# FIITJEE

# ALL INDIA TEST SERIES

## FULL TEST – X

## JEE (Advanced)-2021

### PAPER –1

### TEST DATE: 19-09-2021

# ANSWERS, HINTS & SOLUTIONS

## *Physics* PART – I

### SECTION – A

1. C

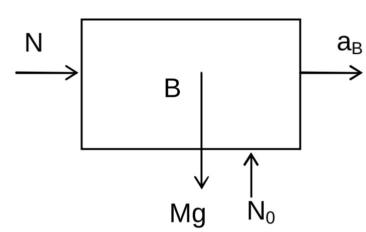
Sol. $a_A\cos\theta = a_B$ …(i)

$N = Ma_B$

$N = Ma_A\cos\theta$ …(ii)

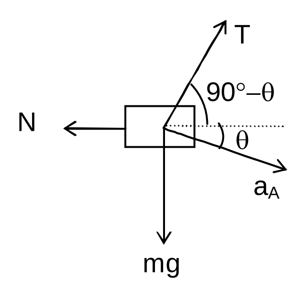
$mg\sin\theta - N\cos\theta = ma_A$

$mg\sin\theta - Ma_A\cos^2\theta = ma_A$




$$a_A = \frac{mg\sin\theta}{(m+M\cos^2\theta)} = \frac{mg\sin 30^\circ}{(m+4m\cos^2 30^\circ)}$$

$$a_A = \frac{g}{8} = \frac{5}{4}\ m/s^2$$



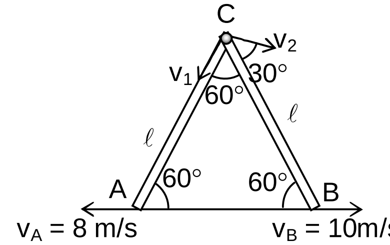

2. A

Sol. $v_1 = v_A\cos 60^\circ = 8\times\frac{1}{2} = 4\ m/s$ …(i)

$v_1\cos 60^\circ + v_2\cos 30^\circ = v_B\cos 60^\circ$

$4\times\frac{1}{2} + v_2\frac{\sqrt{3}}{2} = 10\times\frac{1}{2} \Rightarrow v_2 = 2\sqrt{3}\ m/s$ …(ii)

$v_C = \sqrt{v_1^2+v_2^2} = \sqrt{16+12} = 2\sqrt{7}\ m/s$



3. B

Sol. dQ = dU + PdV

$$C = C_V + \frac{RT}{V}\frac{dV}{dT}$$

$$C_V + \alpha T^2 = C_V + \frac{RT}{V}\frac{dV}{dT}$$

$$\alpha T^2 = \frac{RT}{V}\frac{dV}{dT}$$

$$\frac{\alpha}{R}\int TdT = \int \frac{dV}{V} + \ell nk$$

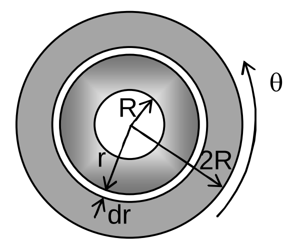
$$\frac{\alpha T^2}{2R} = \ell n(kV)$$

$$Ve^{-\frac{\alpha T^2}{2R}} = \text{constant}$$

4. D

Sol.

$$\int d\tau = \int_R^{2R} \eta\left(\frac{r\theta}{\ell}\right) r2\pi r dr$$




$$\tau = \frac{2\pi\eta\theta}{\ell}\int_R^{2R} r^3 dr$$

$$\tau = \frac{2\pi\eta\theta}{\ell}\left[\frac{r^4}{4}\right]_R^{2R}$$

$$\tau = \frac{15\pi\eta R^4\theta}{2\ell}$$

5. B

Sol. $P_2 \cos\theta = P$ …(i)

$P_2 \sin\theta = P_1$ …(ii)

From (i) and (ii)

$P_2^2 = P^2 + P_1^2$

$2mk_2 = 2 \times 2mk + 2mk_1$

$k_2 = 2k + k_1$

$k_2 = (2 \times 2) + 0.9$

$k_2 = 4.9$ eV

The energy released in the dissociation process is

$\Delta k = (k_1 + k_2) - k$

$\Delta k = (0.9 + 4.9) - 2$

$\Delta k = 5.8 - 2$

$\Delta k = 3.8$ eV

6. A

Sol.

$$\frac{1}{f_{\ell_1}} = (1.5-1)\left(\frac{1}{10} - \frac{1}{40}\right) = \frac{3}{80} \Rightarrow f_{\ell_1} = \frac{80}{3}\text{ cm}$$

$$\frac{1}{f_{\ell_2}} = \left(\frac{4}{3} - 1\right)\left(\frac{1}{40} + \frac{1}{40}\right) = \frac{1}{60} \Rightarrow f_{\ell_2} = 60\text{ cm}$$

$$\frac{1}{f_{\ell_3}} = (1.5-1)\left(\frac{-1}{40}+\frac{1}{20}\right) = \frac{1}{80} \Rightarrow f_{\ell_3} = 80\,\text{cm}$$

Now,

$$-\frac{1}{F} = \frac{2}{f_{\ell_1}} + \frac{2}{f_{\ell_2}} + \frac{2}{f_{\ell_3}} - \frac{1}{f_m}$$

$$-\frac{1}{F} = \frac{3}{40} + \frac{1}{30} + \frac{1}{40} + \frac{1}{10}$$

$$-\frac{1}{F} = \frac{2}{10} + \frac{1}{30}$$

$$-\frac{1}{F} = \frac{7}{30}$$

$$\Rightarrow F = -\frac{30}{7}\,\text{cm}$$

Now, $\frac{1}{v}+\frac{1}{u}=\frac{1}{F}$

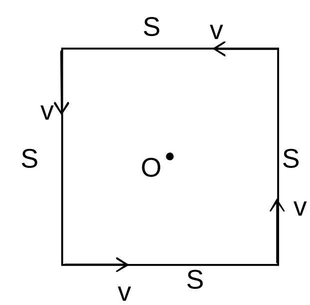
$$\frac{1}{v} + \frac{1}{(-15)} = -\frac{7}{30}$$

$$\frac{1}{v} = \frac{1}{15} - \frac{7}{30} = \frac{2-7}{30}$$

v = –6 cm

7. A, B, D

Sol. Velocity of approach between two successive particles at any time, $-\frac{dS}{dt} = v - 0 = v$




