FIITJEE

ALL INDIA TEST SERIES

FULL TEST – VIII

JEE (Advanced)-2021

PAPER –1

TEST DATE: 04-09-2021

ANSWERS, HINTS & SOLUTIONS

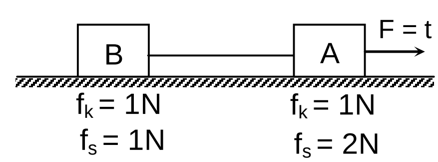
Physics

PART – I

SECTION – A

1. D

Sol. When t = 3s block just about to move and acceleration of block given by $a = \frac{t-2}{1}$ $t > 3$



$$\int_0^v dv = \int_3^{10} (t-2)dt$$

$$v = \frac{t^2}{2} - 2t\Big|_3^{10} = (50-20) - \left(\frac{9}{2} - 6\right)$$

$$= 30 + 1.5 = 31.5 \text{ m/s}$$

2. A

Sol. Sign between x and t is $\oplus$ hence wave is propagating in negative x direction.

wave velocity $= \frac{\text{coefficient of t}}{\text{coefficient of x}} = \frac{3}{1}$

Max value of y is amplitude, y has maximum value when denominator is least i.e. when $(x+3t)^2$ is zero

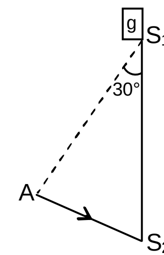
$y = 3m$

shape of pulse is given by $y = \frac{6}{2+x^2}$ $(t = 0)$

shape of pulse is symmetrical out y-axis

3. B
Sol. Apply KVL.

4. D
Sol. Extra phase change in glass = phae change in water of length AS_2.



$$\frac{2\pi t}{\lambda g}-\frac{2\pi}{\lambda\omega}t=\frac{2\pi}{\lambda\omega}\cdot\frac{2}{3}\sin 30^o$$

$$\Rightarrow \quad t\left[\frac{3}{2}-\frac{4}{3}\right]=\frac{4}{3}\times\frac{1}{3}$$

$$\Rightarrow \quad t=\frac{8}{3}mm.$$

5. C
Sol. Potential of centre of sphere = $\frac{Kq}{r}+V_i=\frac{Kq}{r}$

where V_i = potential due to induced charge at centre = 0 [$\therefore$ Σq_i = 0 and all induced charges are equidistance from centre]

$\therefore$ potential at point P = $\frac{Kq}{r}=\frac{Kq}{r_1}+V_i$ (For conductor all points are equipotential)

$$\therefore \quad V_i=K\left(\frac{q}{r}-\frac{q}{r_1}\right)$$

6. A
Sol. Applying Snell's law between the points *O* and *P*, we have

$$2\times\sin 60^0=\left(\sin 90^0\right)\times\frac{2}{\left(1+H^2\right)},\quad 2\times\frac{\sqrt{3}}{2}=1\times\frac{2}{\left(1+H^2\right)}$$

$$\left(1+H^2\right)=\frac{2}{\sqrt{3}},\ H=\sqrt{\left(\frac{2}{\sqrt{3}}-1\right)}$$

7. BD
Sol. Length $\propto G^x c^y h^z$

$$L=\left[M^{-1}L^3T^{-2}\right]^x\left[LT^{-1}\right]^y\left[ML^2T^{-1}\right]^z$$

By comparing the power of M, L and T in both sides we get
$-x+z=0, 3x+y+2z=1$ and $-2x-y-z=0$
By solving above three equations we get

$$x=\frac{1}{2}, y=-\frac{3}{2}, z=\frac{1}{2}$$

8. ABCD
Sol. Let elongation in spring A, B and C be x_1, x_2 and x_3 respectively.
Considering spring forces and constraint relations

$x_2 = 4x_3$ …(i)
$x_2 = 2x_1$ …(ii)
and $x_1 + 2x_2 + x_3 = x$ …(iii)

