FIITJEE

ALL INDIA TEST SERIES

FULL TEST – IX

JEE (Advanced)-2021

PAPER –2

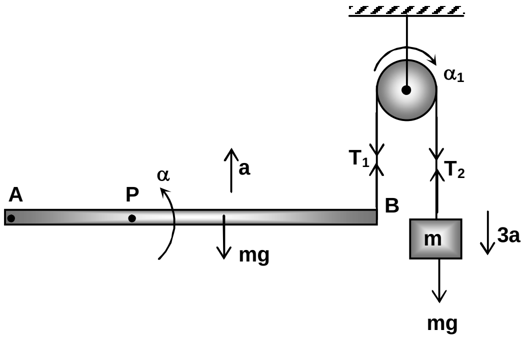
TEST DATE: 11-09-2021

ANSWERS, HINTS & SOLUTIONS

Physics PART – I

SECTION – A

1. A, B, C



Sol. $a = \frac{\ell}{4}\alpha$ …(i)

$3a = R\alpha_1$ …(ii)

$mg - T_2 = 3ma$ …(iii)

$T_2R - T_1R = \frac{mR^2}{2}\alpha_1$

$T_2 - T_1 = \frac{3}{2}ma$ …(iv)

$T_1\left(\frac{3\ell}{4}\right) - mg\left(\frac{\ell}{4}\right) = \frac{7}{48}m\ell^2\alpha$

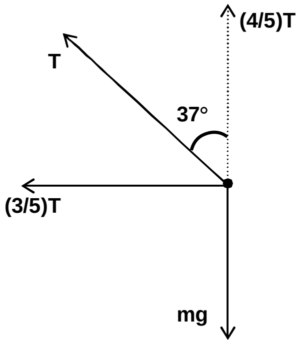
$T_1 - \frac{mg}{3} = \frac{7}{9}ma$ …(v)

From these equation

$T_1 = \frac{41}{95}mg,\ T_2 = \frac{59}{95}mg$

$a = \frac{12g}{95}$ and acceleration of block = $\frac{36}{95}g$

2. A, B, C, D



Sol. $\frac{4}{5}T = mg$

$T = \frac{5}{4}mg = 12.5\text{ N}$

$\Delta\ell = \frac{T}{K} = \frac{12.5}{10} = 1.25$

Natural length $= 5 - 1.25 = 3.75$ m

$\frac{3}{5}T = \frac{mv^2}{r}$

$v = \left(\sqrt{10}\right)\frac{3}{2}$ m/s

Horizontal distance = vt

$\left[\sqrt{10}\left(\frac{3}{2}\right)\right]\left(\sqrt{\frac{2\times 2}{10}}\right) = 3$ m

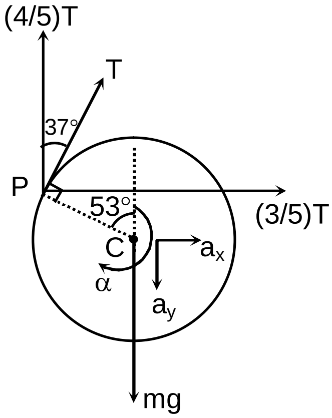
3. A, B, C

Sol. Velocity after impact = $= -\frac{1}{2}(3\hat{i}) + \hat{j}$

Loss in kinetic energy = $\frac{1}{2}m(3^2+1^2) - \frac{1}{2}m\left[\left(\frac{3}{2}\right)^2 + 1^2\right] = 27$ J

$\vec{J} = m\left(-\frac{3}{2}\hat{i} + \hat{j}\right) - m(3\hat{i} + \hat{j}) = -\frac{9}{2}m\hat{i} = -36\hat{i}$ N-sec

4. A, B, C



Sol. $\frac{3}{5}T = ma_x$

$a_x = \frac{3T}{5m}$ …(i)

$mg - \frac{4}{5}T = ma_y$

$a_y = g - \frac{4T}{5m}$ …(ii)

$\tau_C \Rightarrow TR = \frac{1}{2}mR^2\alpha$

$R\alpha = \frac{2T}{m}$ …(iii)

acceleration of point P along the string ‘S_1’ is zero

5. A, B, C

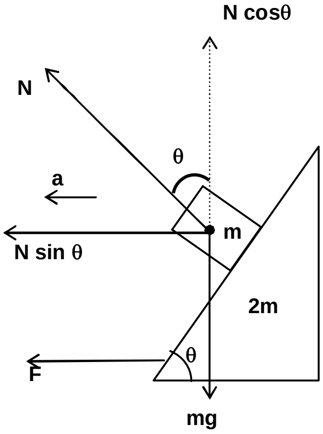
Sol. N = mg secθ

N sinθ = ma

a = g tanθ

F = 3ma

F = 3mg tanθ



6. A, C, D

Sol. $F_x = -\frac{\partial U}{\partial x} = -3, F_y = -\frac{\partial U}{\partial y} = -4$

$\vec{F} = -(3\hat{i} + 4\hat{j})$

When it crosses y-axis, x = 0

$x = u_x t + \frac{1}{2} a_x t^2$

$6 = +\frac{1}{2} \cdot 3t^2 \Rightarrow t = 2\,\text{sec}$

$\vec{v} = 0 - (3\hat{i} + 4\hat{j})2 \Rightarrow |\vec{v}| = 10\,\text{m/s}$

$\Delta x(t = 1) = -1.5$ and $\Delta y = -2$

Co-ordinates = (6 – 1.5, 4 –2) = (4.5, 2)

SECTION – B

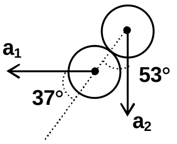
7. 7

Sol. Work performed = $\int_{2R}^{4R} \frac{1}{2}\varepsilon_0 E^2 dV + \int_{6R}^{8R} \frac{1}{2}\varepsilon_0 E^2 dV = \frac{7}{192}\frac{q^2}{\pi\varepsilon_0 R}$

8. 3

Sol. $a_1 \cos 37° = a_2 \cos 53°$

$\frac{a_1}{a_2} = \frac{k}{4}$



9. 4

Sol. va = 4 $\Rightarrow a^2 + v\frac{da}{dt} = 0$

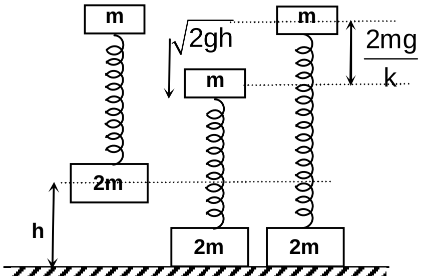
$\Rightarrow \int_{2/3}^{a} 4\frac{da}{a^3} = -\int_{2}^{5} dt$ (at t = 2 sec, a = 2/3)