The time in which the particles will converge,

$$\tau = \frac{a}{v} \quad \text{...(i)}$$

The distance covered by each particle till they coverage

$$v \times \tau = v \times \frac{a}{v} = a$$

The acceleration of each particle when the separation between two successive particles becomes 'S' is

$$a_n = v\omega = v \times \frac{v}{S} = \frac{v^2}{S} \quad \text{...(ii)}$$

At $t = \frac{\tau}{5}$, $S = \frac{4a}{5}$

$$\therefore a_n = \frac{5v^2}{4a}$$

The radius of curvature of the trajectory of each particle is

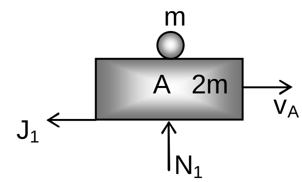
$$r = \frac{v^2}{a_n} = \frac{v^2}{v^2/s} = s \quad \text{...(iii)}$$

At $t = \frac{\tau}{3}$, $s = \frac{2a}{3}$

$$\therefore r = \frac{2a}{3}$$

8. A, C

Sol. $\int N_1 dt = 40\text{ m}$ …(i)




$J_1 = \int \mu_1 N_1 dt = \mu_1 \int N_1 dt$

$J_1 = 0.3 \times 40\text{ m}$

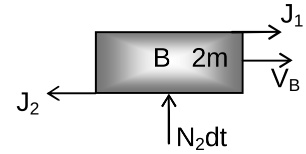
$J_1 = 12\text{ m}$ …(ii)

Now, $J_1 = 30m - 3mv_A$

$12m = 30m - 3mv_A$

$v_A = 6\text{ m/s}$

$\int N_2 dt = 40\text{ m}$ …(iii)




$J_2 = \int \mu_2 N_2 dt = \mu_2 \int N_2 dt$

$J_2 = 0.1 \times 40m = 4m$

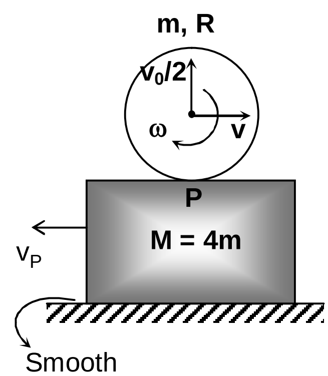
Now, $J_1 - J_2 = 2mv_B$

$12\text{ m} - 4m = 2mv_B$

$v_B = 4\text{ m/s}$

9. A, C, D

Sol. $\int N dt = m\left(\frac{v_0}{2} + v_0\right) = \frac{3mv_0}{2}$ …(i)




$J_{max} = \int \mu N dt = \mu \int N dt = \frac{3mv_0}{4}$ …(ii)

Let the sphere starts pure rolling during collision

$\omega R - v = v_P$ …(iii)

Using conservation of momentum

$mv - Mv_P = 0$

$v_P = \frac{v}{4}$ …(iv)

From (iii) and (iv)

$\omega = \frac{5v}{4R}$ …(v)

Using conservation of angular momentum of sphere about an axis passing through the contact point 'P' and fixed to the ground

$$\frac{2}{5}mR^2\omega_0 = \frac{2}{5}mR^2\omega + mvR$$

$$\frac{2}{5}mR^2\left(\frac{5v_0}{4R}\right) = \frac{2}{5}mR^2\left(\frac{5v}{4R}\right) + mvR$$

$$\frac{mv_0R}{2} = \frac{3mvR}{2} \Rightarrow v = \frac{v_0}{3} \text{ and } v_P = \frac{v_0}{12}$$

From equation (v), $\omega = \frac{5v_0}{12R}$

The impulse due to friction on the sphere during collision is

$$J = mv = \frac{mv_0}{3}$$

10. A, B, D

Sol. $k_{eq} = \left(\frac{k_1 + k_2}{2}\right)$ …(i)

The electric field in both the dielectrics will be same and given by

$$E = \frac{q}{4\pi\varepsilon_0 r^2 k_{eq}}$$

$$E = \frac{q}{2\pi\varepsilon_0 (k_1 + k_2) r^2} \quad \text{…(ii)}$$

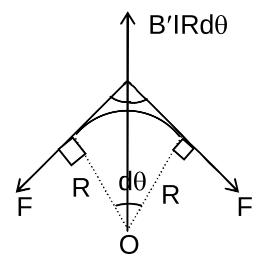
The potential difference between the conducting spheres is

$$V = \int_a^b E dr = \int_a^b \frac{q}{2\pi\varepsilon_0 (k_1 + k_2) r^2} dr$$

$$V = \frac{q(b-a)}{2\pi\varepsilon_0 (k_1 + k_2) ab}$$

11. A, C

Sol. Linear current density, $i = \frac{I}{d}$




$$B_i = \frac{1}{2}\mu_0 i = \frac{\mu_0 I}{2d}$$

$$B' = B_i = \frac{\mu_0 I}{2d} \quad \text{…(i)}$$

Now,

$F d\theta = B' I R d\theta$

$F = B' I R$

$$F = \frac{\mu_0 I^2 R}{2d} \quad \text{…(ii)}$$

The tension developed in the wire is $F = \frac{\mu_0 I^2 R}{2d}$

For rupturing of the coil,

$$F > \frac{\sigma_0 \pi d^2}{4}$$

$$\frac{\mu_0 I^2 R}{2d} > \frac{\sigma_0 \pi d^2}{4}$$

$$I > \sqrt{\frac{\sigma_0 \pi d^3}{2\mu_0 R}}$$

$$I_{min} = \sqrt{\frac{\sigma_0 \pi d^3}{2\mu_0 R}}$$

12. A, D

Sol. $\phi = \int_a^{2a} \frac{\mu_0 I}{2\pi r} a dr = \frac{\mu_0 I a}{2\pi} \ell n 2$

