FIITJEE

ALL INDIA TEST SERIES

FULL TEST – XIII

JEE (Main)-2021

TEST DATE: 27-06-2021

ANSWERS, HINTS & SOLUTIONS

Physics PART – A

SECTION – A

1. B

Sol. For series combination $R = 220^2\left(\frac{1}{200}+\frac{1}{100}\right) = 220^2 \times \frac{3}{200}$

$\therefore \text{Power} = \frac{220^2}{R} = \frac{200}{3}\,\text{W}$

2. C

Sol. As $\gamma > 1$

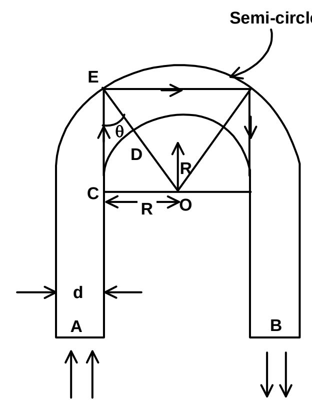
for $TV^{\gamma-1}$ = constant $\frac{dT}{dV} < 0$ $\quad \frac{d^2T}{dV^2} > 0$

and for $T^{\gamma} = KP^{\gamma-1}$

$\frac{dT}{dP} > 0 \quad \frac{d^2T}{dP^2} < 0$

3. A

Sol. As shown in the figure, angle of incidence will be minimum for the ray travelling along the inner edge. So the whole incident light entering the glass through surface A will emerge from the glass through surface B.



$\theta > I_C$

$\sin\theta > \sin I_C$

But as, $\sin I_C = \frac{1}{\mu} = \frac{2}{3}$

And from the geometry of the figure,

$$\sin\theta = \frac{OC}{OE} = \frac{R}{OD+DE} = \frac{R}{R+d}$$

so, $\frac{R}{R+d} > \frac{2}{3}$, i.e. $R > 2d$

i.e., $\frac{d}{R} < \frac{1}{2}$ so $\left(\frac{d}{R}\right) = \frac{1}{2}$

4. D

Sol. $\Delta E = -\left(\frac{GMm}{8R} - \frac{GMm}{4R}\right)$

5. D

Sol. No heat will be produced as no charge flows through S_2 when it is closed.

6. A

Sol. Given $\eta_1 = \frac{1}{6}$, $\eta_2 = \frac{1}{3}$

If the temperatures of the source and the sink between which the cycle is working are T_1 and T_2, then the efficiency in the first case will be

$\eta_1 = 1 - \frac{T_2}{T_1} = \frac{1}{6}$

In the second case $\eta_2 = 1 - \frac{T_2 - 65}{T_1} = \frac{1}{3}$

Solving $T_1 = 390$ K and $T_2 = 325$ K.

7. B

Sol. Particle is at one of its extreme position. Therefore the phase is $\pi/2$.

8. B

Sol. Source and observer are receding with a constant relative velocity.

9. C

Sol. $A_r = \frac{|v_1 - v_2|}{v_1 + v_2} \times A_i$; $v_1 = \sqrt{\frac{T}{\mu}}$; $v_2 = \sqrt{\frac{T}{4\mu}}$; $v = f\lambda$

10. B

Sol. Displacement method

$\frac{u}{D-u}=2$ (I) & D – u – u = 30 cm ...(II)

3u = 2D (I) & D – 2u = 30 cm

And $\frac{1}{f}=\frac{1}{D-u}+\frac{1}{u}$

$\Rightarrow$ f = 20 cm.

11. B

Sol. Initial extension = $\frac{3mg}{k}$. Final extension = $\frac{mg}{k}$.

$\therefore$ Amplitude = $\frac{2mg}{k}$.

12. B

Sol. $dV = -IdR = = -I\rho\frac{dx}{A}$

$\frac{dV}{dx}=-I\frac{\rho}{A}=-k$

$\Rightarrow R = k\ell/I$

II Method:

$V_2 - V_1 = \int\left(\frac{dV}{d\ell}\right)d\ell = kL = IR$

13. A

Sol. The resistivity of pure silicon is 2300 Ω m

and μ_e = 0.135 m^2/v-s, μ_h = 0.048 m^2/v-s.

Using

$\sigma = 1/\rho = (n_I\,\mu_e + n_I\,\mu_h)e$

$(2300)^{-1} = n_I\,(0.135 + 0.048) \times 1.6 \times 10^{-19}$

$n_I = 1.5 \times 10^{16}$ /m^3.

Is the intrinsic electron & hole concentration. The resistivity of a specimen doped with 10^{19} P-atoms/m^3 can be found from :

σ (conductivity) = $n_e\, e.\mu_e$ ($\because n_e = 10^{19}/m^3 >> n_I$)

$= 10^{19} \times 1.6 \times 10^{-19} \times 0.135 = 0.216$ mho / m.

$\rho = \frac{1}{\sigma} = 4.6\ \Omega$ -m

14. D

Sol. $V = 2(x^2 - y^2)$

Equipotential surfaces will be hyperbolic and $\vec{E}$ everywhere will be $\perp$ to them.

