FIITJEE

ALL INDIA TEST SERIES

FULL TEST – VIII

JEE (Advanced)-2021

PAPER –2

TEST DATE: 04-09-2021

ANSWERS, HINTS & SOLUTIONS

Physics

PART – I

SECTION – A

1. ABC

Sol. Assume x-axis along OA and y-axis perpendicular to it in the plane of ring.
After 1st collision

Velocity of particle = $\frac{v}{\sqrt{2}}$ along x-axis

And velocity of ring = $\frac{v}{\sqrt{2}}$ along y-axis

After second collision velocity of ring = $\frac{v}{\sqrt{2}}\hat{i}+\frac{v}{\sqrt{2}}\hat{j}$

Velocity of particle = 0

Time to return at point A = $4\times\frac{R\sqrt{2}}{v}$

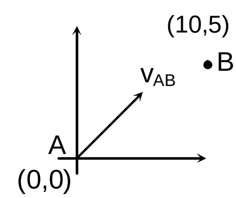
2. AC

Sol. $\vec{v}_{AB}=(3-a)\hat{i}+(3-b)\hat{j}$

$\vec{a}_{AB}=\vec{o}$

$(3-a)\times2=10$ and $(3-b)\times2=5$

a = –2 and b = $\frac{1}{2}$



3. BC

Sol. $v_L = v_C = v_R$;

$\Rightarrow x_L = x_C = R$

when inductor is short circuited

$Z = \sqrt{R^2 + x_C^2} = \sqrt{2}R$

$\therefore I = \frac{30}{Z} = \frac{30}{\sqrt{2}R}$

$\therefore V_L = ix_L = \frac{30}{\sqrt{2}R} \times R = \frac{30}{\sqrt{2}}$

$\therefore$ (A) is incorrect and with similar calculations (B) will be correct.

Here f_0 is the resonance frequency as $v_L = v_C$

$\omega_0 = 2\pi f_0 = \frac{1}{\sqrt{LC}}$

$\frac{x_L}{x_C} = \frac{\omega L}{1/\omega C} = \omega^2 LC$

Given $f = 2f_0$

$\Rightarrow \omega = 2\omega_0$

$\therefore \frac{x_L}{x_C} = 4$

$\therefore$ (C) is also correct.

4. ABC

Sol. $KE = h\nu - \phi$

5. AC

Sol. Velocity of insect w.r.t. strip = $\frac{\ell}{t}$

Let strip moves with speed ν

Initial momentum was 0

$\Rightarrow 0 = m\left(\frac{\ell}{t} + v\right) + Mv$

$v = -\frac{m\ell/t}{M+m}$ i.e. $\frac{m\ell/t}{M+m}$ toward left

$\therefore$ Velocity of insect w.r.t. ground = $\frac{\ell}{t} - \frac{m\ell/t}{M+m} < \frac{\ell}{t}$.

6. BCD

Sol. Apply Faraday law for direction of induced current.

SECTION – B

7. 3

Sol. Let T be the tension in the ideal string and 'a' be the acceleration of the blocks at the instant of release. For the block on the left, the upward acceleration may be found from

$T + k_1x - mg = ma$

For the block on the right, the downward acceleration may be found from

$k_2x + mg - T = ma$
Adding the equations gives the acceleration of the blocks as
$a = (k_1 + k_2)x/(2m)$
However, subtracting the equations gives
$T = mg - (k_1 - k_2)x/2$
for maximum value of k_1T will be zero.
$mg = \left(\frac{k_1 - k_2}{2}\right)x$; $k_1 = 300$.

8. 1

Sol. $F = -\frac{dU}{dr} = -\frac{k}{r}, \frac{k}{r} = \frac{mv^2}{r}$ ..... (i)

$E_n = \frac{1}{2}mv^2 + k\ln r$ ..... (ii)

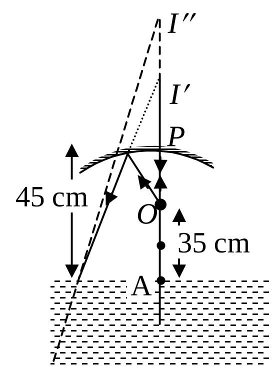
$mvr = \frac{nh}{2\pi}$ ..... (iii)