The charge that will flow through the loop,

$$q = \frac{|\Delta\phi|}{R} = \frac{\mu_0 I a \ell n 2}{2\pi R} \quad \text{…(i)}$$

The current induced in the loop is

$$i = -\frac{1}{R}\frac{d\phi}{dt} = -\frac{\mu_0 a \ell n2}{2\pi R}\frac{dI}{dt}$$

The net attractive force on the loop is

$$F = \left(\frac{\mu_0 Iia}{2\pi a}\right) - \left(\frac{\mu_0 Iia}{4\pi a}\right) = \frac{\mu_0 Ii}{4\pi} \quad \ldots(ii)$$

Now,

$$mv = \int F dt$$

$$mv = \int_I^0 \frac{\mu_0 I}{4\pi}\left(-\frac{\mu_0 a \ell n2}{2\pi R}\frac{dI}{dt}\right)dt$$

$$v = \frac{\mu_0^2 I^2 a \ell n2}{16\pi^2 mR}$$

## SECTION – C

13. 00152.00

Sol.

$$\Delta W = \int_{(OA)} \vec{F}\cdot d\vec{r} + \int_{(AB)} \vec{F}\cdot d\vec{r}$$

$$\Delta W = -\int_0^4 x^3 dx + \int_0^6 12y dy$$

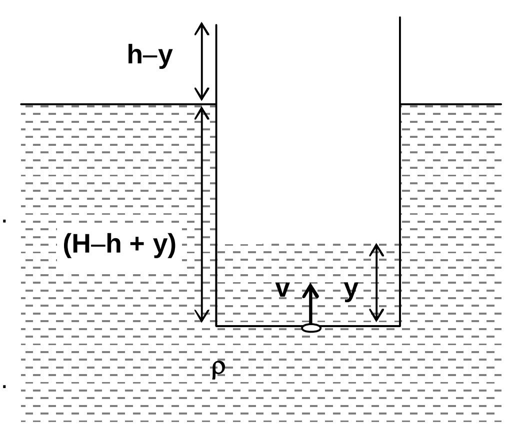
$$\Delta W = -\left[\frac{x^4}{4}\right]_0^4 + 6\left[y^2\right]_0^6$$

$\Delta W = -64 + 216$

$\Delta W = 152$ Joule

14. 00012.50

Sol. Using Bernoulli's equation




$$P_0 + \rho g(H - h + y) = P_0 + \rho g y + \frac{1}{2}\rho v^2$$

$$\rho g(H - h) = \frac{1}{2}\rho v^2$$

$$v = \sqrt{2g(H-h)}$$

Time required to sink the tank,

$$t = \frac{\ell bh}{av}$$

$$t = \frac{\ell bh}{a\sqrt{2g(H-h)}}$$

$$t = \frac{50 \times 20 \times 5 \times 10^{-2}}{2\sqrt{2 \times 10 \times 0.2}} = \frac{25}{2} \text{ sec}$$

t = 12.50 sec

15. 00027.20
(Range: 27.18 to 27.20)

Sol. $f_{max} = \left(\frac{v}{v-\omega A}\right)f_0$

$f_{min} = \left(\frac{v}{v+\omega A}\right)f_0$

Frequency bandwidth registered by the stationary observer is

$\Delta f = f_{max} - f_{min}$

$\Delta f = \left(\frac{v}{v-\omega A}\right)f_0 - \left(\frac{v}{v+\omega A}\right)f_0$

$\Delta f = \frac{vf_0 2\omega A}{v^2 - \omega^2 A^2}$

Since $\omega A << v$

$\Delta f = \frac{2vf_0\omega A}{v^2}$

$A = \frac{v\Delta f}{2f_0\omega}$

$A = \frac{340 \times 40}{2 \times 1000 \times 25} = 0.272$ m

A = 27.2 cm

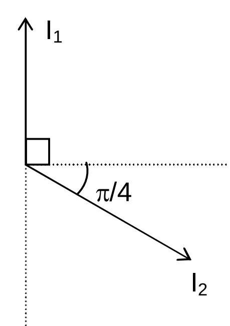
16. 00250.00

Sol. Let the rms current through the capacitance C is $I_1$

Now, $Z_2 = R + \omega Li$

$Z_2 = 20 + 20i$

The rms current through the inductor is




$I_2 = \frac{80}{20\sqrt{2}} = 2\sqrt{2}$ A

Now,

$I^2 = I_1^2 + I_2^2 + 2I_1I_2\cos\left(\frac{3\pi}{4}\right)$

$4 = I_1^2 + 8 + 2I_1 \times 2\sqrt{2}\left(-\frac{1}{\sqrt{2}}\right)$

$I_1^2 - 4I_1 + 4 = 0$

$\Rightarrow (I_1 - 2)^2 = 0 \Rightarrow I_1 = 2A$

$X_C = \frac{80}{I_1} = \frac{80}{2} = 40\Omega$

$X_C = 40\ \Omega$

$\frac{1}{\omega C} = 40$

$C = \frac{1}{100 \times 40} = 250 \times 10^{-6}$ F

$C = 250\ \mu F$

17. 00003.30

Sol. The Q-value of the reaction is

$Q = (3 \times 2.80) - (4 \times 1)$

$Q = 8.40 - 4$

$Q = 4.40$ MeV …(i)