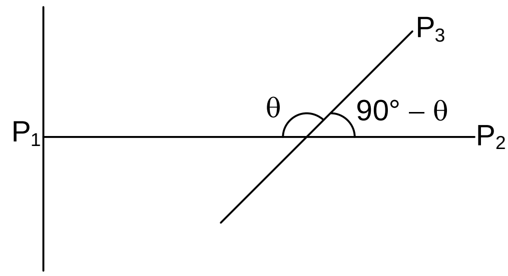
15. C

Sol. $F \propto \frac{m}{r^2}$ (force is conservative like gravitational force)

Velocity decreases after collision so total energy will decrease. Path is bound so, an ellipse inside the circles.

16. A

Sol. No light is emitted from the second polaroid, so P_1 and P_2 are perpendicular to each other



Let the initial intensity of light is I_0. So Intensity of light after transmission from first polaroid $= \frac{I_0}{2}$.

Intensity of light emitted from P_3

$$I_1 = \frac{I_0}{2}\cos^2\theta$$

Intensity of light transmitted from polaroid i.e., from

$$P_2 = I_1\cos^2(90° - \theta) = \frac{I_0}{2}\cos^2\theta\sin^2\theta$$

$$P_2 = \frac{I_0}{8}(2\sin\theta\cos\theta)^2 = \frac{I_0}{8}\sin^2 2\theta$$

17. D

Sol. I_L and I_C will be in opposite phases.

$I_{net} = I_L - I_C$

$= 0.6 - 0.4 = 0.2$ A

18. C

Sol. Maximum velocity $v_{max} = \sqrt{2as}$

$$v_{avg} = \frac{6s}{T} \qquad T = t_1 + t_2 + t_3$$

$$t_1 = \sqrt{\frac{2s}{a}}, \qquad t_2 = \frac{2s}{v_{max}} = \sqrt{\frac{2s}{a}}$$

Retardation $\Rightarrow$ B

$$\beta = \frac{v_{max}^2}{6s}$$

$$\beta = \frac{2a\times s}{6\times s} = \frac{a}{3}$$

$$t_3 = \frac{v_{max}}{\beta} = 3\sqrt{\frac{2a\times s}{a}} = 3\sqrt{\frac{2s}{a}}$$

$$T = 5\sqrt{\frac{2s}{a}}$$

$$v_{avg} = \frac{6}{5}\frac{5}{\sqrt{\frac{2s}{a}}} \;;\; v_{avg} = \frac{6}{5}\sqrt{\frac{as}{2}}$$

$$\frac{v_{avg}}{v_{max}} = \frac{6}{5}\sqrt{\frac{as}{2}} \times \frac{1}{\sqrt{2as}} = \frac{3}{5}$$

19. C

Sol. For closed organ pipe $f_C = (2n-1)\frac{v}{4L_C}$ with n = 1, 2, …

While for open organ pipe $f_O = n\frac{v}{2L_O}$ with n = 1, 2, …

According to given problem

$$f_C = \frac{v}{4L_C} = 110 \qquad \text{i.e.,} \qquad L_C = \frac{330}{4 \times 110} = 0.75 \text{ m}$$

Furthermore

$$\frac{3v}{4L_C} \sim \frac{2v}{2L_O} = 2.2 \qquad \text{or} \qquad \frac{330}{L_O} = 330 \mp 2.2 \qquad \left(\text{as } \frac{3v}{4L_C} = 3f_C = 330\right)$$

i.e., $L_O = \frac{330}{330 \mp 2.2} \Rightarrow L_O = 1.00067$ m or 0.9937 m

20. C

Sol. For charging q = $CE(1 - e^{-t/RC})$

Charge at t = RC $\Rightarrow$ $q_o = CE(1 - e^{-1})$

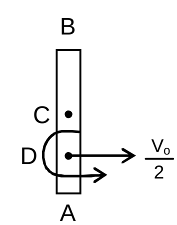
At t = RC discharging starts

$$\Rightarrow q = q_o(e^{-t/RC}) = CE(1 - e^{-1}) \times \frac{1}{e} = CE\left(\frac{1}{e} - \frac{1}{e^2}\right)$$

SECTION – B

21. 00001.25

Sol. By COAM about point D (i.e. about new centre of mass)



$$MV_o\frac{L}{4} = \left[\frac{ML^2}{12} + \frac{ML^2}{16} + \frac{ML^2}{16}\right]\omega$$

or, $w = \frac{6v_o}{5L}$

Now, $t = \frac{\theta}{w} = \frac{(\pi/2)}{w} = \frac{5\pi L}{12v_o} = 1.25$

22. 00001.20

Sol. $\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{f} = \frac{1}{2} + \frac{1}{3} \Rightarrow f = \frac{6}{5}$

23. 00001.50

Sol. muy = mVy + $\frac{3}{2}MR^2\omega$ where u is speed of ball before collision, V speed of ball after collision and ω is angular speed of cylinder after collision.

mu = mV + MRω (COM)

$\therefore$ y = $\frac{3R}{2}$.

