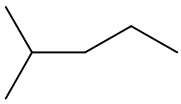
35. The electronegativity of phosphorus is maximum in
(A) PCl_5
(B) $PClF_4$
(C) PCl_2F_3
(D) PCl_3F_2

36. $CH_3CH_2CH_2Cl + CH_3\underset{\underset{Cl}{|}}{C}HCH_3 \xrightarrow[\text{Dry ether}]{Na} Products$

Which of the following is not a product of above reaction?

(A) 

(B) 

(C) 

(D) 

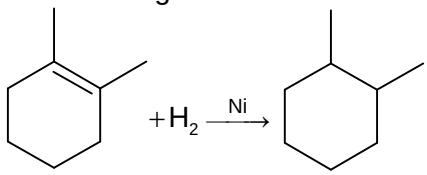
37. The half-life of a chemical reaction does not change by either increasing or decreasing the initial concentration of reactant. What is the order of the reaction?

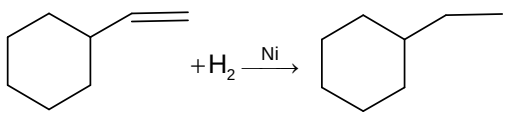
(A) Zero
(B) First
(C) Second
(D) Fractional

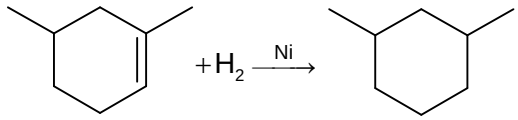
38. K_{sp} of AgCl in water at 25^oC is 1.8×10^{-10}. If 10^{-5} mole of Ag^+ ions are added to this solution, K_{sp} will be:

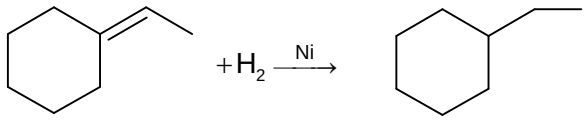
(A) 1.8×10^{-15}
(B) 1.8×10^{-5}
(C) 1.8×10^{-10}
(D) none of these

39. Which of the following reaction liberates maximum amount of heat energy?

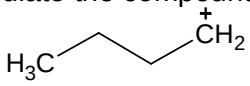
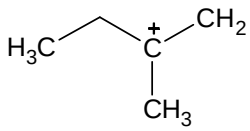
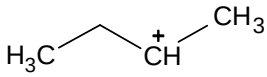
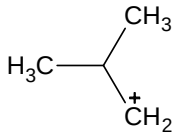
(A) 

(B) 

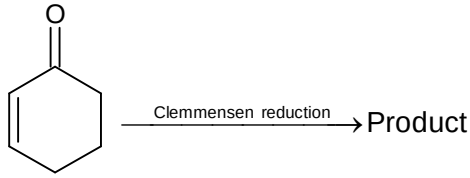
(C) 

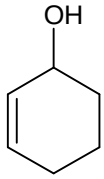
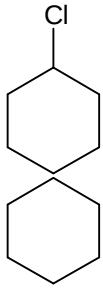
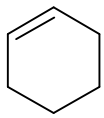
(D) 

40. How much work in kJ mol^{-1} unit is done by reversible and isothermal expansion of 1.2 mole of an ideal gas to 10 times of its original volume at 27^oC?
(A) 4.191
(B) 6.892
(C) -6.892
(D) -4.191

41. Dehydration of 1-butanol gives 2-butene as a major product, by which of the following intermediate the compound 2-butene obtained.
(A) 
(B) 
(C) 
(D) 

42. In which of the following crystals, alternate tetrahedral voids are occupied?
(A) NaCl
(B) ZnS
(C) CaF_2
(D) Na_2O

43. 