Now, $\vec{P}_1 + \vec{P}_2 = 0$

$P_1^2 = P_2^2$

$2m_1k_1 = 2m_2k_2$

$k_1 = 3k_2$ …(ii) ($m_2 = 3m_1$)

Also,

$k_1 + k_2 = Q$

$k_1 + k_2 = 4.40$

$3k_2 + k_2 = 4.40$ [from (ii)]

$k_2 = 1.10$ MeV

$k_1 = 3k_2 = 3.30$ MeV

$k_1 = 3.30$ MeV

18. 00005.94

Sol. Least count = $\frac{P}{N} = \frac{1\,\text{mm}}{100} = 0.01\,\text{mm}$

Diameter of the wire is

$d = 4\text{ mm} + 56 \times \text{L.C.} - 6 \times \text{L.C.}$

$d = 4\text{mm} + 50 \times \text{L.C.}$

$d = 4\text{ mm} + 50 \times 0.01\text{ mm}$

$d = 4.50$ mm

$d = 0.450$ cm

The curved surface area of the wire is

$S = \pi d\ell$

$S = \frac{22}{7} \times 0.450 \times 4.2 = 5.94\,\text{cm}^2$

# Chemistry

## PART – II

### SECTION – A

19. D

Sol. For 2nd excited state to 1st excited state transition
(n = 3) (n = 2)

$\Delta E = 7.56\ \text{eV} = 13.6 \times z^2\left(\frac{1}{2^2}-\frac{1}{3^2}\right)$

$\Rightarrow z = 2$

Now transition from 2nd excited state (n = 3) to ground state (n = 1)

$\Delta E = 13.6 \times z^2 \times \left[\frac{1}{1^2}-\frac{1}{3^2}\right]$

$= 13.6 \times 2^2 \times \frac{8}{9} = 48.36\ \text{eV}$

20. C

Sol.

$XeF_6 \xrightarrow{+\ H_2O(\text{excess})} XeO_3 + HF$

$XeF_6 \xrightarrow{+\ SiO_2(\text{excess})} \underset{(\text{explosive})}{XeO_3} + SiF_4$

$XeO_3 + OH^- \longrightarrow \underset{\text{hydrogen xenate ion}}{HXeO_4^-}$

$2HXeO_4^- + 2OH^- \longrightarrow XeO_6^{4-} + Xe + 2H_2O + O_2$

21. A

Sol. $Fe^{+2}\overset{+3}{C}_2O_4 \longrightarrow Fe^{3+} + 2\overset{+4}{C}O_2$ (oxidation in acidic medium)

n-factor = (3 + 2 × 4) – (2 + 2 × 3) = 3

No. of equivalents of ferrous oxalate = 0.1 × 3 = 0.3

$CaC_2O_4.H_2O \longrightarrow Ca^{2+} + 2CO_2 + H_2O$ (oxidation acidic medium)

n-factor = (2 + 2 × 4) – (2 + 2 × 3) = 2

no. of equivalents of $CaC_2O_4.H_2O$ = $2 \times \frac{29.2}{146}$

= 2 × 0.2 = 0.4

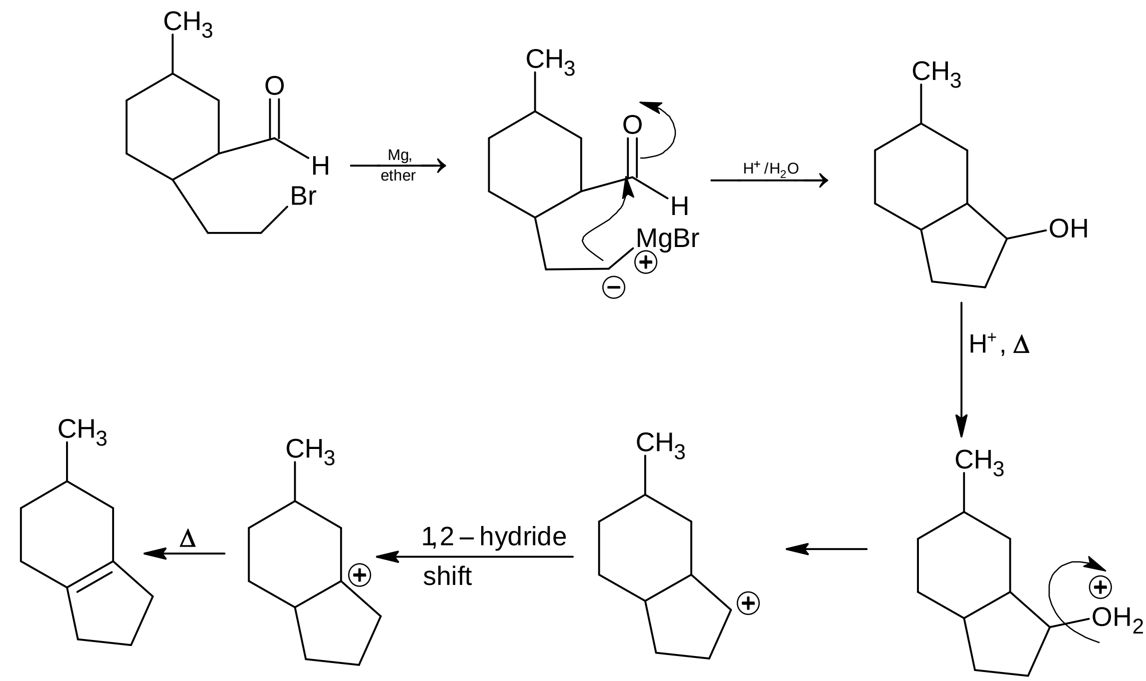
Total equivalent of both oxalates = 0.3 + 0.4 = 0.7

So, 0.035 × 5 × 'x' (L) = 0.7

'x' = 4

22. B

Sol.