$\Rightarrow \quad x_1 = \left(\frac{2}{11}\right)x \quad ; \quad x_2 = \left(\frac{4}{11}\right)x \quad ; \quad x_3 = \left(\frac{1}{11}\right)x$

Also, F = $2K\left(\frac{x}{11}\right)$

$\Rightarrow \quad T = 2\pi\sqrt{\frac{11m}{2k}}$

9. ABD

Sol. $\lambda = v, \frac{v}{2}, \frac{v}{3}$ ……

$\lambda' = 4\ell, \frac{4\ell}{3}, \frac{4\ell}{5}$

$\Rightarrow$ f′ = 50, 150, 250 …..

10. ABC

Sol. $\Delta U = \frac{fR\Delta T}{2}$

$\Delta W = \frac{nR\Delta T}{1-x}$ where PV^x = constant. Here $x = -\frac{1}{2}$

11. ABD

Sol. For ammeter,

$i = \frac{i_{max}\left(R_s + R_A\right)}{\left(R_s\right)}$

$\Rightarrow \quad i = 0.1mA$ for $R_s = 50\Omega$ [as $R_A = 50\Omega$ and $i_{max} = 50\mu A$]

For voltmeter,

$V = i_{max}(R_A + R_V)$

$\Rightarrow \quad V \approx 10V$ for $R_v = 200k\Omega$

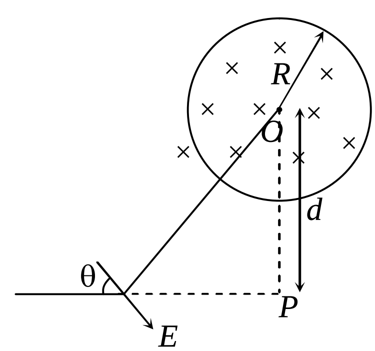
12. ACD

Sol. $\int \vec{E}\cdot d\vec{l} = A\frac{dB}{dt}$

$E2\pi\sqrt{x^2 + d^2} = \pi R^2 k$

$E = \frac{\pi R^2 k}{2\sqrt{x^2 + d^2}}$

$W_{ext} = \int_0^\infty q\vec{E}\cdot dx = \frac{q\pi R^2}{4}k$



SECTION – C

13. 00000.75

Sol. $e = \frac{v_2 - v_1}{u_1 - u_2}$

$1 = \frac{v_2 - (-2)}{u_1}$

$u_1 = v_2 + 2$

$u_1 = 1(-2) + 5(u_1 - 2)$

$u_1 = -2 + 5u_1 - 10$

$u_1 = \frac{12}{4} = 3 m/s$

$v_2 = 1 m/s$

Kinetic energy of the centre of mass = $\frac{1}{2} \times (1+5) \times \left(\frac{3}{1+5}\right)^2 = \frac{3}{4} J$

14. 00002.17

Sol. $\frac{2.17 + 2.17 + 2.18}{3}$ = 2.1733 ….. ≈ 2.17 mm upto 3 significant figure.

15. 00000.89

Sol. 2g – T = 2a …(i)

TR = $I\alpha$ …(ii)

a = $R\alpha$ …(iii)

From (ii) and (iii) T = $\frac{Ia}{R^2}$

$\therefore \quad 2g = a\left(2 + \frac{I}{R^2}\right)$

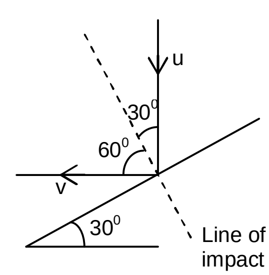
$\Rightarrow \quad a = \frac{2g}{\left(2 + \frac{I}{R^2}\right)} = \frac{2 \times 9.8}{2 + \frac{0.2}{0.01}} = \frac{9.8}{11} \approx 0.89 \text{ m/s}^2$

16. 00000.33

Sol. Let velocity of ball just before collision is u and just after collision is v.