24. 00337.50

Sol. Mass per unit length, $\mu = \frac{m}{\ell} = \frac{\rho A\ell}{\ell} = \rho A$

$\mu_s = \mu_{A\ell} = 78 \times 10^{-4}$ kg/m

$\therefore$ Speed of wave is same in both wire

$$V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{80 \times 10^4}{78}} = \frac{2 \times 10^2}{\sqrt{3.9}}$$

$$\nu_{min} = \frac{V}{\lambda_{max}} = \frac{200}{\sqrt{3.9} \times 0.3} \quad \left[\frac{\lambda_{max}}{2} = 15 \text{ cm for C as a node}\right]$$

$= 337.5$ Hz

25. 00001.30

Sol. Since collision is elastic $\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$

$\Rightarrow v_1^2 + v_2^2 = v^2$ ....(i)

and momentum conservation given $m\vec{v} = m\vec{v}_1 + m\vec{v}_2$

$\Rightarrow \vec{v}_1 + \vec{v}_2 = \vec{v}$ ....(ii)

Equation (i) and (ii) shows that angle between the velocity of both bodes after collision is 90^0 thus angle made by another body = 90 – 15 = 75^0=1.30 rad

26. 00003.20

Sol. $\mu = \frac{3}{2}$; V = 8 ; m = $\frac{1}{4}$

$1 + m = \frac{V}{f} = \frac{V}{2R}$

27. 00001.80

Sol. $m\frac{dv}{dt} = -6\pi\eta rv$

$$(1)\frac{dv}{dt} = 6\pi\left(\frac{1}{18\pi}\right)(1)v$$

$$\frac{dv}{dt} = -\frac{v}{3}$$

$$\frac{dv}{v} = -\frac{dt}{3}$$

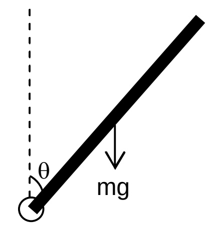
$$[\ln v]_2^{0.5} = \left[-\frac{t}{3}\right]_{t_1}^{t_2}$$

$$\ln 4 = \frac{\Delta t}{3}$$

$$\Delta t = 3 \ln 4$$

28. 00014.70

Sol. The figure shows the rod at an angle θ to the vertical. If we take torques about the pivot we need not be concerned with the force due to the pivot. The torque due to the weight is $1/2 mgL\sin\theta$, so the second law for the rotational motion is



$$\frac{mgL}{2}\sin\theta = \frac{ML^2}{3}\alpha$$

Thus $\alpha = \frac{3g\sin\theta}{2L}$

When the rod is horizontal $\theta = \pi/2$ and $\alpha = 3g/2L$.

The tangential linear acceleration of the free end is

$$a_t = \alpha L = \frac{3g}{2}$$

29. 00002.50

Sol. $x^2 + y^2 = \ell^2$

$$\frac{dy}{dt} = -\frac{x}{y}\left(\frac{dx}{dt}\right)$$

If $\theta = 37°$

$$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt} = -\frac{4}{3}v_o, \quad \vec{v}_{cm} = \frac{mv_o\hat{i} - m\frac{4v_o}{3}\hat{y}}{2m}, \quad \left|\vec{V}_{cm}\right| = \frac{5v_o}{6} = 2.5 \text{ m/s}$$

30. 00002.50

Sol. $\frac{\lambda_R}{\lambda_S} = 1.5$

So, the rate of disintegration of R will be 1.5 times that of S. Thus, the half-life of S will be 1.5 times that of R. So, two half lives of S will be equal to the three half –lives of R.

$$\frac{N_R}{N_S} = \frac{0.625}{0.25} = 2.50$$

Chemistry

PART – B

SECTION – A

31. C

Sol. $KI_3 \longrightarrow K^+ + I_3^-$

$\therefore$ The oxidation number of iodine in I_3^- is $-\frac{1}{3}$

32. B

Sol. $Cu \longrightarrow Cu^{2+} + 2e^-$ ΔG_1^0

$2Cu^{2+} + 2e^- \longrightarrow 2Cu^+$ ΔG_2^0

$\Delta G_f^0 = \Delta G_1 + \Delta G_2$

$\Delta G_f^0 = -2.303RT\log K$

33. B

Sol. At high pressure 'a' is neglected due to repulsion between gas molecules.

$\therefore$ P(V – b) = RT or PV = RT + Pb

34. C

Sol. Milliequivalent of H_2SO_4 = 25.1 × 0.02 = 0.502

Equivalent = 0.502 × 10^{-3}

Eq of H_2SO_4 = Eq of $CaCO_3$

$\therefore$ Mass of $CaCO_3$ = Eq × 50 = 0.502 × 10^{-3} × 50 = 0.0251

100 mL H_2O = 100 g contains 0.0251 g $CaCO_3$

10^6 g of water will contain $\frac{0.0251}{10^2}\times 10^6$ = 251 g of $CaCO_3$

$\therefore$ The degree of hardness = 251 ppm

35. B

Sol. Phosphorus contains maximum positive charge in $PClF_4$

36. D

Sol. The products are due to coupling of different alkyl groups. If $CH_3CH_2CH_2Cl$ is RCl and $CH_3\underset{\displaystyle Cl}{\underset{|}{C}}HCH_3$ is R'Cl, the products will be RR, R'R' and RR'.

37. B

Sol. For first order reaction, the half-life time is given as: $t_{1/2} = \frac{0.693}{k}$

$\therefore$ The half-life is independent of the concentration of reactant.

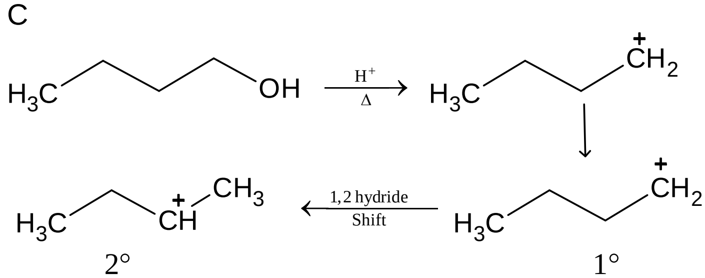
38. C

Sol. K_{sp} changes only by changing temperature.