23. D

Sol. $Pb(NO_3)_2 \xrightarrow{\Delta} \underset{\left(\substack{\text{Buff colour} \\ \text{when hot}}\right)}{PbO} + 2NO_2 + \frac{1}{2}O_3$

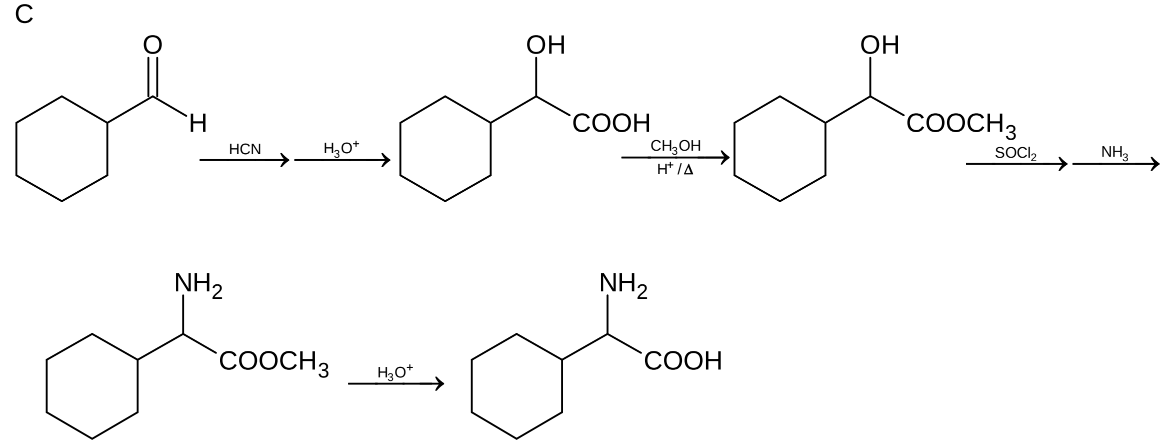
$2AgNO_3 \xrightarrow{\Delta} 2Ag + O_2 + 2NO_2$

$FeC_2O_4 \xrightarrow{\Delta} FeO + CO + CO_2$

$ZnSO_4 \xrightarrow{\Delta} \underset{(\text{yellow when hot})}{ZnO} + SO_2 + O_2$

24. C

Sol.




25. A, B, D

Sol. (A) Adsorption (Physical or chemisorption) is an exothermic process. High energy of activation in chemisorption requires higher temperature (contrary of Le-Chatelier's principle)

(B) Adsorption is a spontaneous process so $\Delta G < 0$ at all temperature.

(C) $x/m = \frac{k_1k_2p}{1+k_1p}$ at low P, $k_1p << 1$

$\therefore\ x/m = k_1k_2p$ or $\left(\frac{x}{m} \propto p\right)$

At high value of p, $1 << k_1p$

Then $\frac{x}{m} = \frac{k_1k_2p}{k_1p} = \text{constant}$

(D) As $\Delta S$ is always negative in adsorption, so the factor ($-T\Delta S$) has a positive value rendering $\Delta G$ to be less negative.

26. A, B, D

Sol. (A) 'S' is not measure of available energy.

(B) $\Delta S$ between the two states is not path dependent whether reversible or irreversible $\Delta S = S_2 - S_1$

(D) $-\Delta G$ is equal to the available energy of the system to do useful work.

27. A, B, C, D

Sol. All are Aluminium alloys.

28. A, B, C, D

Sol. $XeF_6$ reacts with $SiO_2$ present in glass and is converted to $XeO_3$ (explosive), so it cannot be stored in glass bottles.

29. A, B, D

Sol. Sucrose, Glycogen and Starch are non-reducing sugars whereas Maltose is reducing sugar.

30. A, D

Sol. O-acetyl salicyclic acid (Aspirin) is mainly taken for feverish cold, headaches, rheumatism and heart disease. Iodoform is antiseptic.

## SECTION – C

31. 00083.33

Sol. $M_{obs} = \frac{1000\ k_f w}{\Delta T_f \times W}$, w = 5g, W = 95g

$= \frac{1000 \times 5.7 \times 5}{1.50 \times 95} = 200\ \text{g mol}^{-1}$

$i = \frac{M_{normal}}{M_{observed}} = \frac{75}{200} = \frac{3}{8}$

$\alpha = \frac{1-i}{1-\frac{1}{n}} = \frac{1-\frac{3}{8}}{1-\frac{1}{4}} = \frac{5/8}{3/4} = 0.8333$

= 83.33%

32. 00073.89

Sol. Complex is $[Co(NH_3)_5Cl]Cl_2 \rightleftharpoons [Co(NH_3)_5Cl]^{2+} + 2Cl^-$

$i = 1 + (n-1)\alpha$

$i = 1 + (3-1)1 \Rightarrow i = 3$

$\pi = iCST = 3 \times 1 \times 0.0821 \times 300 = 73.89$ atm

33. 00002.05

Sol. $\underset{2.7/27=0.1\,\text{mole}}{Al} + \frac{3}{2}Cl_2 \longrightarrow \underset{0.1\text{ mole}}{AlCl_3}$

At a temperature of 500 K $AlCl_3$ sublimes and exist as dimer

$\underset{0.1\text{ mole}}{2AlCl_3} \longrightarrow \underset{0.05\text{ mole}}{Al_2Cl_6}$