u sin 30° = v sin 60°

$e = \frac{v\cos 60^0}{u\cos 30^0} = \frac{1}{3}$



17. 00337.50

Sol. Mass per unit length, $\mu = \frac{m}{\ell} = \frac{SA\ell}{\ell} = \rho A$

$\mu_s = \mu_{A\ell} = 78 \times 10^{-4}$ kg/m

$\therefore$ Speed of wave is same in both wire

$$V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{80 \times 10^4}{78}} = \frac{2 \times 10^2}{\sqrt{3.9}}$$

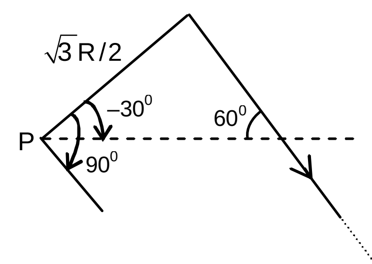
$$\nu_{min} = \frac{V}{\lambda_{max}} = \frac{200}{\sqrt{3.9} \times 0.3} \quad \left[\frac{\lambda_{max}}{2} = 15 \text{ cm for C as a node}\right]$$

$= 337.4\ H_2$

18. 00006.92
Range: 6.92 to 6.93

Sol. $B = \frac{\mu_o I}{4\pi\sqrt{3}\,R/2}\left[\sin 90^o + \sin(-30^o)\right]$

$= \frac{\mu_o I}{4\sqrt{3}\,R}$



Chemistry

PART – II

SECTION – A

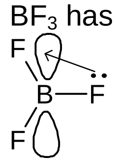
19. B
Sol. It is a first order reaction.

20. B
Sol. Both NO and O_2 are formed at the same time.

21. D
Sol. Relatively lowering will be minimum for $[Co(NH_3)_3Cl_3]$ as i = 1 and hence it will have maximum vapour pressure.

22. B
Sol. $6\overset{+4}{Xe}F_4 + 12H_2O \longrightarrow 4\overset{0}{Xe} + 2\overset{+6}{Xe}O_3 + 24HF + 3O_2$

23. B
Sol. BF_3 has back bonding

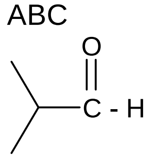


24. B
Sol. Antiaromatic

25. ABC
Sol. O and O_2 contain two unpaired electrons each. NO contains one unpaired electron. N_2 contains no unpaired electrons.

26. AB
Sol. Bases can form silicates with SiO_2(an acidic compound).

27. BCD
Sol. HOBr is Br_2/H_2O.

28. ABC
Sol. 

undergoes aldol condensation.

29. AD
Sol. $AlCl_3 + 3H_2O \xrightarrow{\text{Hydrolysis}} Al(OH)_3 + 3HCl$
B^{3+} cannot exist in aqueous media

30. BC

Sol. $\underset{(s)}{2C} + \underset{(g)}{3H_2} \rightarrow \underset{(g)}{C_2H_6}$

$\Delta H_f^0 = \left[2 \times \Delta H_{sub}^0 + 3 \times B.E(H-H)\right] - \left[B.E.(C-C) + 6 \times B.E(C-H)\right]$

$\Rightarrow -85 = \left[(2 \times 718) + (3 \times 436)\right] - (x + 6y)$

$\therefore x + 6y = 2829$ 1

Similarly for $C_3H_8(g)$

$2x + 8y = 4002$ 2

Solving (1) & (2), x = 345
y = 414

SECTION – C

31. 00006.00

Sol. $Cr_2(SO_4)_3 \xrightarrow{NaOH} Cr(OH)_3 \xrightarrow{Na_2O_2} Na_2CrO_4$

32. 00012.60

Sol. $pH = \frac{1}{2}\left[p_{K_w} + p_{k_a} + \log C\right]$

$or, 12.8 = \frac{1}{2}\left[14 + p_{k_a} + \log 10^{-1}\right]$

On solving, $p^{K_a} = 12.6$

33. 00116.24

Sol. $\Delta T_f = K_f \times m$

or, $0 - T_f = 1.86 \times 2 \times \frac{1000}{W}$

or, $0 - (-4.8) = 1.86 \times \frac{2000}{W}$

On solving, W = 775 g

$\therefore$ Mass of ice formed = 891.24 – 775 = 116.24 g

34. 00370.65

Sol. $2.303 \log \frac{P_2}{P_1} = \frac{\Delta H}{R}\left(\frac{T_2 - T_1}{T_1T_2}\right)$