39. B
Sol. Heat of hydrogenation of the least substituted alkene is maximum.

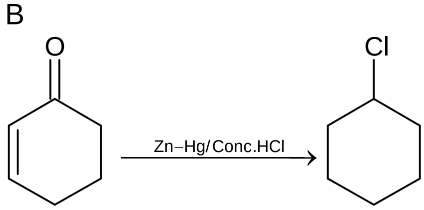
40. C
Sol. $W = -2.303n\ RT \log \frac{V_2}{V_1} = -2.303 \times 1.2 \times 8.314 \times 10^{-3} \times 300 \log \frac{10V_1}{V_1} = -6.892\ kJ\ mol^{-1}$

41. C
Sol.



42. B
Sol. S^{2-} makes the CCP lattice where Zn^{2+} ions are present at the alternate tetrahedral voids.

43. B
Sol.



44. C
Sol. Octahedral complex with C.N number 6.

45. B
Sol. $SiO_2 + 2NaOH \longrightarrow Na_2SiO_3 + H_2O$

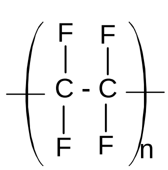
$SiO_2 + Na_2CO_3 \longrightarrow Na_2SiO_3 + CO_2$

46. B
Sol. In H-atom, the orbitals(five 3d, three 3p and one 3s) have same energy. So, they are called degenerate orbitals.

47. C
Sol. $pH = p_{Ka} + \log\frac{[CH_3COONa]}{[CH_3COOH]} = 5 + \log\frac{200 \times 0.4}{400 \times 0.2} = 5$

48. D
Sol. In chlorobenzene, Cl exerts –I and +R effect. In other compounds, it only exerts –I effect.

49. C
Sol.

Teflon is 

50. B

Sol. The products are

$$CH_3-CH=CH-\overset{\overset{\large O}{||}}{C}-CH_3 \text{ and } CH_3-\underset{\underset{\large CH_3}{|}}{C}=CH-CHO$$

SECTION – B

51. 00520.00

Sol. $P_A^o = 400$ mm, $P_B^o = 600$ mm

$X_A = \dfrac{4}{10}$, $X_B = \dfrac{6}{10}$

$\therefore$ P = $X_A P_A^o + X_B P_B^o$ = $(0.4 \times 400) + (0.6 \times 600)$ = 160 + 360 = 520 mm of Hg

52. 00273.50

Sol. $CH_4 + Cl_2(\text{excess}) \longrightarrow CH_3Cl + CH_2Cl_2 + CHCl_3 + CCl_4$

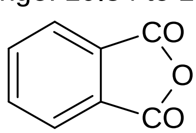
Most acidic(P) = $CHCl_3$ = 119.5 g mol^{-1}

Most heaviest(Q) = CCl_4 = 154 g mol^{-1}

$\therefore$ Molar mass of P + Q = 273.5 g mol^{-1}

53. 00010.84

[Range: 10.84 to 10.90]

Sol. P =  , Q = H_2O

% of Q = 10.84

54. 00240.80

Sol. $C_{rms} = \sqrt{\dfrac{3RT}{M}}$

$\therefore C_{rms} \ \alpha \ \sqrt{T}$

55. 00009.60

Sol. Moles of HCl = $\dfrac{M \times V}{1000} = \dfrac{1200 \times 0.2}{1000} = 0.24$

$\therefore$ Moles of NaOH = 0.24

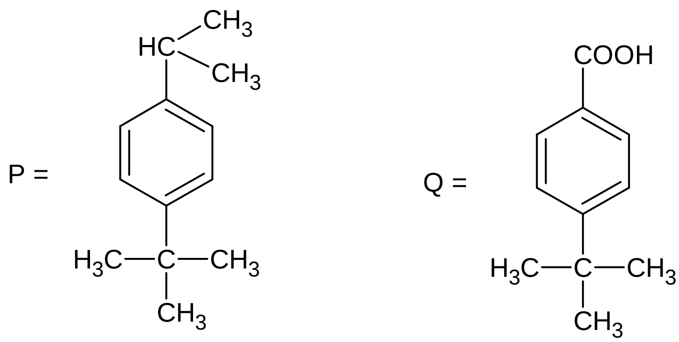
Mass of NaOH = 0.24×40 = 9.6 g

56. 00007.50

Sol. $pH = \dfrac{1}{2}\left(p^{K_{a_1}} + p^{K_{a_2}}\right) = \dfrac{1}{2}(4.5 + 10.5) = 7.5$

57. 00003.00

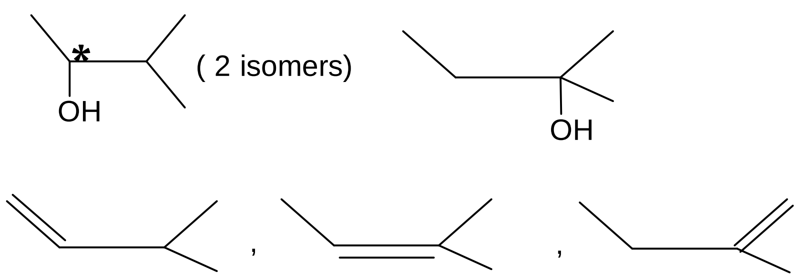
Sol.