Volume of gaseous dimer = $\frac{0.05 \times 0.082 \times 500}{1}$ = 2.05 L

34. 00002.31

Sol. Volume of resulting solution = 0.35 + 0.15 = 0.50 L

$[MgCl_2] = [Mg^{2+}] = \frac{0.15 \times 0.1}{0.5} = 0.03\text{ M}$

$[NH_3] = \frac{0.35 \times 0.1}{0.5} = 0.07\text{ M}$

$[OH^-]$ for just precipitation of $Mg(OH)_2$ = $\sqrt{\frac{K_{sp}}{[Mg^{2+}]}} = \sqrt{\frac{1.2 \times 10^{-11}}{0.03}}$ = $2.0 \times 10^{-5}$ M

$NH_3 + H_2O \rightleftharpoons NH_4^+ + OH^-$

$k_b = 2 \times 10^{-5} = \frac{[NH_4^+][OH^-]}{[NH_3]}$

$\Rightarrow [NH_4^+] = \frac{2 \times 10^{-5} \times 0.07}{2 \times 10^{-5}} = 0.07\text{ M}$

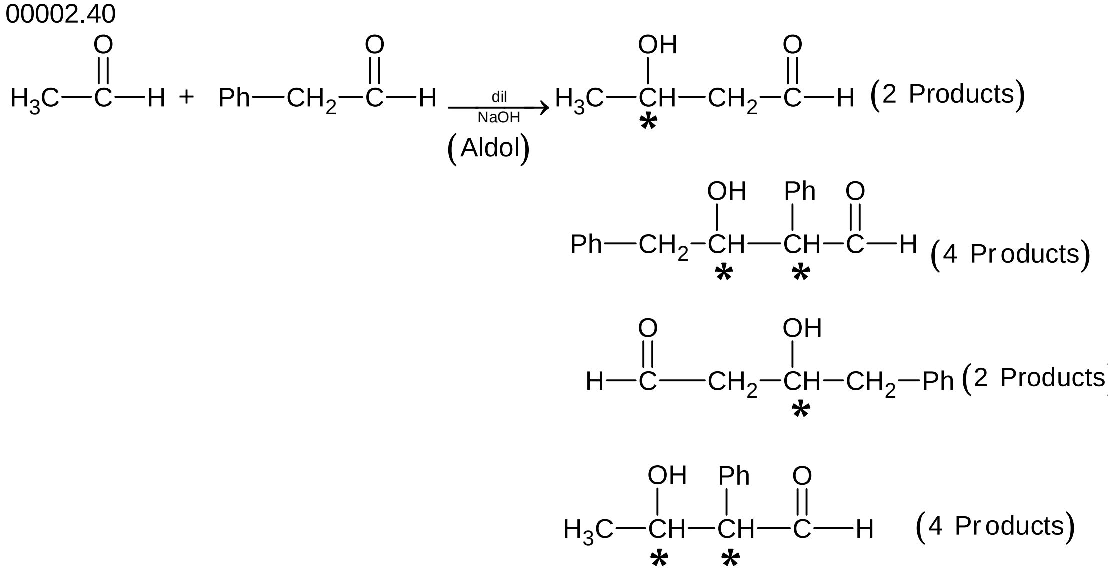
Then $[(NH_4)_2SO_4] = \frac{0.07}{2} = 0.035\text{ M}$

= $0.035 \times 132.0$ = 4.62 g/L

$\therefore$ Amount required for 0.5 L = $\frac{4.62}{2}$ = 2.31 g

35. 00002.40

Sol.



$\therefore$ X = 12 and hence $\frac{X}{5} = \frac{12}{5} = 2.4$

36. 00012.22

Sol. Volume of $N_2$ at STP $V_2 = \frac{P_1V_1}{T_1}\frac{T_2}{P_2} = \frac{756\times 48.6}{300}\times\frac{273}{760}$ = 43.99 ml

Percentage of nitrogen in compound = $\frac{28}{22400}\times\frac{V_2}{w}\times 100$

$= \frac{28}{22400}\times\frac{43.99}{0.45}\times 100$

= 12.22%

# Mathematics PART – III

## SECTION – A

37. D

Sol. We first make the change of variable $t = x^{-\frac{1}{5}}u \Rightarrow dt = x^{-\frac{1}{5}}du$

Hence, $\lim\limits_{x\to 0}\int_0^x \frac{\sin(xt^5)}{x^7}dt = \lim\limits_{x\to 0}\frac{x^{-\frac{1}{5}}\int_0^{x^{6/5}}\sin(u^5)}{x^7}du = \lim\limits_{x\to 0}\frac{\int_0^{x^{6/5}}\sin(u^5)}{x^{36/5}}du$

Apply L' Hospital's rule

$$\lim_{x\to 0}\frac{\sin(x^6)\cdot\frac{6}{5}x^{\frac{1}{5}}}{\frac{36}{5}x^{31/5}}\ ;\ \frac{1}{6}\lim_{x\to 0}\frac{\sin(x^6)}{(x^6)} = \frac{1}{6}\times 1 = \frac{1}{6}$$

38. D

Sol. $x_1 + x_2 + x_3 + x_4 < 2021 \Rightarrow x_1 + x_2 + x_3 + x_4 + A = 2020$

Number of integral solution = ${}^{2024}C_4$ and $x_1 + x_2 + x_3 + x_4 \le 221$

$x_1 + x_2 + x_3 + x_4 + B = 221$. Number of integral solution = ${}^{225}C_4$

So, total number of integral solution ${}^{2024}C_4 - {}^{225}C_4$

39. A

Sol. Hyperbola $2x^2 - 2y^2 - 4x + 41 = 0$ and director circle $x^2 + y^2 = a^2 + b^2$ meets only two points (In this case touches)