$a = \sqrt{\frac{4}{15}}$

10. 4

Sol. By the conservation of energy

$$\frac{1}{2}m\left(\sqrt{2gh}\right)^2 = \frac{1}{2}k\left(\frac{2mg}{k}\right)^2 + mg\left(\frac{2mg}{k}\right)$$



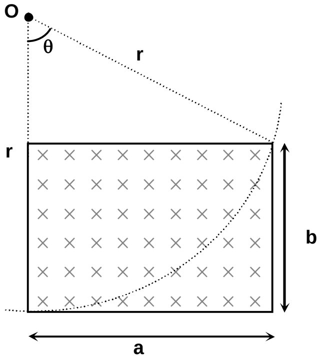
11. 8

Sol. $r = \frac{mv}{qB}$

$\sin\theta = \frac{a}{r},\ \cos\theta = \frac{r-b}{r}$

$\left(\frac{a}{r}\right)^2 + \left(\frac{r-b}{r}\right)^2 = 1 \Rightarrow r = \frac{(a^2+b^2)}{2b} = \frac{mv}{qB}$

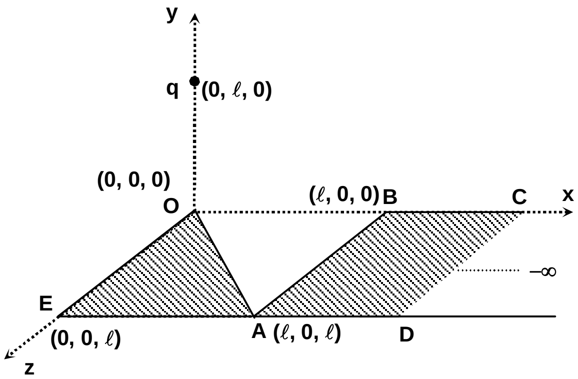
$v = \frac{qB(a^2+b^2)}{2mb} = 8\text{ m/s}$



12. 2

Sol. $\phi = \phi_{OAE} + \phi_{ABCD}$

$= \frac{q}{48\varepsilon_0} + \frac{q}{48\varepsilon_0} = \frac{q}{24\varepsilon_0}$



SECTION – C

13. 00001.60

Sol. deviation, $\delta = \alpha = (i-r)+(i-r)$

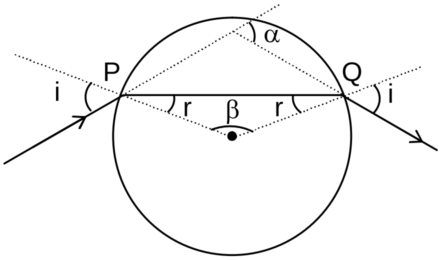
$i = \left(r + \frac{\alpha}{2}\right)$ …(i)

$r = \left(\frac{\pi}{2} - \frac{\beta}{2}\right)$ …(ii)

$i = \left(\frac{\pi}{2} - \frac{\beta}{2} + \frac{\alpha}{2}\right)$

Using snell's law

$\sin i = \mu \sin r$



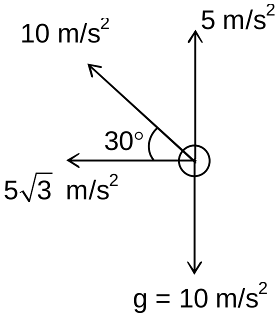
$$\mu = \frac{\sin\left[\frac{\pi}{2} - \left(\frac{\beta - \alpha}{2}\right)\right]}{\sin\left(\frac{\pi}{2} - \frac{\beta}{2}\right)}$$

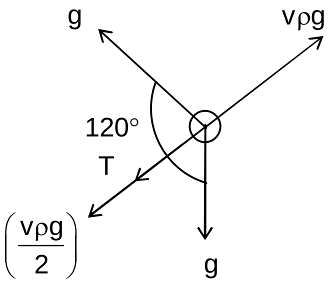
$$\mu = \frac{\cos\left(\frac{\beta - \alpha}{2}\right)}{\cos\left(\frac{\beta}{2}\right)} = \frac{\cos 37^\circ}{\cos 60^\circ} = 1.60$$

14. 00025.00

Sol. $T = v\frac{\rho}{2}g$

$T = mg$





15. 00100.00

Sol. $P_{av} = VI\cos\phi = V\left(\frac{V}{z}\right)\left(\frac{R}{z}\right) = \frac{V^2R}{Z^2}$

It means Z has to be same in both cases

$Z_1 = Z_2 \Rightarrow \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (X_C - X_L)^2}$

$2X_L = X_C$

$\Rightarrow C = \frac{1}{2\omega^2 L} = 10^{-6} = 1 \times 10^{-6}$

16. 00045.75

Sol. $8V = 6S$

$V = \frac{3}{4}\text{mm}$

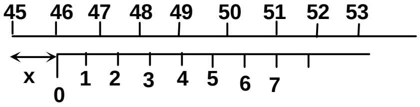
$7V = 7 \times \frac{6}{8} = 5.25$ mm

Hence 7th division of vernier coincides with 51th division of main scale

Hence 51S = X + 45S + 7V

X = 6S − 7V = 0.75 mm

So measured length = 45.75 mm



17. 00040.00

Sol. Applying snell's law

$$1\sin 53^\circ = \frac{4}{3}\sin r_1$$

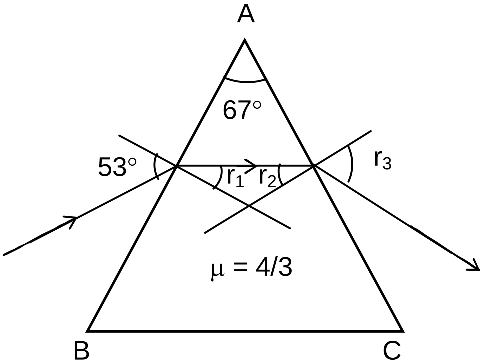
$r_1 = 37^\circ$

So, $r_2 = 67^\circ - 37^\circ = 30^\circ$

$$\frac{4}{3}\sin 30^\circ = \sin r_3$$

$r_3 = 42^\circ$

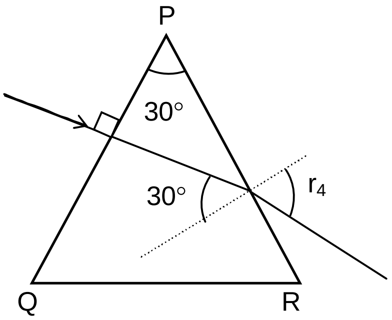
So, $\delta_1 = (53^\circ - 37^\circ) + (42^\circ - 30^\circ) = 28^\circ$