$T_2 = 100^oC$ P_2 = 760 mm (1 atm)
= 373 K

T_1 = ? P_1 = 700 mm

$\Delta H = 540 \times 18$ cal/ mol

$2.303 \log \frac{760}{700} = \frac{540 \times 18}{2}\left(\frac{373 - T_1}{T_1 \times 373}\right)$

T_1 = 370.65 K

35. -00089.40

Sol. $\underset{N\equiv N}{N_2} + \frac{1}{2}\underset{O=O}{O_2} \longrightarrow N=N=0$

$\Delta H = \sum BE_R - \sum BE_P$

$= \left(946.2 + \frac{497}{2}\right) - (418 + 605.3) = 171.4\,kJ/mol$

RE = ΔH_{exp} - ΔH_{cal} = 82 – 171.4 = -89.4 kJ

36. 00005.70

Sol. $pH = -\log K_a + \log\frac{[\text{Conj.base}]}{[\text{Acid}]}$

For - I : $4 = -\log 10^{-5} + \log\frac{[\text{Conj.base}]}{0.5}$

Conjugate base = 0.05 M = salt

For – II: 6 = $-\log 10^{-5}$ + $\log\frac{[\text{Conj.base}]}{0.5}$

[C.base] = [Salt] = 5 M

On mixing equal volumes

New conc. of salt NaA = $\frac{0.05\times V + 5\times V}{2V} = \frac{5.05}{2}M$

New conc. of HA in mixed buffer = $\frac{0.5V + 5V}{2V}$

$pH = -\log 10^{-5} + \log\frac{5.05}{2\times 0.5} = 0.5M = 5.70$

Mathematics

PART – III

SECTION – A

37. C

Sol. 4 circles touch sides of any given triangle formed by 3 tangents.

38. C

Sol. a, b > 0 and c < 0

angle between OA and OB is $\frac{\pi}{2}$.

Since z_2 lie in 2nd quadrant

$\Rightarrow \bar{z}_2$ will lie in 3rd quadrant.

$\Rightarrow \sqrt{2} < |z_1 - \bar{z}_2| \le 2$.

Also true if z_2 lies in 3rd quadrant.

39. B

Sol. Writing r as linear combination of a, b and $a \times b$, we have $r = xa + yb + z(a \times b)$ for solats x, y, z

$0 = r.a = x|a|^2 + ya.b$

$1 = r.b = xa.b + y|b|^2$

Solving we get $y = \frac{|a|^2}{|a|^2|b|^2 - (a.b)^2} = |a|^2$ and $x = \frac{a.b}{(a.b)^2 - |a|^2|b|^2} = a.b$

Also $1 = [r \quad a \quad b] = z|a \times b|^2$

$\Rightarrow \qquad z = \frac{1}{|a \times b|^2}$

Thus $r = \left((a.b)a - |a|^2 b\right) + \frac{a \times b}{|a \times b|^2}$

$= a \times (a \times b) + \frac{a \times b}{|a \times b|^2}$

40. D

Sol. $\sum_{r=0}^{n-1}\left(\frac{n+1}{n}\right)\left(\frac{r.\,{}^nC_r\,{}^nC_{r+1}}{r+2}\right)$

$= \sum_{r=0}^{n-1}\left(\frac{n+1}{n}\right)\left(r.\,{}^nC_r\right)\left(\frac{{}^nC_{r+1}}{r+2}\right)$

$= \sum_{r=0}^{n-1}\left(\frac{n+1}{n}\right)\left(n.\,{}^{n-1}C_{r-1}\right)\left(\frac{{}^{n+1}C_{r+2}}{n+1}\right)$