Solving these $E_n = \frac{k}{2}\left(1 + \ln\left(\frac{n^2h^2}{4\pi^2 mk}\right)\right)$

required ratio = $\frac{E_2 - E_1}{E_4 - E_2} = 1.$

9. 1

Sol. For the reflection at the concave mirror,
$u = -10$ cm; v = ?; $f = -15$ cm
From the mirror formula, we have
$v = \frac{uf}{u-f} = \frac{(-10)\times(-15)}{-10+15} = \frac{150}{5}$ = + 30 cm
The positive sign indicates that the image is formed on the other side of the concave mirror,
Now, the image formed by the concave mirror serves as a virtual object for refraction at water surface which takes placed from air to water. So,



$\mu = \frac{\text{Apparent height}}{\text{Real height}}$

$\therefore$ AI″ = Apparent height = μ × real height = $\frac{4}{3}\times 75$ = 100 cm.

10. 2

Sol. $\frac{q_1}{C_1} = \frac{q_2}{C_2}$; $q_1 + q_2 = 2Q_0$

$C_1 = \frac{\varepsilon_0 A}{d_0 + vt}$; $C_2 = \frac{\varepsilon_0 A}{d_0 - vt}$

$\frac{q_1}{q_2} = \frac{d_0 - vt}{d_0 + vt}$, $q_2\left(\frac{d_0 - vt}{d_0 + vt}\right) + q_2 = 2Q_0$

$q_2\left[\frac{2d_0}{d_0+vt}\right]=2Q_0$, $q_2=\frac{2Q_0}{2d_0}\left(d_0+vt\right)$

$I=\frac{dq_2}{dt}=\frac{Q_0 v}{d_0}$ = 2 amp

11. 3

Sol. Initially the rod will be in equilibrium if

$2T_o = Mg$ with $T_o = kx_o$ …(i)

when the current I is passed through the rod, it will experience a force

F = BIL vertically up,

In equilibriums

2T + BIL = Mg with T = kx …(ii)

from (i) & (ii)

$\frac{T}{T_o}=\frac{Mg-BIL}{Mg}$ i.c. $\frac{x}{x_o}=1-\frac{BIL}{Mg}$

or, $B=\frac{Mg\left(x_o-x\right)}{I\ \ L\ \ x_0}$

Putting the values we get $B=1.5\times10^{-2}\,T$.

12. 4

Sol. Here 3rd maxima is shifted by 3×10^{-4} m. It indicates fringe width increases by 1×10^{-4} m.

Hence $\beta=\frac{\lambda\left(D+0.5\right)}{d}=\frac{\lambda D}{d}+1\times10^{-4}$

or $\frac{0.5\lambda}{d}=1\times10^{-4}$ m or $\lambda=\frac{2\times10^{-3}\times1\times10^{-4}}{0.5}=4\times10^{-7}$ m = 400 nm

SECTION – C

13. 00003.75

Sol. T = $\frac{3}{8}M\,\omega^2L^2$

Stress = $\frac{F_{\perp}}{(A/\cos\theta)}=\frac{3}{8}\frac{M\omega^2L^2\cos^2\theta}{A}$

14. 00002.00

Sol. We have relation $M\frac{dv}{dt} = F_{ext} + v_{rel}\frac{dM}{dt}$.

In the question, we have the force by the gravity as the external force. As the rocket is to lift from the ground the minimum acceleration of the rocket to do so is, o, beyond which, with a slight increase it will zoom into the sky.

Therefore, $\frac{dv}{dt} = 0$

Putting these into the equation, we obtain

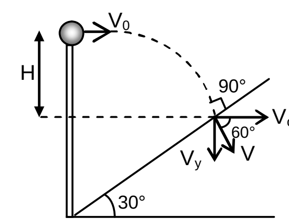
$0 = Mg + v_{rel}\left(-\frac{dM}{dt}\right)$

($\frac{dM}{dt}$ is rate of decrease of mass hence negative. Mg is in the direction of the relative velocity of the fuel with respect to the rocket)

Hence, $\frac{dM}{dt} = \frac{M}{V}g = \frac{400 \times 10}{2 \times 1(10^3)} = 2 \text{ kg/sec}$

15. 00000.67
Range: 0.66 to 0.67

Sol. $v_y = \sqrt{2gH}$



$$\frac{v_y}{v_x} = \frac{\sqrt{2gH}}{v_o} = \tan 60^o = \sqrt{3}$$

$$v_0 = \sqrt{\frac{2gH}{3}}$$

16. 00000.50

Sol. The system is equivalent to a binary system as shown in the diagram.

$$r_1 = \frac{M \times 3R}{M + 2M} = R$$

$$r_2 = 2R$$

Considering the circular motion of point mass m

$$M\omega_2^2 r_2 = \frac{GM^2}{36R^2} \quad ; \quad \omega_2 = \sqrt{\frac{GM}{72R^3}} = \omega_1$$

17. 00002.75

Sol. Equivalent resistance across capacitor (after shorting the battery) = $\frac{11R}{4}$.