$\frac{4}{3}\sin 30^\circ = 1\sin r_4 \Rightarrow r_4 = 42^\circ$

$\delta_2 = 42^\circ - 30^\circ = 12^\circ$

So, net deviation $\delta = \delta_1 + \delta_2 = 40^\circ$

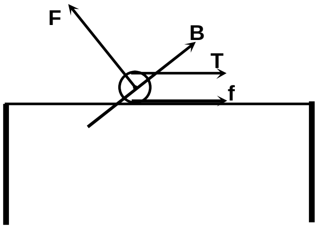


18. 00003.50

Sol. $$F_{min} = \frac{\mu mg + mg}{\sqrt{1+\mu^2}}$$

$$i\ell B_{min} = \frac{(\mu+1)}{\sqrt{1+\mu^2}} mg$$

$B_{min} = 3.50$ Tesla



Chemistry

PART – II

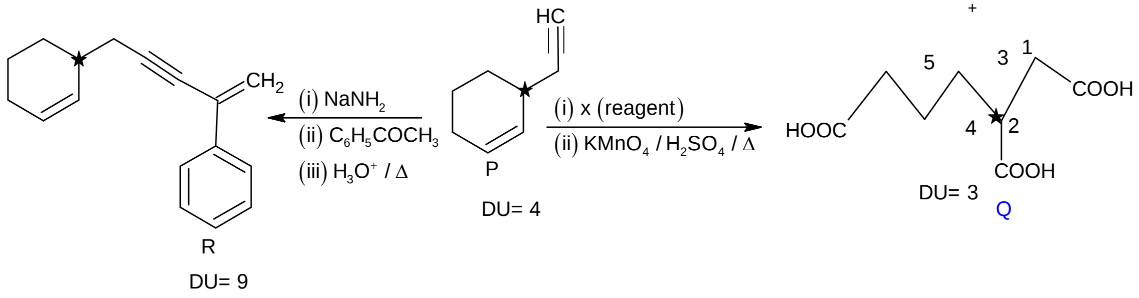
SECTION – A

19. A

Sol. At constant pressure, the addition of inert gas shifts the equilibrium towards larger no. of moles.

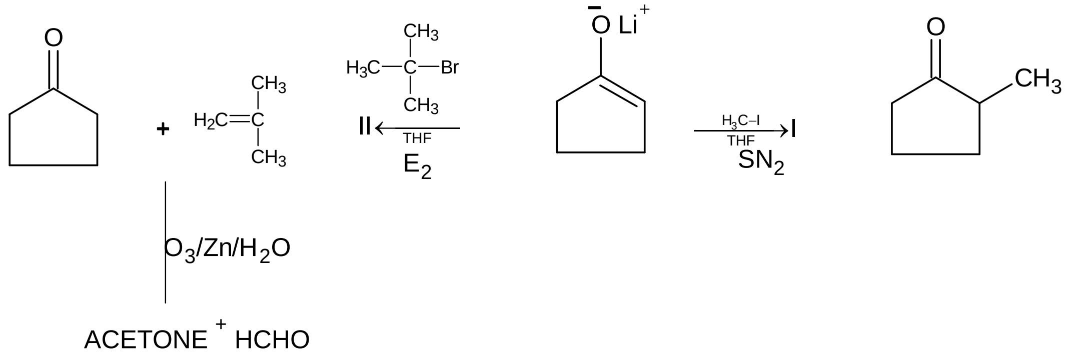
20. A,B,C,D

Sol.



21. B,C, D

Sol.



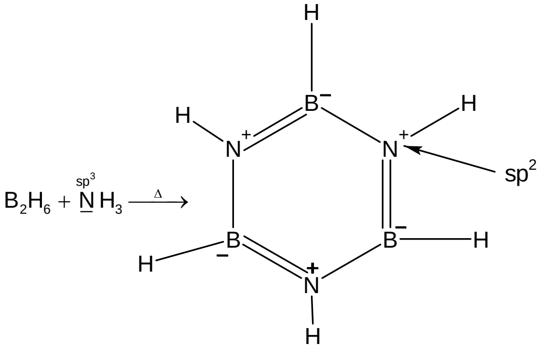
22. A, B, C, D

Sol. (A)

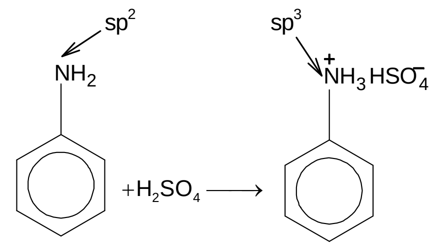
$$Na\overset{sp}{B}O_2 + H_2O_2 + H_2O \longrightarrow Na_2\left[(HO)_2\overset{sp^3}{B}(O-O)_2\overset{sp^3}{B}(OH)_2\right].6H_2O$$

(sodium peroxoborate)

(B)



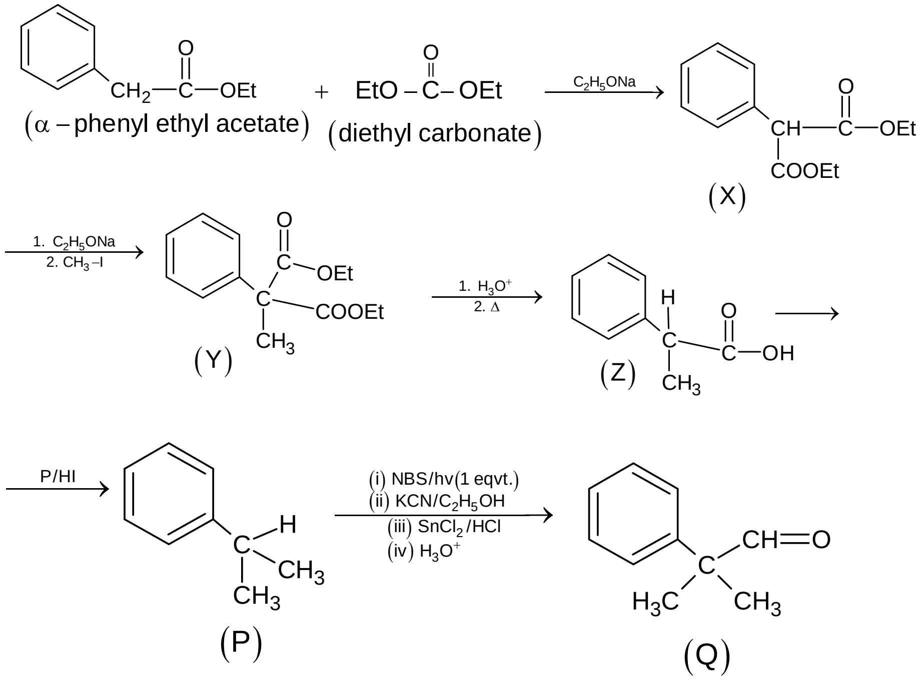
(C)