Hence number of CH_3 groups = 3

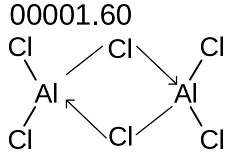
58. 00006.00

Sol. The products are



59. 00001.60

Sol.



$\therefore$ x = 6, y = 2, z = 0

$$\therefore \left(\frac{x+y+z}{5}\right) = \frac{8}{5} = 1.6$$

60. 00003.50

Sol. P = $NaHSO_3$, Q = Na_2SO_3

R = Na_2SO_4, S = $BaSO_4$

X = 2 + 1 + 4 = 7

$$\therefore \frac{X}{2} = 3.5$$

Mathematics PART – C

SECTION – A

61. B

Sol. $f(\alpha_i) = \tan\alpha_i \ e^{-\alpha_i}\left(f(\alpha_i) + \frac{1}{f(\alpha_i)}\right) = \frac{2e^{-\alpha_i}}{\sin 2\alpha_i} = 2$

$$\sum_{i=1}^{4} e^{-\alpha_i}\left(f(\alpha_i) + \frac{1}{f(\alpha_i)}\right) = 8$$

62. A

Sol. Let $y = \frac{ax^4 - bx^3 + cx^2 - bx + a}{\left(x^2+1\right)^2} \Rightarrow y = \frac{a\left(x^2 + \frac{1}{x^2}\right) - b\left(x + \frac{1}{x}\right) + c}{\left(x + \frac{1}{x}\right)^2}$

$y(t) = y = \frac{a\left(t^2 - 2\right) - bt + c}{t^2}$ where $t = x + \frac{1}{x}$

$y(t) = y = a - \frac{b}{t} + \frac{c-2a}{t^2}$, y(t) attains minimum value at $x = \frac{1}{2}$ or

$x = 3 \Rightarrow t = x + \frac{1}{x} = \frac{5}{2}$

$y(t)$ attains minimum value at $t = \frac{5}{2} \Rightarrow y'\left(\frac{5}{2}\right) = 0 \Rightarrow \frac{b}{t^2} - \frac{2(c-2a)}{t^3} = t = \frac{5}{2} = 0$

$\Rightarrow 5b = 4(c-2a) a, b, c \in N$, Hence (A) is correct.

63. A

Sol. $\sqrt{x \pm \sqrt{20x - 100}} = \sqrt{5} \pm \sqrt{x-5}$

$$= \int_5^{10}\left(\sqrt{5} + \sqrt{x-5}\right)dx + \int_5^{10}\left(\sqrt{5} - \sqrt{x-5}\right)dx = 10\sqrt{5}$$

64. B

Sol Let $I = \int \frac{x\ e^x}{\left(1+e^x\right)} dx$

$$= \int x . \frac{e^x}{\sqrt{\left(1+e^x\right)}} dx$$

$$= x.2\sqrt{\left(1+e^x\right)} - \int 1.2\sqrt{\left(1+e^x\right)}\,dx$$

$$= 2x\sqrt{\left(1+e^x\right)} - 2\int\sqrt{\left(1+e^x\right)}\,dx$$

In second integral

Put $1+e^x = t^2$

$\therefore \quad dx = \dfrac{2t\,dt}{t^2-1}$

Then, $= 2x\sqrt{(1+e^x)} - 4\int \dfrac{t^2-1+1}{(t^2-1)}dt$

$$= 2x\sqrt{(1+e^x)} - 4\int\left(1+\frac{1}{t^2-1}\right)dt$$

$$= 2x\sqrt{(1+e^x)} - 4\left\{t+\frac{1}{2}\ln\left(\frac{t-1}{t+1}\right)\right\} + c$$

$$= 2x\sqrt{(1+e^x)} - 4\sqrt{(1+e^x)} - 2\ln\left(\frac{\sqrt{(1+e^x)}-1}{\sqrt{(1+e^x)}+1}\right) + c$$

$$= (2x-4)\sqrt{(1+e^x)} - 2\ln\left(\frac{\sqrt{(1+e^x)}-1}{\sqrt{(1+e^x)}+1}\right) + c$$

On comparing

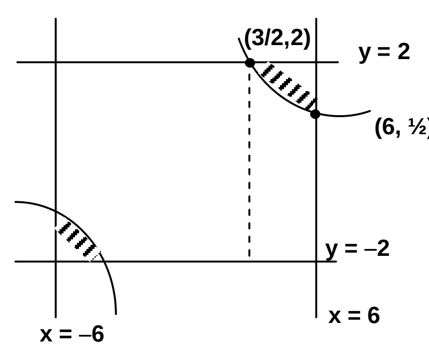
$$f(x) = 2x-4,\ g(x) = \frac{\sqrt{(1+e^x)}-1}{\sqrt{(1+e^x)}+1}$$

65. C

Sol. Area $= 2\left\{4x\dfrac{9}{2} - \int_{3/2}^{6}\dfrac{3}{x}dx\right\}$

$= 36 - 6\,\ell n\,4$

$= 12\,(3 - \ell n\,2)$



66. A

Sol. $\Rightarrow y\dfrac{dy}{dx} = \dfrac{x(1+y^2)}{1+x^2}$

$\Rightarrow 5(y^2+1) = 1+x^2$

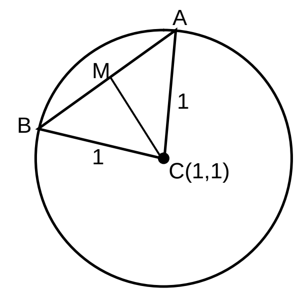
67. A

Sol. The given circle is $(x-1)^2 + (y-1)^2 = 1$.