Voltage across capacitor in steady state = $\frac{3E}{4}$.

$$q = \frac{CE}{4}\left(1 - e^{\frac{-4t}{11RC}}\right)$$

18. 00000.75

Sol. $\tau = \frac{L}{Req} = \frac{2}{8} = \frac{1}{4}$

at steady state inductor short circuited

Hence i = 0.75

Chemistry

PART – II

SECTION – A

19. BCD
Sol. These concentrated terms don't depend upon volume.

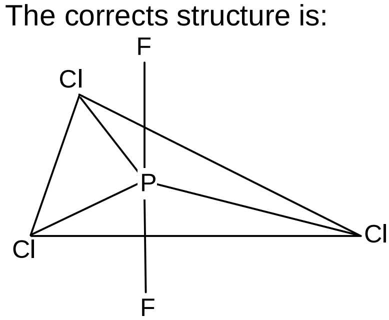
20. AB
Sol. In peroxy acid anti-addition takes place.

21. ACD
Sol. It is benzoin condensation.

22. AC
Sol. CN^- and NO^+ total no. of electrons is 14. Hence bond order is 3.

23. BD
Sol. $(BN)_x$ and H_3BO_3 form polymers.

24. ABC
Sol. The corrects structure is:



SECTION – B

25. 7
Sol. X = $(NH_4)_2Cr_2O_7$, Y = Cr_2O_3, Z = N_2, the liquid is H_2O.

26. 4
Sol. $\Delta T_b = K_b \times i \times m$
or, $4.16 = 0.52 \times i \times 2$
or $i = 4$

27. 3
Sol. $E_{Cell} = E^o_{Cell} - \frac{0.0591}{n}\log\frac{[Zn^{2+}]}{[H^+]^2}$

or, $0.6418 = [0-(-0.76)] - \frac{0.0591}{n}\log\frac{10^{-2}}{[H^+]^2}$

on solving, $[H^+]^2 = 10^{-6}$
$\therefore [H^+] = 10^{-3}$, pH = 3

28. 6
Sol. $P_4 + 3NaOH + 3H_2O \longrightarrow PH_3 + 3NaH_2PO_2$
x + y = 6

29. 4
Sol. Nylon-6 & Teflon are homopolymer rest all are co-polymers

30. 8
Sol. $\frac{r_+}{r_-}$ = radius ratio = $\frac{1.7}{1.8}$ = 0.944
Hence C.No. 8

SECTION – C

31. 00016.00
Sol. The electron configuration of Cu^+ is :
$1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10}$
$\therefore$ Number of electrons with l = 0 is 6
Number of electrons with l = 2 is 10
Total number of electrons = 16

32. 00002.50
Sol. The balanced equation is:
$$2MnO_4^- + 5C_2O_4^{2-} + 16H^+ \longrightarrow 2Mn^{2+} + 10CO_2 + 8H_2O$$

33. 00003.20
Sol. Moles of HCl = $\frac{200 \times 0.4}{1000}$ = 0.08
$\therefore$ Moles of NaOH = 0.08
Mass of NaOH = 0.08 $\times$ 40 = 3.2 g

34. 00002.85
Sol. $\left[H^+\right] = \sqrt{K_a C} = \sqrt{0.08 \times 2.5 \times 10^{-5}} = 1.41 \times 10^{-3}$
pH = -1 eq[H^+]
= 3 – log 1.41 = 2.85

35. 00009.24
Sol. $NH_3 + HCl \longrightarrow NH_4Cl$

Initial	50×0.1	25×0.1	
After neutralization	2.5	–	2.5

pOH = $p^{K_b} + \log\frac{\left[NH_4^+\right]}{\left[NH_3\right]}$
$= 4.76 + \log\frac{2.5}{2.5} = 4.76$
pH = 14 – 4.76 = 9.24

36. 00006.72
Sol. 3N of H_2O_2 = 3 $\times$ 5.6 = 16.8 volume of O_2
Volume of O_2 produced by H_2O_2 = 200 $\times$ 16.8 = 3360 mL
Same volume will be produced by $KMnO_4$ = 3360 mL
Total volume of O_2 = 6720 mL = 6.72 L

Mathematics

PART – III

SECTION – A

37. CD

Sol. Clearly $p_1 = \frac{1}{2}$.