$\Rightarrow 2x^2 - 2(a^2 + b^2 - x^2) - 4x + 41 = 0$

$4x^2 - 4x + 41 - 2(a^2 + b^2) = 0$ roots are real and equal D = 0

$16 = 4 \times 4(41 - 2(a^2 + b^2))$

$a^2 + b^2 = 20$

$a, b \in N$

a = 4, b = 2 or a = 2, b = 4

Hence, |a – b| = 2

40. A

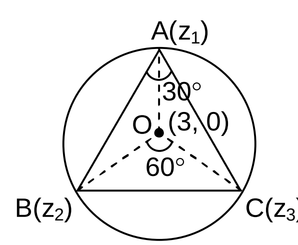
Sol. In equilateral $\Delta BOC$

$z_0^2 + z_2^2 + z_3^2 = z_0z_2 + z_2z_3 + z_3z_0$

But $z_0 = (3, 0)$

$\Rightarrow z_2^2 + z_3^2 + 9 = 3z_2 + z_2z_3 + 3z_3$

$\Rightarrow z_2^2 + z_3^2 - 3z_2 - 3z_3 - z_2z_3 + 9 = 0$




41. D

Sol. Equation can be written as $\tan^2\left(\frac{\pi}{4}-\frac{A}{4}\right)+\tan^2\left(\frac{\pi}{4}-\frac{B}{4}\right)+\tan^2\left(\frac{\pi}{4}-\frac{C}{4}\right)=1$

$$\Rightarrow\left(\tan\left(\frac{\pi}{4}-\frac{A}{4}\right)-\tan\left(\frac{\pi}{4}-\frac{B}{4}\right)\right)^2+\left(\tan\left(\frac{\pi}{4}-\frac{B}{4}\right)-\tan\left(\frac{\pi}{4}-\frac{C}{4}\right)\right)^2$$
$$+\left(\tan\left(\frac{\pi}{4}-\frac{C}{4}\right)-\tan\left(\frac{\pi}{4}-\frac{A}{4}\right)\right)^2=0$$

$\Rightarrow$ A = B = C
$\Rightarrow R = \sqrt{3}$

42. A
Sol. Let A = $(1 + x)^{300}$, B = $(1 + x^2)^{200}$, C = $(1 + x^3)^{150}$
n(A) = 301, n(B) = 201, n(C) = 151
$n(A \cap B) = 151$, $n(B \cap C) = 67$, $n(A \cap C) = 101$, $n(A \cap B \cap C) = 51$
$\therefore n(A \cup B \cup C) = 385$

43. B, C
Sol. $\frac{d}{dx}g(x) = \frac{d}{dx}\frac{1}{x}\int_0^x f(t)dt$

$\frac{d}{dx}g(x) = \frac{1}{x}f(x) + \int_0^x f(t)dt\left(-\frac{1}{x^2}\right)$ ..... (1)

and as g′(2) = 0, we must have $-\frac{1}{4}\int_0^2 f(t)dt + \frac{f(2)}{2} = 0$

$\Rightarrow \int_0^2 f(t)dt = 2f(2) = 10$

Now, $\frac{d^2}{dx^2}g(x) = \frac{d}{dx}\left(-\frac{1}{x^2}\int_0^x f(t)dt + \frac{1}{x}f(x)\right) = \frac{2}{x^3}\int_0^x f(t)dt - \frac{1}{x^2}f(x) - \frac{1}{x^2}f(x) + \frac{1}{x}f'(x)$

$\frac{d^2}{dx^2}g(2) = \frac{1}{4}\int_0^2 f(t)dt - \frac{f(2)}{2} + \frac{f'(2)}{2} = -\frac{3}{2} < 0$

Hence, g(x) has a local maximum at x = 2

44. A, B, C, D
Sol. Equation of normal is $y = mx - 2m - m^3$ and point (9, –6) lies on it $m^3 - 7m - 6 = 0$
Roots are –1, –2, 3

$\therefore A = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 3 \end{bmatrix} \Rightarrow |A| = 8$

(A) $|adj(adj\ A)| = |A|^{(n-1)^2} = (8)^4 = 2^{12}$

(B) $|adj\ A| = |A|^{n-1} = 8^2 = 2^6$

(C) $\det|A^{-1}| = \frac{1}{\det A} = \frac{1}{8}$

(D) $\left|(adj\ A^{-1})^{-1}\right| = \frac{1}{|adj(A^{-1})|} = \frac{1}{(A^{-1})^{3-1}} = \frac{1}{|A^{-1}|^2} = |A|^2 = 8^2$

45. A, B, C
Sol. $\because$ $P_1$, $P_2$, $P_3$ intersect xy plane
For point P put z = 0

$\left.\begin{array}{l} 2x + y = 1 \\ x - y = 2 \\ 4x - y = 5 \end{array}\right\} P = (1, -1, 0)$

For point Q (yz plane put x = 0)

$\left.\begin{array}{r} y+z=1 \\ -y+z=2 \end{array}\right\}$ $Q=\left(0,-\frac{1}{2},\frac{3}{2}\right)$ and $-y + \alpha z = 5$

$\frac{1}{2}+\alpha\frac{3}{2}=5 \Rightarrow \frac{3\alpha}{2}=\frac{9}{2} \Rightarrow \alpha = 3$

(A) Let D.R. of line is (a, b, c)

Hence, equation of line passing through P is $\frac{x-1}{a}=\frac{y+1}{b}=\frac{z}{3}$

and DR of plane $P_3$ is (4, –1, 3)

Hence, plane is $\frac{x-1}{4}=\frac{y+1}{-1}=\frac{z}{3}$

(B) Projection of $\vec{a}$ on $\vec{b}$ is $\vec{a}\cdot\hat{b} \Rightarrow$ Projection of $\overrightarrow{PQ}$ an x-axis is $\overrightarrow{PQ}\cdot\hat{i} = 1$

(C) Centroid $\frac{0+1+0}{3}, \frac{0-1-\frac{1}{2}}{3}, \frac{0+0+\frac{3}{2}}{3}=\left(\frac{1}{3},-\frac{1}{2},\frac{1}{2}\right)$