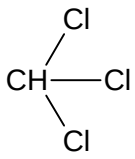
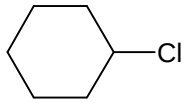
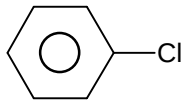
The product of above reaction is:
(A) 
(B) 
(C) 
(D)

44. The shape of $[Co(NH_3)_6]^{3+}$ is:
(A) Tetrahdedral
(B) Trigonal bipyramidal
(C) Octahedral
(D) Square planar

45. Which of the following two reagents form silicates when react with silica (SiO_2)?
(A) NaCl and Na_2S
(B) NaOH and Na_2CO_3
(C) Na_2SO_4 and $NaNO_3$
(D) NaBr and $NaHSO_4$

46. The correct statement regarding the third orbit of hydrogen atom is
(A) it can accommodate a maximum of nine electrons
(B) it contains nine degenerate atomic orbitals
(C) it contains nine atomic orbitals having three sets of degeneracy
(D) it contains nine sub-shells

47. 200 mL of 0.4 M solution of CH_3COONa is mixed with 400 mL of 0.2 M solution of CH_3COOH. After complete mixing, 400 mL of 0.1 M NaCl is added to it. What is the pH of the resulting solution? [K_a of $CH_3COOH = 10^{-5}$]
(A) 5.4
(B) 6
(C) 5
(D) 6.2

48. In which of the following compound, chlorine exerts the maximum number of electronic effects?
(A) 
(B) 
(C) $CH_2 = CH - CH_2Cl$
(D) 

49. Which of the following linkage is present in “Teflon”?
(A) H – C
(B) F – F
(C) C – C
(D) C = C

50. $CH_3CHO + CH_3COCH_3 \xrightarrow{\text{dil.NaOH}} \text{Products}$
How many product(s) containing five carbon atom(s) is/are formed in above reaction? [Do not consider stereoisomers]
(A) 1
(B) 2
(C) 3
(D) 4

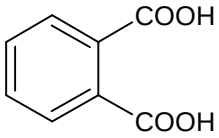
SECTION – B
(Numerical Answer Type)

This section contains **10** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. XXXXX.XX).

51. The vapour pressure of two solvents A and B are respectively 400 and 600 mm of Hg. A solution was prepared by mixing 4 moles of A and 6 moles of B. What is the vapour pressure of the resulting solution in mm of Hg unit?

52. $CH_4 + Cl_2\,(\text{excess}) \longrightarrow P + Q + R + S$

P is the most acidic product
Q is the most heaviest product
What is the sum of the molar masses of P and Q in g mol^{-1} unit?

53. 

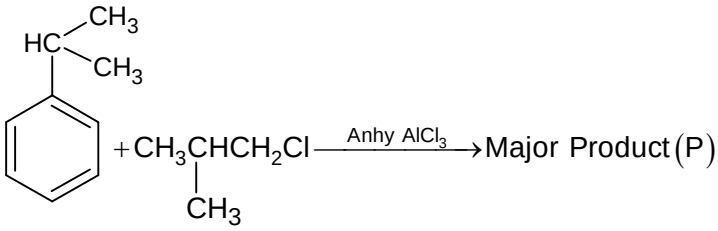
$\xrightarrow{\text{Heat}} (P) + (Q)$

In above reaction product (P) contains benzene ring and (Q) is not an organic compound? What mass% of the reactant is converted to (Q)?

54. The root mean square velocity of an ideal gas is 120.4 cm s^{-1} at a certain temperature. What will be its velocity in cm s^{-1} unit if the temperature is increased four times of its original value?

55. How much gram of sodium hydroxide can be completely neutralized by 1.2 L of 0.2 M HCl solution?

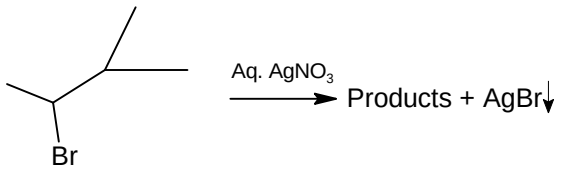
56. H_2A is a weak dibasic acid. The K_{a_1} and K_{a_2} values of it, are $10^{-4.5}$ and $10^{-10.5}$ respectively. What is the pH of 0.1 M of solution of salt NaHA?