$p_2 = p(HH) + p(T) = \frac{3}{4}$

Now, the score n can be obtained in two distinct ways.
I. By throwing head when the score is n – 1
II. By throwing tail when the score is n – 2

$\Rightarrow p_n = (p_{n-1})\frac{1}{2} + (p_{n-2})\frac{1}{2} = \frac{1}{2}(p_{n-1} + p_{n-2})$

Also, from this $P_4 = \frac{11}{16}$.

38. ABC

Sol. $8\sin x = \frac{\sqrt{3}}{\cos x} + \frac{1}{\sin x}$

$\Rightarrow 8\sin^2 x\cos x = \sqrt{3}\sin x + \cos x$

$4(1-\cos 2x)\cos x = \sqrt{3}\sin x + \cos x$

$3\cos x - 2(\cos 3x + \cos x) = \sqrt{3}\sin x$

$\cos x - 2\cos 3x = \sqrt{3}\sin x$

$\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x = \cos 3x$

$\cos\left(x + \frac{\pi}{3}\right) = \cos 3x$

$3x = 2nx \pm \left(x + \frac{\pi}{3}\right)$

(+)	(–)
$3x = 2x\pi + x + \frac{\pi}{3}$	$3x = 2n\pi - x - \frac{\pi}{3}$
$2x = 2n\pi + \frac{\pi}{3}$	$4x = 2n\pi - \frac{\pi}{3}$
$x = nx + \frac{\pi}{6}$	$x = \frac{n\pi}{2} - \frac{\pi}{12}$
$[0, 2\pi]$	
$\frac{\pi}{6}$	$\frac{\pi}{2} - \frac{\pi}{12} = \frac{6\pi - \pi}{12} = \frac{5\pi}{12}$

$\frac{5\pi}{6}$ $\quad$ $\pi-\frac{\pi}{12}=\frac{11\pi}{12}$

$\frac{3\pi}{2}-\frac{\pi}{12}=\frac{18\pi-\pi}{12}=\frac{17\pi}{12}$

$2\pi-\frac{\pi}{12}=\frac{23\pi}{12}$

39. ACD

Sol. $g'(0)=\lim\limits_{h\to\infty}\frac{g(0+h)-g(0)}{h}=0$

$\Rightarrow g(x)$ is differentiable $\forall x\in R$

$$g'(x)=\begin{cases}2x\sin\left(\frac{\pi}{x}\right)-\pi\sin\left(\frac{\pi}{x}\right)+2(x-1)\sin\left(\frac{\pi}{x-1}\right)-\pi\sin\left(\frac{\pi}{x-1}\right); & x\neq 1\\ 0\ ; & x=0,1\end{cases}$$

But $\lim\limits_{x\to 0}g'(x)=$ does not exist $\neq g'(0)\Rightarrow g'(x)$ is discontinuous at x = 0

Similarly $\lim\limits_{x\to 1}g'(x)=$ does not exist.

40. AB

Sol. $\det(A+B)=-\det(A)\det(B)\det(A+B)$

$=-\det\left(A^T\right)\det\left(B^T\right)\det(A+B)$

$=-\det\left(A^T(A+B)B^T\right)$

$=-\det\left(A^T(A+B)B^T\right)$

$=-\det\left(A^TAB^T+A^TBB^T\right)$

$=-\det\left(B^T+A^T\right)$

$=-\det(A+B)^T$

$=-\det(A+B)$

$\Rightarrow \det(A+B)=0$

41. AB

Sol. The point of intersection of diagonal is (0, 0) and one vertex A (2, 1)