46. A, B

Sol. $2\sqrt{m}-2<\sum_{r=1}^{m}\frac{1}{\sqrt{r}}<2\sqrt{m}-1$

47. A, B, C

Sol. $1 + 2x + 3x^2 + ..... + 11x^{10} + 10x^{11} + ..... x^{20} = (1 + x + ..... + x^{10})^2$

$\Rightarrow \left(\frac{1-x^{11}}{1-x}\right)^2=0 \Rightarrow x=\cos\frac{2k\pi}{11}+i\sin\frac{2k\pi}{11}$, k = 1, 2, ..... 10

48. A, B, C

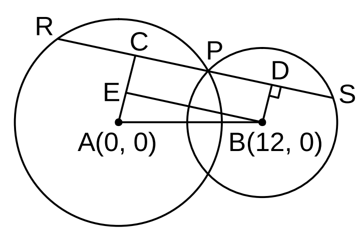
Sol. Let PR = 2a, AC = b, BD = c

$\Delta AEB \Rightarrow 144 = (b - c)^2 + 4a^2$ ..... (1)

$\Delta ACP \Rightarrow 64 = a^2 + b^2$ ..... (2)

$\Delta PBD \Rightarrow 36 = a^2 + c^2$ ..... (3)

From equation (1), (2) and (3), we get $2a=\sqrt{130}$




## SECTION – C

49. 00002.82

Sol. Let $a^2 = \tan\alpha$, $b^2 = \tan\beta$ and $c^2 = \tan\gamma$

Then, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \Rightarrow \sin^2\alpha = \cos^2\beta + \cos^2\gamma$

Use AM $\geq$ GM

$\sin^2\alpha = \cos^2\beta + \cos^2\gamma = 2(\cos^2\beta\cos^2 x)^{1/2}$

$\Rightarrow \sin^2\alpha \geq 2\cos\beta\cos x$ ..... (1)

Similarly, $\sin^2\beta \geq 2\cos\alpha\cos\gamma$ ..... (2)

and $\sin^2\gamma \geq 2\cos\alpha\cos\beta$ ..... (3)

Multiplying 1, 2, 3

$\sin^2\alpha\sin^2\beta\sin^2\gamma \geq 8\cos^2\alpha\cos^2\beta\cos^2\gamma$

$\Rightarrow \tan^2\alpha\tan^2\beta\tan^2\gamma \geq 8 \Rightarrow \tan\alpha\tan\beta\tan\gamma \geq 2\sqrt{2}$

$\Rightarrow a^2 b^2 c^2 \geq 2\sqrt{2}$ minimum value $2\sqrt{2}=2.82$

50. 00216.00

Sol. Probability of defective screw $p = \frac{1}{10}$ and non-defective screw $q = \frac{9}{10}$

Probability that a pocket of 10 screws would have to be replaced = probability that pocket contains three or more defective screw

$p(x) = 1 - p(x < 3) = 1 - \{p(x = 0) + p(x = 1) + p(x = 2)\}$

$= 1-\left\{{}^{10}C_0\left(\frac{1}{10}\right)^0\left(\frac{9}{10}\right)^{10} + {}^{10}C_1\left(\frac{1}{10}\right)^1\left(\frac{9}{10}\right)^9 + {}^{10}C_2\left(\frac{1}{10}\right)^2\left(\frac{9}{10}\right)^8\right\}$

$= 1-\left\{\frac{9^{10}+10(9^9)+45(9)^8}{10^{10}}\right\} = 1-\frac{9^8(81+90+45)}{10^{10}} = 1-\frac{9^8(216)}{10^{10}}$

Hence, $\lambda = 216$

51. 00015.00

Sol. $f(x) = (x - 2)g(x) + 13$

Let $\alpha$ be integral root then $f(\alpha) = 0$

$\therefore (\alpha - 2)g(\alpha) + 13 = 0 \Rightarrow g(\alpha) = -\frac{13}{\alpha-2}$ as $g(\alpha)$ is integral, so $\alpha = 1, 3, 15, -11$

Same $f(x) = (x - 10)h(x) + 5$ and $f(\alpha) = 0 \Rightarrow h(\alpha) = -\frac{5}{\alpha-10} \Rightarrow \alpha = 15$

So, $\alpha$ satisfy both that is 15

52. 00001.00

Sol. If $a \geq 1$, min (f(x) = f(1) = 1 $\Rightarrow a = 1$

If $a \leq -1$, min (f(x) = f(–1) = 1 $\Rightarrow a = -\frac{5}{3}$

If $-1 < a < 1$, min (f(x) = f(a) $\Rightarrow$ no values of a

$\therefore\ s = 1-\frac{5}{3} = -\frac{2}{3}, n = 2$

$\therefore [n + s] = 1$

53. 00003.00

Sol. Let $y = \frac{1}{1+f(x)}$, then equation becomes $\frac{dy}{dx}+\frac{g'(x)}{g(x)}y = g'(x)$ on solving

We get $\frac{g(x)}{1+f(x)} = \frac{g(x)^2}{2}+c$

$\therefore f(2) = 3$

54. 00003.00

Sol. $|\vec{a}-\vec{b}|^2 + |\vec{b}-\vec{c}|^2 + |\vec{c}-\vec{a}|^2 = 2\left(|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2\right) - 2\left(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}\right)$

$= 52 - 2\left(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}\right) \leq 78$

Because, $|\vec{a}+\vec{b}+\vec{c}|^2 \geq 0$

$\Rightarrow |\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2 + 2\left(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}\right) \geq 0$

$\Rightarrow 2\left(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}\right) \geq -26$

Hence, $12k + \lambda = 78$ where $k = 2$, $\lambda = 54$

$k = 3$; $\lambda = 42$ and $k = 5$; $\lambda = 18$

Thus, three values