$= \sum_{r=0}^{n-1} {}^{n-1}C_{r-1}\,{}^{n+1}C_{r+2}$

$$=\sum_{r=0}^{n-1} {}^{n-1}C_{n-r}\ {}^{n+1}C_{r+2}$$
$$={}^{n-1}C_n\ {}^{n+1}C_2+{}^{n-1}C_{n-1}\ {}^{n+1}C_3+.....+{}^{n-1}C_1\ {}^{n+1}C_{n+1}$$
$$={}^{2n}C_{n+2}={}^{2n}C_{n-2}$$

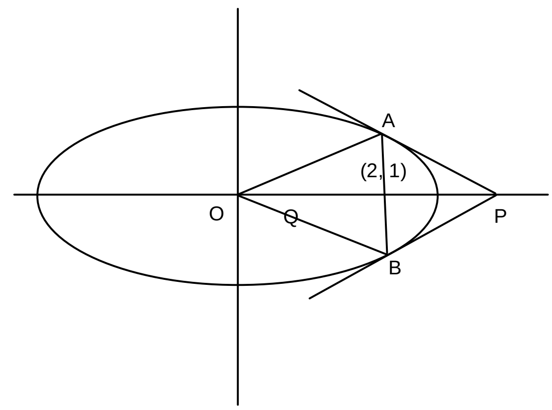
41. D

Sol. Let the point of intersection of tangents at A and B be P (h, k).

The equation of AB is $\frac{xh}{4}+\frac{yk}{1}=1$ ...(1)

Homogenizing the equation of ellipse using (1), we have

$$\frac{x^2}{4}+\frac{y^2}{1}=\left(\frac{xh}{4}+\frac{yk}{1}\right)^2$$
$$\Rightarrow\ x^2\left(\frac{h^2-4}{16}\right)+y^2\left(k^2-1\right)+\frac{2hk}{4}xy=0$$



Given equation of OA and OB is $x^2+4y^2+\alpha xy=0$

Equation (2) and (3) represent same pair of straight lines,

Hence, $\frac{h^2-4}{16}=\frac{k^2-1}{4}=\frac{hk}{2\alpha}$

$\Rightarrow\ h^2-4=4\left(k^2-1\right)$

$\Rightarrow\ h^2-4k^2=0$

$\Rightarrow\ \left(h-2k\right)\left(h+2k\right)=0$

Therefore, locus is $\left(x-2y\right)\left(x+2y\right)=0$.

42. C

Sol. Let P (h, k) be the centre of the circle.

Since it passes through (1, 0), its radius $=\sqrt{\left(h-1\right)^2+k^2}$

Circle touches the line $x+y=0$

$\therefore\ \frac{h+k}{\sqrt{2}}=\pm\sqrt{\left(h-1\right)^2+k^2}$

Hence, locus of the centre P (h, k) is $S=x^2+y^2-2xy-4x+2=0$

Here, $h^2=ab$ and $\Delta\neq 0$.

So, it represents a parabola.

43. CD

Sol. $f_n=$number of subset in which n appears + number of subset in which n does not appears.

$S=\{1,2,3,........(n-2),(n-1),n\}$ when n appears obviously (n – 1) will not appear and when n does not appear up to (n – 1) will appear

$\Rightarrow\quad f_n=f_{n-2}+f_{n-1}$

$f_1=2$

$f_2=3$

$\Rightarrow \quad f_3 = f_1 + f_2 = 2 + 3 = 5$

$f_4 = f_2 + f_3 = 3 + 5 = 8$

44. AD

Sol. By intermediate value property $\frac{f(0)+f(2)}{2} = f(c),\ 0 < c < 2$

By mean value theorem

$f(1) - f(0) = f'(c_1),\ 0 < c_1 < 1$

$f(2) - f(1) = f'(c_2),\ 1 < c_2 < 2$

By subtraction

$f(0) + f(2) - 2f(1) = f'(c_2) - f'(c_1)$

$= (c_2 - c_1) f''(c),\ c_1 < c < c_2 \Rightarrow f(0) + f(2) - 2f(1) < 0$

$\Rightarrow f(0) + f(2) < 2f(1)$

45. BC

Sol. (A) $(s-b)(s-c) = bc \Rightarrow a = b + c \Rightarrow$ Not possible

(B) $b^2 \sin 2c + c^2 \sin 2B = 4R^2\left[\sin^2 B.2\sin C\cos C + \sin^2 C.2\sin B\ \cos B\right]$