$\Rightarrow$ Vertex is (–2, –1) verties B and D are

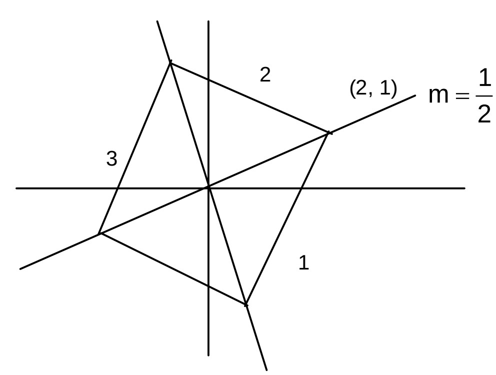
$\pm i(2+i)$

$=-1+2i, 1-2i$

$\Rightarrow B(-1,2)$ and $D(1,-2)$

$\Rightarrow$ Equation of the sides are

$x+3y=\pm 5,\ 3x-y=\pm 5$.



42. ABC

Sol. $\left(2x-3y+z\right)^{10}$

General term $= \sum_{\alpha+\beta+\gamma=10} \dfrac{10!\left(2x\right)^{\alpha}\left(-3y\right)^{\beta}\left(z\right)^{\gamma}}{\alpha!\beta!\gamma!}$ ……….(1)

$\therefore$ Number of terms $= \dfrac{12!}{10!2!} = \dfrac{12\times 11}{2} = 66$

Put $x=y=z=1$ in (1), we get

Sum of all the coefficients 0.

Put $\alpha=2,\ \beta=3,\ \gamma=5$ in (1),

We get coefficient of $x^2y^3z^5 = \dfrac{10!}{2!.3!.5!}\left(2\right)^2\left(-3\right)^3 = \dfrac{-10!\times 9}{5!}$

SECTION – B

43. 9

Sol. Given,

$a+19d=\log_{10}20$ ……….(1)

$a+31d=\log_{10}32$ ……….(2)

(2) – (1)

$12d=\log_{10}\dfrac{32}{20}=\log_{10}16-1$

$12d=4\log_{10}2-1$

$\log_{10}2=\dfrac{12d+1}{4}$ ……….(A)

Again (2) + (1)

$2a+50d=\log_{10}640=6\log_{10}2+1$

$\log_{10}2=\dfrac{2a+50d-1}{6}$ ……….(B)

$\therefore \dfrac{12d+1}{4}=\dfrac{2a+50d-1}{6} \Rightarrow 36d+3=4a+100d-2$

$4a+64d=5$

$\underbrace{a+16d}_{17^{th}\text{ term}}=\dfrac{5}{4}$

Hence, 17th term is rational and its value is $\dfrac{5}{4}=\dfrac{p}{q}\Rightarrow\left(p+q\right)=9$

44. 1

Sol. $\lim\limits_{t\to x+1}\dfrac{t^2f\left(x+1\right)-\left(x+1\right)^2f\left(t\right)}{f\left(t\right)-f\left(x+1\right)}=1$

Using LH rule

$\lim\limits_{t\to x+1}\dfrac{2t\,f\left(x+1\right)-\left(x+1\right)^2f'\left(t\right)}{f'\left(t\right)}=1$

$$\frac{2(x+1)f(x+1)-(x+1)^2 f'(x+1)}{f'(x+1)}=1$$

$$2xf(x)-x^2f'(x)=f'(x)$$

$$2xy=(1+x^2)\frac{dy}{dx}, \qquad \frac{2x}{1+x^2}dx=\frac{dy}{y}$$

$$\ell n(1+x^2)=\ell n|y|$$

$$1+x^2=cy$$

$$f(0)=1\Rightarrow c=1$$

$$\Rightarrow y=1+x^2$$

$$\Rightarrow \lim_{x\to 1}\frac{\ell n(1+x^2)-\ell n 2}{x-1}=\lim_{x\to 1}\frac{2x}{1+x^2}=1$$

45. 3

Sol. Given, $z+\omega=i$ ……………(1)

and $z^2+\omega^2=1$ …………….(2)

$\therefore$ From (1), on squaring we get

$z^2+\omega^2+2z\omega=-1 \quad \Rightarrow \quad 1+2z\omega=-1$ [Using (2)]

$\Rightarrow zw=-1$ …………..(3)

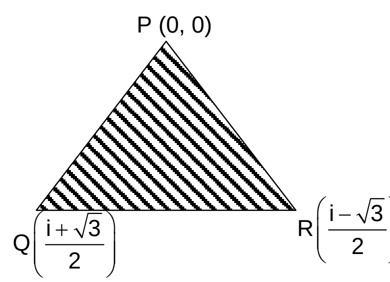
Now, let us consider a quadratic equation in x whose roots are z and ω.