(D) $NaF + \underset{\underset{sp^2}{\uparrow}}{BCl_3} \longrightarrow \underset{\underset{sp^3}{\uparrow}}{NaBFCl_3}$

23. B

Sol.



24. A, C, D

Sol. Radicals: $NH_4^+, Cl^-, Fe^{+2}, SO_4^{--}$

Blue colour is $Fe_3[Fe(CN)_6]_2$ in which Fe(II) and Fe(III) are present.

$BaSO_4$ is soluble in conc. H_2SO_4 as it forms $Ba(HSO_4)_2$.

Red vapours formation is due to Cl^- in chromyl chloride

SECTION – B

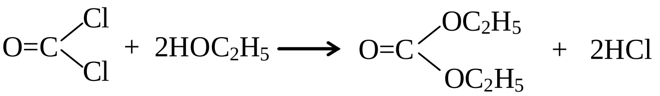
25. 4

Sol. $CCl_3–CH(OH)_2 + NaOH \longrightarrow CHCl_3 + HCOO^-Na^+ + H_2O$

Chloral hydrate (A)

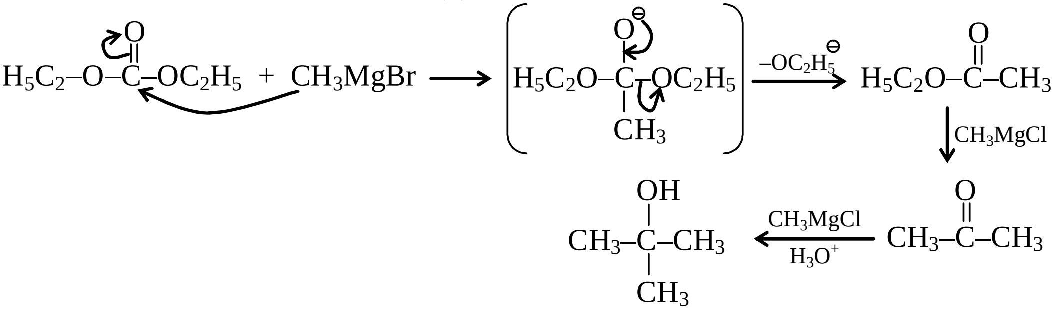
$$CHCl_3 + \frac{1}{2}O_2 \xrightarrow{h\nu} COCl_2 + HCl$$

(A) (phosgene) (B)



$$O{=}C\begin{smallmatrix}Cl\\Cl\end{smallmatrix} + 2HOC_2H_5 \longrightarrow O{=}C\begin{smallmatrix}OC_2H_5\\OC_2H_5\end{smallmatrix} + 2HCl$$

(diethyl carbonate)
(C)



26. 9

Sol.

$$\underset{\substack{(1\,\text{mole})\\(H_3PO_3\text{ is dibasic})}}{PCl_3} + 3H_2O \longrightarrow \underset{(1\,\text{mole})}{H_3PO_3} + \underset{(3\text{ mole})}{3HCl}$$

No. of moles of NaOH required = (1 × 2) + (3 × 1) = 5

$$\underset{\substack{(0.5\text{ mole})\\(H_3PO_4\text{ is tribasic})}}{PCl_5} + 4H_2O \longrightarrow \underset{(0.5\text{ mole})}{H_3PO_4} + \underset{(2.5\text{ mole})}{5HCl}$$

No. of moles of NaOH required = (0.5 × 3) + (2.5 × 1) = 4
Total number of moles of NaOH = 5 + 4 = 9.

27. 7

Sol. Dacron, Buna-S-rubber, Bakellite, Glyptal are COPOLYMERS and rest are homopolymers.

28. 1

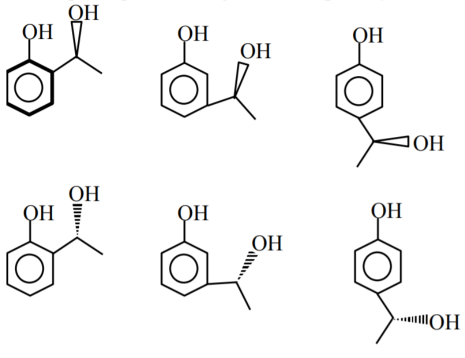
Sol. According to given data four atoms are F,Ne,Na and Mg which have atomic number 9,10,11 and 12. Hence F has 1 unpaired electron in 2p subshell.

29. 9

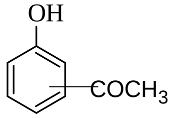
Sol. $C_8H_{10}O_2$ → Gives $FeCl_3$ test means Phenol derivative

↓

Rotate plane polarized light means optically active



All compound on oxidation with PCC will give following compounds which give positive iodoform test.



(o,m and p isomer)

30. 6

Sol. All the statements except (v) are true in option (v) $K_2CrO_4(aq)$ when react with $Ba(NO_3)_2(aq)$ Forms a yellow coloured precipitate $BaCrO_4$.

SECTION – C

31. 00001.03

Sol. By observing given reactions, we find n factor for $KBrO_3$ is 5 and 6 respectively.