In the figure MC = $1 \times \frac{\sqrt{3}}{2}$

$\Rightarrow$ locus of the point M is

$(x-1)^2 + (y-1)^2 = \frac{3}{4}$.



68. D

Sol. Let the coordinates of P be (α, β).

Then $PQ = 2\beta$ and $OP = \sqrt{\alpha^2 + \beta^2}$

Since OPQ is an equilateral triangle OP = PQ

$\Rightarrow \quad \alpha^2 + \beta^2 = 4\beta^2 \Rightarrow \alpha^2 = 3\beta^2$

$\Rightarrow \quad \alpha = \pm\sqrt{3}\beta$

Also, since (α, β) lies on the given hyperbola,

$\frac{\alpha^2}{a^2} - \frac{\beta^2}{b^2} = 1$

$\Rightarrow \quad \frac{3\beta^2}{a^2} - \frac{\beta^2}{b^2} = 1 \Rightarrow \frac{3}{a^2} - \frac{1}{b^2} = \frac{1}{\beta^2} > 0 \Rightarrow \frac{b^2}{a^2} > \frac{1}{3}$

$\Rightarrow \quad e^2 - 1 > \frac{1}{3} \Rightarrow e^2 > \frac{4}{3} \Rightarrow e > \frac{2}{\sqrt{3}}$

69. C

Sol. Take P $\equiv \left(at^2, 2at\right)$, $S = (a, 0)$, $C = \left(2a + at^2, 0\right)$

Let Q $\equiv (\alpha, \beta)$

$\Rightarrow \frac{\alpha + a}{2a} = \left(1 + t^2\right)$ and $\frac{\beta}{2a} = t$

$\Rightarrow$ required locus is $2a(x + a) = (4a^2 + y^2)$

70. D

Sol. $\dfrac{\frac{x}{y}\tan A + \tan B}{\left(1 + \frac{x}{y}\right)} = \dfrac{\sin A + \sin B}{\cos A + \cos B} = \tan\left(\dfrac{A+B}{2}\right) = 1$

71. A

Sol. Putting x = 1, y= 2 then

$g(1)g(2) = g(1) + g(2) + g(2) - 2$

$\Rightarrow \; 5g(1) = 8 + g(1) \qquad \left(\because g(2) = 5\right)$

$\therefore \quad g(1) = 2$

Also, replacing y by $\frac{1}{x}$ in the given relation, then

$$g(x)g\left(\frac{1}{x}\right)=g(x)+g\left(\frac{1}{x}\right)+g(1)-2$$

or $g(x)8\left(\frac{1}{x}\right)=g(x)+g\left(\frac{1}{x}\right)$

$\therefore \quad g(x)=1\pm x^n$

$\Rightarrow \quad g(2)=1\pm 2^n=5$

Taking +ve sign

$2^n=2^2$

$\therefore \quad n=2$

$\Rightarrow \quad g(x)=1+x^2$

$\therefore \quad g(3)=1+3^2=10$

72. C

Sol. $P_n-P_{n+1}=\frac{1}{l_1.l_2....l_n}-\frac{1}{l_1.l_2...l_nl_{n+1}}$

$\Rightarrow P_n-P_{n+1}=\frac{(l_{n+1}-1)}{l_1.l_2.....l_nl_{n+1}}=\frac{l}{l_{n+1}};$

$H_{n+1}-H_n=\left(\frac{l}{l_1}+\frac{l}{l_2}+.....+\frac{l}{l_n}+\frac{l}{l_{n+1}}\right)-\left(\frac{l}{l_1}+\frac{l}{l_2}+....+\frac{l}{l_n}\right)$

$\Rightarrow H_{n+1}-H_n=\frac{l}{l_{n+1}}$

$\Rightarrow H_{n+1}-H_n=P_n-P_{n+1}$

$\Rightarrow P_{n+1}+H_{n+1}=P_n+H_n$

$\Rightarrow P_{n+1}+H_{n+1}=P_1+H_1=2,\ \forall n\in N$

73. C

Sol. $\frac{m}{m-1}+\frac{m+1}{m}=\frac{-b}{a}$ ……….(1)

$\left(\frac{m}{m-1}\right)\left(\frac{m+1}{m}\right)=\frac{c}{a}$

$\Rightarrow m=\frac{c+a}{c-a}$

Substitute m in (1)

$\frac{c+a}{2a}+\frac{2c}{c+a}=\frac{-b}{a}$

$(a+b+c)^2=b^2-4ac$

74. D

Sol. Let the A.P be $x, x+y, x+y, x+2y, ...$ Then

$a = x+(p-1)y; b = x+(q-1)y; c = x+(r-1)y$

$\therefore (b-c) = (q-r)y, (c-a) = (r-p)y, (a-b) = (p-q)y$

Let the G.P. be $u, uv, uv^2.......$ Then $a = uv^{p-1}, b = uv^{q-1}, c = uv^{r-1}$

Now, $\log\left\{a^{b-c}b^{c-a}c^{a-b}\right\} = (b-c)\log a + (c-a)\log b + (a-b)\log c$

$= (q-r)y\log\left(uv^{p-1}\right) + (r-p)y\log\left(uv^{q-1}\right) + (p-q)y\log\left(uv^{r-1}\right)$

$= 0.\log u + 0.\log v = 0$

$\therefore a^{b-c}.a^{c-a}.c^{a-b} = 1$

$\therefore a^b b^c c^a = a^c b^a c^b$

75. B

Sol. Let $t \geq 0$ be such that $x+y+z+t = 29$. Put $x = x_1+1, y = y_1+2, z = z_1+3$ where $x_1, y_1 \geq 0$, the equation becomes $x_1+y_1+z_1+t = 23$.