$\Rightarrow \quad x^2-ix-1=0$

$\therefore \quad x=\frac{i\pm\sqrt{i^2+4}}{2}=\frac{i\pm\sqrt{3}}{2}$

Let x

$\omega=\frac{i+\sqrt{3}}{2}$ $\qquad$ $z=\frac{i-\sqrt{3}}{2}$



$$\text{So, ar.}(\Delta PQR)=\frac{1}{2}\left\|\begin{matrix}0 & 0 & 1\\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 1\\ \frac{-\sqrt{3}}{2} & \frac{1}{2} & 1\end{matrix}\right\|=\frac{1}{2}\left(\frac{\sqrt{3}}{4}+\frac{\sqrt{3}}{4}\right)=\frac{\sqrt{3}}{4}$$

46. 8

Sol. Area $=\int_0^1(6-f(x))dx+\int_{-1}^0(f(x)-(-2))dx$

$=\frac{5}{4}$

47. 3

Sol. $g(1) = e^2$

$\lim_{x\to 1^-} g(x) = \frac{h(1)+1}{6}, \quad \lim_{x\to 1^+} g(x) = \frac{f(1)}{2}$

Since, $g(x)$ is continuous at $x = 1$

$g(1) = \lim_{x\to 1^-} g(x) = \lim_{x\to 1^+} g(x)$

$\Rightarrow 2g(1) + 2f(1) - h(1) + 2 = 3$

48. 2

Sol. $I_1 = \int_0^1 x f''(2x)\,dx$

Putting $t = 2x$, i.e. $dx = \frac{dt}{2}$, we get

$I_1 = \frac{1}{4}\int_0^2 t\,f''(t)\,dt$

$= \frac{1}{4}\left[t f'(t)\Big|_0^2 - \int_0^2 f'(t)\,dt \right]$ [Integrating by parts]

$= \frac{1}{4}\left[t f'(t)\Big|_0^2 - f(t)\Big|_0^2 \right]$

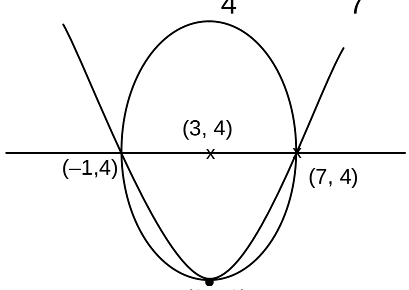
$\Rightarrow I_1 = \frac{1}{4}\left[2f'(2) - f(2) + f(0)\right]$

$= \frac{1}{4}(10 - 3 + 1) = 2$

SECTION – C

49. 00005.00

Sol. Given ellipse $\frac{(x-3)^2}{4^2} + \frac{(y-4)^2}{7^2} = 1$ (vertical ellipse)



Parabola can be taken as

$(x-3)^2 = A\,(y+3)$

It passes through (-1, 4)

$\Rightarrow 16 = 7A \Rightarrow A = 16/7$

$\therefore$ parabola is $7(x-3)^2 = 16y + 48$

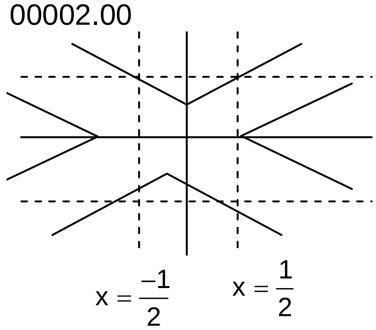
$\therefore \quad 16y = 7(x-3)^2 - 48$

$\therefore$ A = 7, H = 3, K = 48

$\therefore\ \frac{A}{7}+\frac{H}{3}+\frac{K}{16}=5$

50. 00002.00

Sol.



51. 00004.50

Sol. R cos A = $\frac{3}{\sqrt{5}}$ $\Rightarrow$ – R cos B cos C + R sin B sin C = $\frac{3}{\sqrt{5}}$ and 2R cos B cos C = $\frac{3}{\sqrt{5}}$

$\Rightarrow$ R sin B sin C = $\frac{3}{\sqrt{5}}+\frac{3}{2\sqrt{5}}=\frac{9}{2\sqrt{5}}$

sin B sin C = $\frac{9}{2\sqrt{5}}\times\sqrt{\frac{5}{61}}=\frac{9}{2\sqrt{61}}$.