Initial moles of $KBrO_3 = 20 \times \frac{1}{6} \times 10^{-3}$

$\therefore$ Equivalent of $KBrO_3 = \left(20 \times \frac{1}{6} \times 10^{-3}\right) \times 5$

$= 16.6 \times 10^{-3}$

Equivalents of AsO_2^- reacted with excess of $KBrO_3 = 5.1 \times \frac{1}{25} \times 10^{-3} \times 2$ (n factor of AsO_2^- is 2)

$= 4.08 \times 10^{-4}$

$\therefore$ Equivalents of $KBrO_3$ in excess $= 4.08 \times 10^{-4}$

$\therefore$ moles of excess of $KBrO_3$ $= 4.08 \times \frac{10^{-4}}{6}$

$= 6.8 \times 10^{-5}$

$\therefore$ Equivalents of $KBrO_3$ in excess(of n – factor 5) $= 5 \times 6.8 \times 10^{-5} = 3.4 \times 10^{-4}$

$\therefore$ Equivalents of $KBrO_3$ reacted with SeO_3^{-2} $= (16.60 - 0.34) \times 10^{-3}$

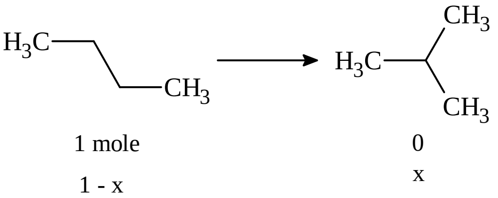
$= 16.26 \times 10^{-3}$

$\therefore$ Equivalents of $SeO_3^{-2} = 16.26 \times 10^{-3}$

$\therefore$ Mass of $SeO_3^{-2} = 16.26 \times 10^{-3} \times \frac{127}{2} = 1.03$ g

32. 00000.91

Sol.



$\Delta G^\circ = \Delta G_f^0(\text{iso-butane}) - \Delta G_f^0(\text{n}-\text{butane})$

$= -21.39 + 15.69 = -5.7 = 2.303$ RT logK

5.7 = 5.7 log K

log K = 1, K = 10

$\frac{x}{1-x} = 10$

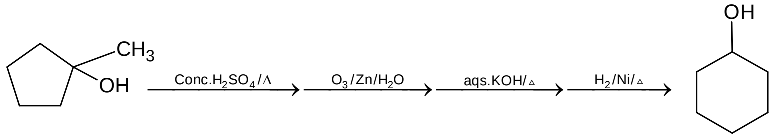
x = 10 – 10x

11x = 10

$x = \frac{10}{11} = 0.91$

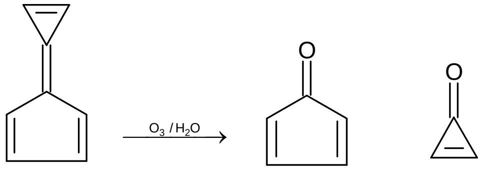
33. 03145.00

Sol.

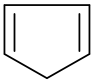


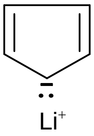
34. 00078.00

Sol.

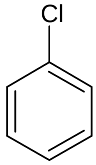


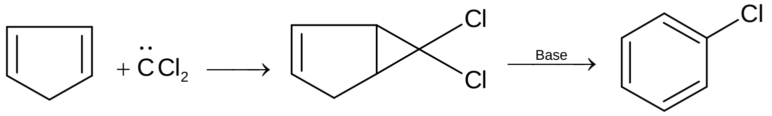
(X) (Y)
aromatic

A -


B -


$+ CH_4 \uparrow$

D -




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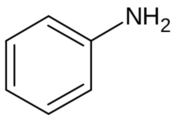

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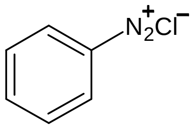

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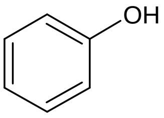

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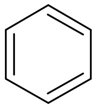
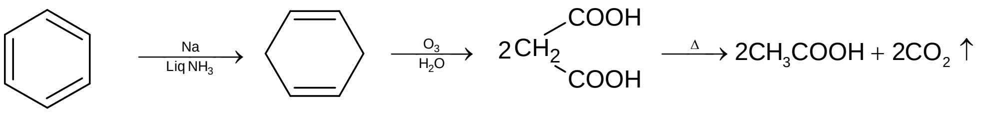
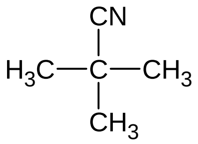




35. 00004.00

Sol. Compound A is

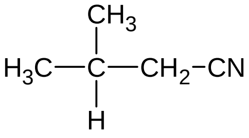
$CH_3 - CH_2 - CH_2 - CH_2 - CN$ or
(A)



$$H_3C-\overset{CN}{\underset{CH_3}{C}}-CH_3$$

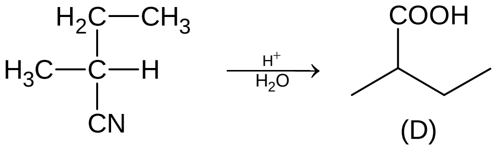
(A)

or



$$H_3C-\overset{CH_3}{\underset{H}{C}}-CH_2-CN$$

Compound B is

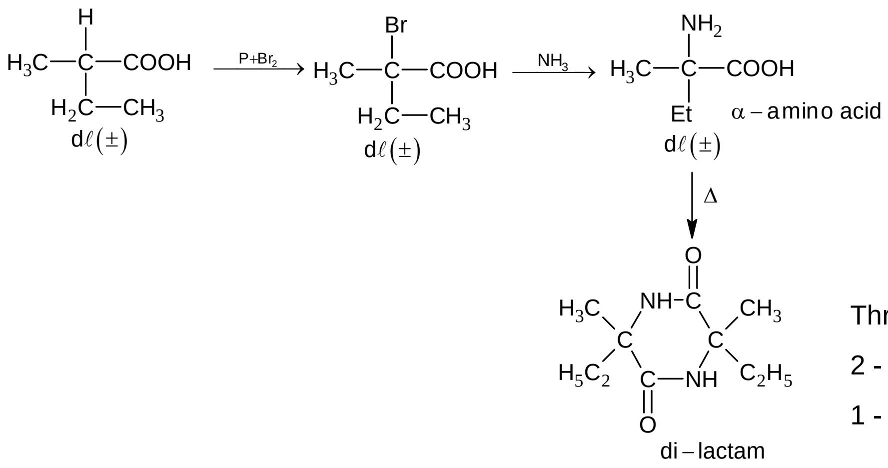


(B) (D)

Meq. of NaOH = Meq. of C or D

$$0.5 \times y = \frac{0.102}{102} \times 1000$$

y = 2



Three isomers

2 - optical active

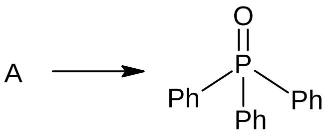
1 - optical inactive

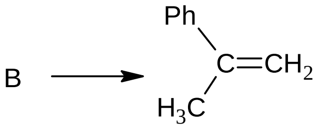
x = 2

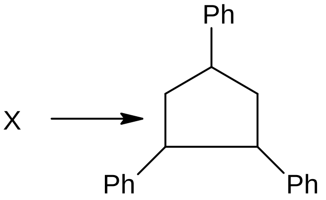
x + y = 4

36. 00017.00

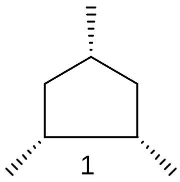
Sol.