57. 

$(P) \xrightarrow[\text{Heat}]{KMnO_4/OH^-} (Q)$

How many CH_3 group(s) is/are present in (Q)?

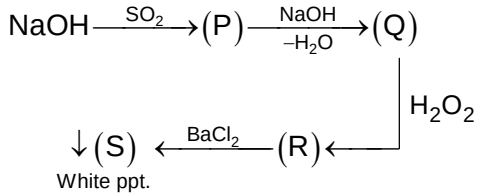
58.



Total number of all possible products including stereoisomers are

59. For Al_2Cl_6 dimer
If x = Number of 2c – 2e covalent bonds present in it.
y = Number of co-ordinate covalent bonds present in it
and z = number of 3c – 2e bonds present in it, then what is the value of $\left(\frac{x+y+z}{5}\right)$?

60.



The precipitate(S) in insoluble in all mineral acids. If the sum of number of atoms present in one molecule of sodium salt(R) is X, then what is $\left(\frac{X}{2}\right)$?

Mathematics

PART – C

SECTION – A
(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

61. Let α_i be the solution of equation $\sin 2x = e^{-x}, \alpha_i \in [0, 2\pi]$ and $i = 1,2,3,4$ also $f(x) = (x + \ln \sin 2\alpha_1)(x + \ln \sin 2\alpha_2)(x + \ln \sin 2\alpha_3)(x + \ln \sin 2\alpha_4) + \tan x$

The value of $\sum_{i=1}^{4} e^{-\alpha_i}\left(f(\alpha_i) + \frac{1}{f(\alpha_i)}\right)$ is.

(A) 4
(B) 8
(C) 12
(D) 16

62. If a, b, c are natural numbers and $\frac{ax^4 - bx^3 + cx^2 - bx + a}{(x^2+1)^2}$ attains minimum value at $x = 2$ or $x = \frac{1}{2}$, then the possible values of a, b, c are respectively

(A) 1, 4, 7
(B) 1, 8, 11
(C) 1, 4, 9
(D) 1, 2, 3

63. The value of $\int_5^{10}\left(\sqrt{x + \sqrt{20x - 100}} + \sqrt{x - \sqrt{20x - 100}}\right)dx$ is

(A) $10\sqrt{5}$
(B) $5\sqrt{5}$
(C) $10\sqrt{2}$
(D) $8\sqrt{2}$

64. If $\int \frac{xe^x}{\sqrt{(1+e^x)}}dx = f(x)\sqrt{(1+e^x)} - 2\ln g(x) + c$, then

(A) $f(x) = x - 1$
(B) $g(x) = \frac{\sqrt{(1+e^x)} - 1}{\sqrt{(1+e^x)} + 1}$

(C) $g(x)=\dfrac{\sqrt{(1+e^x)}+1}{\sqrt{(1+e^x)}-1}$

(D) $f(x) = x - 2$

65. Area bounded by $|x - 3y| + |x + 3y| \le 12$ and $xy \ge 3$ is

(A) $4(3 - \ell n2)$

(B) $8(3 - \ell n2)$

(C) $12(3 - \ell n2)$

(D) none of these

66. The conic section $y = g(x)$ passing through (3, 1) and having the property that the normal at any point Q(x, y) on $y = g(x)$ meet the x-axis at N and $NR = \dfrac{x(1+y^2)}{1+x^2}$, where R is the foot of the ordinate at Q. Equation of $y = g(x)$ is