$= 4R^2.2\sin B \sin C \sin A$

$= 8R^2.\frac{b}{2R}.\frac{a}{2R}\sin c$

$= 2ab\sin C$

possible if $C = 30^o$

(C) $\cos C = \frac{3^2 + 5^2 - 7^2}{2(3)(5)} = \frac{9 + 25 - 49}{30} = -\frac{1}{2} = \cos\frac{2\pi}{3}$

Hence possible

(D) $\cos\frac{A-C}{2} - \cos\frac{A+C}{2} = 0 \Rightarrow \sin\frac{A}{2}\sin\frac{C}{2} = 0$

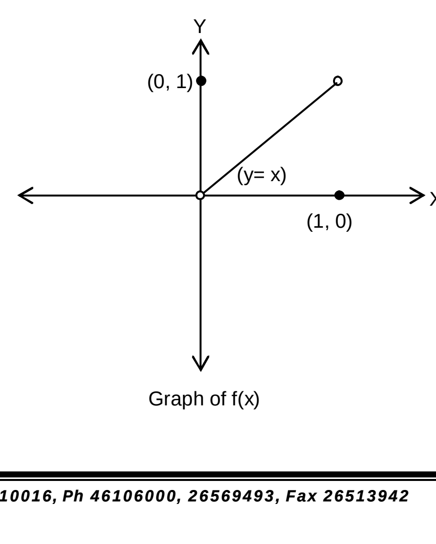
Not possible

46. BCD

Sol. (A) False, $f(x) = \begin{bmatrix} x^2 \sin\frac{1}{x^2} & x \neq 0 \\ 0 & x = 0 \end{bmatrix}$

(B) This statement is true

For example: $f(x) = \begin{cases} x & 0 < x < 1 \\ 1 & x = 0 \\ 0 & x = 1 \end{cases}$



Graph of f(x)

(C) $f(x) = x^2 - \cos \pi x + 4$

Now $f(-1) = 1 - (-1) + 4 = 6$

and $f(2) = 4 - 1 + 4 = 7$

As f is continuous on $[-1, 2]$, so $f(c) = 2\pi$, for some $c \in (-1, 2)$

(By using intermediate value theorem).

(D) $(f^{-1})'(4) = \dfrac{1}{f'(0)} = \dfrac{1}{1+e^x}\Bigg|_{x=0} = \dfrac{1}{2}$

47. BD

Sol. Equation of required plane is

$(x - y + 1) + \lambda(2y + z - 6) = 0$

$x + (2\lambda - 1)y + \lambda z + 1 - 6\lambda = 0$

Since it makes an angle of 30^o with x + y + z = 5

$\Rightarrow \dfrac{|1 + (2\lambda - 1) + \lambda|}{\sqrt{3} \cdot \sqrt{1 + \lambda^2 + (2\lambda - 1)^2}} = \dfrac{\sqrt{3}}{2}$

$\Rightarrow |6\lambda| = 3\sqrt{5\lambda^2 - 4\lambda + 2}$

$\Rightarrow 4\lambda^2 = 5\lambda^2 - 4\lambda + 2$

$\Rightarrow \lambda = (2 \pm \sqrt{2})$

48. ABD

Sol. Volume $= \left|\left[2\vec{b} \times \vec{c} \quad 3\vec{c} \times \vec{a} \quad 4\vec{a} \times \vec{b}\right]\right| = 18$