Its number of solutions is ${}^{26}C_3 = 2600$.

76. C

Sol. $f(x) = \sum_{r=1}^{n}\left((r+1)^2\, {}^nC_r - r^2\, {}^nC_{r-1}\right)$

$= \left(2^2\, {}^nC_1 - 1^2\, {}^nC_0\right) + \left(3^2\, {}^nC_2 - 2^2\, {}^nC_1\right) + \left(4^2\, {}^nC_3 - 3^2\, {}^nC_2\right) + ...(n+1)^2\, {}^nC_n - n^2\, {}^nC_{n-1}$

$= (n+1)^2\, {}^nC_n - 1 = n^2 + 2n$

77. C

Sol. $\overrightarrow{BA} \times F = (2i+4j+8k) \times (2i+2j+5k) = \begin{vmatrix} i & j & k \\ 2 & 4 & 8 \\ 2 & 2 & 5 \end{vmatrix}$

$4i + 6j - 4k = 4i + 6j + 2\lambda k$ (given).

$\therefore \lambda = -2$

78. B

Sol. The total number of cases is $6 \times 6 \times 6 = 6^3 = 216$. The number of favorable ways

= coefficient of x^k in $\left(x + x^2 + ... + x^6\right)^3$

= coefficient of x^{k-3} in $\left(1-x^6\right)^3 (1-x)^{-3}$

= coefficient of x^{k-3} in $\left(1-3x^6\right)\left(1 + {}^3C_1x + {}^4C_2x^2 + {}^5C_3x^3 + ...\right)$

[Note that $6 \leq k-3 \leq 11$]

$= {}^{k-3+2}C_{k-3} - (3)\ {}^{k-3+2-6}C_{k-3-6}$

$= {}^{k-1}C_2 - (3)\, {}^{k-7}C_2$

Now, $P(k) = \dfrac{21k - k^2 - 83}{216}$

$\therefore\ S = \dfrac{35}{54}$.

79. A

Sol. Given, $\sin\beta = \sqrt{\sin\alpha\cos\alpha} \Rightarrow \sin^2\beta = \sin\alpha\sin\alpha$

Now, $\cos 2\beta = 1 - 2\sin^2\beta = 1 - 2\sin\alpha\cos\alpha$

$= (\sin\alpha - \cos\alpha)^2$

$= 2\sin^2\left(\dfrac{\pi}{4} - \alpha\right)$ or $2\cos^2\left(\dfrac{\pi}{4} + \alpha\right)$

80. A

Sol. $\begin{vmatrix} 1-3 & -1-[K] & 0 \\ 1 & 2 & 1 \\ 2 & 3 & 4 \end{vmatrix} = 0$

$[K] = 4$

SECTION – B

81. 00004.00

Sol. Let $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\left|\vec{r}.\dfrac{2\hat{i} + \hat{j}}{\sqrt{5}}\right| = \left|\vec{r}.\dfrac{(-2\hat{i} + \hat{j})}{\sqrt{5}}\right| = |\vec{r}.\hat{k}|$

$\Rightarrow |2x + y| = |-2x + y| = \sqrt{5}|z|$

$\Rightarrow \vec{r} = \left(0, \dfrac{\sqrt{5}}{\sqrt{6}}, \dfrac{1}{\sqrt{6}}\right), \left(0, \dfrac{\sqrt{5}}{\sqrt{6}}, -\dfrac{1}{\sqrt{6}}\right), \left(\dfrac{\sqrt{5}}{3}, 0, \dfrac{2}{3}\right), \left(\dfrac{\sqrt{5}}{3}, 0, -\dfrac{2}{3}\right)$

82. 00005.00

Sol. $f(x) = \sqrt{\cos^{-1}(2x) + 2} + \sqrt{1 - \cos^{-1}(2x)}$

$1 - \cos^{-1}(2x) \ge 0$

$\Rightarrow -1 \le 2x \le 1$ $\qquad$ $\cos^{-1}(2x) \le 1$

$\Rightarrow -\dfrac{1}{2} \le x \le \dfrac{1}{2}$ $\qquad$ $2x \ge \cos(1)$ $\qquad$ $x \ge \dfrac{1}{2}\cos 1$

Domain is $\frac{1}{2}\cos 1 \le x \le \frac{1}{2}$ and Range : $\left[\sqrt{3}, \sqrt{2}+1\right]$

$a + b = 5$

83. 00006.40

Sol. $\frac{\frac{d^3y}{dx^3}}{\frac{d^2y}{dx^2}} = 8$

$\Rightarrow \quad \ln\frac{d^2y}{dx^2} = 8x + c$

or $\quad \ln y_2 = 8x + c$

Putting x = 0, we have $c = 0$.

$\therefore \quad \ln y_2 = 8x \Rightarrow y_2 = e^{8x}$

$\therefore \quad y_1 = \frac{e^{8x}}{8} + D$

Again putting $x = 0$, $D = -\frac{1}{8}$.