(A) $5(1+y^2)=1+x^2$

(B) $8 + y^2 = x^2$

(C) $y^2 + 2y + 6 = x^2$

(D) None of these

67. The locus of mid points of the chords of the circle $x^2 - 2x + y^2 - 2y + 1 = 0$ which are of unit length is

(A) $(x - 1)^2 + (y - 1)^2 = \dfrac{3}{4}$

(B) $(x - 1)^2 + (y - 1)^2 = 2$

(C) $(x - 1)^2 + (y - 1)^2 = 4$

(D) none of these

68. If PQ is a double ordinate of the hyperbola $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$ such that OPQ is an equilateral triangle, O being the centre of the hyperbola. Then the eccentricity e of the hyperbola, satisfies

(A) $1<e<\dfrac{2}{\sqrt{3}}$

(B) $e=\dfrac{2}{\sqrt{3}}$

(C) $e=\dfrac{\sqrt{3}}{2}$

(D) $e>\dfrac{2}{\sqrt{3}}$

69. P is variable point different from vertex parabola $y^2 = 4ax$. Tangent at P cuts tangent at vertex and axis at A and B. Normal at P cuts axis at C. PM is perpendicular form P to the directrix and S is focus. Rhombus PSCQ is completed, then the locus of Q is
(A) $y^2 = 4ax$
(B) $2a(x + a) = y^2$
(C) $2a(x + a) = y^2 + 4a^2$
(D) none of these

70. In triangle ABC, if $\angle c = \frac{\pi}{2}$ and $\frac{x}{y} = \frac{\cos A}{\cos B}$ then $\frac{x\tan A + y\tan B}{x + y} =$
(A) 4
(B) 8
(C) 2
(D) 1

71. If $g(x)$ is a polynomial satisfying $g(x)g(y) = g(x) + g(y) + g(xy) - 2$ for all real x and y and $g(2) = 5,$ then $g(3)$ is equal to
(A) 10
(B) 24
(C) 21
(D) none of these

72. Let a sequence of integers $I_1, I_2, I_3, ...$ be defined by $I_1 = 1$ and $I_{n+1} = 1 + I_1.I_2.I_3.....I_n \; \forall n \geq 1$. If $H_n = \sum_{k=1}^{n} \frac{1}{I_k}$ and $P_n = \prod_{k=1}^{n} \frac{1}{I_k}$ then value of $H_{2022} + P_{2022} -$
(A) 4
(B) 8
(C) 2
(D) 1

73. If the roots of quadratic equation $ax^2 + bx + c = 0$ are $\frac{m}{m-1}$ and $\frac{m+1}{m},$ then $(a + b + c)^2$ is equal to
(A) $b^2 - 2ac$
(B) $2b^2 - ac$
(C) $b^2 - 4ac$
(D) $2(b^2 - 2ac)$

74. If p^{th}, q^{th} and r^{th} terms of both an A.P. and a G.P. be respectively a, b and c, then $\left(\frac{b}{c}\right)^a\left(\frac{c}{a}\right)^b\left(\frac{a}{b}\right)^c$ is

(A) 4
(B) 3
(C) 2
(D) 1

75. Find the number of integral solutions of $x+y+z\le 29$ such that $x>0, y>1, z>2$.

(A) 2900
(B) 2600
(C) 2500
(D) 2400

76. $f(n)=\sum_{r=1}^{n}\left(r^2\left({}^nC_r-{}^nC_{r-1}\right)+\left((2r+1){}^nC_r\right)\right)$ then $f(30)$ equals

(A) 900
(B) 930
(C) 960
(D) 980

77. Let $F=2i+2j+5k$ and $\overline{A}=(1,2,5), \overline{B}=(-1,-2,-3)$ and $\overrightarrow{BA}\times F=4i+6j+2\lambda k$, then the value of $|\lambda|$ is

(A) 4
(B) 3
(C) 2
(D) 1

78. Three six – faced fair die are thrown together let P (k) denote the probability that the sum of the numbers appearing on the dice is $k(9\le k\le 14)$. Find 54S, where $S=\sum_{k=9}^{14}P(k)$