$\Rightarrow \quad 24\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]^2 = 18 \qquad \Rightarrow \quad \left|\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]\right| = \dfrac{\sqrt{3}}{2}$

$$\text{Now, } \left[\vec{a} \quad \vec{b} \quad \vec{c}\right] = \begin{vmatrix} (1+\sin\theta) & \cos\theta & \sin 2\theta \\ \sin\left(\theta + \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) & \sin\left(2\theta + \frac{4\pi}{3}\right) \\ \sin\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta - \frac{2\pi}{3}\right) & \sin\left(2\theta - \frac{4\pi}{3}\right) \end{vmatrix}$$

Applying $R_1 \to R_1 + R_2 + R_3$ and expanding

$\left|\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]\right| = \sqrt{3}\,|\cos 3\theta| = \dfrac{\sqrt{3}}{2}$

$\Rightarrow \cos 3\theta = \pm\dfrac{1}{2} \Rightarrow 3\theta = \dfrac{\pi}{3}, \dfrac{2\pi}{3}, \dfrac{4\pi}{3} \Rightarrow \theta = \dfrac{\pi}{9}, \dfrac{2\pi}{9}, \dfrac{4\pi}{9}$

SECTION – C

49. 00020.83

Sol. E_1 : First bag is chosen, $P(E_1) = \frac{1}{2}$.

E_2 : Second bag is chosen, $P(E_2) = \frac{1}{2}$.

A : Drawn number is 4.

Now, $P(A) = P(E_1).P\left(\frac{A}{E_1}\right) + P(E_2).P\left(\frac{A}{E_2}\right)$

$= \frac{1}{2}.\frac{1}{4} + \frac{1}{2}.\frac{1}{6} = \frac{5}{24}$

50. 00001.50

Sol. $\because f(x) = x^3 + x^2 f'(1) + xf''(2) + f'''(3)$

$\therefore\ f'(x) = 3x^2 + 2xf'(1) + f''(2)$

Put $x = 1$

$\therefore\ f'(1) + f''(2) = -3$ .....(i)

Again, $f''(x) = 6x + 2f'(1)$, $f'''(x) = 6$

Put $x = 2$

$\therefore\ f''(2) = 12 + 2f'(1)$ .....(ii)

Solving equation (i) and (ii), we get

$f'(1) = -5$ and $f''(2) = 2$

$\therefore\ f(x) = x^3 - 5x^2 + 2x + 6$

$\therefore\ f(2) - f(1) = -6$

$= -f(0)$

51. 00007.75

Sol. $t_r = \frac{2r^2}{2r^2 - 60r + 900}$

$= \frac{2r^2}{(r-30)^2 + r^2}$

$\Rightarrow t_r + t_{30-r} = 2$

52. 00002.25

Sol. We have $f(x) - f(-x) = 6x$

$\therefore\ f(4) - f(-4) = 24$

$\Rightarrow\ N = f(4) = 24 + 2286 = 2310 = 2.3.5.7.11$

Hence number of divisors $= 2^{n-1} = 2^{5-1} = 16$

53. 00247.50

Sol. $a_r = \frac{r(r+1)}{2}$

$\sum_{r=1}^{k} \frac{1}{a_r} = \frac{2k}{k+1}$

$T_k = \frac{k}{\sum_{r=1}^{k} \frac{1}{a_r}} = \frac{k+1}{2}$

$S_{30} = \sum_{r=1}^{k} T_k = \frac{1}{2}.\left(\frac{31.31}{2} - 1\right) = \frac{495}{2}$

54. 00002.83
(range 2.81 to 2.84)

Sol. $a = \sin\theta\cos\phi$

$b = \cos\theta$

$c = \sin\theta\sin\phi$

WLOG $a \ge b \ge c$

$\Rightarrow E = 2(a - c)$

$= 2\sin\theta(\cos\phi - \sin\phi)$

$= 2\sqrt{2}\sin\theta\cos\left(\phi + \frac{\pi}{4}\right) \le 2\sqrt{2}$