52. 00005.25

Sol . $|\vec{a}|=1, |\vec{b}|=1, |\vec{a}+\vec{b}|=\sqrt{3}$

$3=1+1+2\vec{a}.\vec{b}\Rightarrow\vec{a}.\vec{b}=\frac{1}{2}$

$\vec{C}=\vec{a}+2\vec{b}-3\left(\vec{a}\times\vec{b}\right)$

$p=\left(\vec{a}\times\vec{b}\right)\times\vec{c}\Rightarrow\left(\vec{a}\times\vec{b}\right)\times\left(\vec{a}+2\vec{b}-3\left(\vec{a}\times\vec{b}\right)\right)$

$=\left(\vec{a}\times\vec{b}\right)\times\vec{a}+2\left(\vec{a}\times\vec{b}\right)\times\vec{b}$

$=\left(\vec{a}.\vec{a}\right)\vec{b}-\left(\vec{b}.\vec{a}\right)\vec{a}+2\left(\left(\vec{a}.\vec{b}\right)\vec{b}-\left(\vec{b}.\vec{b}\right).\vec{a}\right)$

$=\vec{b}-\frac{1}{2}\vec{a}+2\times\left(\frac{1}{2}\vec{b}-\vec{a}\right)$

$=2\vec{b}-\frac{5}{2}\vec{a}$

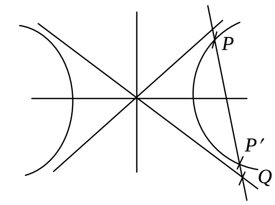
$|\vec{p}|=\sqrt{4+\frac{25}{4}+10\vec{a}.\vec{b}}=\sqrt{\frac{41}{4}-5}$

$=\frac{\sqrt{21}}{2}$

53. 00000.02

Sol. For any line in case of hyperbola we know that $PQ = P'Q'$

$$\frac{PQ}{P'Q'} + \frac{PQ'}{P'Q} = 1 + \frac{PP' + P'Q'}{PP' + PQ} = 2$$



54. 00013.33

Sol. Let $\vec{r} = \lambda\vec{b} + \mu\vec{c}$ and $\vec{c} = \pm(xi + yj)$. Since $\vec{c}$ and $\vec{b}$ are perpendicular, we have

$$4x + 3y = 0 \qquad \Rightarrow \qquad \vec{c} = \pm x\left(i - \frac{4}{3}j\right)$$

$$\pm 1 = \text{ proj. of } \vec{r} \text{ on } \vec{b} = \frac{\vec{r}.\vec{b}}{|\vec{b}|} = \frac{(\lambda\vec{b} + \mu\vec{c}).\vec{b}}{|\vec{b}|} = \frac{\lambda\vec{b}.\vec{b}}{|\vec{b}|}$$

$$\left[\because \vec{b}.\vec{c} = 0\right]$$

$= \lambda|\vec{b}| = 5\lambda$. Hence $\lambda = \pm\frac{1}{5}$

Also, $\pm 2 = \text{proj. of } \vec{r} \text{ on } \vec{c} = \frac{\vec{r}.\vec{c}}{|\vec{c}|}$

$$= \frac{(\lambda\vec{b} + \mu\vec{c}).\vec{c}}{|\vec{c}|} = \mu|\vec{c}| = \frac{5}{3}\mu x$$

Thus, $\mu x = \pm\frac{6}{5}$. Therefore,

$$\vec{r} = \frac{1}{5}(4i + 3j) + \frac{6}{5}\left(i - \frac{4}{3}j\right) = \pm(2i - j)$$

$$\vec{r} = \frac{1}{5}(4i + 3j) - \frac{6}{5}\left(i - \frac{4}{3}j\right) = \pm\left(-\frac{2}{3}i + \frac{11}{5}j\right)$$

Thus there are four such vectors

$$\sum_{i=1}^{4}|\vec{r}_i|^2 = 2|2i - j|^2 + 2\left|-\frac{2}{5}i + \frac{11}{5}j\right|^2 = 20$$