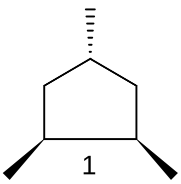


DU = 13



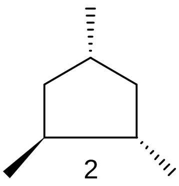
1

Optical inactive



1

Optical inactive



2

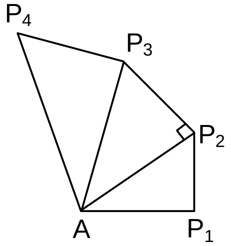
Optical active

Mathematics

PART – III

SECTION – A

37. B, C, D



Sol. $AP_2 = \sqrt{1^2+1^2}$; $AP_3 = \sqrt{1^2+(2^\circ)^2+(2)^2}$

$$AP_4 = \sqrt{1^2+(2^\circ)^2+2^2+(2^2)^2}$$

$$AP_5 = \sqrt{1^2+(2^\circ)^2+2^2+(2^2)^2+(2^3)^2}$$

$$\Rightarrow AP_{n+1} = \sqrt{1+\sum_{r=1}^{n}(2^{r-1})^2} = \sqrt{1+\sum_{r=1}^{n}4^{r-1}} = \sqrt{1+\frac{4^n-1}{4-1}}$$

$$\Rightarrow AP_{n+1} = \sqrt{\left(\frac{4^n+2}{3}\right)}$$

In ΔP_nAP_{n+1}

$$P_nP_{n+1} = 2^{n-1},\ AP_{n+1} = \sqrt{\frac{4^n+2}{3}} \Rightarrow \sin(\angle P_nAP_{n+1}) = \frac{P_nP_{n+1}}{AP_{n+1}} = \frac{2^{n-1}}{\sqrt{\frac{4^n+2}{3}}} = \sqrt{\left(\frac{3\cdot 4^n}{4^{n+1}+8}\right)}$$

$$\Rightarrow \lim_{n\to\infty}\sin(\angle P_nAP_{n+1}) = \lim_{n\to\infty}\sqrt{\frac{3}{4+8.4^{-n}}} = \frac{\sqrt{3}}{2} \Rightarrow \lim_{n\to\infty}\angle P_nAP_{n+1} = \frac{\pi}{3}$$

38. A, C

Sol. Curve C will be radical axis through the points A(4, 5) and B(7, 11)

$\Rightarrow y-5 = \frac{11-5}{7-4}(x-4) \Rightarrow y = 2x - 3$

Line touches the parabola $x^2 - 4x + 2y + k = 0$

$\Rightarrow x^2 - 4x + 2(2x - 3) + k = 0$

$\Rightarrow x^2 + k - 6 = 0$ has repeated roots

$\Rightarrow k = 6$

39. A, B, C, D

Sol.
$$\lim_{x\to\infty}\sum_{k=1}^{x} f\left(\frac{10k}{x}\right)\cdot\frac{2}{x} = \int_0^1 f(10x)\cdot 2dx = 30$$

$$\Rightarrow 2\int_0^1 f(10x)dx = \frac{1}{5}\int_0^{10} f(t)dt = 30 \Rightarrow \int_0^{10} f(t)dt = 150$$

$$\lim_{x\to\infty}\sum_{k=1}^{x} f\left(2-\frac{2k}{x}\right)\frac{1}{2x} = 12 \Rightarrow \lim_{x\to\infty}\sum_{k=0}^{(x-1)} f\left(\frac{2k}{x}\right)\cdot\frac{1}{2x} = 12 \Rightarrow \int_0^1 \frac{1}{2}\cdot f(2x)dx$$

$$\Rightarrow \frac{1}{4}\int_0^2 f(t)dt = 12 \Rightarrow \int_0^2 f(t)dt = 48$$

$$\lim_{x\to\infty}\sum_{k=1}^{x} 6f\left(2+\frac{8k}{x}\right)\cdot\frac{1}{x} = \int_0^1 6f(2+8x)dx = \frac{6}{8}\int_2^{10} f(t)dt = \frac{3}{4}\left(\int_0^{10} f(t)dt - \int_0^2 f(t)dt\right) = \frac{153}{2}$$

40. A, C, D

Sol. We know that $e^x = \sum_{r=0}^{\infty} \frac{x^r}{r!}$, let $x = e^{i\theta} \cdot \cos\theta$

Let $e^{\cos\theta \cdot e^{i\theta}} = \sum_{r=0}^{\infty} \left(\frac{\cos^r \theta \cdot e^{i(r\theta)}}{r!} \right) \Rightarrow e^{\cos\theta(\cos\theta + i\sin\theta)} = \sum_{r=0}^{\infty} \frac{\cos^r \theta \cdot (\cos r\theta + i\sin r\theta)}{r!}$

$\Rightarrow \sum_{r=0}^{\infty} \frac{\cos r\theta \cdot \cos^r \theta}{r!} + i\sum_{r=0}^{x} \frac{\cos^r \theta \sin r\theta}{r!} = e^{\cos^2\theta} \cdot e^{i(\sin\theta\cos\theta)}$

$\Rightarrow x_2 = e^{\cos^2\theta} \cos(\sin\theta\cos\theta) \Rightarrow x_1 = e^{\cos^2\theta} \sin(\sin\theta\cos\theta)$

$\Rightarrow x_1^2 + x_2^2 = e^{2\cos^2\theta}$; $2x_1x_2 = e^{2\cos^2\theta} \sin(2\sin\theta\cos\theta)$

41. A, B

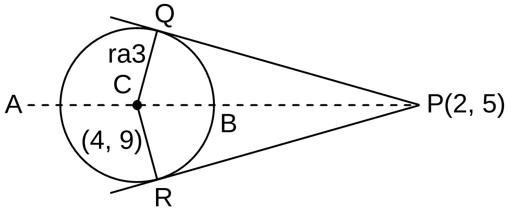
Sol. $x^4 - 3x^2 + 9 = (x^2 + 3)^2 - 9x^2 = (x^2 - 3x + 3)(x^2 + 3x + 3)$