$\Rightarrow \quad y_1 = \frac{e^{8x}}{8} - \frac{1}{8}$

$\Rightarrow \quad y = \frac{e^{8x}}{64} - \frac{x}{8} + E$

Putting $x = 0$, $E = \frac{1}{8} - \frac{1}{64} = \frac{7}{64}$.

Hence, $y = \frac{e^{8x}}{64} - \frac{x}{8} + \frac{7}{64} = \frac{e^{8x} - 8x + 7}{64}$.

84. 00004.00

Sol. $3f(x) + 2f\left(\frac{x+59}{x-1}\right) = 10x + 30$

For x = 7, 3f (7) + 2f (11) = 100

For x = 11, 3f (11) + 2f (7) = 140

Solving, we get f (7) = 4.

85. 00001.00

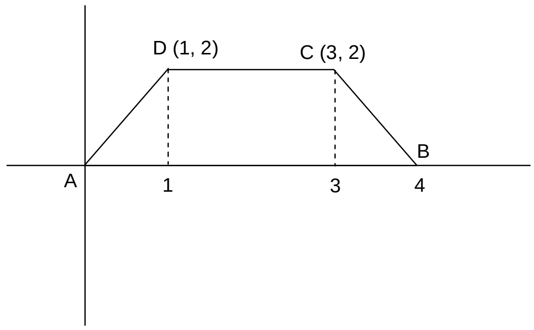
Sol. $C_1 : |z - 5| = 1$

$C_2 : |z - 5| = 2$

Distance between C_1 and $C_2 = 1$

86. 00006.00

Sol.



Area bounded by $|y+1| = f(x+4)$ is equal to area bounded by $|y| = f(x)$

87. 00012.00

Sol. Let first term of G.P. be a then

$$t_1 + t_2 + ... + t_{109} = t_1 + t_2 + ... + t_{100} + 12$$

$$t_{101} + t_{102} + ... + t_{109} = 12$$

$$aq^{100} + aq^{101} + ... + aq^{108} = 12 \qquad ......(i)$$

$$a + aq + aq^2 + ... + aq^8 = \frac{\lambda}{q^{100}}$$

$$a\left[1 + q + q^2 + ... + q^8\right] = \frac{\lambda}{q^{100}} \qquad ....(ii)$$

$$\frac{\lambda}{q^{100}} = \frac{12}{q^{100}}\lambda = 12$$

88. 00065.00

Sol. $(a+b)^2 + (b+c)^2 + (c+a)^2 \le 0$

$$(a+b) = (b+c) = (c+a) = 0 \Rightarrow a = b = c = 0$$

$$0 = \begin{vmatrix} 4 & 0 & 1 \\ 1 & 4 & 0 \\ 0 & 1 & 4 \end{vmatrix} = 65$$

89. 00009.60

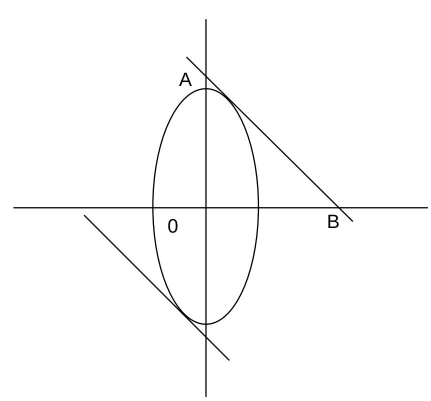
Sol. $y = mx \pm \sqrt{a^2m^2 + b^2}$

$$y = -\frac{4}{3} \times \pm\sqrt{18 \times \frac{16}{9} + 32}$$

$$y = -\frac{4}{3} \times \pm 8$$

Distance between tangents

$$= \frac{16}{\sqrt{1 + \frac{16}{9}}} = \frac{16 \times 3}{5} = \frac{48}{5}$$



90. 00005.25

Sol. $\left|\vec{a}\right| = 1, \left|\vec{b}\right| = 1, \left|\vec{a}+\vec{b}\right| = \sqrt{3}$

$3 = 1+1+2\vec{a}.\vec{b} \Rightarrow \vec{a}.\vec{b} = \frac{1}{2}$

$\vec{C} = \vec{a} + 2\vec{b} - 3\left(\vec{a}\times\vec{b}\right)$

$p = \left(\vec{a}\times\vec{b}\right)\times\vec{c} \Rightarrow \left(\vec{a}\times\vec{b}\right)\times\left(\vec{a}+2\vec{b}-3\left(\vec{a}\times\vec{b}\right)\right)$

$= \left(\vec{a}\times\vec{b}\right)\times\vec{a} + 2\left(\vec{a}\times\vec{b}\right)\times\vec{b}$

$= \left(\vec{a}.\vec{a}\right)\vec{b} - \left(\vec{b}.\vec{a}\right)\vec{a} + 2\left(\left(\vec{a}.\vec{b}\right)\vec{b} - \left(\vec{b}.\vec{b}\right).\vec{a}\right)$

$= \vec{b} - \frac{1}{2}\vec{a} + 2\times\left(\frac{1}{2}\vec{b} - \vec{a}\right)$

$= 2\vec{b} - \frac{5}{2}\vec{a}$

$\left|\vec{p}\right| = \sqrt{4 + \frac{25}{4} + 10\vec{a}.\vec{b}} = \sqrt{\frac{41}{4} - 5}$

$= \frac{\sqrt{21}}{2}$