(A) 30
(B) 35
(C) 40
(D) 45

79. If $\sin\beta$ is the geometric mean between $\sin\alpha$ and $\cos\alpha$, then $\cos 2\beta$ is equal to

(A) $2\sin^2\left(\frac{\pi}{4}-\alpha\right)$

(B) $2\cos^2\left(\frac{\pi}{4}-\alpha\right)$

(C) $2\sin^2\left(\frac{\pi}{4}+\alpha\right)$

(D) None of these

80. If lines $\frac{x-3}{1}=\frac{y-[k]}{2}=\frac{z-1}{1}$ and $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ intersect, (where [.] denotes greatest integer function) then K can be

(A) 4

(B) $\frac{13}{4}$

(C) 5

(D) $\frac{37}{6}$

SECTION – B
(Numerical Answer Type)

This section contains **10** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. XXXXX.XX).

81. A unit vector $\vec{r}$ has equal length of projections along the vectors $\frac{2\hat{i}+\hat{j}}{\sqrt{5}},\frac{-2\hat{i}+\hat{j}}{\sqrt{5}}$ and $\hat{k}$. If $\vec{r}.\left(\hat{i}+\hat{j}+\hat{k}\right)$ is positive then number of such vectors $\vec{r}$ will be

82. If the range of real – valued function $f(x)=\sqrt{\cos^{-1}(2x)+2}+\sqrt{1-\cos^{-1}(2x)}$ is $\left[\sqrt{a},\sqrt{b}+1\right]$ (where a and b are positive integers) then $(a+b)$ is equal to

83. The solution of the differential equation $\frac{d^3y}{dx^3}-8\frac{d^2y}{dx^2}=0$ satisfying $y(0)=\frac{1}{8}$, $y_1(0)=0$ and $y_2(0)=1$ is $y=\frac{e^{8x}-8x+7}{\lambda}$. The numerical value of $\frac{\lambda}{10}$ is

84. The function 'f' satisfies the functional equation $3f(x)+2f\left(\frac{x+59}{x-1}\right)=10x+30$ for all real $x\neq 1$. The value of f (7) is

85. Let $z_1=3$ and $z_2=7$ represent two points A and B respectively on complex plane. Let the curve C_1 be the locus of point P(z) satisfying $|z-z_1|^2+|z-z_2|^2=10$ and the curve C_2 be the locus of points P (z) satisfying $|z-z_1|^2+|z-z_2|^2=16$. Least distance between curves C_1 and C_2 is:

86. If $f(x)=\begin{cases}2x, & 0\le x<1\\ 2, & 1\le x\le 3\\ 8-2x, & 3<x\le 4\end{cases}$ then half of the area of the region bounded by the curve $|y+1|-f(x+4)=0$ is

87. In a geometric progression with common ratio q, the sum of the first 109 terms exceeds the sum of the first 100 terms by 12. If sum of the first nine terms of the progression is $\frac{\lambda}{q^{100}}$ then find the value of λ.

88. $a^2+b^2+c^2+ab+bc+ca\le 0,$ then value of determinant
$$\begin{vmatrix} (a+b+2)^2 & a^2+b^2 & 1 \\ 1 & (b+c+2)^2 & b^2+c^2 \\ c^2+a^2 & 1 & (c+a+2)^2 \end{vmatrix}$$ is

89. Distance between the parallel tangents having slopes $-\frac{4}{3}$ to the ellipse $\frac{x^2}{18}+\frac{y^2}{32}=1,$ is

90. Let $|\vec{a}|=1, |\vec{b}|=1$ and $|\vec{a}+\vec{b}|=\sqrt{3}$. If $\vec{c}$ be a vector such that $\vec{c}=\vec{a}+2\vec{b}-3(\vec{a}\times\vec{b})$ and $p=|(\vec{a}\times\vec{b})\times\vec{c}|,$ then p^2 is equal to