$\Rightarrow 2x^4 - 6x^3 + 9x^2 - 9x + 9 = (x^2 - 3x + 3)(2x^2 + 3)$

Hence, $P(x) = x^2 - 3x + 3$

42. A, B, C, D

Sol. $S = x^2 + y^2 - 8x - 18y = 88 = 0$



(A) Length of tangent

$L = \sqrt{4 + 25 - 16 - 90 + 88} = \sqrt{11}$

(B) Angle between two tangents is α

$\tan\alpha = \frac{2RL}{L^2 - R^2} = \frac{2 \times 3 \times \sqrt{11}}{11 - 9} = 3\sqrt{11} = \sqrt{99}$

(C) Area of $\Delta PQR = \frac{3 \times (11)^{3/2}}{20}$

(D) Equation of the circum circle

ΔPQR is $(x - 4)(x - 2) + (y - 9)(y - 5) = 0$

$\Rightarrow x^2 + y^2 - 6x - 14y + 53 = 0$

SECTION – B

43. 1

Sol. $(x^{2021} - 1) = (x - 1)(x - \alpha)(x - \alpha^2)(x - \alpha^3).....(x - \alpha^{2020})$

$\Rightarrow \ln(x^{2021} - 1) = \sum_{r=1}^{2021} \ln(x - \alpha^{r-1})$ differentiating both side w.r.t x, we get

$\left(\frac{2021 \cdot x^{2020}}{x^{2021} - 1} \right) = \left(\frac{1}{x - 1} \right) + \sum_{r=1}^{2020} \left(\frac{1}{x - \alpha^r} \right)$

Put x = 2, we get $\left(\frac{2021 \cdot 2^{2020}}{2^{2021} - 1} \right) = 1 + \sum_{r=1}^{2020} \left(\frac{1}{2 - \alpha^r} \right)$

$$\Rightarrow \left(2^{2021}-1\right)+\left(2^{2021}-1\right)\sum_{r=1}^{2020}\left(\frac{1}{2-\alpha^{r}}\right)=2021\cdot 2^{2020}$$

$$\Rightarrow \left(2^{2021}-1\right)\sum_{r=1}^{2020}\left(\frac{1}{2-\alpha^{r}}\right)=2019\cdot 2^{2020}+1$$

$\Rightarrow$ a = 2019, b = 2020

44. 2

Sol. $x_{n+1}=\frac{3}{5}x_n+\frac{4}{5}$

$$\Rightarrow \text{Let } x_n=\left(\frac{3}{5}\right)t_n \Rightarrow \left(\frac{3}{5}\right)^{n+1}t_{n+1}=\left(\frac{3}{5}\right)^{n+1}t_n+\frac{4}{5}$$

$$\Rightarrow t_{n+1}-t_n=\frac{4}{5}\left(\frac{5}{3}\right)^{n+1} \Rightarrow \sum_{n=1}^{n-1}\left(t_{n+1}-t_n\right)=\frac{4}{5}\sum_{n=1}^{n-1}\left(\frac{5}{3}\right)^{n+1}$$

$$\Rightarrow t_n-t_1=\frac{4}{5}\left(\frac{5}{3}\right)^{2}\left(\frac{\left(\frac{5}{3}\right)^{n-1}-1}{\frac{5}{3}-1}\right) \Rightarrow x_1=\frac{3}{5}t_1=-7$$

$$\Rightarrow t_1=-\frac{35}{3} \Rightarrow t_n+\frac{35}{3}=2\left(\left(\frac{5}{3}\right)^{n}-\frac{5}{3}\right) \Rightarrow t_n=2\left(\frac{5}{3}\right)^{n}-15$$

$$\Rightarrow x_n=\left(\frac{3}{5}\right)^{n}\left(2\left(\frac{5}{3}\right)^{n}-15\right)=2-15\left(\frac{3}{5}\right)^{n} \Rightarrow \lim_{n\to\infty}x_n=2$$

45. 4

Sol. a cos B – b cos A = $\frac{3}{5}c \Rightarrow a\left(\frac{a^2+c^2-b^2}{2ac}\right)-b\left(\frac{b^2+c^2-a^2}{2bc}\right)=\frac{3}{5}c$

$$\Rightarrow a^2-b^2=\frac{3}{5}c^2 \Rightarrow \frac{\tan A}{\tan B}=\frac{\sin A\cos B}{\sin B\cos A}=\frac{a\cdot\left(\frac{c^2+a^2-b^2}{2ac}\right)}{b\cdot\left(\frac{b^2+c^2-a^2}{2bc}\right)}$$

$$\Rightarrow \frac{c^2+a^2-b^2}{b^2+c^2-a^2}=\frac{\frac{8}{5}c^2}{\frac{2}{5}c^2}=4$$

46. 1

Sol. We can select 1, 2, 3, 4 ..... (n – 1) anywhere, the value of f(n) will be uniquely determined
Hence number of functions = 2^{n-1}

47. 2

Sol.
$$\begin{vmatrix}(x^2+1)^2 & (xy+1)^2 & (xz+1)^2\\ (xy+1)^2 & (y^2+1)^2 & (yz+1)^2\\ (xz+1)^2 & (yz+1)^2 & (z^2+1)^2\end{vmatrix}=\begin{vmatrix}1 & x & x^2\\ 1 & y & y^2\\ 1 & z & z^2\end{vmatrix}\begin{vmatrix}1 & 1 & 1\\ 2x & 2y & 2z\\ x^2 & y^2 & z^2\end{vmatrix}$$

48. 2

Sol. $N = 2^{n-1}p$ where $p = (2^n - 1)$

Hence, all possible divisors of N are $1, 2, 2^2, 2^3, \ldots, 2^{n-1}, p, 2p, 2^2p, 2^3p, \ldots, 2^{n-1}p$

Neglecting $2^{n-1}p$ since $N = 2^{n-1}p$

$\Rightarrow$ Sum of division (other than $N = 2^{n-1}p$)

$= 1 + 2 + 2^2 + 2^3 + \ldots + 2^{n-1} + p + 2p + 2^2p + \ldots + 2^{n-2}p$

$= (1 + 2 + 2^2 + \ldots + 2^{n-1}) + p(1 + 2 + 2^2 + \ldots + 2^{n-2})$

$= \frac{(2^n-1)}{2-1} + p\frac{(2^{n-1}-1)}{2-1} = 2^n - 1 + p(2^{n-1}-1) = 2^n - 1 + 2^{n-1}p - p$

$= 2^{n-1}(2^n - 1) + 2^n - 1 - (2^n - 1) = 2^{n-1}(2^n - 1) = N$

$\Rightarrow \sum_{r=1}^{k} x_r = 2N$

SECTION – C

49. 00030.50

Sol. Let $b_n = \frac{1}{a_n}$; n = 0, 1, 2, ….. Then $\left(6-\frac{1}{b_{n+1}}\right)\cdot\left(12+\frac{1}{b_n}\right) = 72$

$\Rightarrow b_{n+1} = 2b_n + \frac{1}{6}$ or $b_{n+1} + \frac{1}{6} = 2\left(b_n + \frac{1}{6}\right)$; $\left\{b_n + \frac{1}{6}\right\}$ is a G.P. with common ratio 2

Then $b_n + \frac{1}{6} = 2^n\left(b_0 + \frac{1}{6}\right) = 2^n\left(\frac{1}{a_0} + \frac{1}{6}\right) = 2^n\left(\frac{1}{3} + \frac{1}{6}\right) = 2^{n-1}$

$b_n = 2^{n-1} - \frac{1}{6}$; $\frac{1}{2}\sum_{i=0}^{5} 2^i - \frac{1}{6}\sum_{i=0}^{5} 1 = \frac{1}{2}\cdot\frac{1(2^6-1)}{2-1} - 1 = \frac{63}{2} - 1 = \frac{61}{2} = 30.50$

50. 00010.50

Sol. Let sin x = t. The expression is changed in $f(t) = (at^2 + a - 3)t$; $f(t) = -at^3 + (a - 3)t$

$\Rightarrow -at^3 + (a - 3)t \geq -3 \Rightarrow -at(t^2 - 1) - 3(t - 1) \geq 0$

$\Rightarrow (t - 1)(-at(t + 1) - 3) \geq 0 \Rightarrow$ as $t - 1 \leq 0 \Rightarrow -at(t + 1) - 3 \leq 0$

$\Rightarrow a(t^2 + t) \geq -3$ ….. (1)

When t = 0, –1 it always hold

When $0 < t \leq 1$; $0 \leq t^2 + t \leq 2$

When $-1 < t < 0$; $-\frac{1}{4} \leq t^2 + t < 0$ therefore, $-\frac{3}{2} \leq a \leq 12$

51. 00000.22

Sol. $xy + z - 1 = xy + 2 - x - y - 1 = xy + 1 - x - y = (x - 1)(y - 1)$

$\because S = \frac{1}{(x-1)(y-1)} + \frac{1}{(y-1)(z-1)} + \frac{1}{(z-1)(x-1)} = \frac{x+y+z-3}{(x-1)(y-1)(z-1)}$

$= \frac{-1}{(x-1)(y-1)(z-1)} = \frac{-1}{xyz-(xy+yz+zx)+x+y+z-1}$

$= \frac{-1}{5-(xy+yz+zx)} = \frac{-1}{5-\frac{1}{2}} = \frac{-2}{9}$

Also, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx)$

$\Rightarrow xy + yz + zx = \frac{1}{2}$

52. 00000.50

Sol. $$I = \int_0^{\pi/2} \frac{x\cos x - \sin x}{x^2 + \sin^2 x}dx = \int_0^{\pi/2} \frac{\frac{x\cos x - \sin x}{x^2}}{1+\left(\frac{\sin x}{x}\right)^2}dx$$

$$\Rightarrow I = \tan^{-1}\left(\frac{\sin x}{x}\right)\Bigg|_0^{\frac{\pi}{2}} = \tan^{-1}\left(\frac{2}{\pi}\right) - \frac{\pi}{4}$$

53. 00004.00

Sol. The expression is $2^{\frac{1}{2}} \cdot 2^{\frac{2}{4}} \cdot 2^{\frac{3}{8}} ..... 2^{\frac{n}{2^n}}$

$$2^{\left(\sum_{r=1}^{n}\frac{r}{2^r}\right)} \Rightarrow 2^{\sum_{r=1}^{\infty}\frac{r}{2^r}}$$

Now, $$\sum_{r=1}^{\infty}\frac{r}{2^r} = \sum_{n=1}^{\infty}\sum_{k=n}^{\infty}\left(\frac{1}{2^k}\right) = \sum_{n=1}^{\infty}\left\{\frac{\frac{1}{2^n}}{1-\frac{1}{2}}\right\} = \sum_{n=1}^{\infty}\frac{1}{2^{n-1}} = \frac{\frac{1}{2^{n-1}}}{1-\frac{1}{2}} = 2$$

54. 00002.00

Sol. $$\frac{8n}{n^4 - 2n^2 + 5} = \frac{2\left[(n+1)^2 - (n-1)^2\right]}{4 + (n+1)^2(n-1)^2} = \frac{\left(\frac{n+1}{\sqrt{2}}\right)^2 - \left(\frac{n-1}{\sqrt{2}}\right)^2}{1+\left(\frac{n-1}{\sqrt{2}}\right)^2\left(\frac{n+1}{\sqrt{2}}\right)^2}$$

$$\Rightarrow \tan^{-1}\left(\frac{8n}{n^4 - 2n^2 + 5}\right) = \tan^{-1}\left(\left(\frac{n+1}{\sqrt{2}}\right)^2\right) - \tan^{-1}\left(\frac{n-1}{\sqrt{2}}\right)^2$$

$$\Rightarrow \sum_{n=1}^{\infty}\tan^{-1}\left(\left(\frac{n+1}{\sqrt{2}}\right)^2\right) - \tan^{-1}\left(\left(\frac{n-1}{\sqrt{2}}\right)^2\right)$$

$$\Rightarrow \lim_{N\to\infty}\left[\tan^{-1}\left(\frac{N+1}{\sqrt{2}}\right)^2 + \tan^{-1}\left(\frac{N}{\sqrt{2}}\right)^2 - \tan^{-1}(0) - \tan^{-1}\left(\frac{1}{2}\right)\right]$$

$$\Rightarrow \pi - \tan^{-1}\left(\frac{1}{2}\right) \Rightarrow \pi - \cot^{-1}(2) \Rightarrow \cot^{-1}(-2)$$